Predictions of Collins effects at BEPC

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Outlines

- Energy evolution in TMD factorization
- Collins asymmetry at BELLE and BABAR
- Predictions of Collins asymmetries at BEPC in a trivial TMD factorization framework
- Summary

QCD k_T resummation

- Consider the production process $h_1 h_2 \rightarrow Z+X$

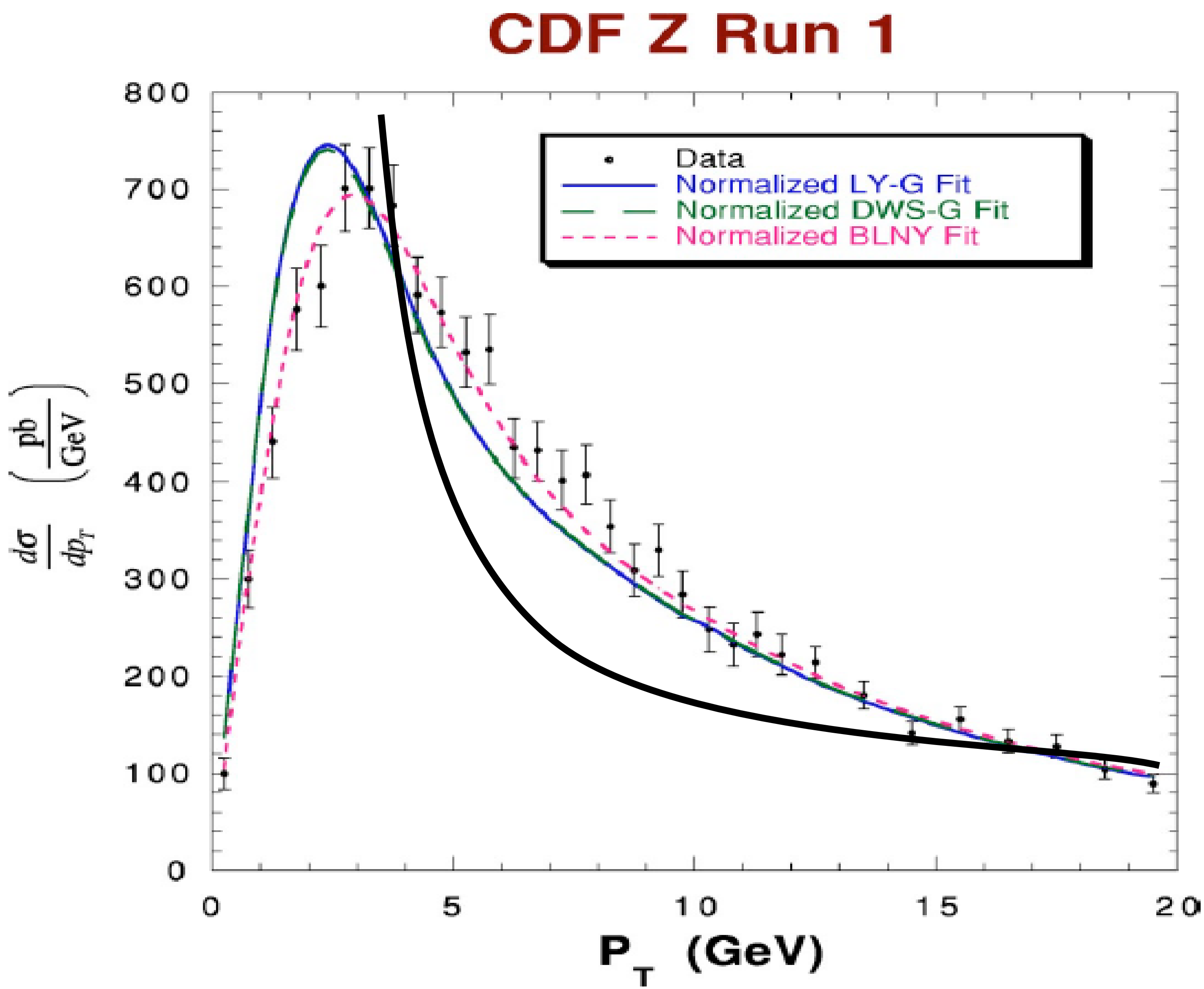
$$\frac{d\sigma}{dQ_T^2} \sim \frac{1}{Q_T^2}\{\, \alpha_S(L+1) \;\; +\alpha_S^2(L^3+L^2) \;\; +\alpha_S^3(L^5+L^4)+\alpha_S^4(L^7+L^6)+\ldots$$
$$+\alpha_S^2(\,L+\;\;1) \;\; +\alpha_S^3(L^3+L^2)+\alpha_S^4(L^5+L^4)+\ldots$$
$$+\alpha_S^3(\,L+\;\;1)+\alpha_S^4(L^3+L^2)+\ldots\,\}$$

