# TMD Factorization and Collins Effect

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# 1. Introduction about QCD factorization

The foundation for using perturbation theory of QCD are QCD factorization theorems. There are two types of factorizations for inclusive processes:

- ◆ Collinear Factorization
- ◆ Transverse Momentum Dependent (TMD) Factorization

With the two factorizations, one can extract different information about inner structure of hadrons from experimental results.

Hadronic scattering with high energy scale Q: (according to power of 1/Q)

$$d\sigma \sim H_2 \otimes \langle O_2 \rangle + H_3 \otimes \langle O_3 \rangle + H_4 \otimes \langle O_4 \rangle + \cdots$$

$H_i$ : Perturbative coefficient functions

$\langle O_i \rangle$ : Hadronic Matrix elements of QCD operators of twist i=2,3,4....

The matrix elements are nonperturbative.....

# Measurements in Quantum Mechanics:

With proven QCD factorization, one can "measure" hadrons in initial states with operators $O_i$

$$n^\mu = (0,1,0,0) \qquad l^\mu = (1,0,0,0)$$

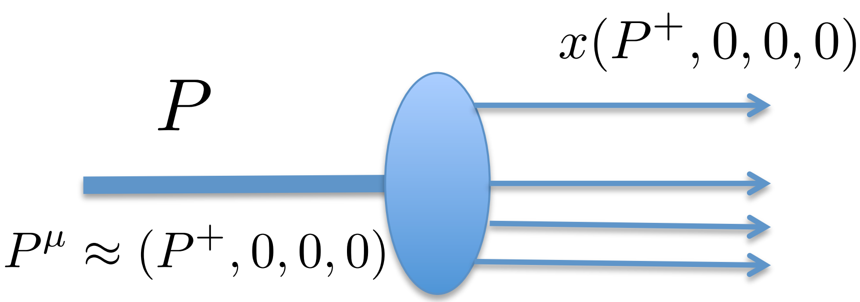
Collinear factorization at twist-2 or leading power:

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle P, s|\bar{\psi}_i(\lambda n)\psi_j(0)|P, s\rangle$$

Quark density matrix at leading twist

$$= \frac{1}{2}\left[ f_1(x)\gamma^- + s_L g_1(x)\gamma^- + h_1(x)\gamma_5\gamma\cdot s_\perp\gamma^- \right]_{ji}$$

One can only extract three functions of nonperturbative effects




The operators only find a quark with the momentum fraction, spin, flavors.. but no transverse momenta!

Only one-dim. Information……

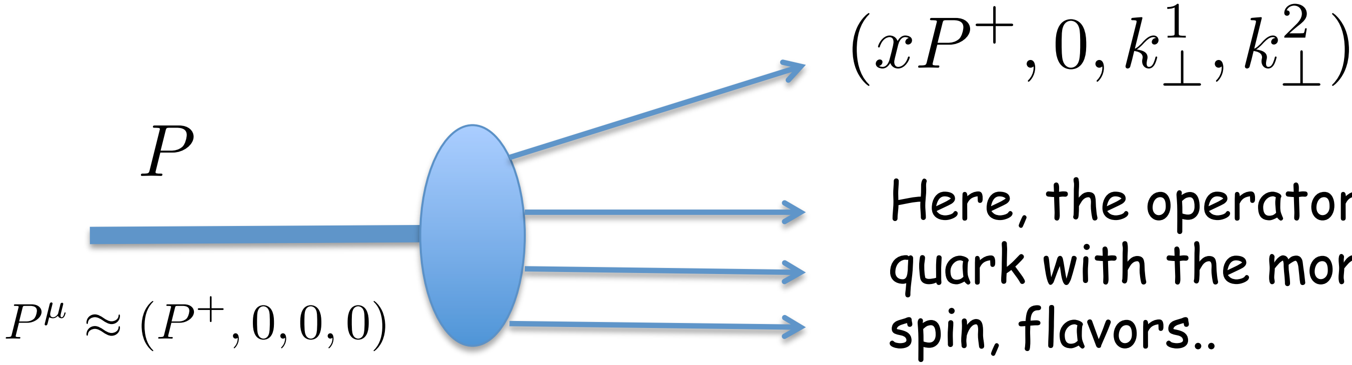
The quark density matrix in TMD factorization:

$$\mathcal{M}_{ij}(x,k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-ix\xi^- P^+ + ik_\perp\cdot\xi_\perp} \langle P,s|\bar{\psi}_j(\xi)\psi_i(0)|P,s\rangle\Big|_{\xi^+=0},$$

Eight TMD parton distribution functions at leading power, more information!

| Quark \ Nucleon | Unpol. | Long. | Trans. |
|---|---|---|---|
| Unpol. | $q(x, k_\perp)$ | | $q_T(x, k_\perp)$ |
| Long. | | $\Delta q_L(x, k_\perp)$ | $\Delta q_T(x, k_\perp)$ |
| Trans. | $\delta q(x, k_\perp)$ | $\delta q_L(x, k_\perp)$ | $\delta q_T(x, k_\perp)$<br>$\delta q_T'(x, k_\perp)$ |

Boer, Mulders, Tangerman et al.




