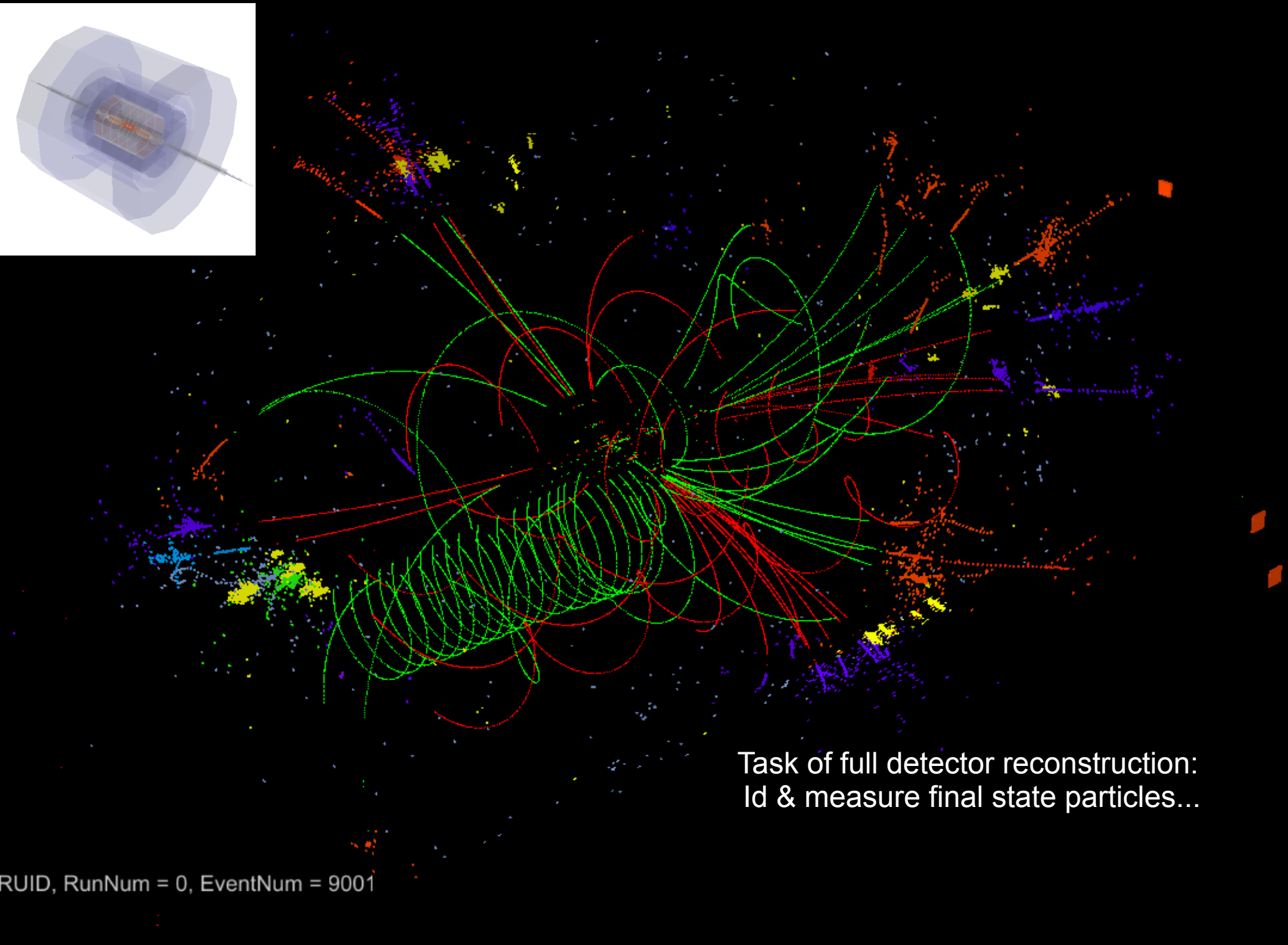
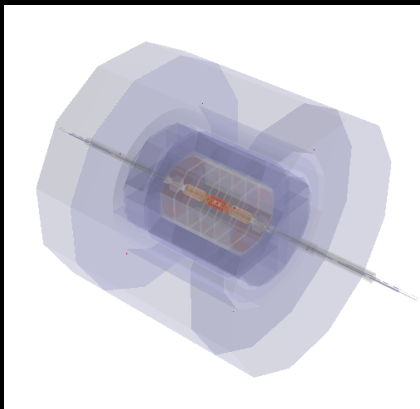




Fractal dimension analysis in a highly granular calorimeter

Manqi RUAN



Task of full detector reconstruction:
Id & measure final state particles...

Revolution of Calorimeter

Development of micro electronics: ultra-high granularity!

#channels, 10^4 - 10^5 (CMS) \rightarrow 10^8 channels (ILC calorimeters)

Imaging calorimeter in 3-D (or even 5-D) in a high DAQ rate...

Role of calorimeter

Measure the incident energy

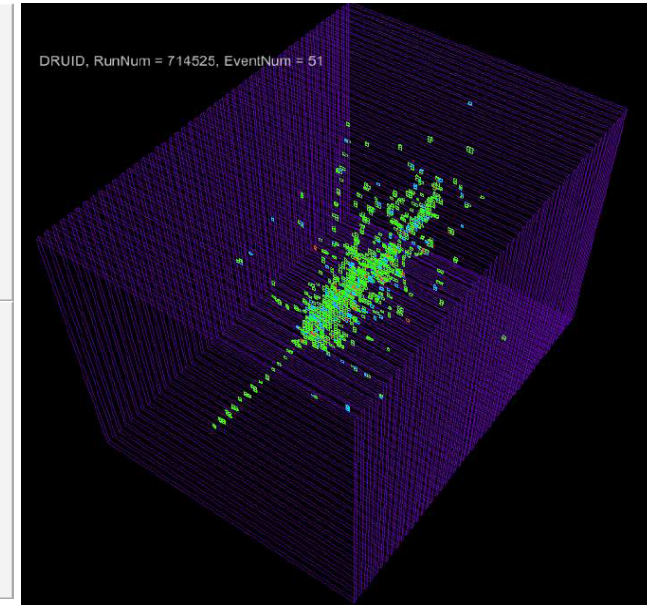
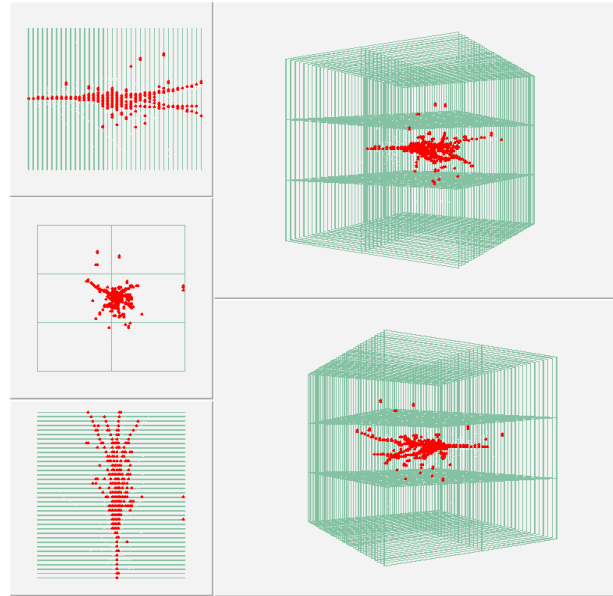
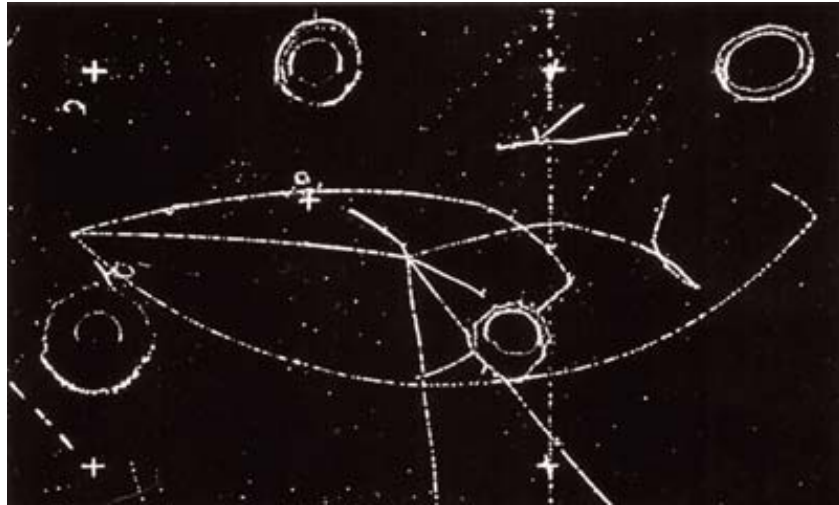
Identify and measure each incident particles with sufficient energy