Where Q_T is the transverse momentum, and Q the mass of Z, and $L = \mathrm{Log}[Q^2 / Q_T^2]$.

- We have to resum these large logs to make reliable predictions

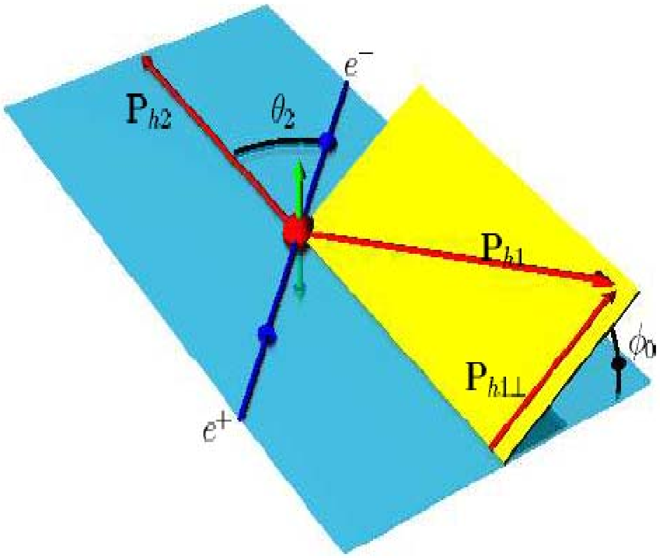
$$\frac{d^3\sigma(M^2, P_\perp, y)}{d^2P_\perp dy} = \sigma_0 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{-iP_\perp \cdot b_\perp} W(x_1, x_2, b, M^2)$$

$$W(Q,b) = e^{-\int_{1/b}^{Q} \frac{d\mu}{\mu}\left(\ln\frac{Q}{\mu}A+B\right)} C \otimes f_1 C \otimes f_2$$



C. P. Yuan et al PRD 67, 073016

Energy evolution in TMD factorization



For the process

$$e^+ + e^- \to H_1 + H_2 + X$$

The cross section can be written as

$$\frac{d\sigma}{dz_{h1}dz_{h2}d^2P_{h\perp}d\theta} = \frac{2\pi N_c\alpha^2}{4Q^2}\left[\left(1+\cos^2\theta\right)Z_{uu} + \sin^2\theta\left(2\hat{e}_x^\alpha\hat{e}_x^\beta - g_\perp^{\alpha\beta}\right)Z_{collins}^{\alpha\beta}\right]$$

$\Downarrow$

$\cos(2\phi_0)$

Energy evolution from TMD factorization

In the TMD factorization, at the small transverse momentum region

$\zeta^2/\rho = Q^2 \times z^2$

$$\widetilde{Z}_{uu} = D(z_1, b_\perp, \zeta_1; \mu) D(z_2, b_\perp, \zeta_2; \mu) H_{uu}^{e^+e^-}(Q;\mu) S(b_\perp, \rho; \mu) \ ,$$

$$\widetilde{Z}^{\alpha\beta}_{\rm collins} = \widetilde{H}_1^{\perp\alpha}(z_1, b_\perp, \zeta_1; \mu) \widetilde{H}_1^{\perp\beta}(z_2, b_\perp, \zeta_2; \mu) H^{e^+e^-}_{\rm collins}(Q;\mu) S(b_\perp, \rho; \mu)$$

By a Fourier transformation

$P_{h\perp}/z$

Z_{uu} and $Z_{collins}$ satisfy CSS evolution equation

$$\frac{\partial}{\partial \ln Q^2} \widetilde{Z}_{uu}(Q;b) = \left(K(b,\mu) + G(Q,\mu)\right) \widetilde{Z}_{uu}(Q;b)$$