Here, the operators not only find a quark with the momentum fraction, spin, flavors..
but also transverse momentum!

The eight parton distribution functions provides more information!

Three-dimensional picture of a hadron (proton)

In general, two factorizations are not equivalent, they can be used in different kinematical regions

TMD factorization has been examined in e+e-, SIDIS, Drell-Yan by J. Collins, D. Soper, G. Sterman, X.D. Ji, J.P. Ma and F. Yuan.

Which factorization should be used?

$$q^\mu = (q^+, q^-, q_\perp^1, q_\perp^2)$$

E.g., Drell-Yan process with measured $q_\perp$

$$h_A(P_A) + h_B(P_B) \to \gamma^*(q) + X \to \ell^- + \ell^+ + X,$$

- $q_\perp \gg \Lambda_{QCD}, q_\perp \sim Q$ Collinear factorization
- $q_\perp \sim \Lambda_{QCD}$ TMD factorization
- $q_\perp \ll Q,\ q_\perp \gg \Lambda_{QCD}$ Both factorizations, TMD offers the possibility to sum large log's $\ln \frac{q_\perp}{Q}$ of perturbative expansion in collinear factorization , CSS resummation……

Small $q_\perp$ is generated by transverse momenta of partons inside hadrons.

➔ One should take these momenta into account

E.g., one of the structure functions for the unpolarized case at tree-level:

$$F_{UU} \sim \int d^2k_{A\perp} d^2k_{B_\perp} q(x, k_{A\perp}) \bar{q}(y, k_{B\perp}) \delta^2(k_{A\perp} + k_{B\perp} - q_\perp) + (q \leftrightarrow \bar{q})$$

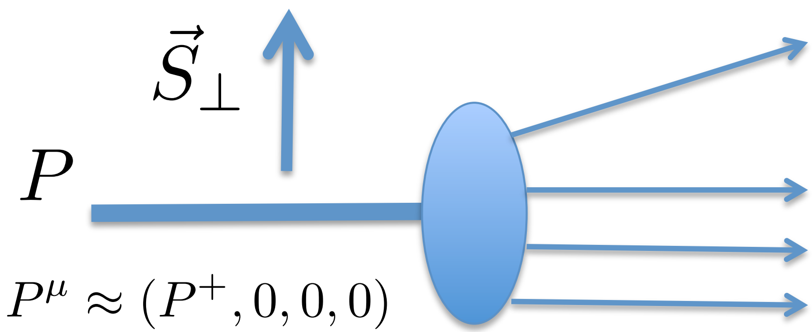
$$q^\mu = (xP_A^+, yP_B^-, q_\perp^1, q_\perp^2)$$

At leading twist there are 24 structure functions.

J.P. Ma and G.P. Zhang, arXiv:1308.2044

The 24 perturbative coefficients are the same!

One of TMD pdf's is of particular interest:




The hadron is transversely polarized, the struck quark is unpolarized.

$\vec{k}_\perp$ : transverse momentum of the struck quark can be correlated with the transverse spin!

The correlation is described by Sivers function:

$$\frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-ix\xi^- P^+ + ik_\perp \cdot \xi_\perp} \langle P, S_\perp | \bar{\psi}(\xi)\gamma^+\psi(0) | P, S_\perp \rangle \Big|_{\xi^+=0},$$

$$= q(x) + q_T(x, k_\perp)\epsilon_\perp^{\mu\nu} S_{\perp\mu} k_{\perp\nu}$$

Unpolarized TMD

Helicity-flip interaction

Absorptive part of amplitude or S-operator

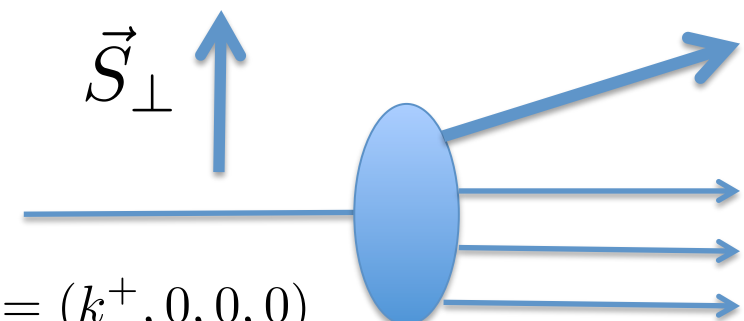
If Sivers function is nonzero from experiment

➔ the quark has nonzero orbital angular momentum in the transversely polarized hadron (proton) !!