DRUID, RunNum = 0, EventNum = 23

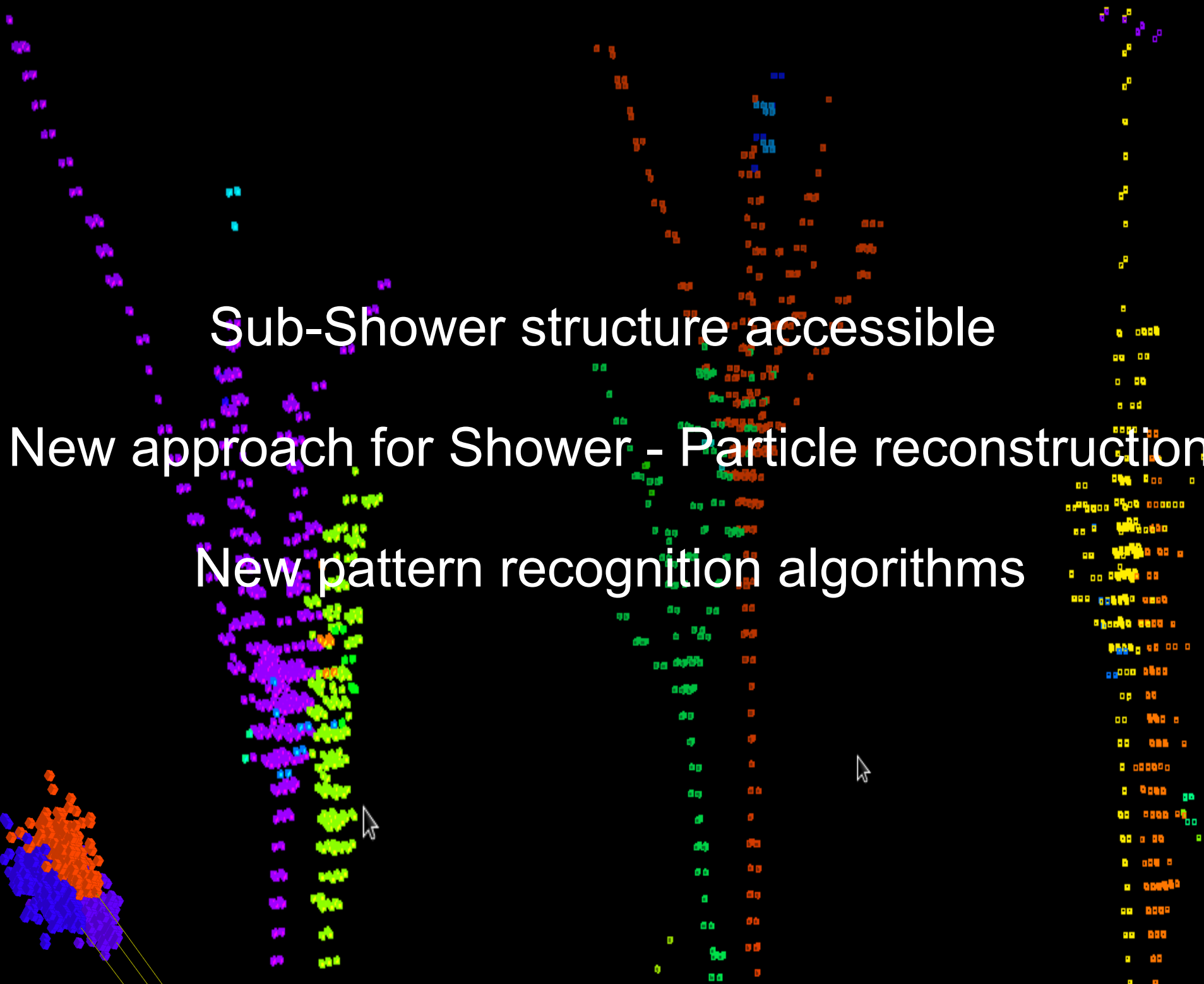
20 GeV Klong reconstructed @ ILD Calo

10cm

Ultra-high granularity...



Granularity $\sim 1 \text{ cm}^{-3}$. 3d, 4d or 5d image...



Sub-Shower structure accessible

New approach for Shower - Particle reconstruction

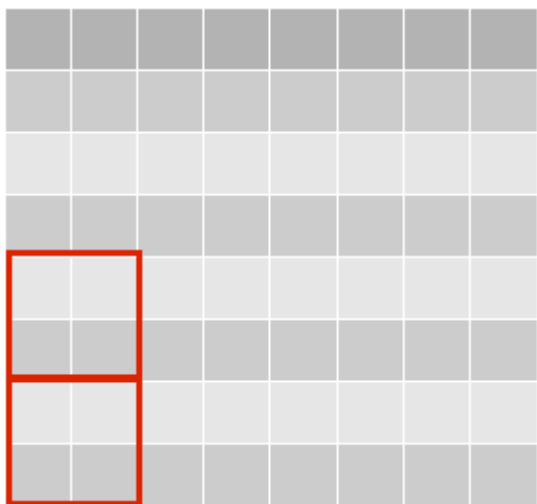
New pattern recognition algorithms

Fractal dimension of particle shower



$$FD_{\beta} = \left\langle \frac{\log(R_{\alpha,\beta})}{\log(\alpha)} \right\rangle + 1.$$

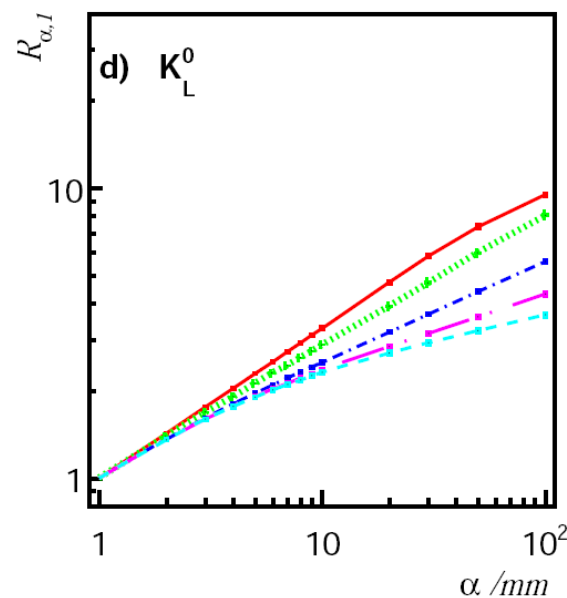
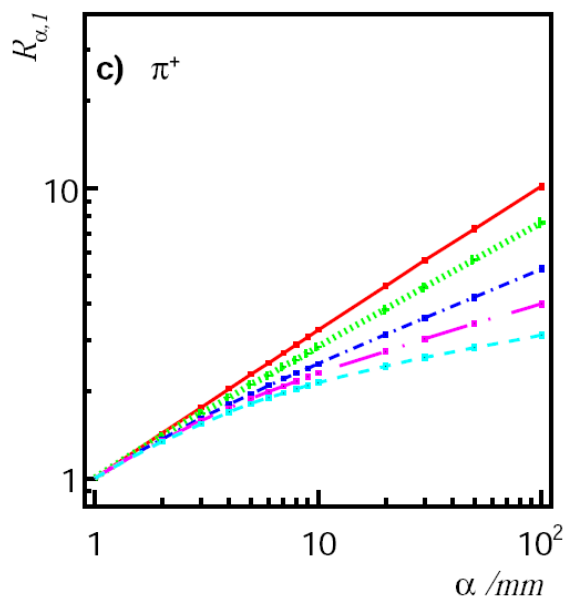
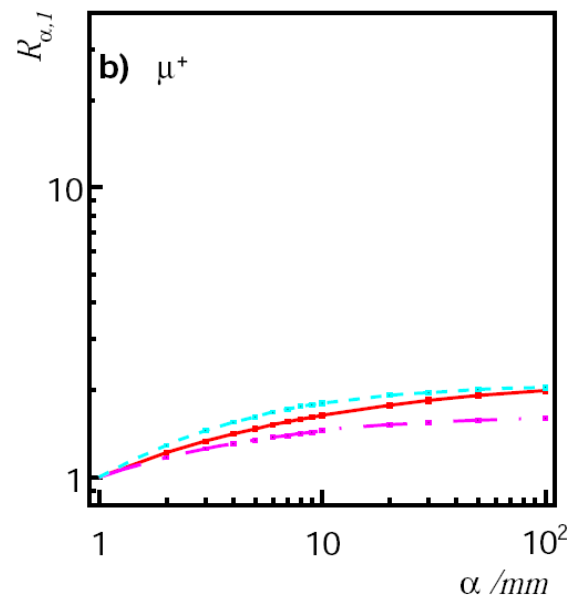
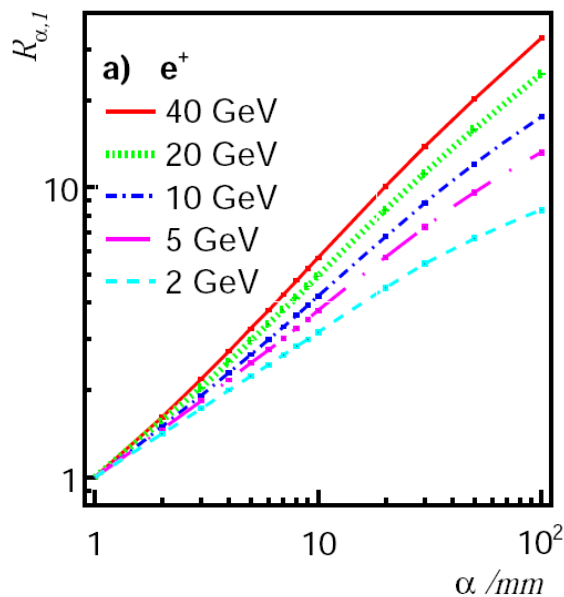
$$R_{\alpha,\beta} = N_{\beta}/N_{\alpha}.$$



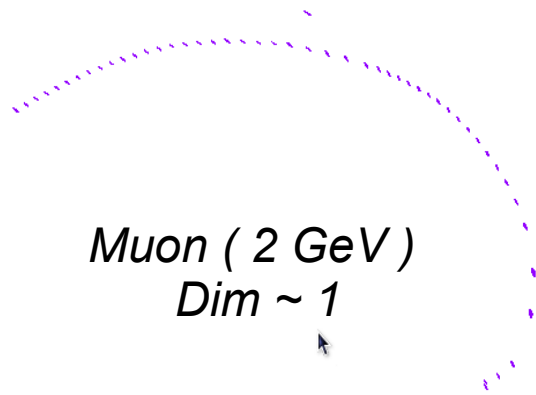
Ultimate cell size: 1mm

Resize cell: 2 – 10, 20, 30,
50, 60, 90, 120, 150 mm.