At one-loop order

$$K(b,\mu) = -\frac{\alpha_s C_F}{\pi} \ln \frac{b^2\mu^2}{c_0^2} \qquad G(Q,\mu) = -\frac{\alpha_s C_F}{\pi}\left(\ln \frac{Q^2}{\mu^2} - \frac{3}{2}\right)$$

Substituting the above result into the evolution equation, and taking into account the running effects in K

$$\widetilde{Z}_{uu}(Q;b) = e^{-S_{pert}(Q^2,b_*)-S_{NP}^{e^+e^-}(Q,b)} \Sigma_q D_q(z_1, C_0/b) D_{\bar{q}}(z_2, C_0/b) \,,$$

$C_0 = 2e^{-\gamma} \approx 1$

$$\widetilde{Z}^{\alpha\beta}_{\text{collins}}(Q;b) = \left(\frac{-ib_\perp^\alpha}{2}\right)\left(\frac{-ib_\perp^\beta}{2}\right) e^{-S_{pert}(Q^2,b_*)-S^{e^+e^-}_{\text{collins}}(Q,b)} \Sigma_q \hat{H}_{1q}(z_{h1}, C_0/b) \hat{H}_{1\bar{q}}(z_{h2}, C_0/b)$$

$C_0 = 2e^{-\gamma} \approx 1$

$$e^{-S_{pert}(Q^2,b_*)-S^{e^+e^-}_{\text{collins}}(Q,b)}$$

For perturbative part:

$$S_{pert}(Q,b) = \int_{c_0/b}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \left[A \ln \frac{Q^2}{\bar{\mu}^2} + B\right]$$

S_{pert} has universality

where $A = C_F \times \alpha_s(\bar{\mu})/\pi$, $B = 3/2 \times \alpha_s(\bar{\mu})/\pi$

We have to make a cutoff for the b in S_{pert}

$$b \Rightarrow b_* = b/\sqrt{1 + b^2/b_{max}^2} \,, \quad b_{max} < 1/\Lambda_{QCD}$$

