

# Quark to Hadron Fragmentation Functions

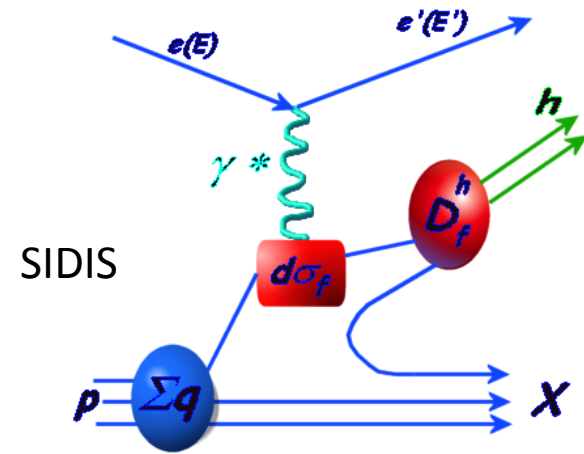
## Relation of BES-III Measurement to JLab-12 GeV SIDIS Experiments

Discussions at IHEP, Nov. 1<sup>st</sup>, 2013.

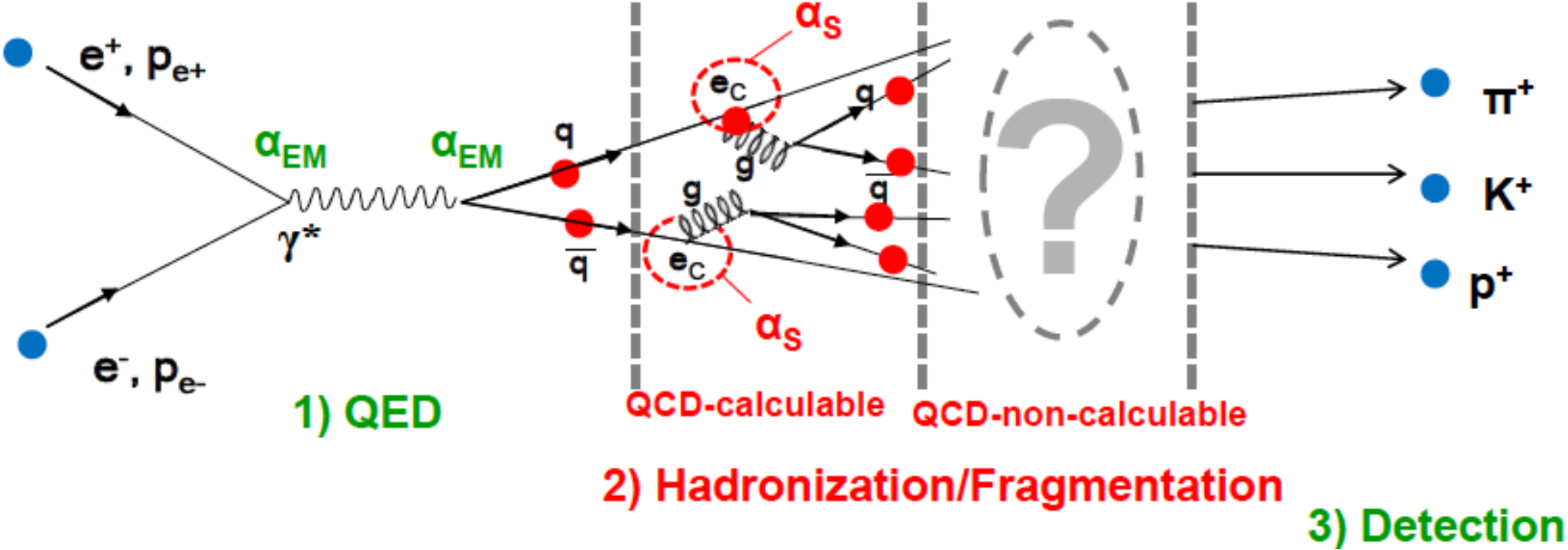
Xiaodong Jiang

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- Spin independent Fragmentation Functions
- Spin dependent Fragmentation Functions



Universality ?  
 Vacuum vs inside a nucleon ?

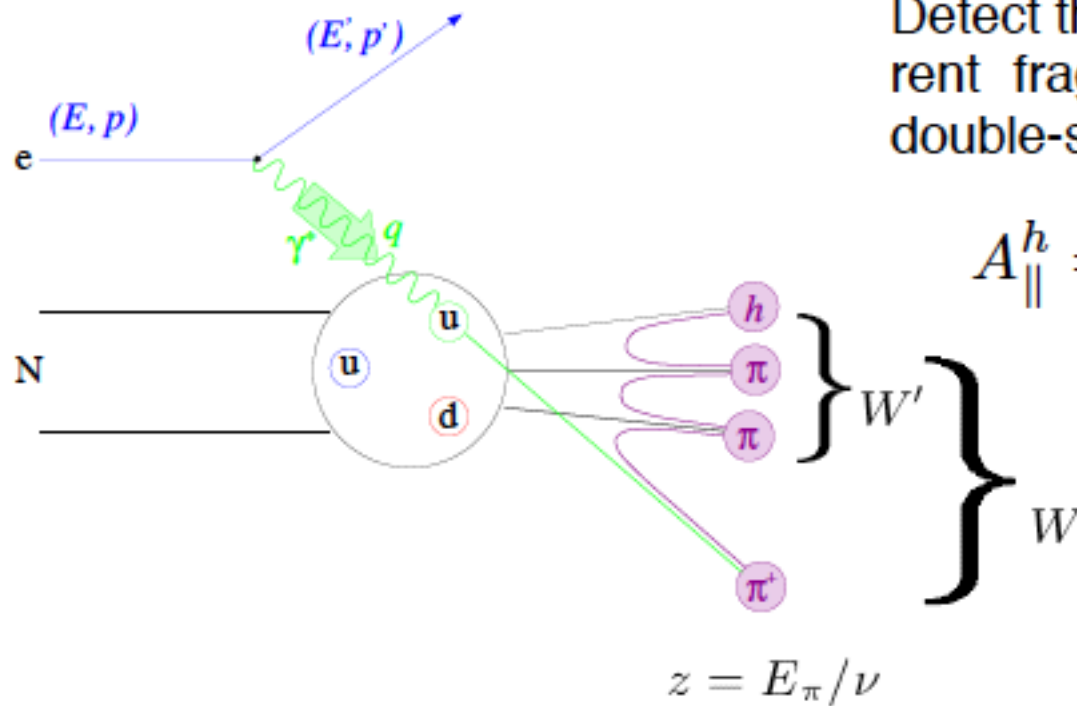


# Four Types of Frag. Func.

- Spin independent Fragmentation Functions
- Spin dependent Fragmentation Functions  
(i.e. initial quark spin to final hadron azimuthal).
- Spin Transfer Fragmentation Functions  
(i.e. initial quark spin to final hadron spin)
- Fragmentation Function of Induced hadron polarization  
(i.e. unpolarized quark to polarized final hadron)

$$\begin{aligned}
N^U(\phi) &= \frac{d\sigma(e^+e^- \rightarrow \pi^\pm \pi^\mp X)}{d\Omega dz_1 dz_2} \propto \frac{5}{9} D^{\text{fav}}(z_1) \bar{D}^{\text{fav}}(z_2) + \frac{7}{9} D^{\text{dis}}(z_1) \bar{D}^{\text{dis}}(z_2) \\
N^L(\phi) &= \frac{d\sigma(e^+e^- \rightarrow \pi^\pm \pi^\pm X)}{d\Omega dz_1 dz_2} \propto \frac{5}{9} D^{\text{fav}}(z_1) \bar{D}^{\text{dis}}(z_2) + \frac{5}{9} D^{\text{dis}}(z_1) \bar{D}^{\text{fav}}(z_2) + \frac{2}{9} D^{\text{dis}}(z_1) \bar{D}^{\text{dis}}(z_2) \\
N^C(\phi) &= \frac{d\sigma(e^+e^- \rightarrow \pi\pi X)}{d\Omega dz_1 dz_2} = N^U(\phi) + N^L(\phi) \propto \frac{5}{9} [D^{\text{fav}}(z_1) + D^{\text{dis}}(z_1)] [\bar{D}^{\text{fav}}(z_2) + \bar{D}^{\text{dis}}(z_2)] + \frac{4}{9} D^{\text{dis}}(z_1) \bar{D}^{\text{dis}}(z_2)
\end{aligned}$$

# Flavor Tagging in Semi-Inclusive DIS



Detect the leading hadron from the current fragmentation and measure the double-spin asymmetry:

$$A_{\parallel}^h = f^h P_B P_T \cdot \mathcal{P}_{kin} \cdot A_{1N}^h$$

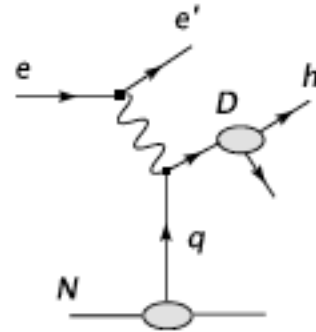
Assume leading order naive  $x$ - $z$  separation:

$$A_{1N}^h(x, Q^2, z) \equiv \frac{\Delta\sigma^h(x, Q^2, z)}{\sigma^h(x, Q^2, z)} = \frac{\sum_f e_f^2 \Delta q_f(x, Q^2) \cdot D_f^h(z, Q^2)}{\sum_f e_f^2 q_f(x, Q^2) \cdot D_f^h(z, Q^2)}$$

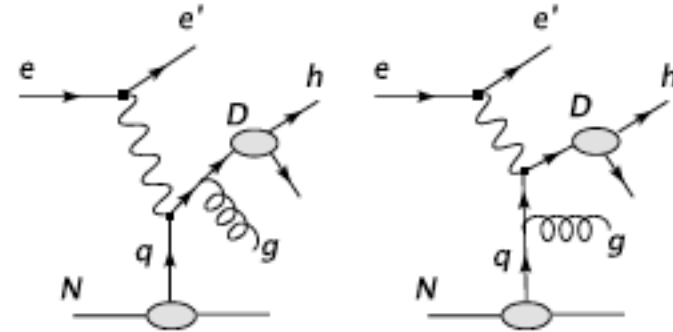
Each asymmetry measurement provides an independent constrain on  $\Delta q_f$ .