Sample: particle gun events
at ILD SDHCAL

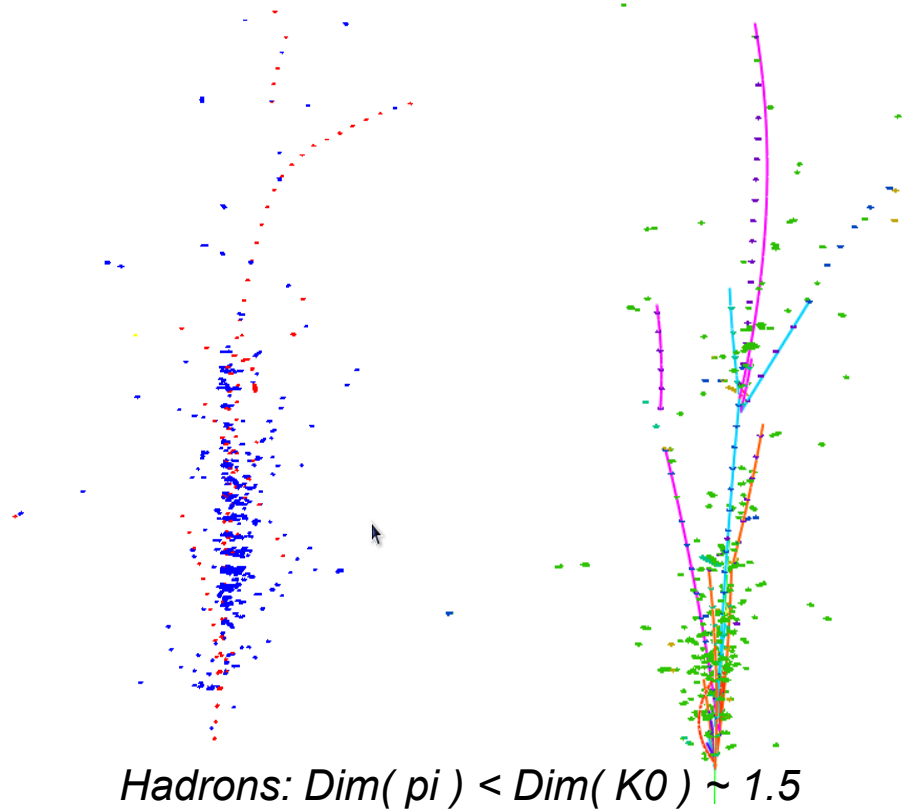


Fractals in Nature

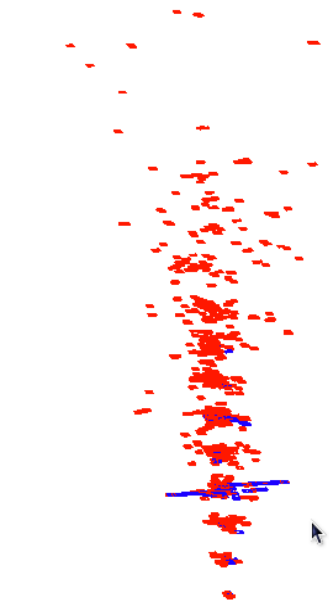


Muon (2 GeV)
Dim ~ 1

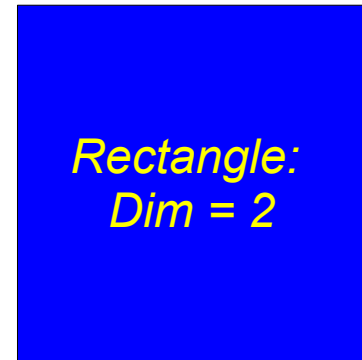
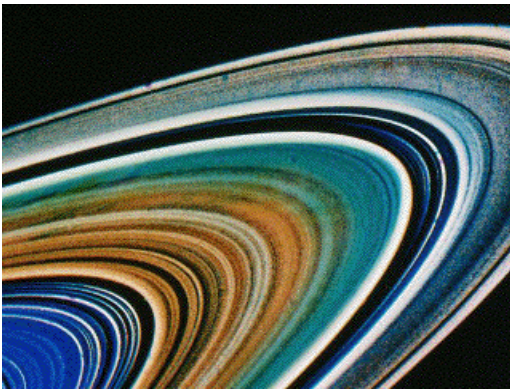
Straight line:
Dim = 1



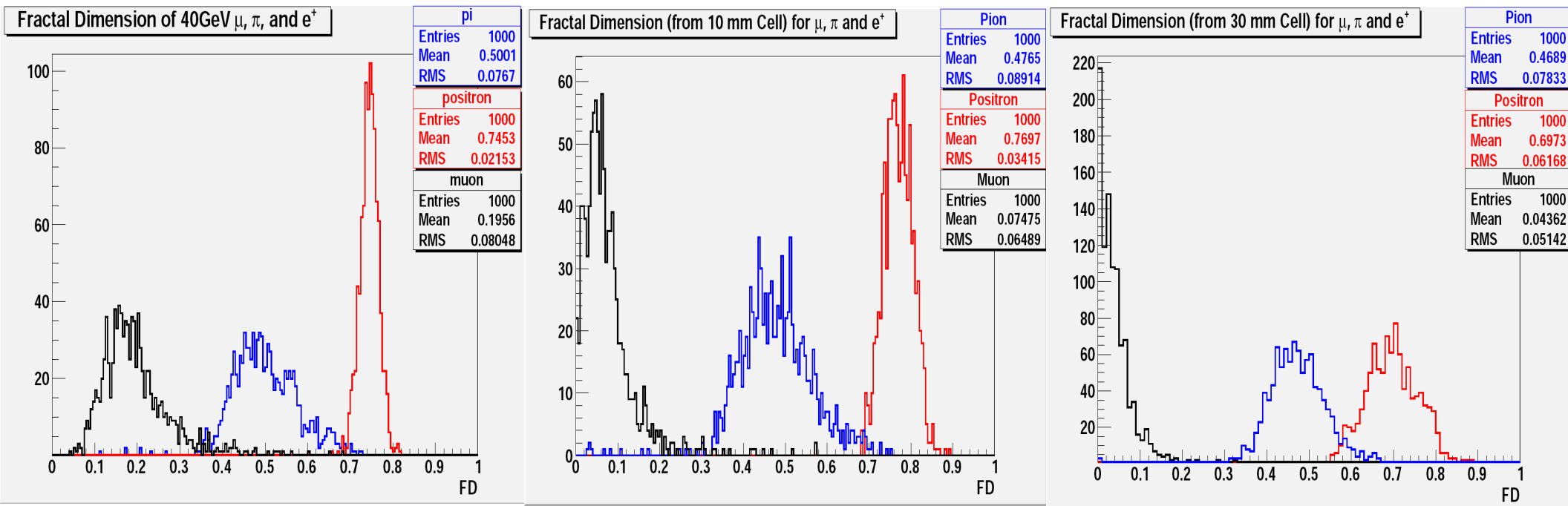
Hadrons: $Dim(\pi) < Dim(K0) \sim 1.5$



Positron (40GeV)
Dim ~ 1.75



FD @ different size



From FD(1mm) to FD(10/30mm):

Positron Peak Smeared

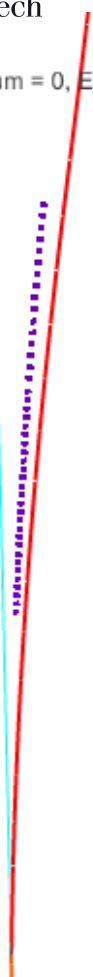
Better $\mu - h$ separation: μ acts more like a line ($FD = 1$); (Anyhow we can create large cells from small ones...)

Hadronic shower: continuous distribution between MIP and EM shower

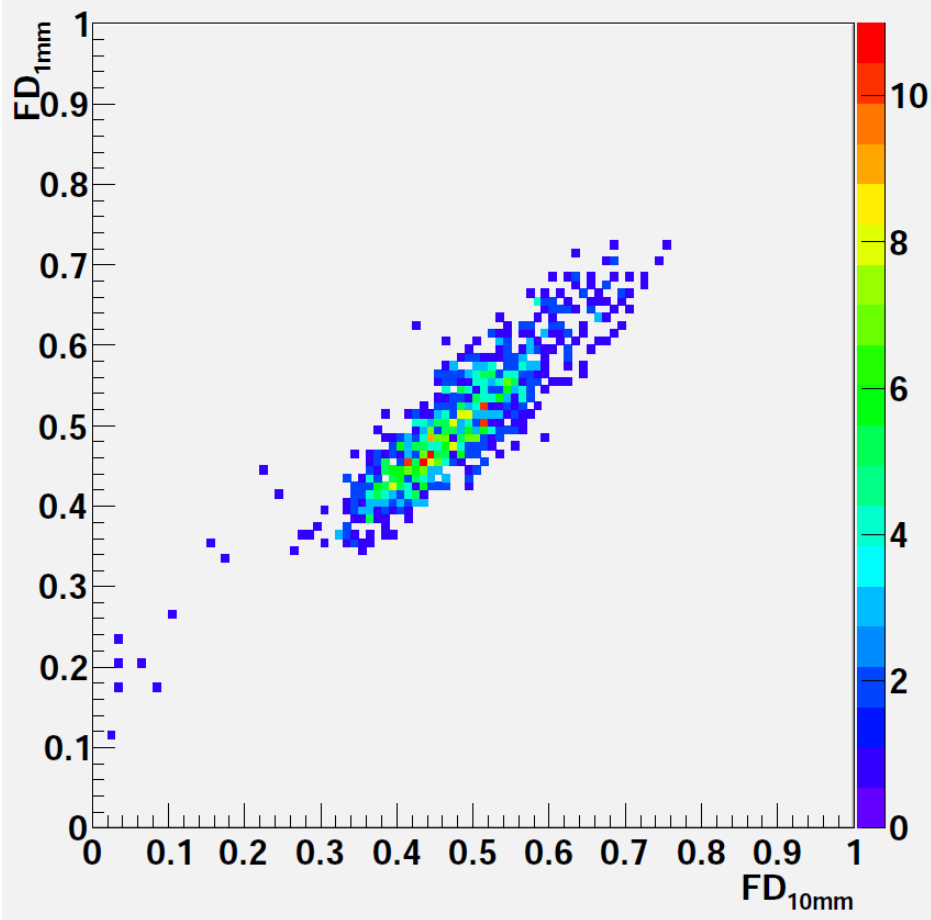
Extreme Cases: Pion



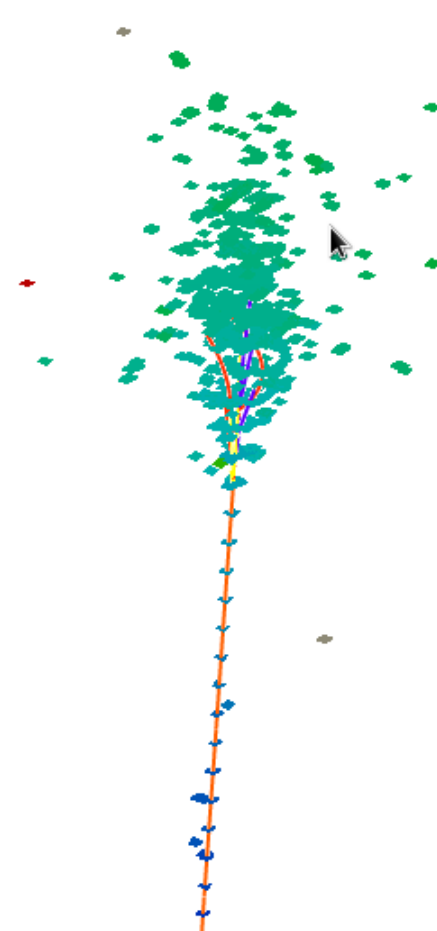
DRUID, RunNum = 0, EventNum = 388



Fractal Dimensions (-1) for 40GeV π



DRUID, RunNum = 0, EventNum = 112



- Pion: MIP, Pion decay;
- EM interaction ($\pi + N = P + \pi^0$); partially identified by interaction point tagging