- For Sudakov factor
  - There are two parts in the Sudakov factor

$$\mathcal{S}_{sud} \Rightarrow \mathcal{S}_{pert}(Q;b_*) + S_{NP}(Q;b)$$

  - Gaussian assumption for the nonperturbative part

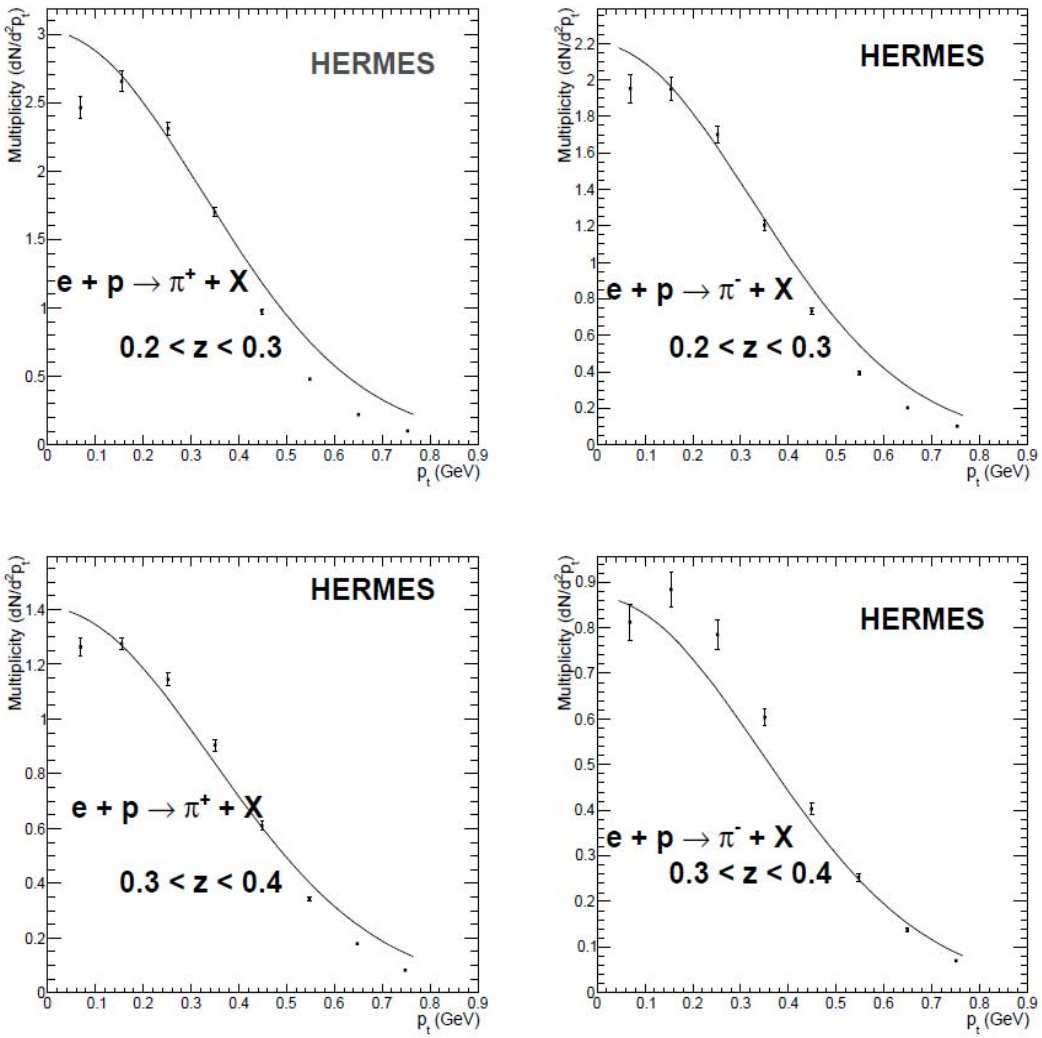
$$S_{NP}^{e^+e^-} = g_q b^2 \boxed{\ln(Q/Q_0)} + g_h \boxed{b^2} \left(1/z_{h1}^2 + 1/z_{h2}^2\right)$$

$$S_{\text{collins}}^{e^+e^-}(Q,b) = g_q b^2 \boxed{\ln(Q/Q_0)} + g_c b^2 \left(1/z_{h1}^2 + 1/z_{h2}^2\right)$$

We assume the Q dependence always satisfies CSS equation.