Experiment: It is not zero!

There are similar things in fragmentation:

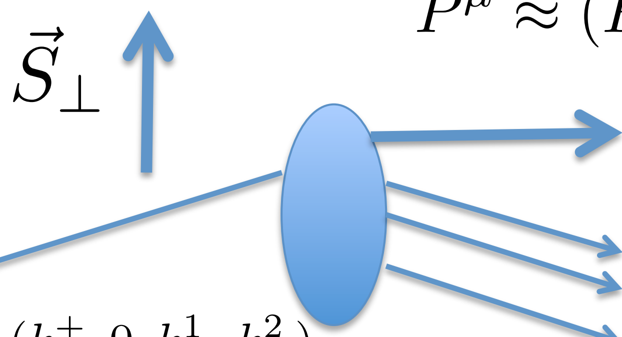
$$P^\mu \approx (P^+, 0, P_\perp^1, P_\perp^2)$$




$$k^\mu = (k^+, 0, 0, 0)$$

A transversely polarized quark fragments into a unpolarized hadron.
The hadron has a nonzero transverse momentum.

T-odd interaction is needed, or nonzero absorptive part.




$k^\mu = (k^+, 0, k_\perp^1, k_\perp^2)$

$$P^\mu \approx (P^+, 0, 0, 0), \quad P^+ = zk^+,$$

$$\vec{P}_\perp = -z\vec{k}_\perp,$$

Operator definition:

$$\mathcal{D}(z, k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{i\xi^- k^+ + ik_\perp \cdot \xi_\perp} \sum_X \langle 0|\psi(\xi)|P, X\rangle\langle X, P|\bar{\psi}(0)|0\rangle$$

$$= z\Big[D(z, k_\perp)\gamma^- + i\sigma^{\mu -} k_{\perp\mu} H_1^\perp(z, k_\perp)\Big],$$

TMD unpolarized fragmentation function

Collins fragmentation function

Transverse spin

Corresponding to collinear factorization, there is only one FF.

The importance to study Collins fragmentation function or Collins effect at e+e- colliders:

One place to study SSA or Sivers function and Collins effect is SIDIS.

In SIDIS, to clearly see SSA from Sivers function and Collins effect, one needs to study the distribution of two azimuthal angles.

In some integrated observables, two effects are mixed.

At e+e- machine, one has only Collins effect.

## 2. Collins effect at e+e- colliders

Inclusive production of two hadrons:

$$e^+ + e^- \to \gamma^*(q) \to h_a(P_a) + h_b(P_b) + X$$

Unpolarized initial states, spin of final states is not observed.

The two momenta are measured.

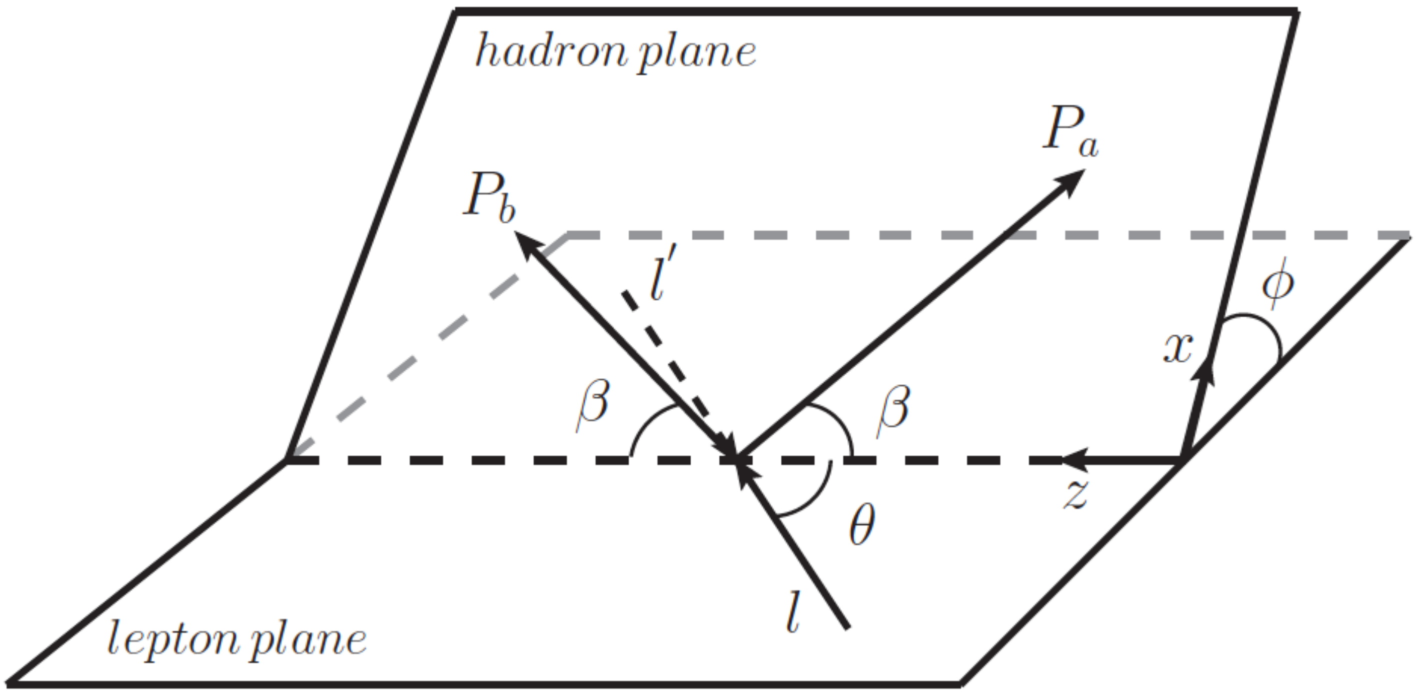
Only four structure functions can be measured in general.