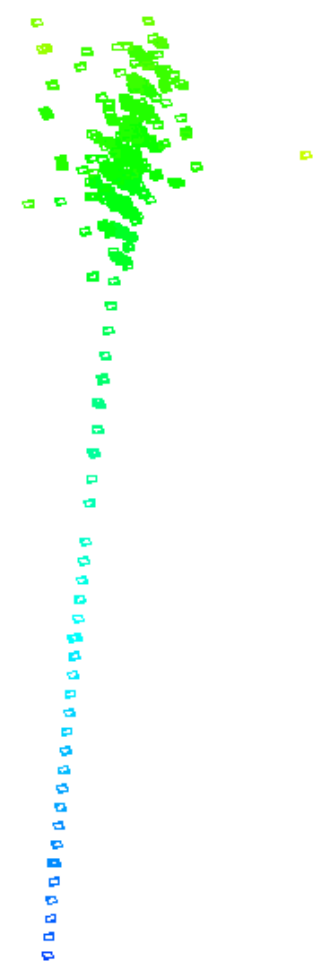
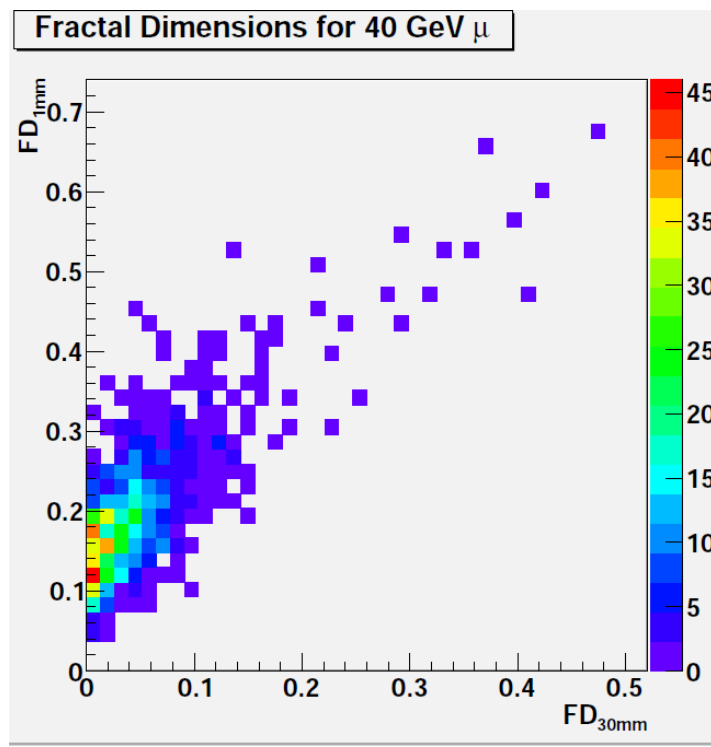
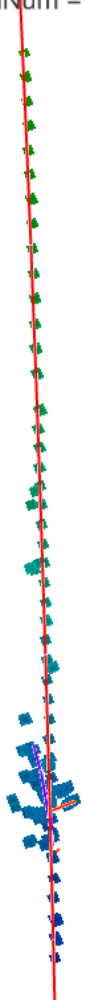
Extreme Cases: Muon



DRUID, RunNum = 0, EventNum = 535

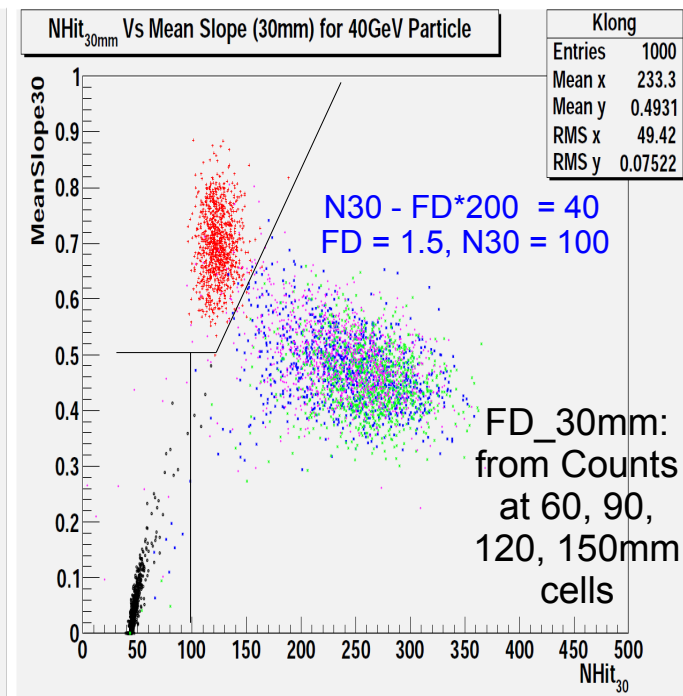
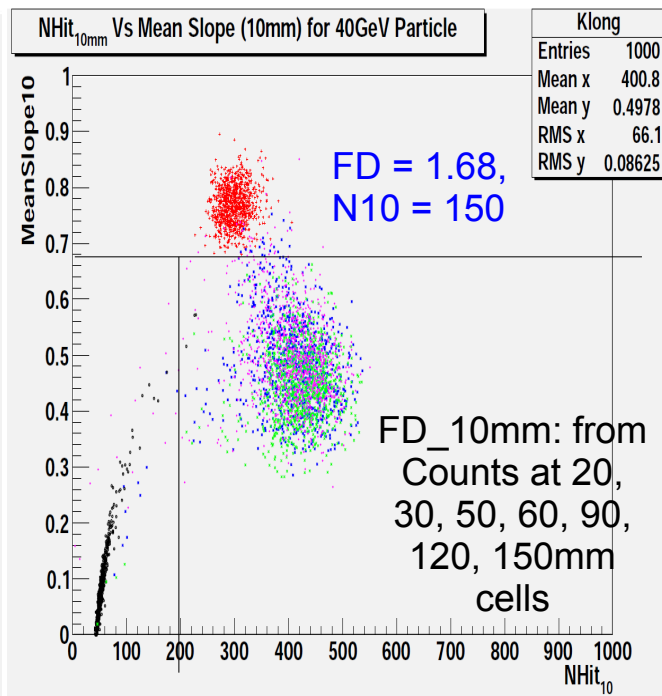
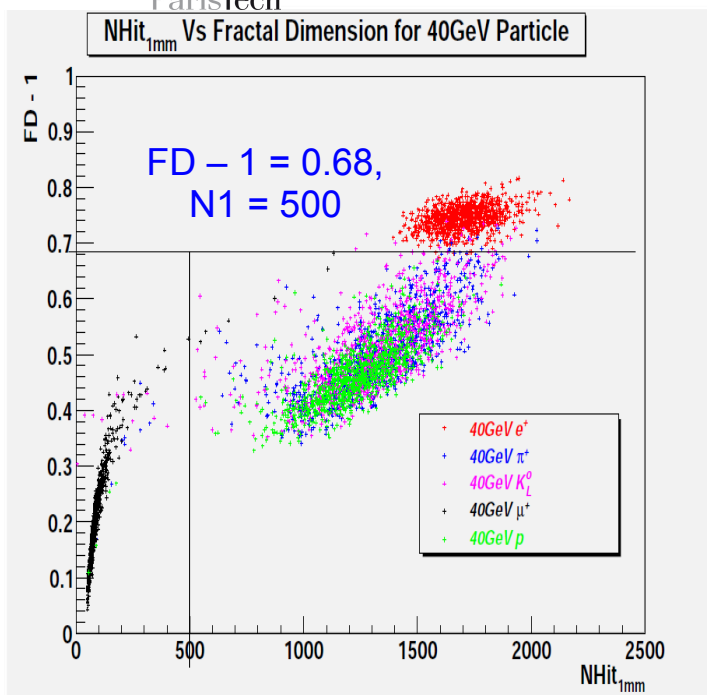
DRUID, RunNum = 0, EventNum = 547

DRUID, RunNum = 0, EventNum = 367



Together with Nhit information: to identify Muon radiation & String noise...