The g_q is universal to Drell-Yan, SIDIS, and $e^+e^- \to hh$

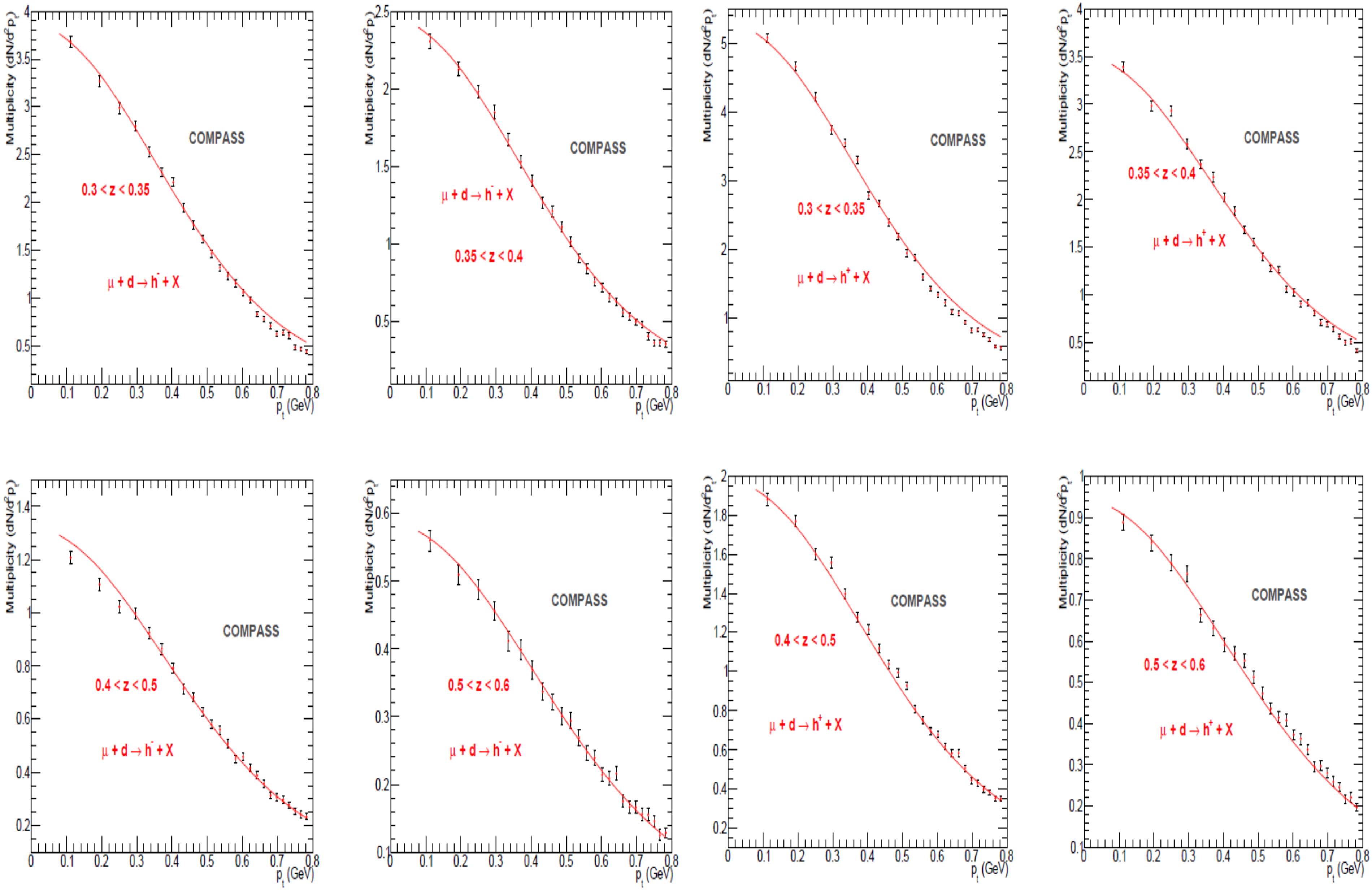
SIDIS



SIDIS at HERMES

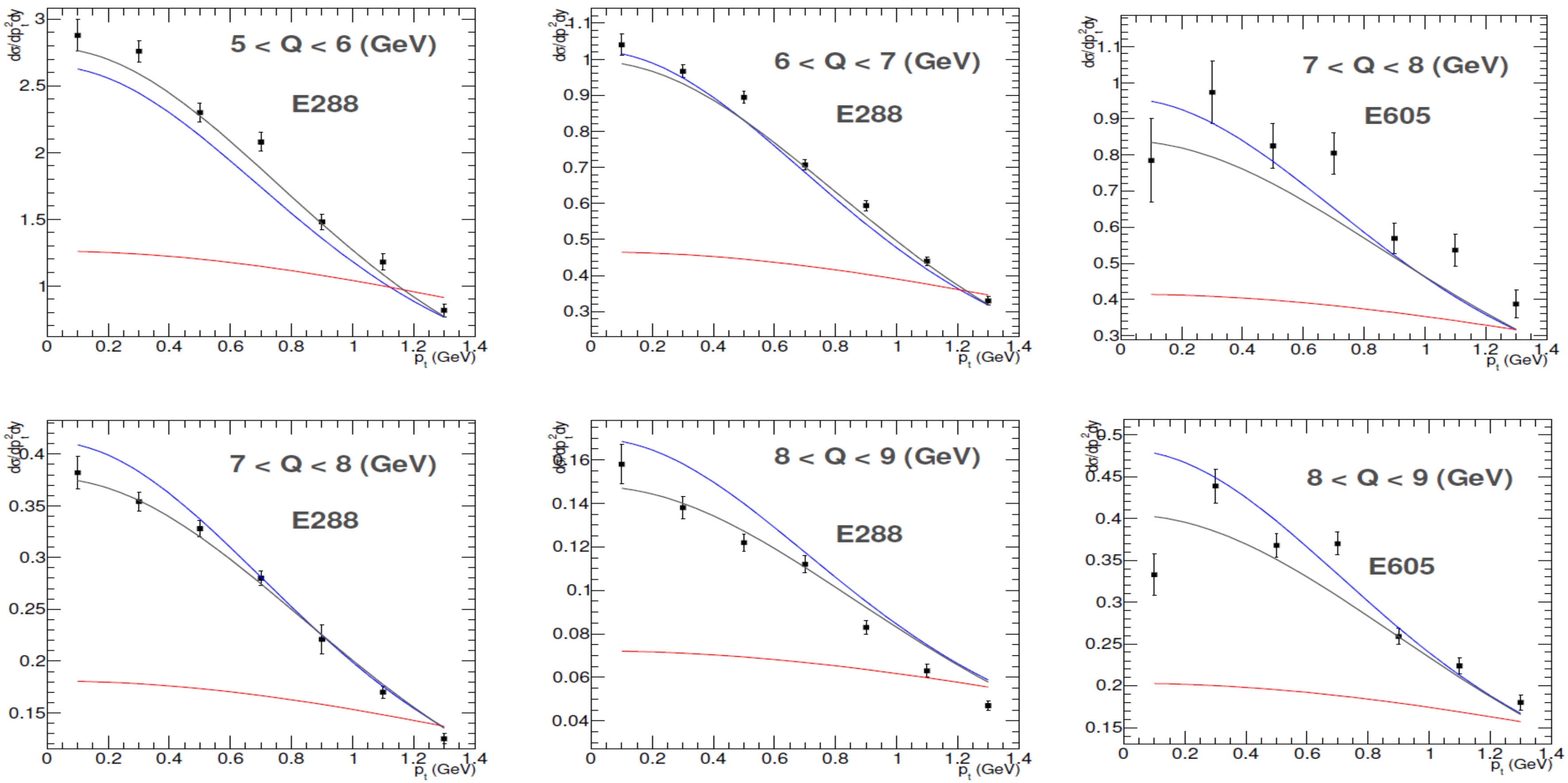
We can get the g_q , g_h from the p_t distribution of cross section for SIDIS and Drell-Yan processes.

Here, these curves are from our fitting in a trivial TMD factorization framework.