# Semi-inclusive DIS at NLO

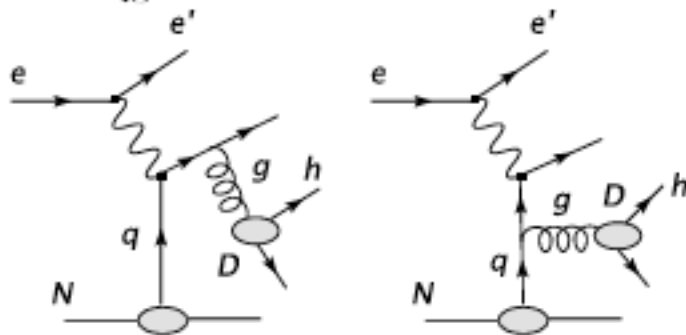
LO:



NLO-qq:



NLO-qg:



NLO-gg:

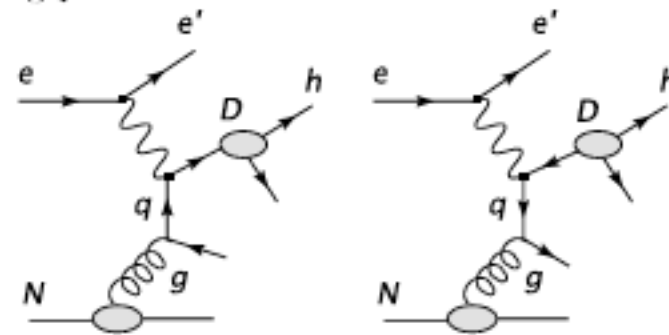


Figure 3: Semi inclusive deep inelastic scattering diagrams at leading order (LO) and the next-to-leading order (NLO).

$$\sigma^h(x, z) = \sum_f e_f^2 q_f \left[ 1 + \otimes \frac{\alpha_s}{2\pi} C_{qq} \otimes \right] D_{qf}^h + \left( \sum_f e_f^2 q_f \right) \otimes \frac{\alpha_s}{2\pi} C_{qg} \otimes D_G^h + G \otimes \frac{\alpha_s}{2\pi} C_{gq} \otimes \left( \sum_f e_f^2 D_{qf}^h \right),$$

## SIDIS Cross Section at the Next-to-Leading-Order

$$q(x, Q^2) \cdot D(z, Q^2) \Rightarrow \int \frac{dx'}{x'} \int \frac{dz'}{z'} q\left(\frac{x}{x'}\right) C(x', z') D\left(\frac{z}{z'}\right) = q \otimes C \otimes D$$

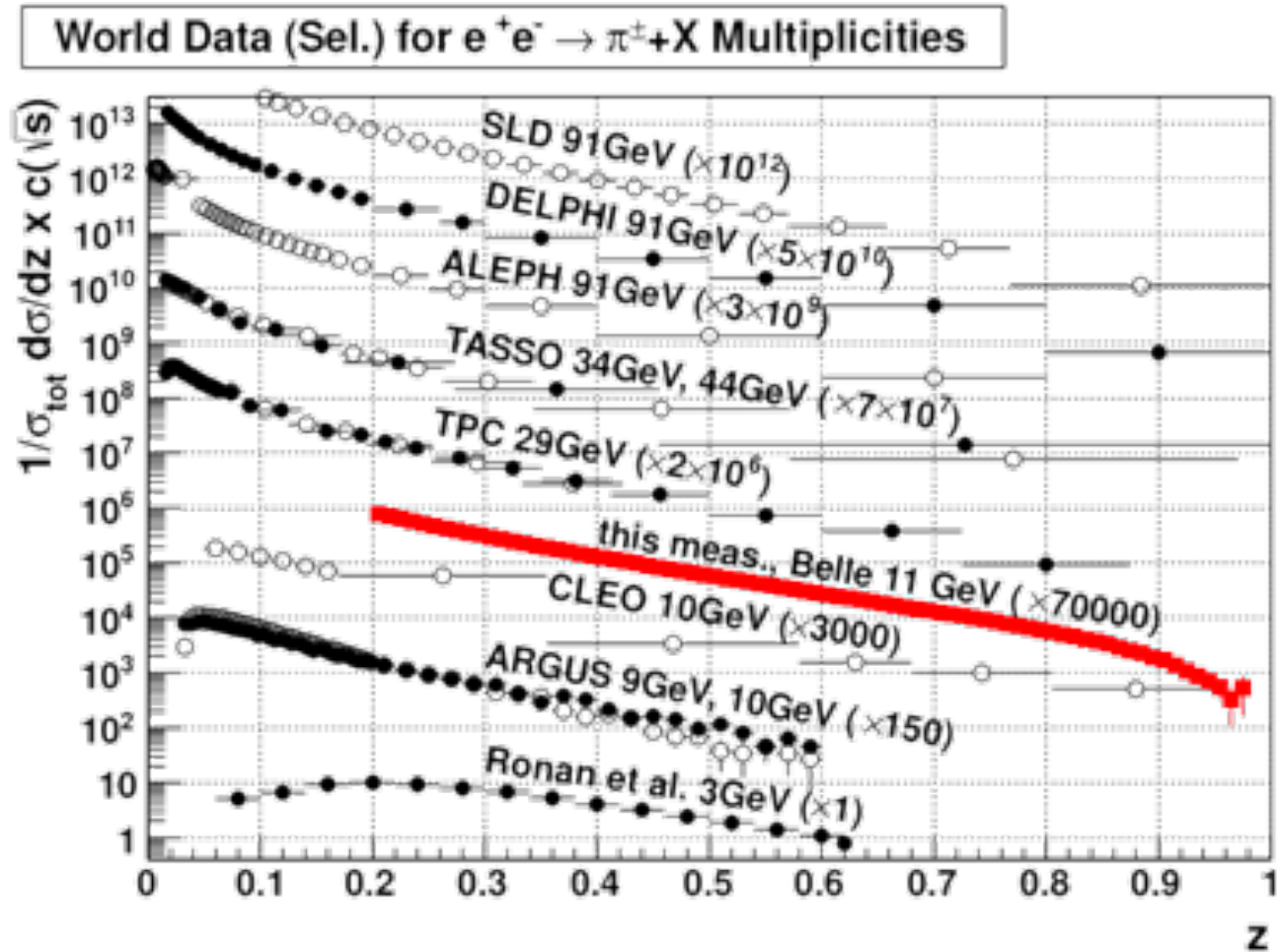
$C$  are well-known Wilson coefficients (D. Graudenz, NPB432, 351(1994)).

$$\begin{aligned} \Delta\sigma^h &= \sum_i e_i^2 \Delta q_i \left[ 1 + \otimes \frac{\alpha_s}{2\pi} \Delta C_{qq} \otimes \right] D_{q_i}^h \\ &+ \left( \sum_i e_i^2 \Delta q_i \right) \otimes \frac{\alpha_s}{2\pi} \Delta C_{qg} \otimes D_G^h + \Delta G \otimes \frac{\alpha_s}{2\pi} \Delta C_{gq} \otimes \left( \sum_i e_i^2 D_{q_i}^h \right) \end{aligned}$$

Isospin symmetry and charge conjugation:  $D_G^h = D_{\bar{G}}^h$ ,  $\sum_i e_i^2 D_{q_i}^h = \sum_i e_i^2 D_{\bar{q}_i}^h$ .

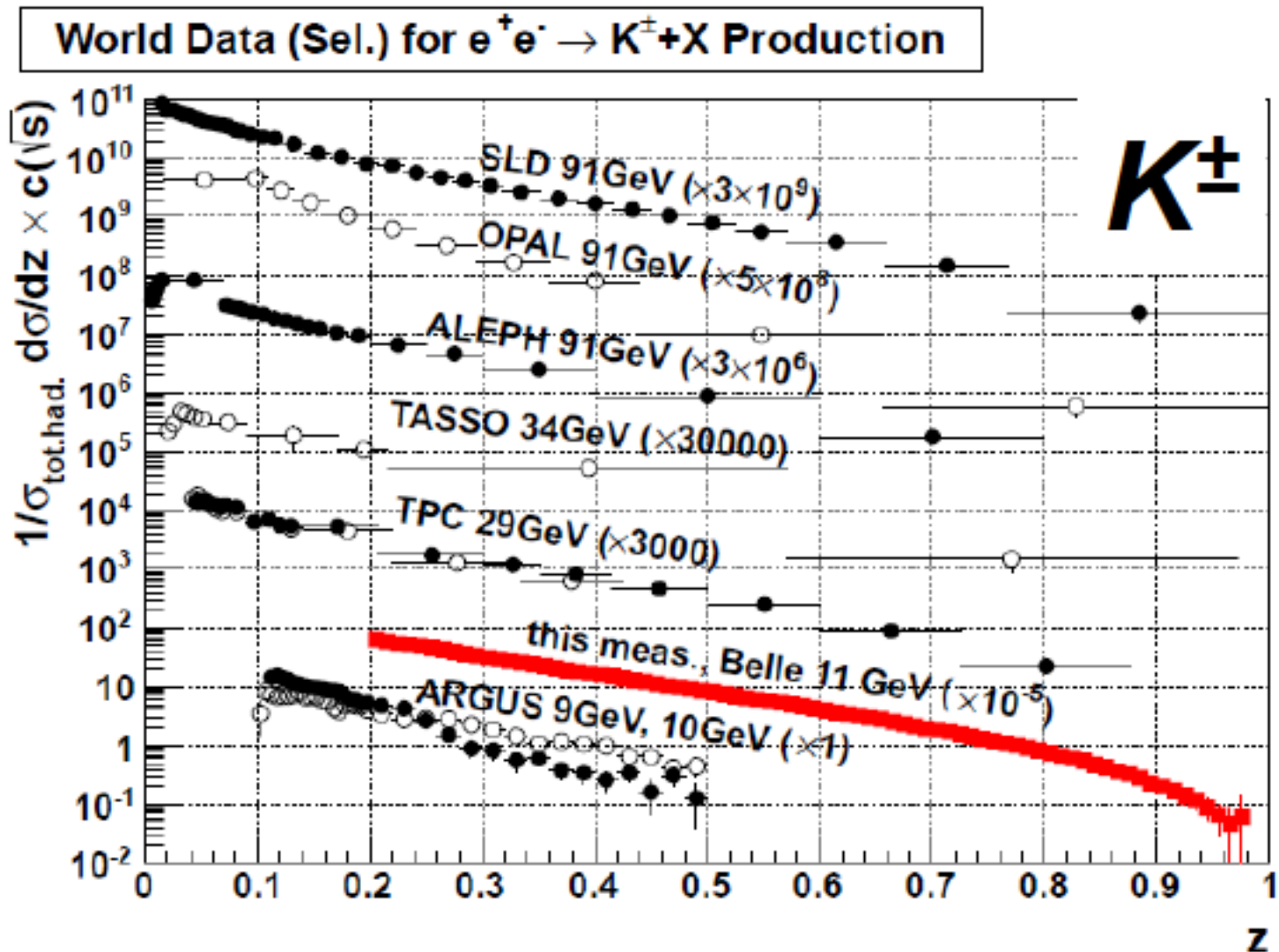
- $C$  and  $\Delta C$  are very similar, NLO effects could cancel out in  $A_{1N}^{\pi^+ + \pi^-}$ .
- NLO global QCD-fit with inclusive and SIDIS data constrain  $\Delta q$  and  $\Delta G$ .