Potential tool for PID



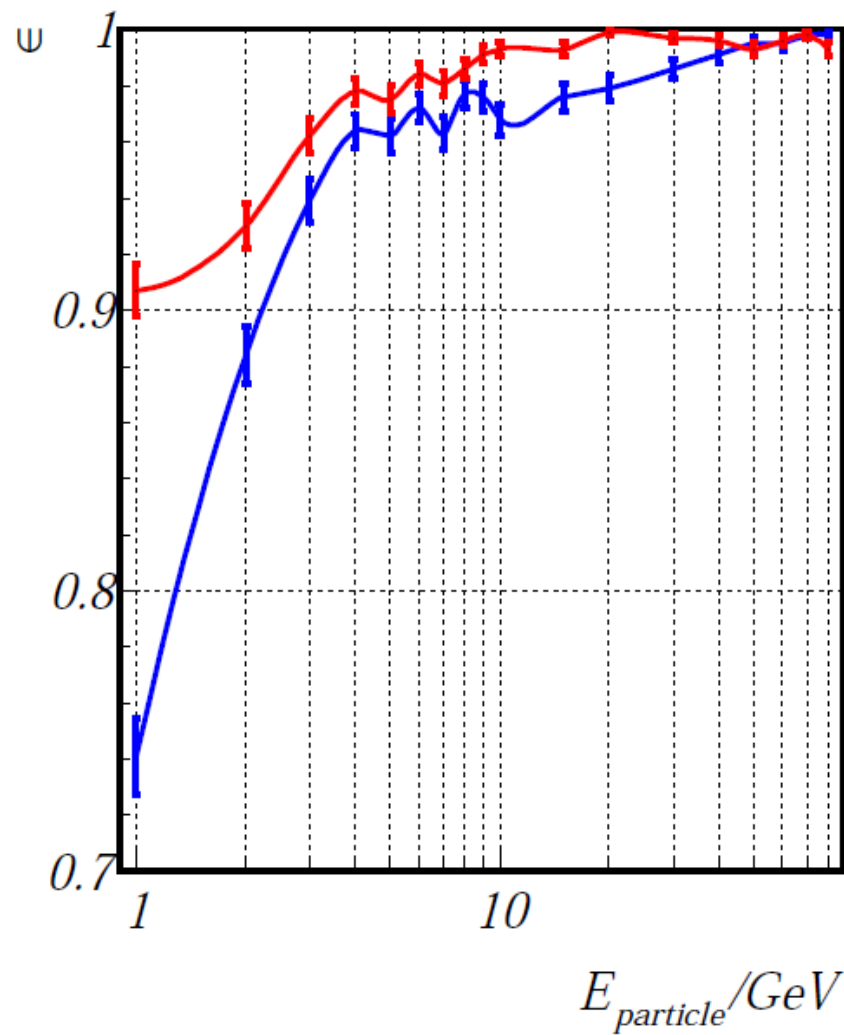
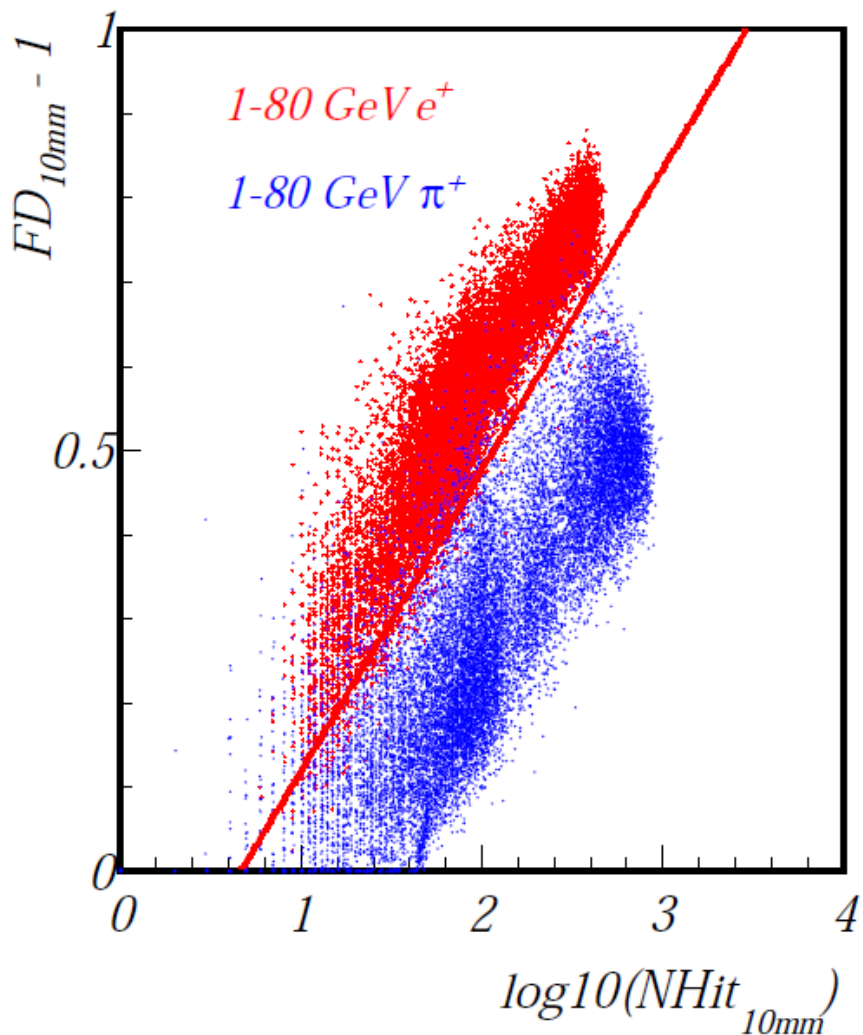
FD together with hit counts: Clear separation at different scales

1mm	e+	u	h
e+	998	0	2
u	1	994	5
h	15	14	971

10mm	e+	u	h
e+	1000	0	0
u	0	995	5
h	17	14	969

30mm	e+	u	h
e+	1000	0	0
u	0	996	4
h	18	11	971

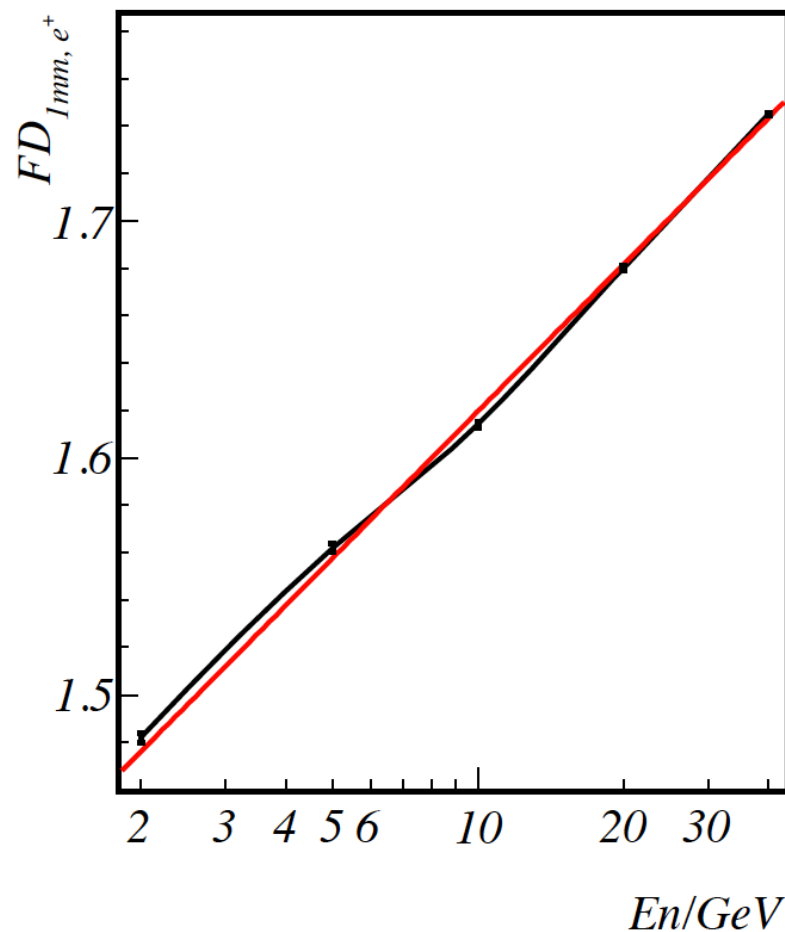
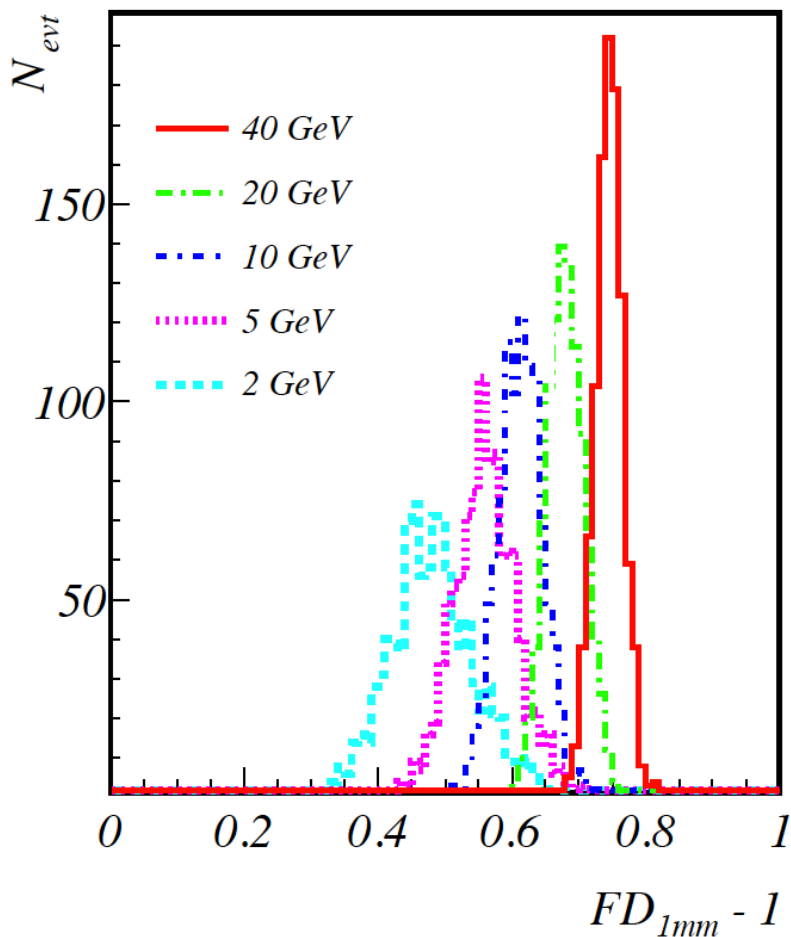
Without knowing energy information



Remark: FD is only the transverse shower information...