Intuitively, one may think at tree-level of QCD:

$$e^+ + e^- \to \gamma^*(q) \to q + \bar{q}$$

The quark and antiquark are back-to-back........

A frame like Collins-Soper frame for Drell-Yan: $(\theta, \phi)$




$$l^\mu_{CS} = \frac{q}{2}\left(1,\ \sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta\right),$$

$$l'^\mu_{CS} = \frac{q}{2}\left(1,\ -\sin\theta\cos\phi,\ -\sin\theta\sin\phi,\ -\cos\theta\right),$$

$$P^\mu_{a\perp} = P^0_{a,CS}(\sin\beta, 0),$$

$$P^\mu_{a,CS} \approx P^0_{a,CS}\left(1,\ \sin\beta,\ 0,\ -\cos\beta\right),$$

$$P^\mu_{b,CS} \approx P^0_{b,CS}\left(1,\ \sin\beta,\ 0,\ \cos\beta\right),$$

$$q^\mu_\perp \approx q(\sin\beta, 0)$$

$$q^\mu_{CS} = \left(q,\ 0,\ 0,\ 0\right),$$

The general angular distribution: $(z_a, z_b, s)$

$$d\mathcal{N} \sim (1+\cos^2\theta)F_{UU}^1 + (1-\cos^2\theta^2)F_{UU}^3$$
$$+(\sin 2\theta\cos\phi)F_{UU}^{\cos\phi} + (\sin^2\theta\cos 2\phi)F_{UU}^{\cos 2\phi}$$

For small transverse momenta, i.e., $q_\perp/q \ll 1$

There is TMD factorization:

$$F_{UU}^1 = H\left[\sum_q e_q^2 \int d^2p_{a\perp}d^2p_{b\perp}D(z_a,p_{a\perp})\bar{D}(z_b,p_{b\perp})\delta^2(\vec{p}_{a\perp}+\vec{p}_{b\perp}-\vec{q}_\perp)\right.$$
$$\left.+(D\leftrightarrow\bar{D})\right]\left(1+\mathcal{O}(\Lambda^2/s)\right) \qquad H = 1+\mathcal{O}(\alpha_s)$$

$$z_{a,b} = \frac{2E_{a,b}}{\sqrt{s}} \qquad \Lambda \sim \Lambda_{QCD}, q_\perp, ..$$

Collins effect:

$$F_{UU}^{\cos 2\phi} = H\left[\sum_q e_q^2 \int d^2p_{a\perp} d^2p_{b\perp} f(p_{a\perp}, p_{b\perp}) H_1^\perp(z_a, p_{a\perp}) \bar{H}_1^\perp(z_b, p_{b\perp}) \delta^2(\vec{p}_{a\perp} + \vec{p}_{b\perp} - \vec{q}_\perp) + (H_1^\perp \leftrightarrow \bar{H}_1^\perp)\right]\left(1 + \mathcal{O}(\Lambda^2/s)\right)$$

$$f(p_{a\perp}, p_{b\perp}) = -\vec{p}_{a\perp} \cdot \vec{p}_{b\perp} + 2\vec{q}_\perp \cdot \vec{p}_{a\perp} \vec{q}_\perp \cdot \vec{p}_{b\perp} / \vec{q}_\perp \cdot \vec{q}_\perp$$

Other two are zero at leading power.

In fact, the Boer-Mulders function (TMD pdf) corresponds more closely to the Collins function.

➔ Violation of Lam-Tung relation in Drell-Yan processes

## 3. Outlook

If BES can study the angular distribution and find Collins effect, it will be a great help for understanding hadronization ➔ QCD