# BELLE





# Belle results vs. World Data Selection



# SIDIS HERMS Charged Hadron Multiplicities on a Proton Target

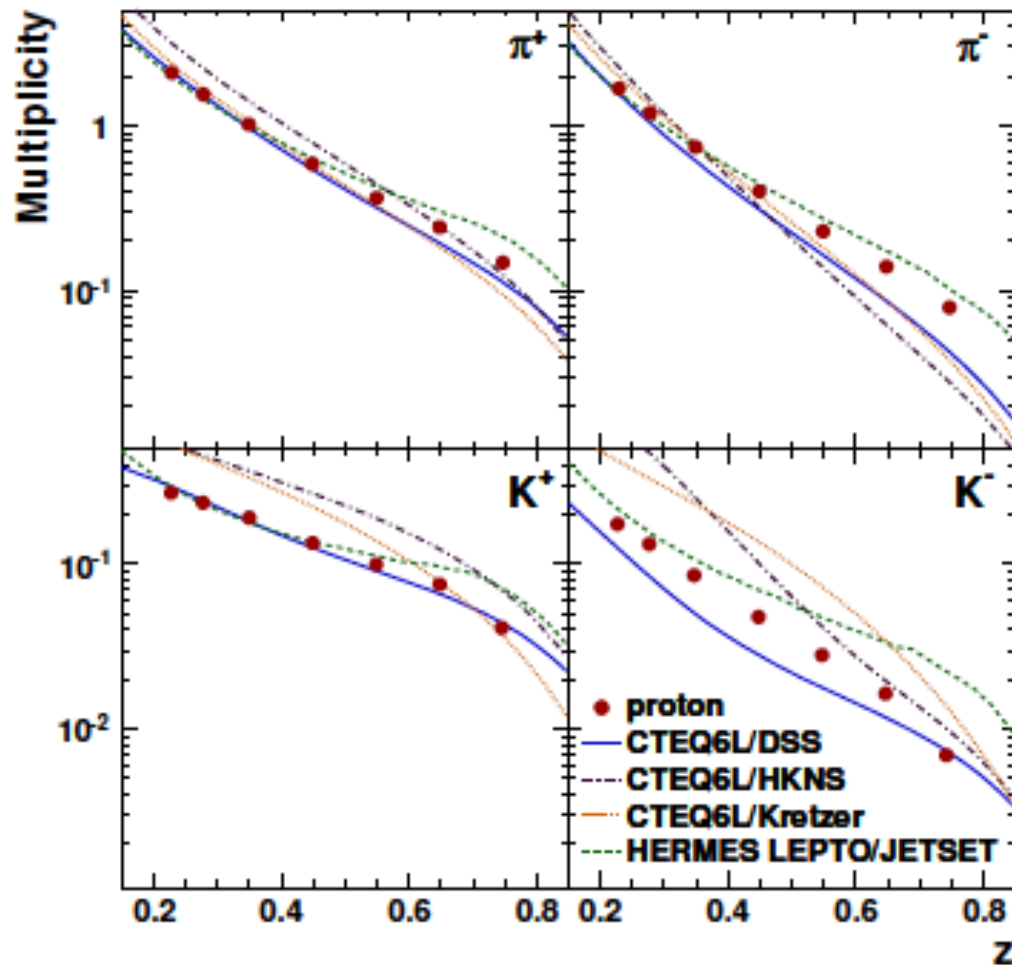
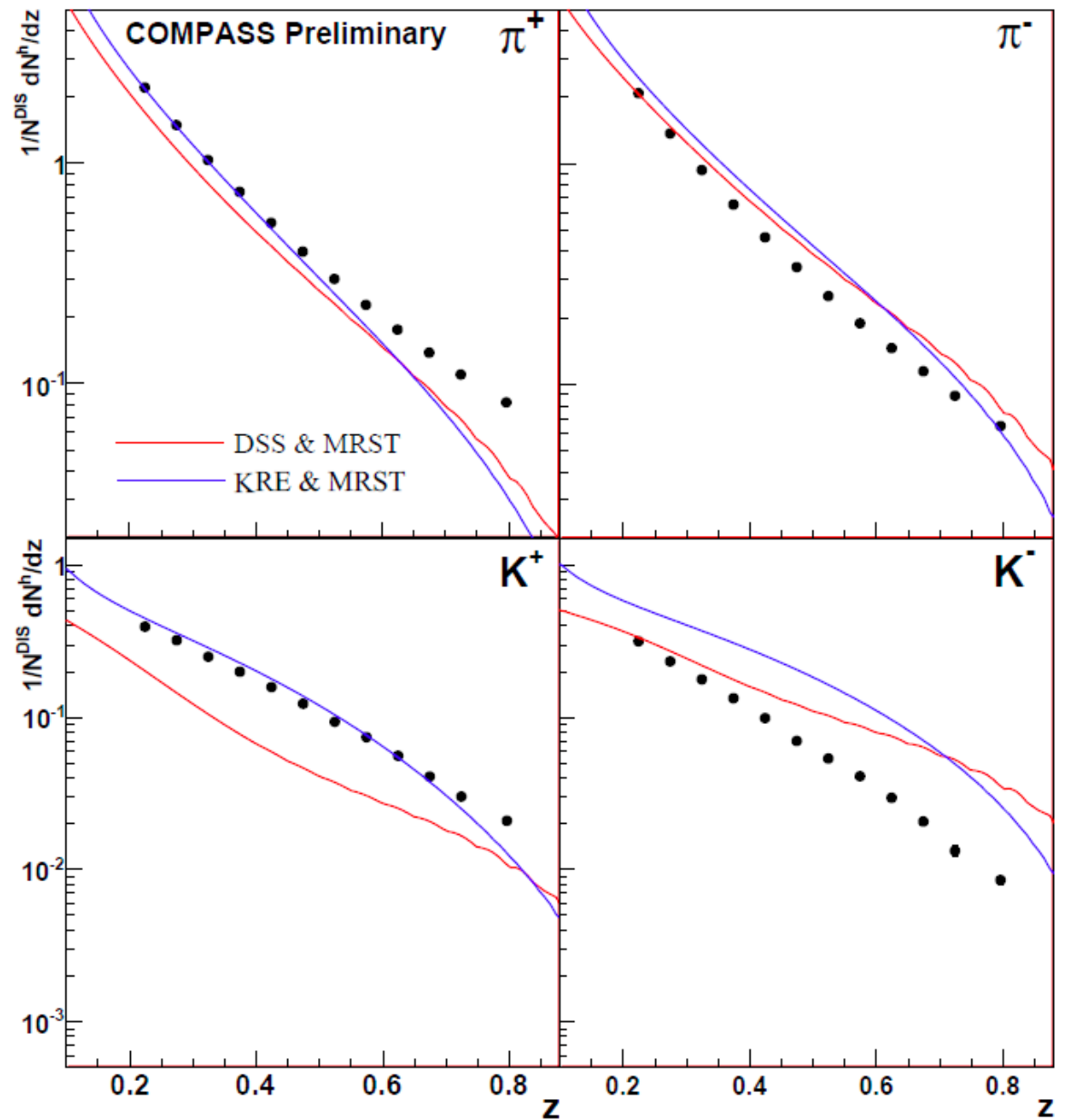


FIG. 9. Comparison of the vector-meson-corrected multiplicities measured on the proton for various hadrons with LO calculations using CTEQ6L parton distributions [45] and three compilations (see text) of fragmentation functions. Also shown are the values obtained from the HERMES Lund Monte Carlo. The statistical error bars on the experimental points are too small to be visible.

# SIDIS COMPASS Charged Hadron Multiplicities on a Proton Target



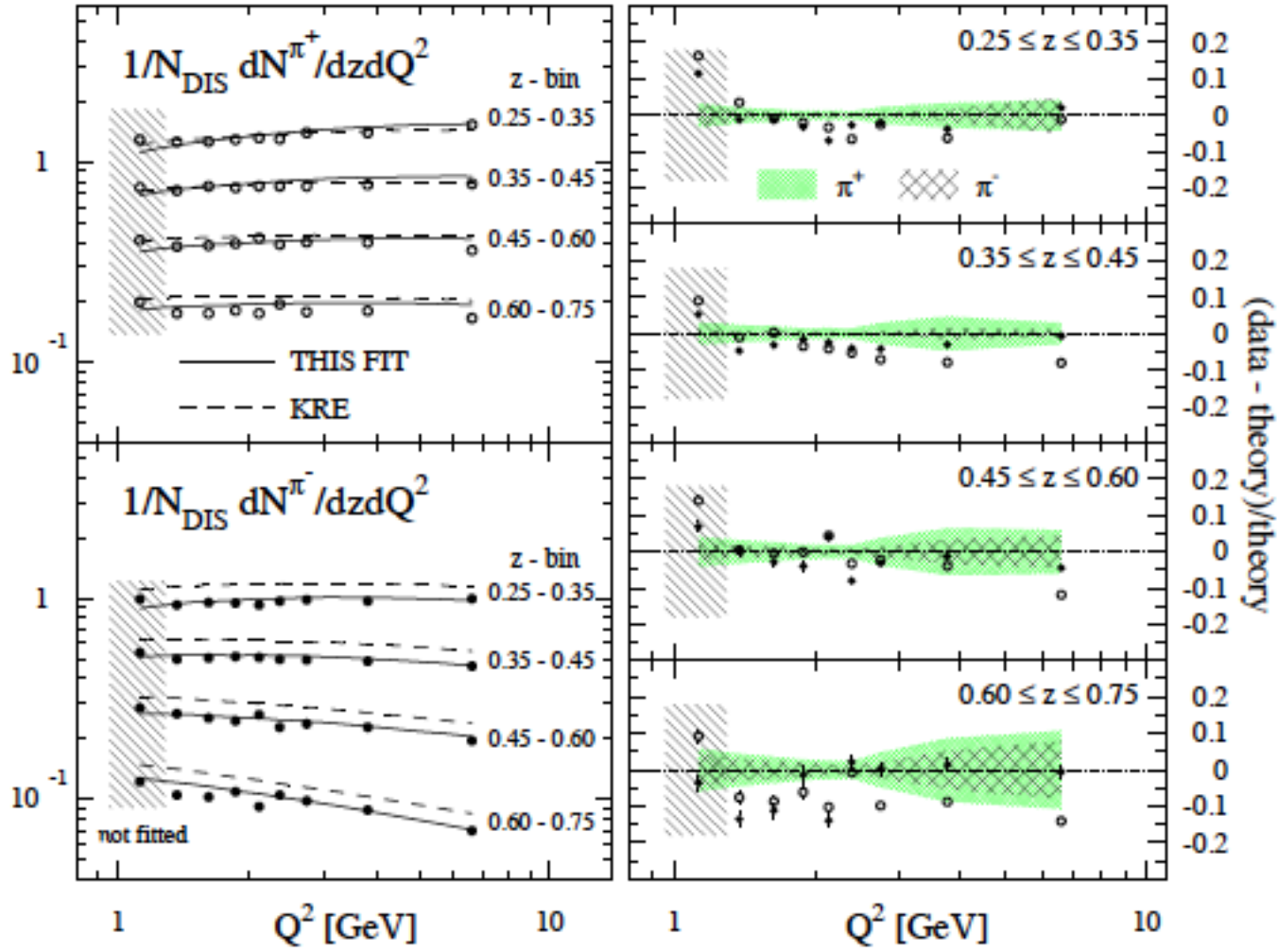


FIG. 4: **L.h.s.** comparison of our NLO results for charged pion multiplicities in SIDIS,  $(1/N_{DIS})dN^{\pi^{\pm}}/dzdQ^2$ , with preliminary HERMES data [18]. Also shown are the results obtained with the KRE [7] parametrization. **R.h.s.** “(data-theory)/theory” for our NLO results, open and full circles denote  $\pi^+$  and  $\pi^-$  multiplicities, respectively. The shaded bands indicate estimates of theoretical uncertainties due to finite bin-size effects (see text).