Promising PID over full energy range

Logarithmic dependence on particle energy



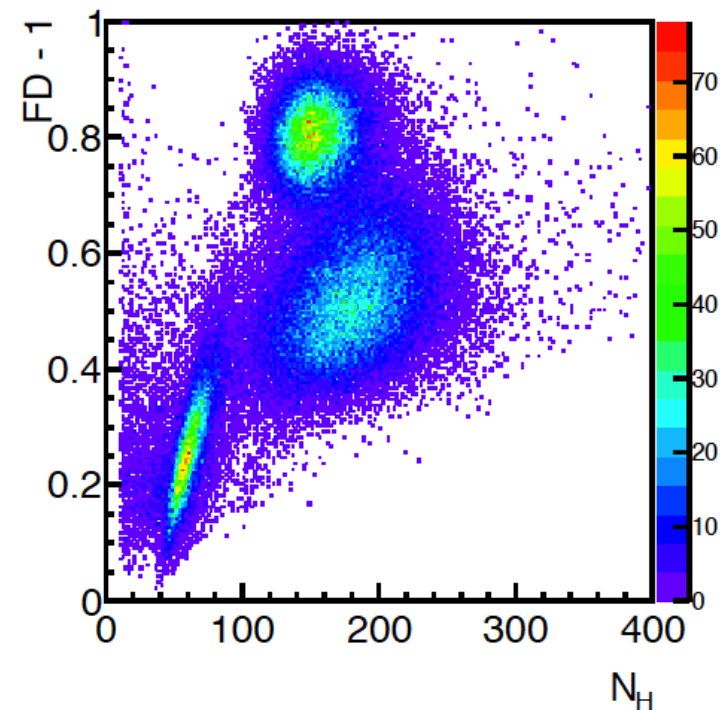
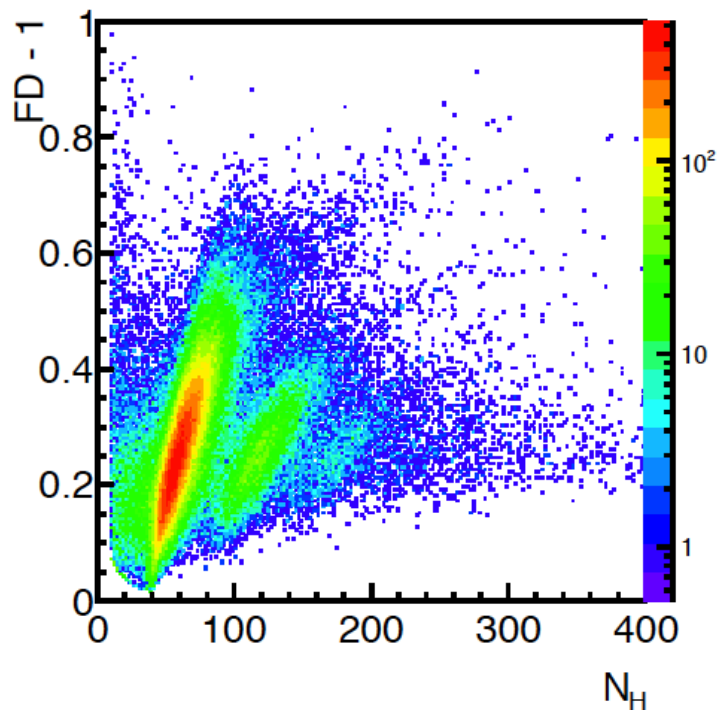
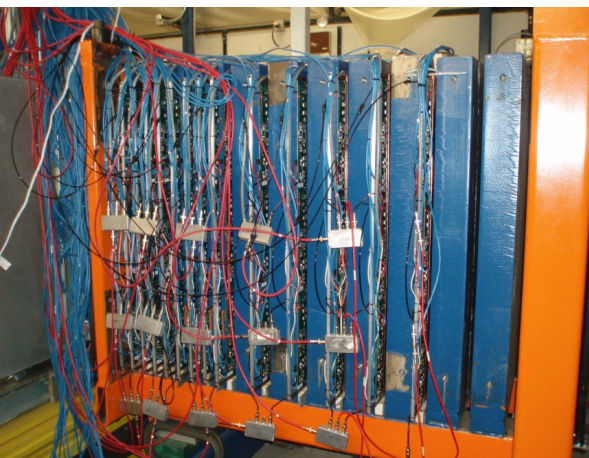
$$FD_{1mm}^{em}(E) = 1.41 + 0.21 \times \log_{10}(E/GeV)$$

$$FD_{1mm}^{had}(E) = 1.24 + 0.15 \times \log_{10}(E/GeV)$$

$$CEPC \quad FD_{1mm}^{mip}(E) = 1.2$$



In Real data?

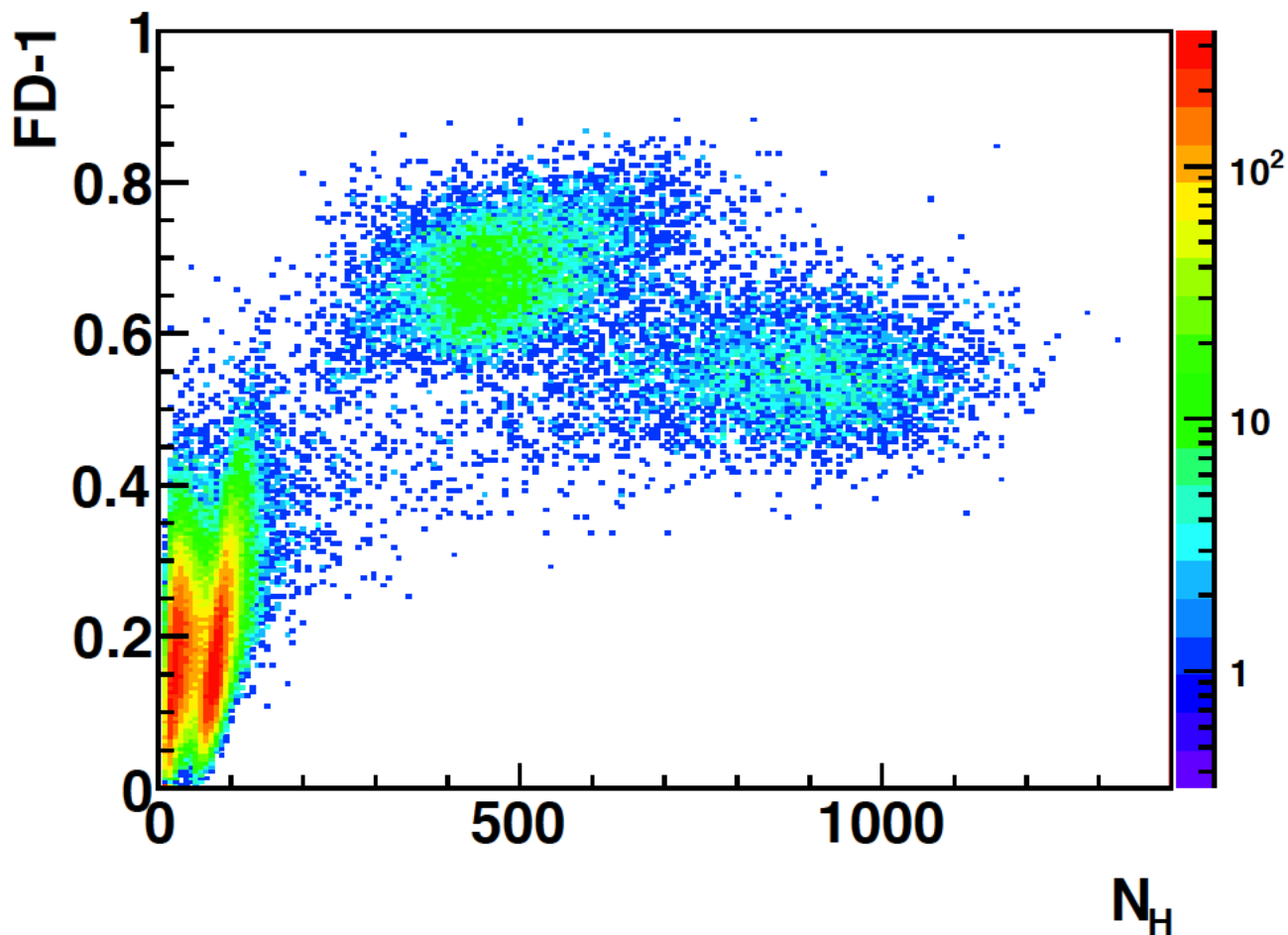


Fermi lab test beam, multiple muon events

Signal triggered by pairs of scintillator tile located in the front/end of the prototype.



In Real data?