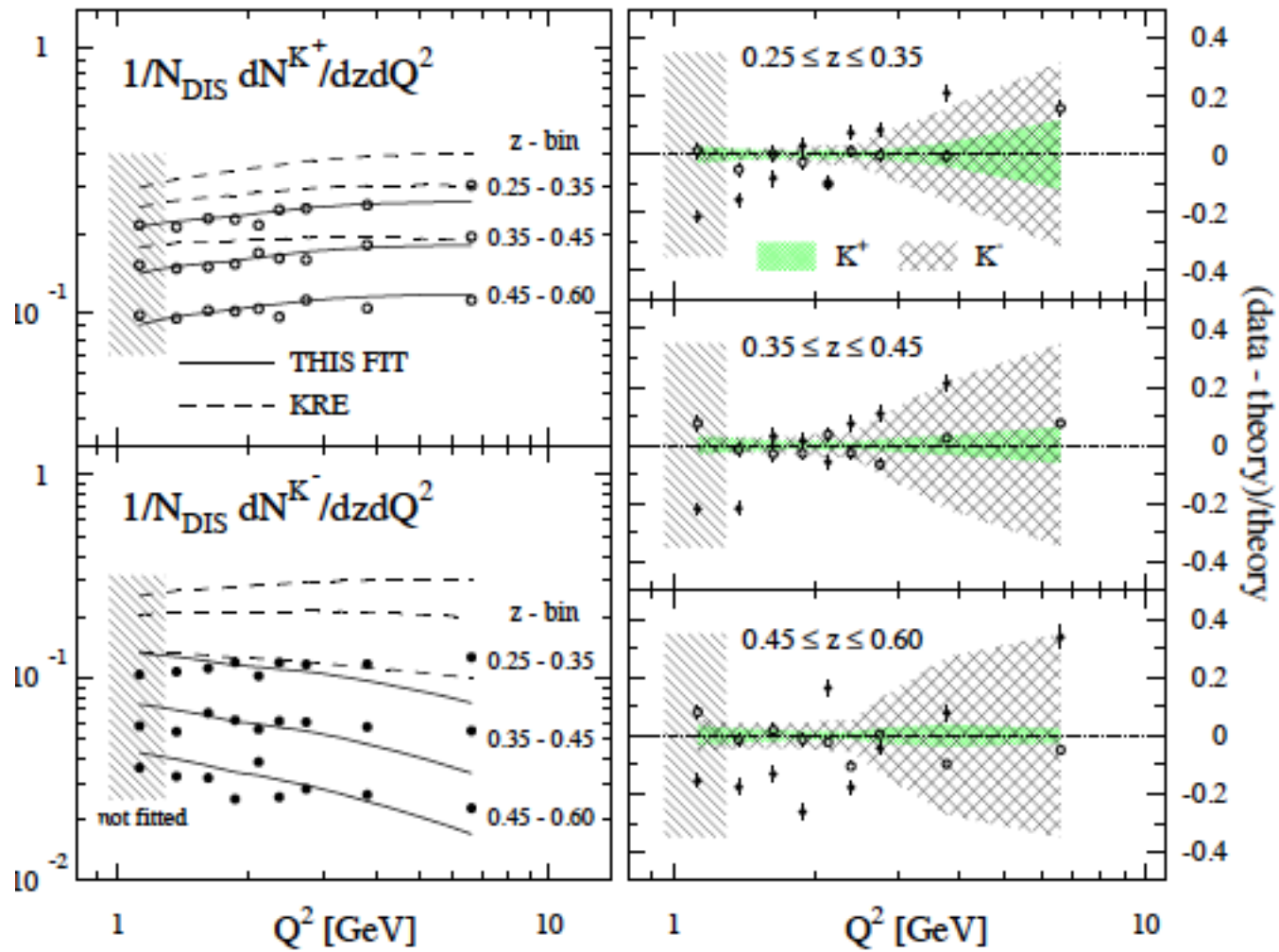
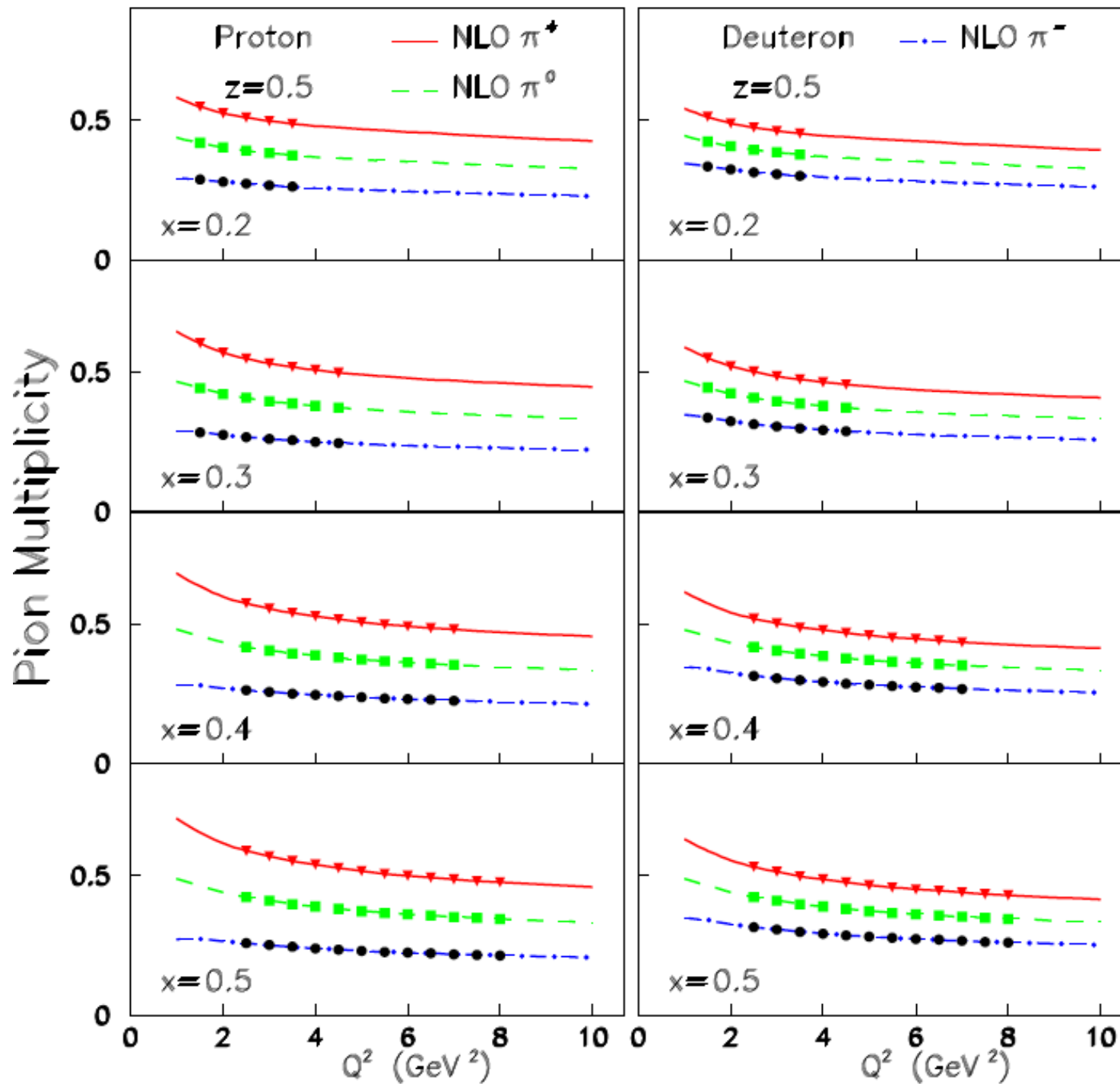


FIG. 13: Same as in Fig. 4 but now for charged kaons.



Projected  
precision of  
JLab-12 GeV  
sidis  
measurements

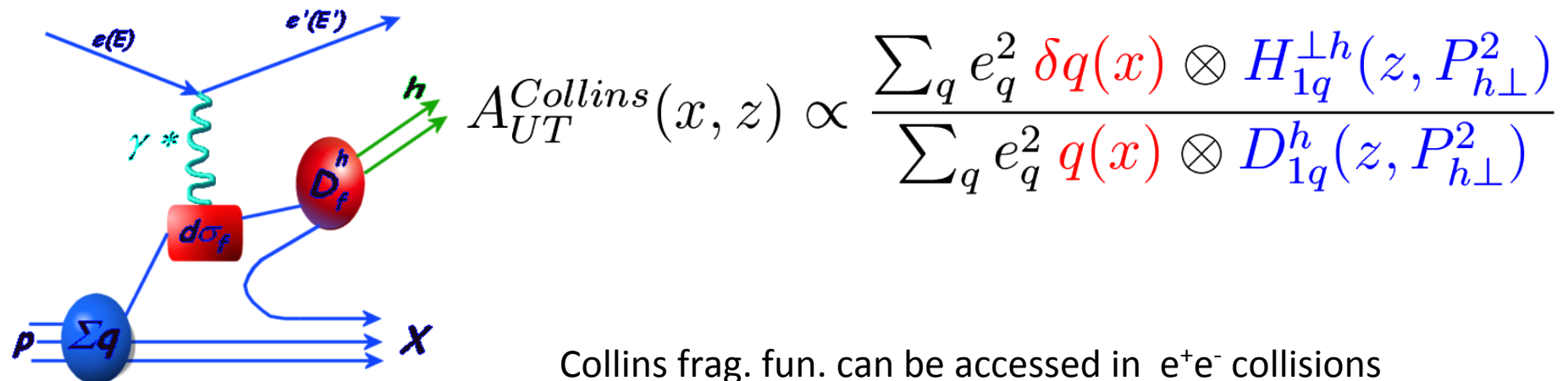
# Spin dependent frag. Func.

## to access quark transversity distributions ...

- Transversity distribution is chiral-odd, not accessible through inclusive deep-inelastic scattering. Need to be combined with another chiral-odd object, i.e. Collins fragmentation function.

Through target single spin asymmetry in **semi-inclusive DIS**.

J.C. Collins, NPB 396, 161(1993).



Collins frag. fun. can be accessed in  $e^+e^-$  collisions (BELLE experiment at KEK).



# Collins-Soper Frame

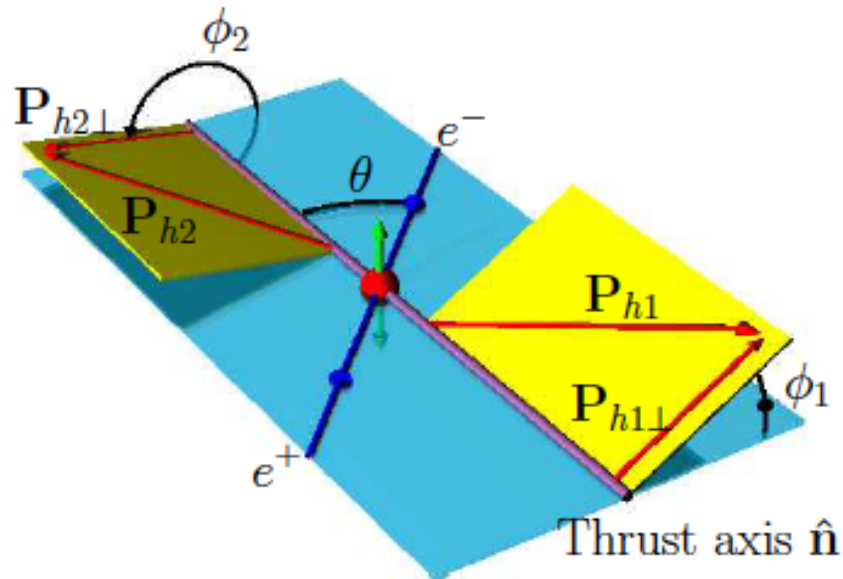
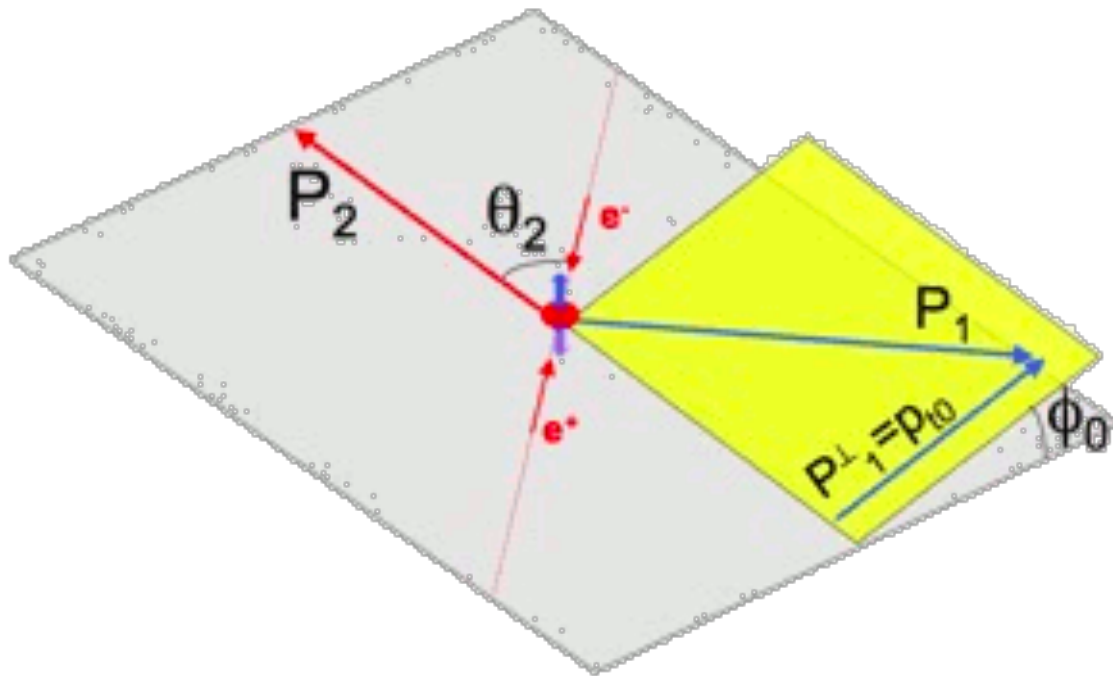


FIG. 1: Definition of the azimuthal angles of the two hadrons. In each case,  $\phi_i$  is the angle between the plane spanned by the lepton momenta and the thrust axis  $\hat{n}$ , and the plane spanned by  $\hat{n}$  and the hadron transverse momentum  $P_{hi\perp}$ .

$$A_{12} \propto \cos(\phi_1 + \phi_2) \frac{H_1^{\perp,[1]} \bar{H}_1^{\perp,[1]}}{D_1^{[0]} \bar{D}_1^{[0]}}$$



# Gottfried-Jackson Frame



$$A_0 \propto \cos(2\phi_0) \frac{\mathcal{F}[WH_1^\perp \bar{H}_1^\perp]}{\mathcal{F}[D_1 \bar{D}_1]}$$

# BELLE

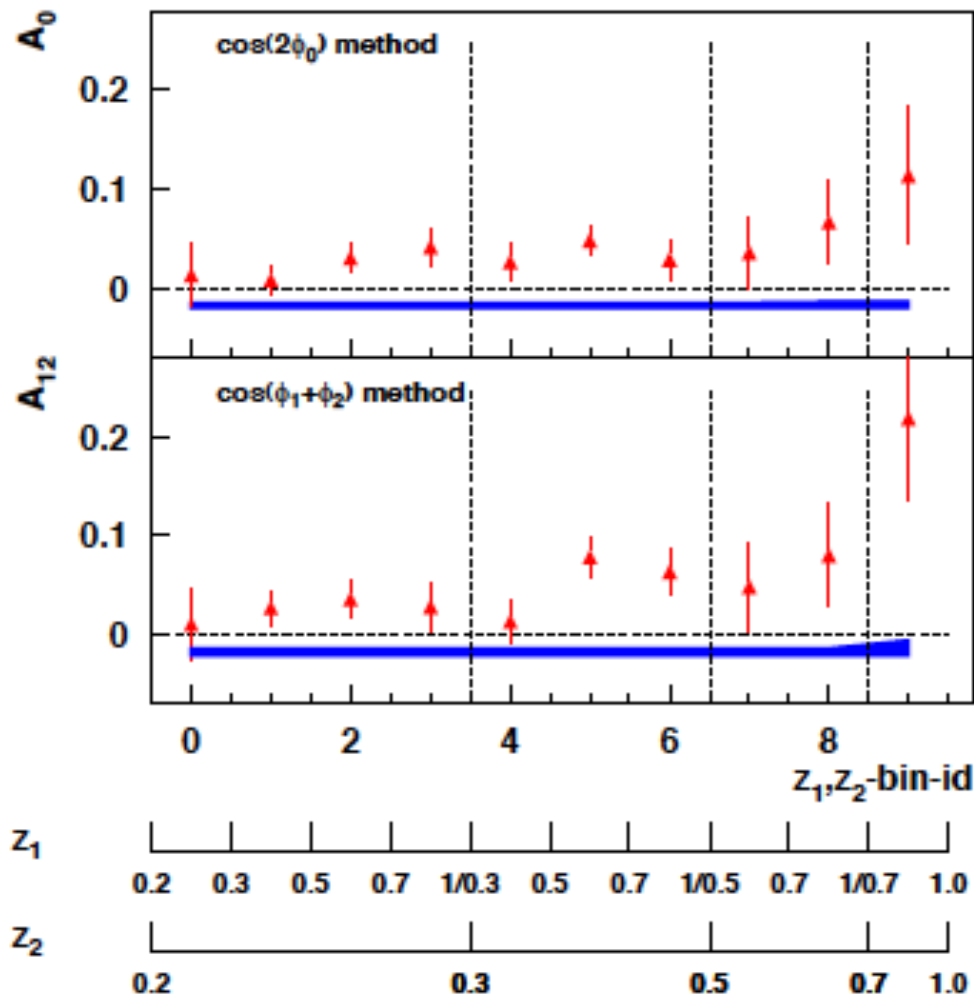


FIG. 3: Values of  $A_0$  and  $A_{12}$  as functions of  $z$ , corrected for the contribution of charm events. The lower scales show the boundaries of the bins in  $z_1$  and  $z_2$ ; see the text. The shaded band shows the size of the systematic errors.

# BARBAR

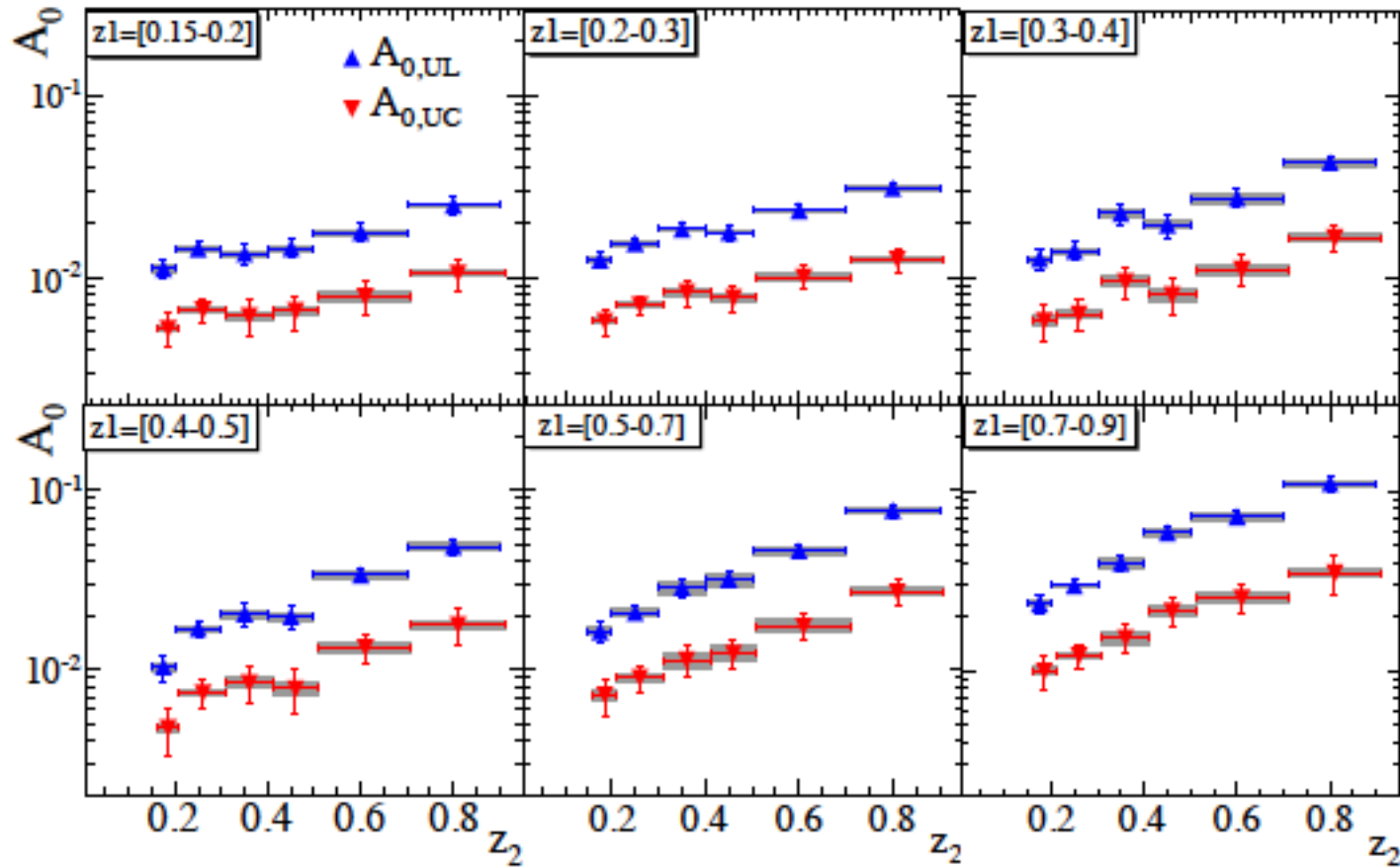
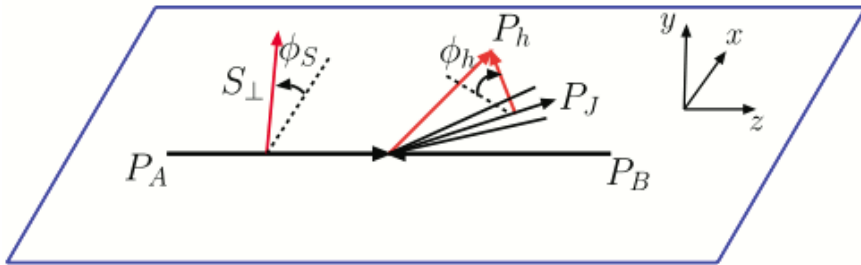


FIG. 15: (color online). Collins asymmetries for light quarks measured in bins of fractional energies  $(z_1, z_2)$ , in RF0. Asymmetries for the UL (up triangles) and UC (down triangles) ratios are reported, with statistical error bars and systematic uncertainties represented by the bands around the points.