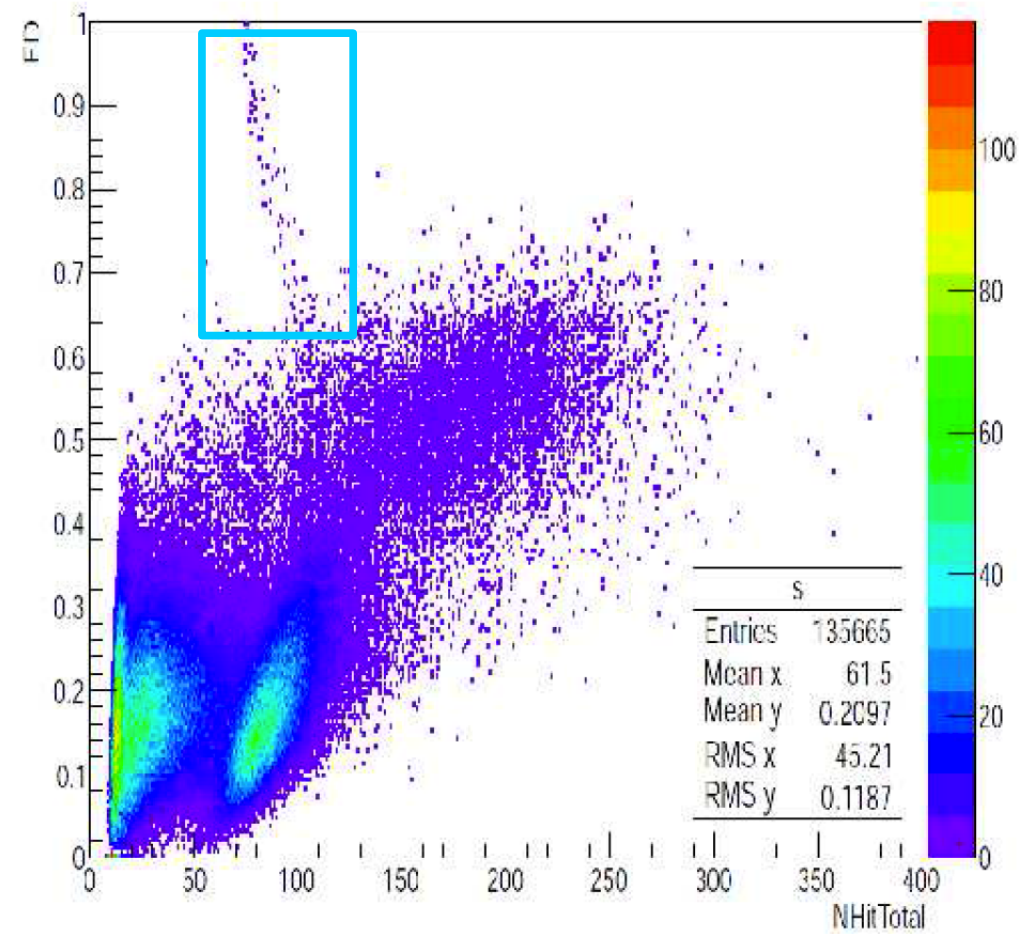
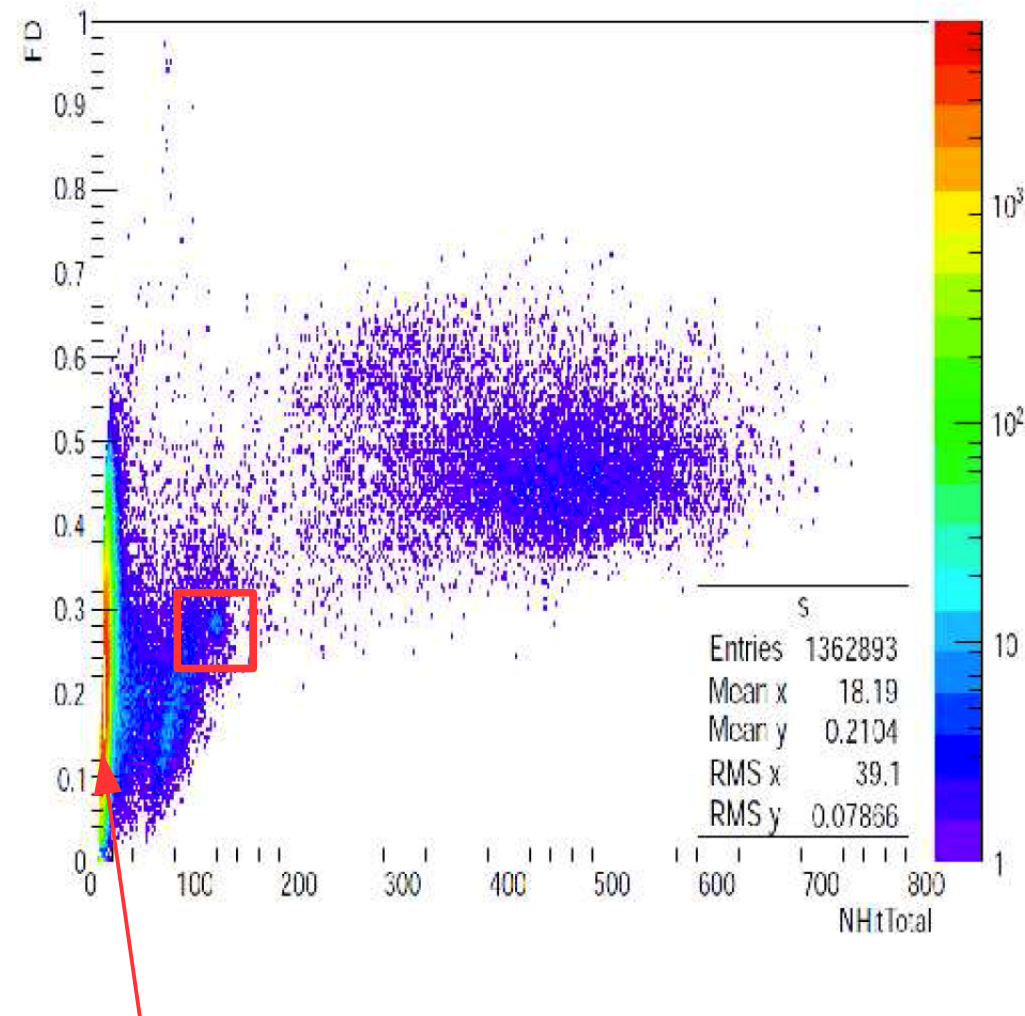


CERN SPS Test beam experimental data

Extremely low noise rate - triggerless mode: Significant cosmic ray component



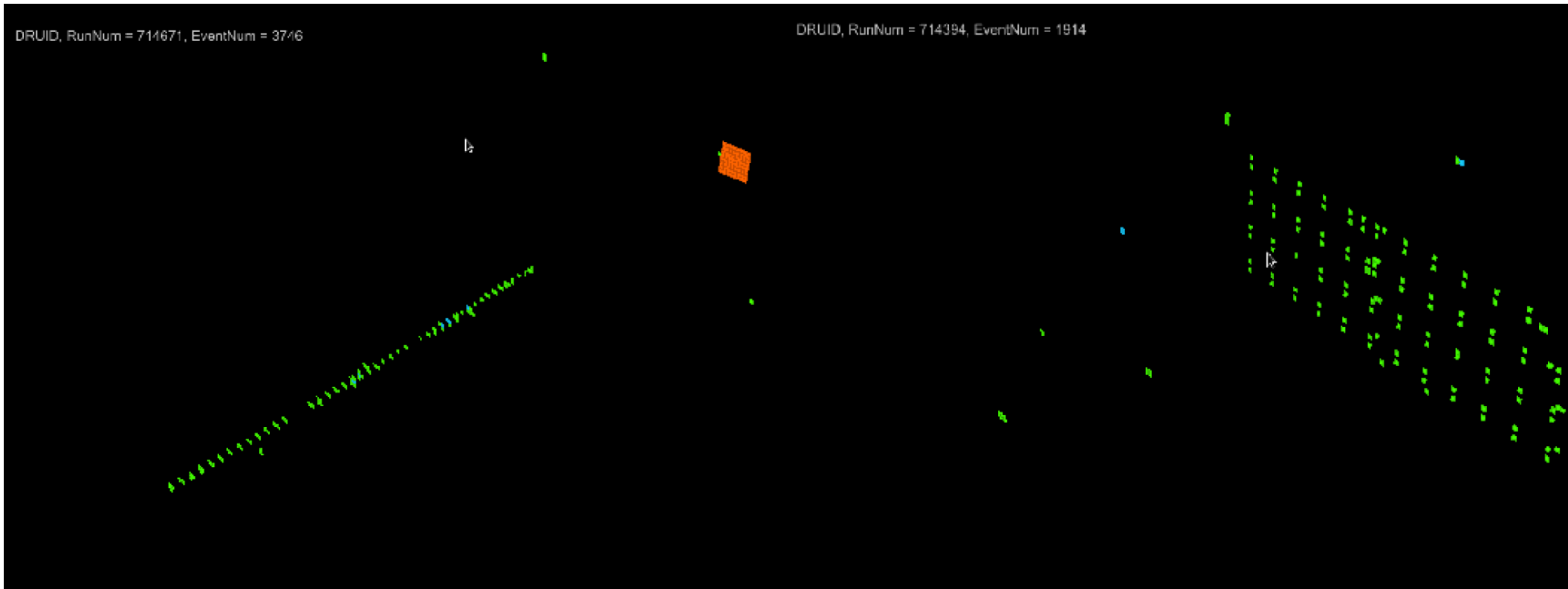
In Real data?



NOISE PATTERN...



In Real data?



Source code...



```
int NHScaleV2( LCCollection *inputHit, std::vector<CalorimeterHit*> clu0, int RatioX, int RatioY, int RatioZ )
{
    int ReScaledNH = 0;
    int NumHit = clu0.size();
    int tmpI = 0;
    int tmpJ = 0;
    int tmpK = 0;
    float tmpEn = 0;
    int NewCellID0 = 0;

    CellIDDecoder<CalorimeterHit> idDecoder(inputHit);          //Input Hits here refer to AllCleanHits collection

    std::map <double, float> testIDtoEnergy;

    for(int i = 0; i < NumHit; i++)
    {
        CalorimeterHit *hit = dynamic_cast<CalorimeterHit*>( clu0[i]);

        tmpI = idDecoder(hit) ["I"] / RatioX;
        tmpJ = idDecoder(hit) ["J"] / RatioY;
        tmpK = (idDecoder(hit) ["K-1"] + 1) / RatioZ;
        tmpEn = hit->getEnergy();

        NewCellID0 = (tmpK << 24) + (tmpJ << 12) + tmpI;

        if(testIDtoEnergy.find(NewCellID0) == testIDtoEnergy.end() )
        {
            testIDtoEnergy[NewCellID0] = tmpEn;
        }
        else
        {
            testIDtoEnergy[NewCellID0] += tmpEn;
        }
    }

    ReScaledNH = testIDtoEnergy.size();

    return ReScaledNH;
}
```