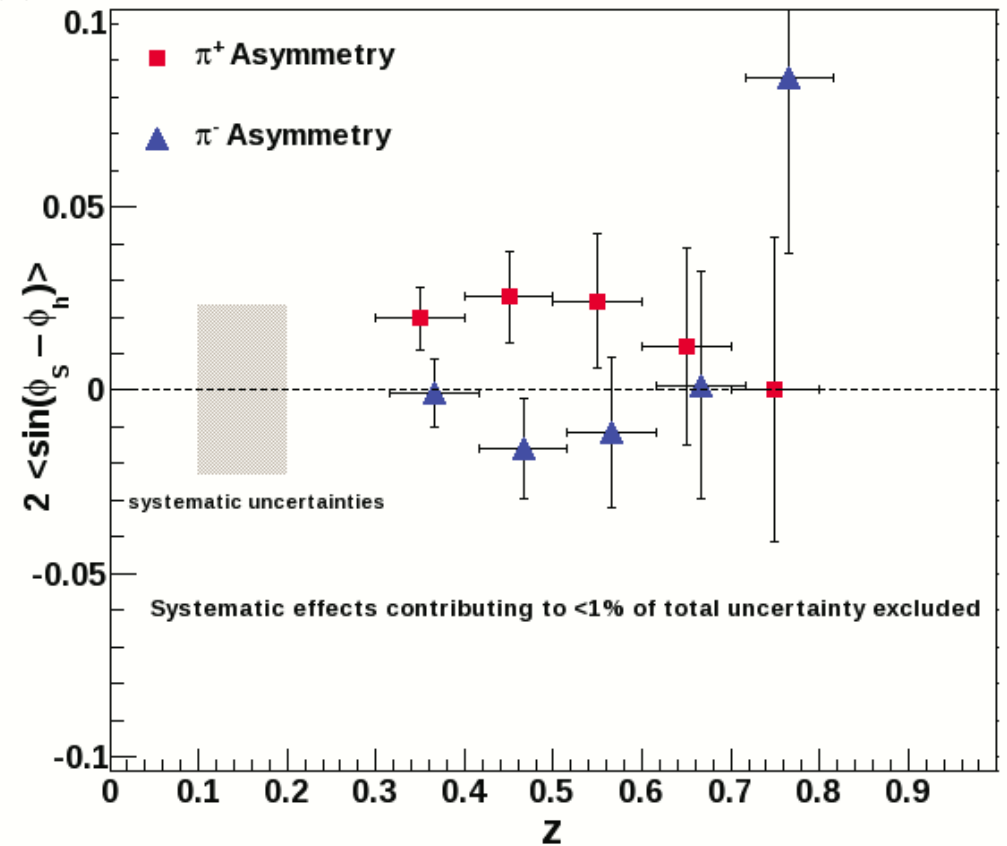
# STAR 2006 Preliminary p+p -> hadron azimuthal distribution

Midrapidity Collins:  $A_N$  is related to  $\Delta_T f(x) \otimes \Delta D(z)$

Measure an azimuthal modulation of a  $\pi$  in a jet

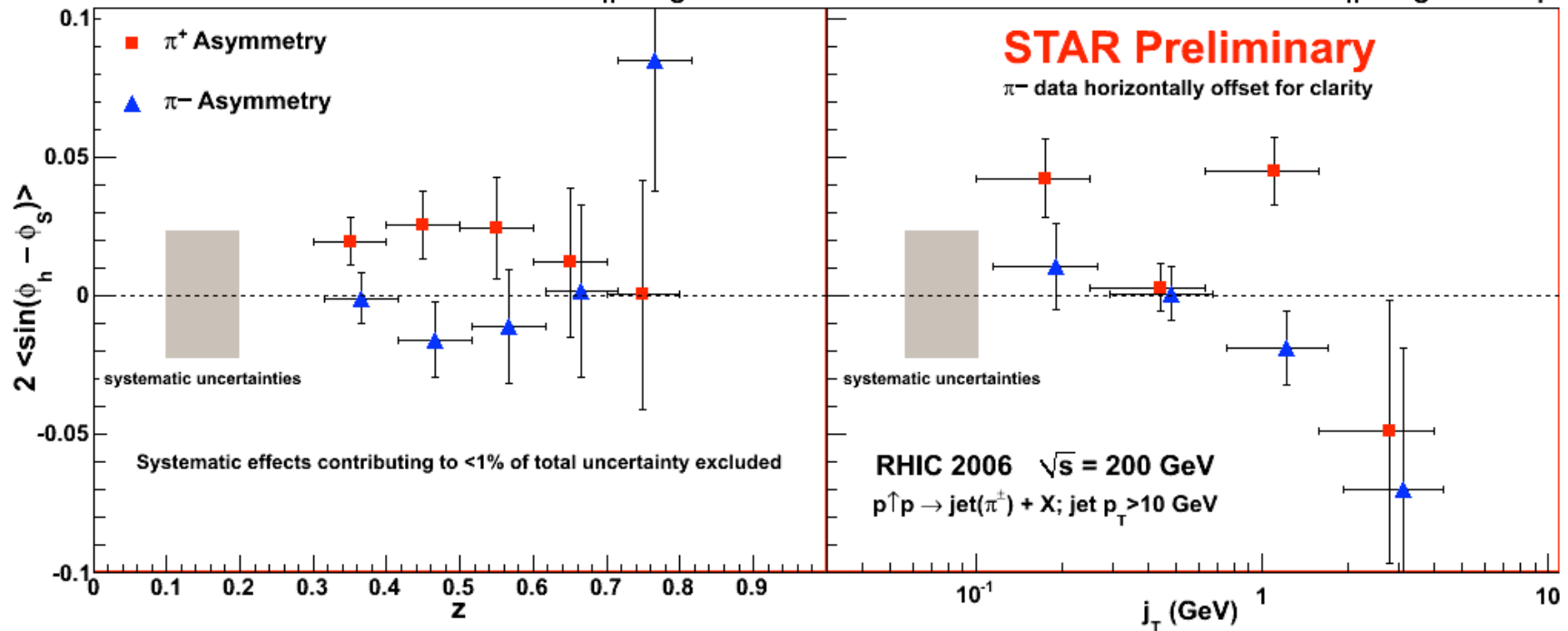


Collins Asymmetry  $A \equiv 2 \langle \sin(\phi_s - \phi_h) \rangle$  vs.  $z$



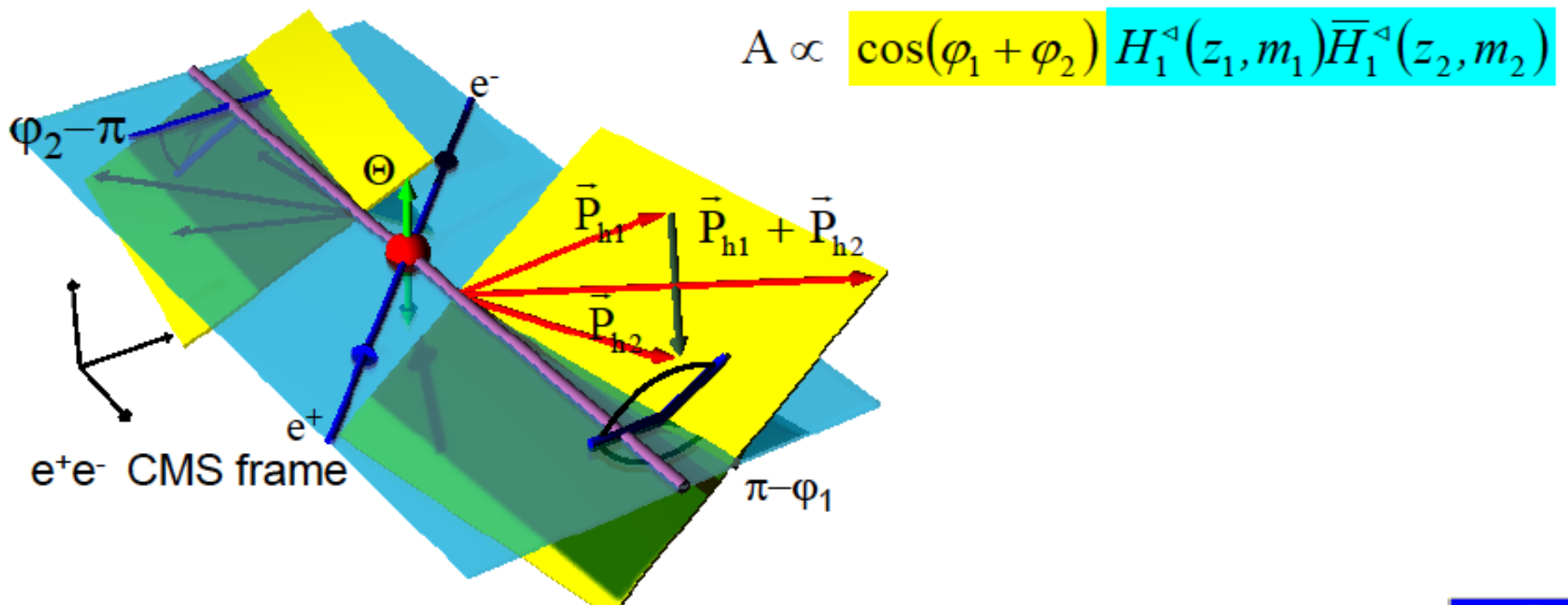
Collins Asymmetry  $A \equiv 2 \langle \sin(\phi_h - \phi_s) \rangle$  vs.  $z$

Collins Asymmetry  $A \equiv 2 \langle \sin(\phi_h - \phi_s) \rangle$  vs.  $j_T$



## Di-hadron Correlation

- > measure simultaneous **azimuthal modulation** of two pion pair yields in **opposing hemispheres**



- Transverse quark spin distributions  $\delta q$  can be extracted from measurements in **pp** and **SIDIS** experiments via:

- transversity x Collins FF:  $\delta q \times H_1^\perp$

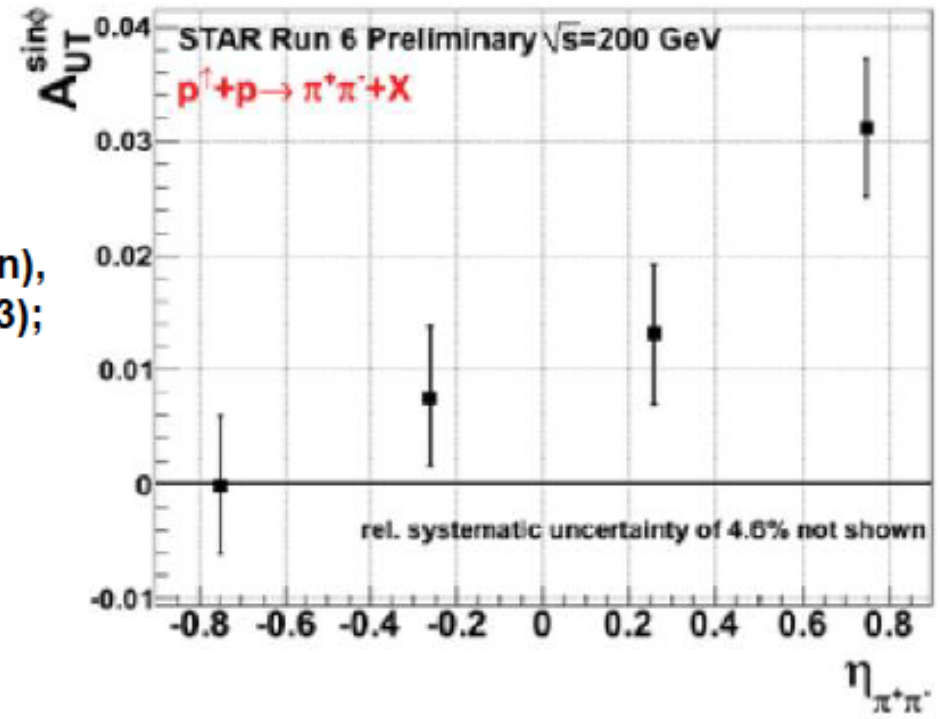
- transversity x Interference FF (IFF):  $\delta q \times H_1^{\sphericalangle}$

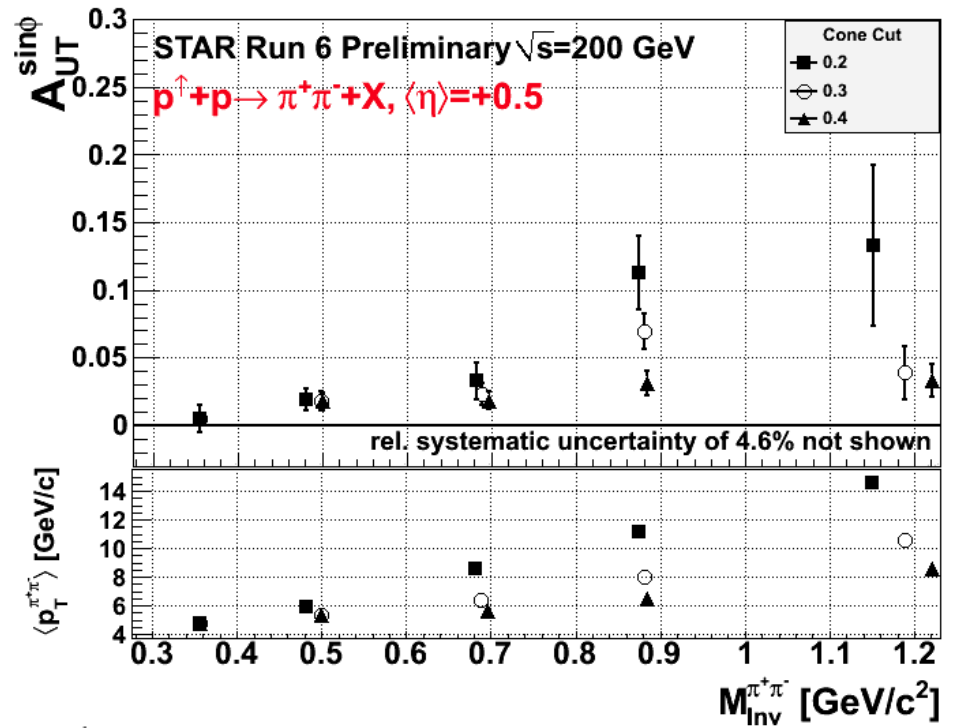
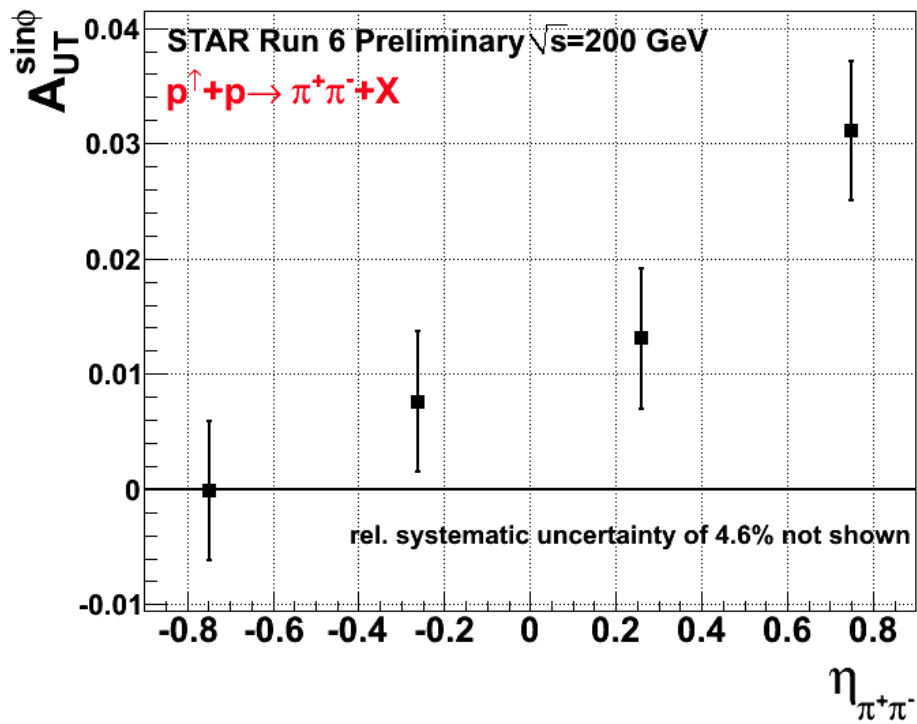
Belle can measure hadrons in two hemispheres

**STAR**  $\pi^+ \pi^-$  IFF

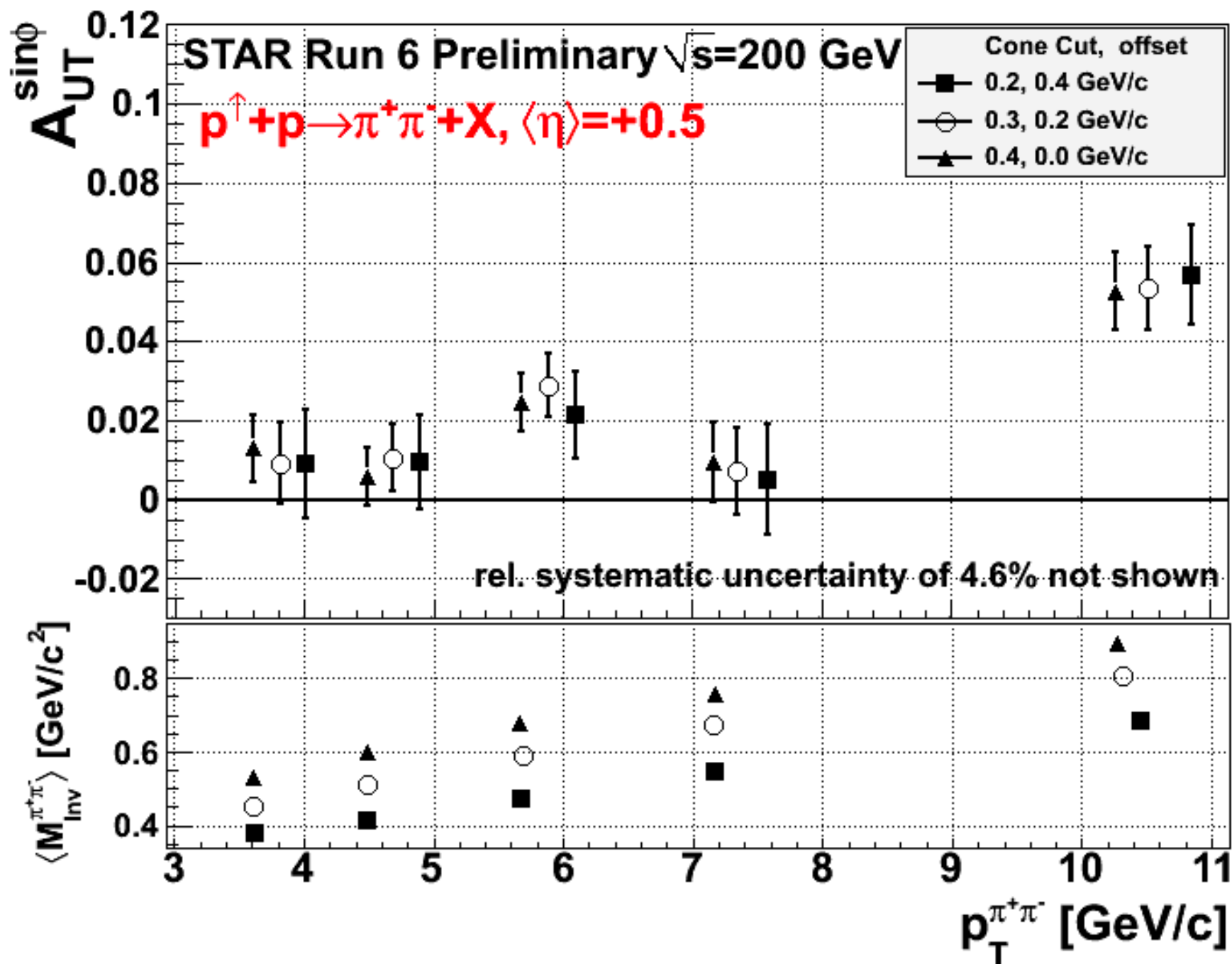
yielding two/four  $\rightarrow (hh)(hh)+X$ :

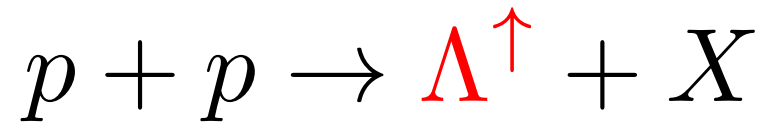
G. Igo et al (STAR collaboration), AIP Conf. Proc. 1523, 188 (2013);











**The puzzle of large induced polarization of hyperon in p+p**

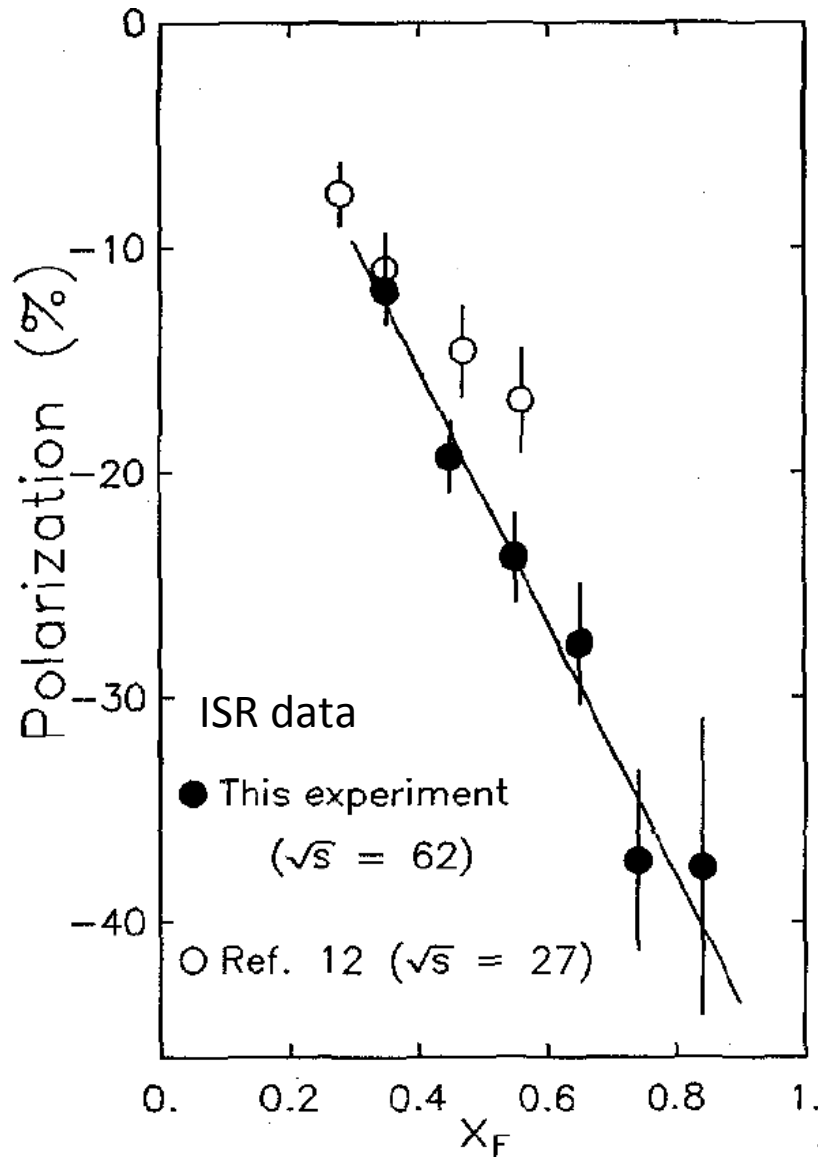


Fig. 3.  $P_{\Lambda^0}$  versus  $x_F$  for those data in fig. 2 with  $p_t > 0.96$  GeV/c (the average  $p_t = 1.14$  GeV/c). The straight line is the fit discussed in the text. The open circles are Fermilab results from ref. [12] with  $p_t > 0.95$  GeV/c.