</afs/ihep.ac.cn/users/m/manqi/Analysis/Arbor/ArborF1/src/ArborTool.cc>

Source code...



```
float FDV2( std::vector<CalorimeterHit*> clu, LCCollection *HitCollection )
{
    float FractalDim = 0;
    int NReSizeHit[10] = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
    int Scale[10] = {2, 3, 4, 5, 6, 7, 8, 9, 10, 20};
    int OriNHit = clu.size();

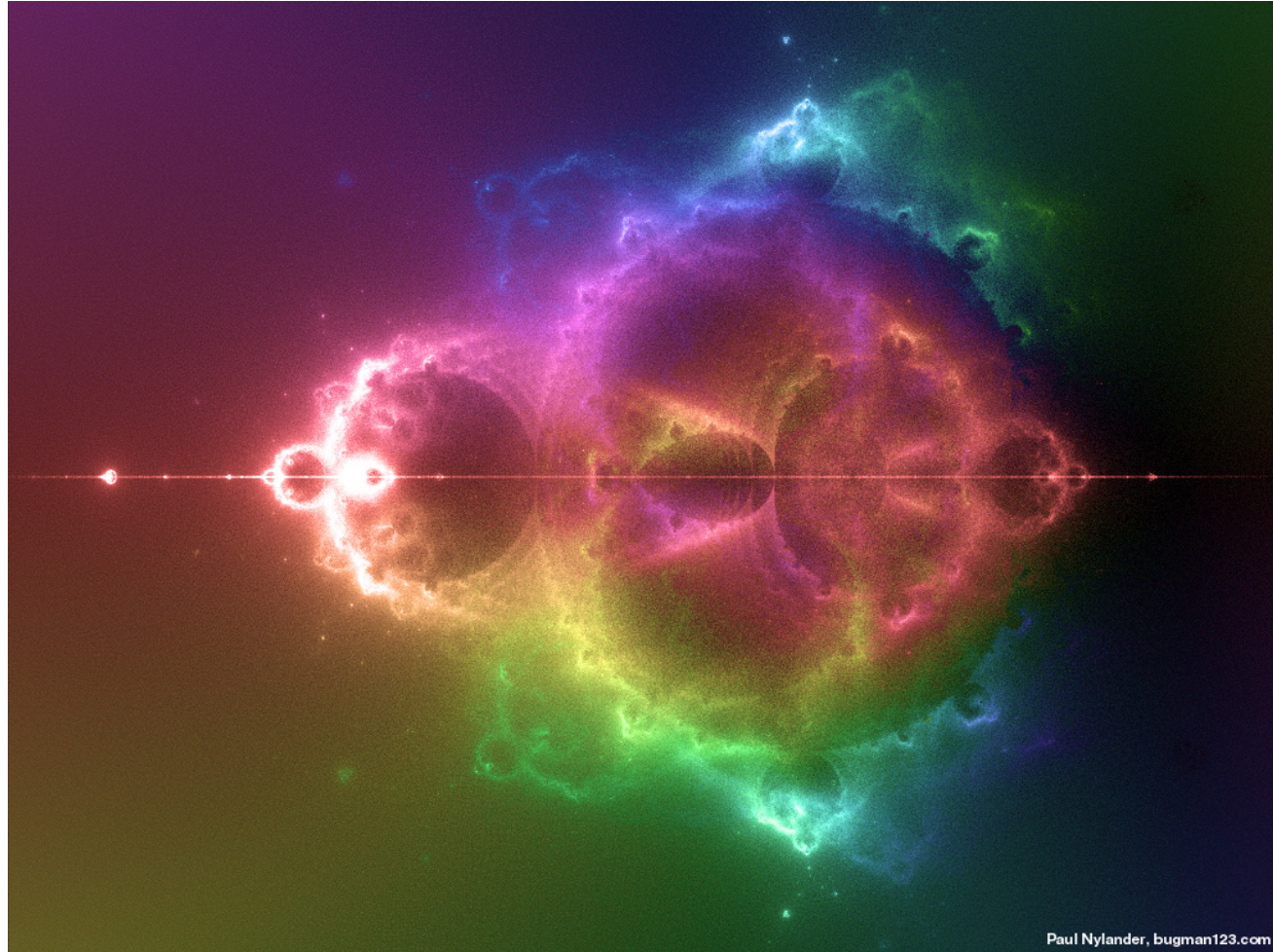
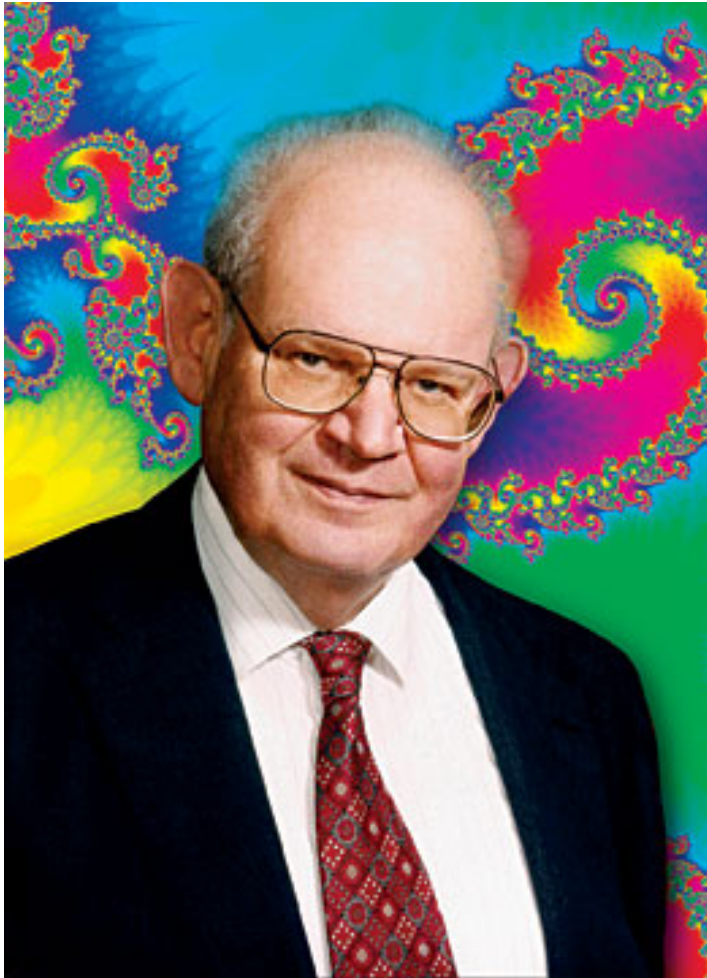
    for(int j = 0; j < 10; j++)
    {
        NReSizeHit[j] = NHScaleV2(HitCollection, clu, Scale[j], Scale[j], 1);
        FractalDim += 0.1 * TMath::Log(float(OriNHit)/NReSizeHit[j])/TMath::Log(float(Scale[j]));
    }

    return FractalDim;
}
```

Summary

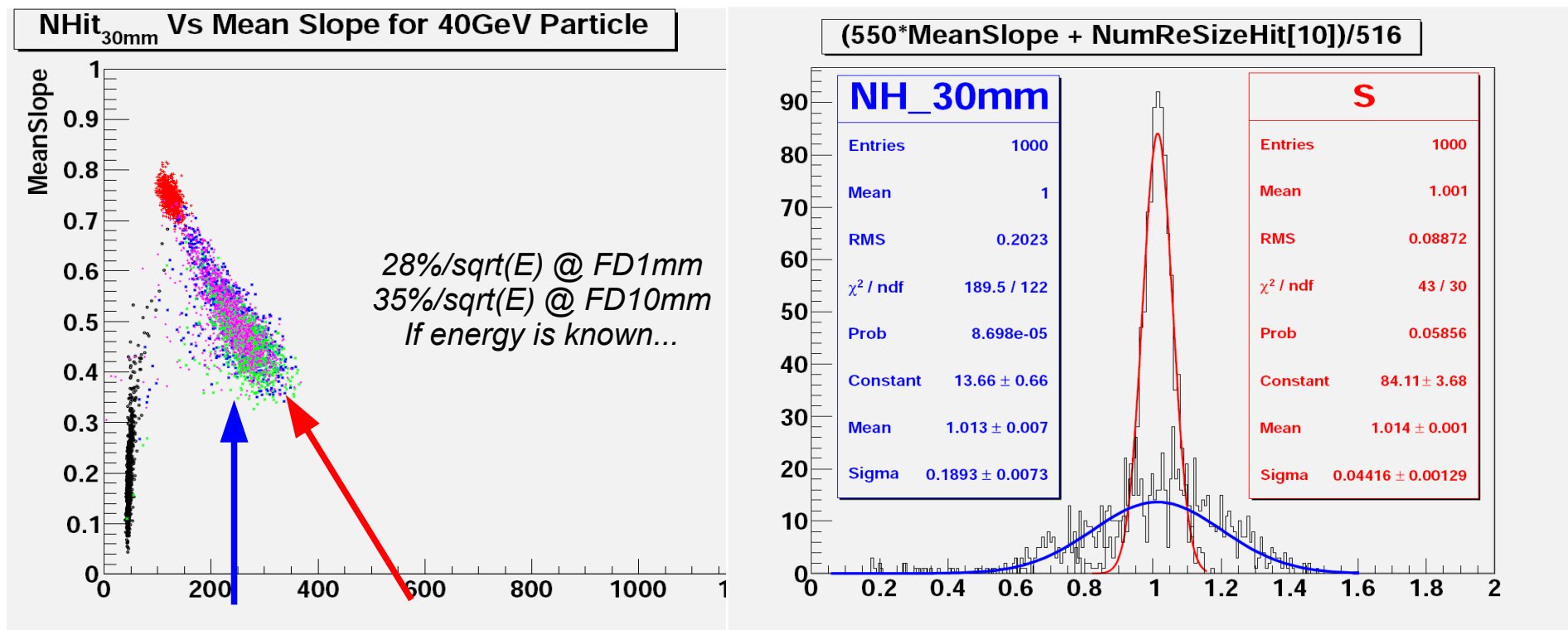


- Impaction of Moore's law: boosted granularity in future calorimeter
- Huge potential for reconstruction at high granularity
 - Shower Fractal Dimension: Highly dependent on Particle type & energy
 - Promising tool for PID
 - Investigating into sub-shower structure
 - Potentially be used for energy measurement
 - Group pattern tagging – Noise tagging
 - [Dream algorithms...](#)



Special Thanks to ...

FD for Energy Estimation



- For example: Compensation based on the correlation of NH_30mm & FD1mm:

$$E = a * \text{NH}_{30} + b * \text{FD} \sim 30\%/\sqrt{E}! \text{ But...}$$
- Correlation coefficient depending on Energy: $b \sim 0.0266 * E$. To measure cluster energy of charged particle (with track info): [check matching](#)
- A set of energy independent (LO) estimator: $E = a' * \text{NH}_x / (1 - \text{FD} * b')$