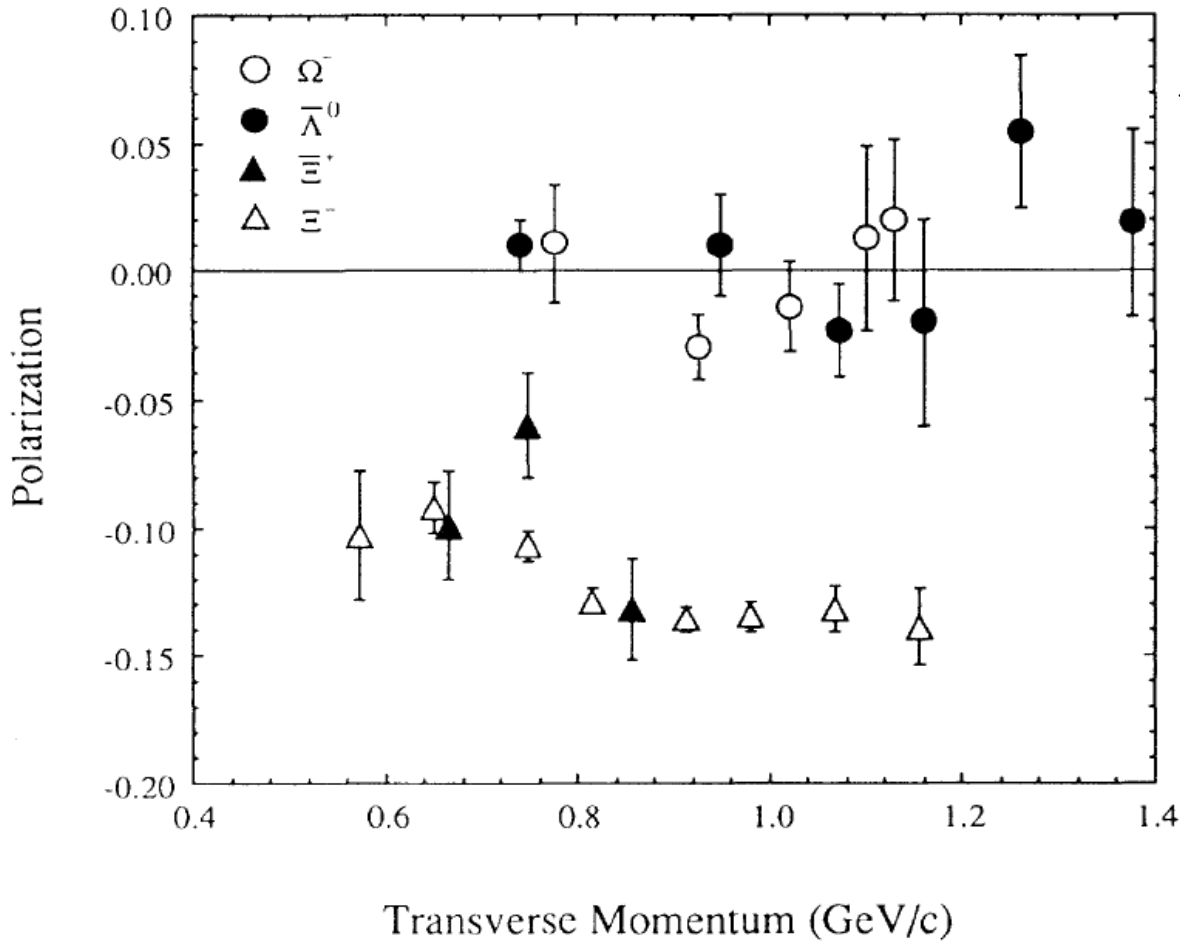


FIG. 3. Comparison of the  $\Omega^-$  polarization with those of the  $\bar{\Xi}^+$ ,  $\bar{\Lambda}^0$ , and  $\Xi^-$  as a function of the transverse momentum. For a given transverse momentum, the average  $x_F$  of the  $\bar{\Lambda}^0$  is slightly lower than that of the  $\Omega^-$ . The  $\Xi^-$  and  $\bar{\Xi}^+$  data are from this experiment (see Refs. [4] and [10]), and the 400 GeV/c  $\bar{\Lambda}^0$  results are taken from Ref. [8].

# Left-Right Asymmetry of Lambda

E704. PRL 75, 3073(1995).

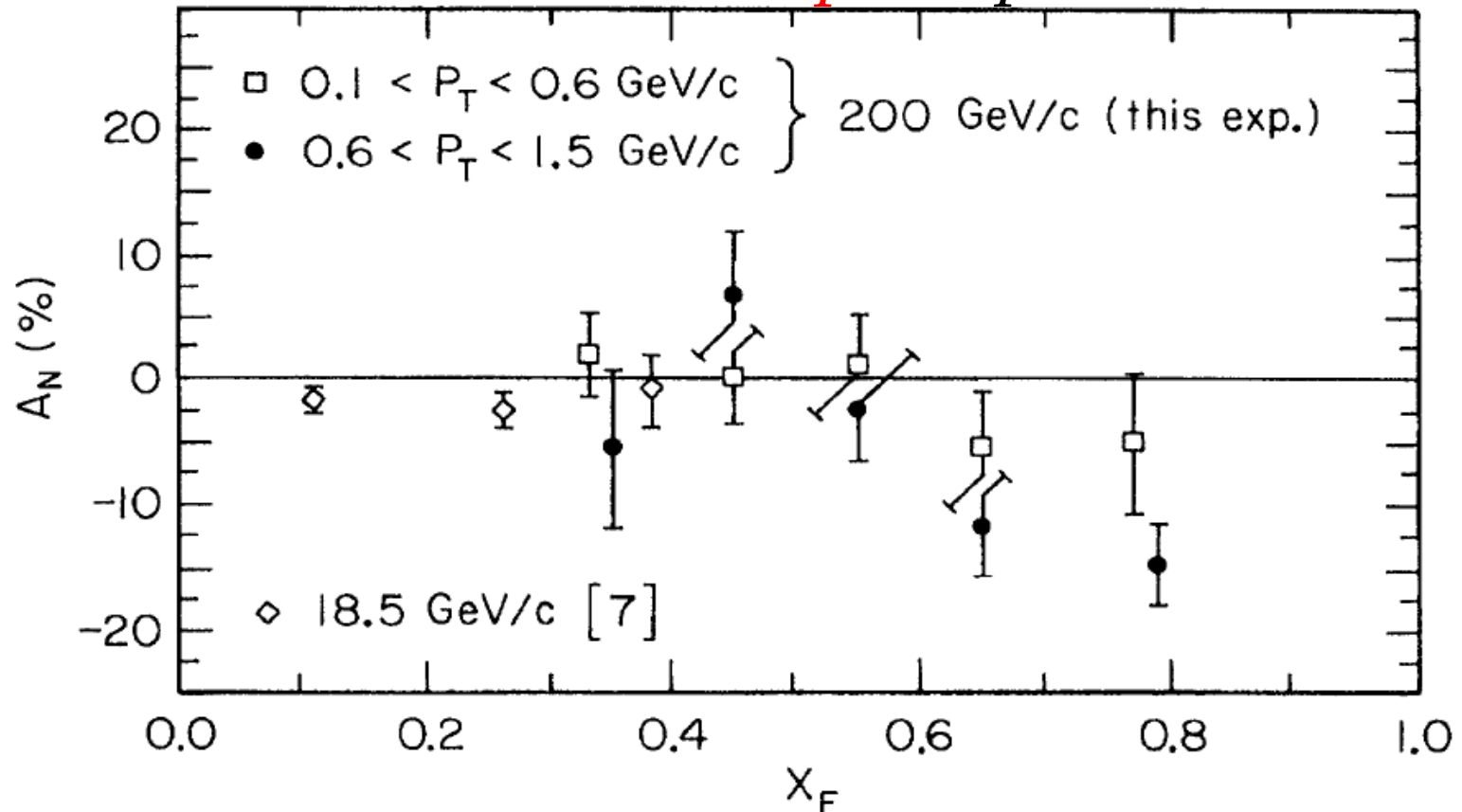
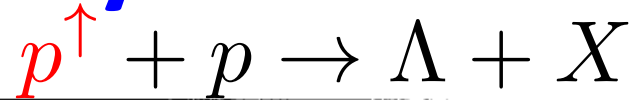
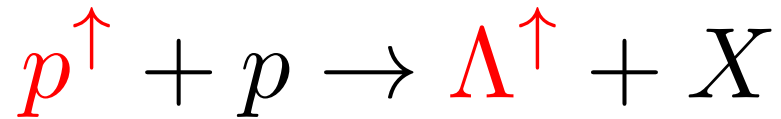


FIG. 3.  $A_N$  data for  $p^\uparrow + p \rightarrow \Lambda^0 + X$  as a function of  $x_F$ . Data at 18.5 GeV/c [7] (◇) are also shown.

# $D_{NN}$ : Transverse Spin Transfer to Lambda



(transversity)\*(final-spin dependent Frag. Func.)

E704: PRL 78, 4003 (1997)

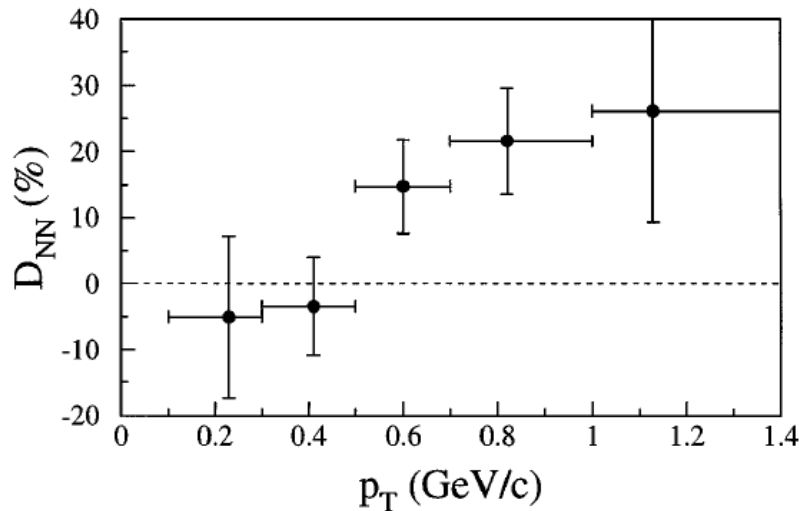


FIG. 1. Depolarization  $D_{NN}$  data as a function of  $p_T$  in  $p^\uparrow + p \rightarrow \Lambda^\uparrow + X$  at 200 GeV/c. The errors shown are statistical only.

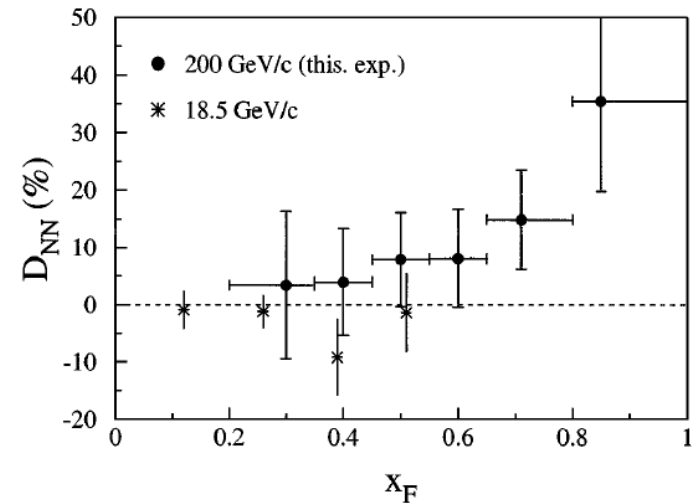


FIG. 2. Depolarization  $D_{NN}$  data as a function of  $x_F$ . The errors shown are statistical only. Also shown are  $D_{NN}$  measurements at 18.5 GeV/c from Ref. [12] ( $\langle p_T \rangle \sim 1.0$  GeV/c for the plotted points).