SIDIS at COMPASS, Q^2=7.75GeV^2, x=0.1

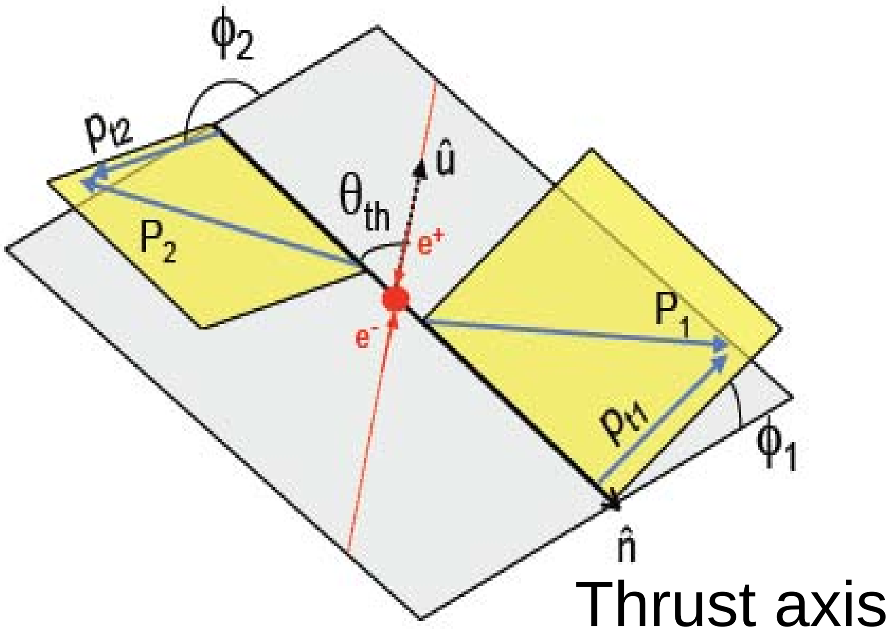
Drell-Yan



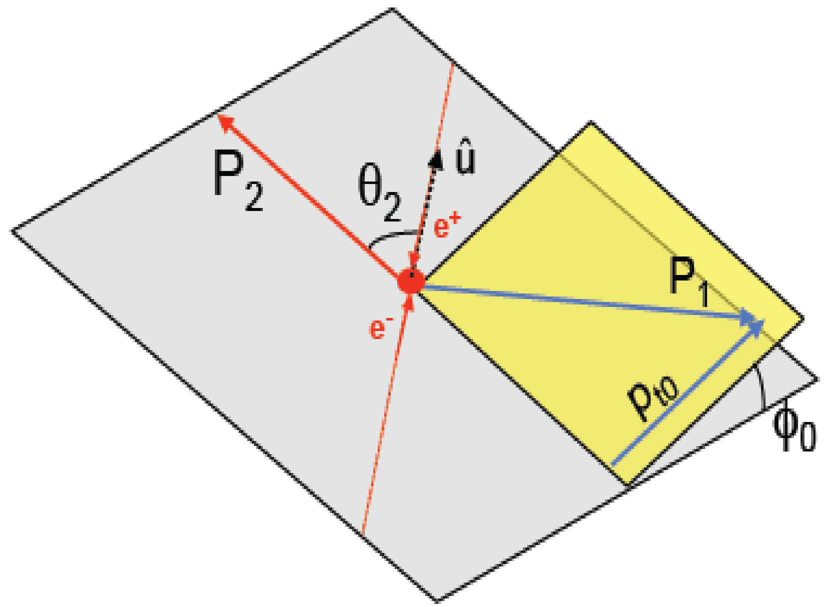
The red line is from S. Mert Aybat and Ted C. Rogers

The black line is from C.P. Yuan and P. Nadolsky and their collaborators

Collins asymmetries in $e^+e^- \rightarrow hh+X$ at BELLE and BABAR



A_{12}



A_0

The Collins asymmetries is proportional to $\cos(\phi_1+\phi_2)$ or $\cos(2\phi_0)$

- Besides Collins effect, the gluon radiation effect also can contribute to the term which is proportional to $\cos(\phi_1+\phi_2)$ or $\cos(2\phi_0)$

$$R_{12} = \frac{N(\phi_1+\phi_2)}{\langle N_{12}\rangle}$$

$$\propto \left[(1+\cos^2\theta)\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2) + \sin^2\theta\cos(\phi_1+\phi_2)\left[\sum_q e_q^2 f(H_1^{\perp}(z_1)\overline{H}_1^{\perp}(z_2))\right.\right.$$

$$\left.\left.+\, C\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2)\right]\right]\cdot\left[(1+\cos^2\theta)\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2)\right]^{-1}$$

$$= 1 + \frac{\sin^2}{1+\cos^2\theta}\cos(\phi_1+\phi_2)\left[\frac{\sum_q e_q^2 f(H_1^{\perp}(z_1)\overline{H}_1^{\perp}(z_2))}{\sum_q e_q^2 D_1(z_1)\overline{D}_1(z_2)} + C\right].$$

$$e^+e^- \to q\bar{q}g \to h_1h_2X$$

$$\frac{dN}{d\Omega} \propto \frac{Q_t^2}{Q^2+Q_t^2}$$

For BELLE and BABAR, they choose $Q_t < 3.5$GeV to suppress this background.

Firstly, we define:

Valence quarks go to pion

$$N^U(\phi) = \frac{d\sigma(e^+e^- \to \pi^\pm\pi^\mp X)}{d\Omega dz_1 dz_2} \propto \frac{5}{9} D^{fav}(z_1)\overline{D}^{fav}(z_2) + \frac{7}{9} D^{dis}(z_1)\overline{D}^{dis}(z_2)$$

Sea quarks go to pion

$$N^L(\phi) = \frac{d\sigma(e^+e^- \to \pi^\pm\pi^\pm X)}{d\Omega dz_1 dz_2} \propto \frac{5}{9} D^{fav}(z_1)\overline{D}^{dis}(z_2) + \frac{5}{9} D^{dis}(z_1)\overline{D}^{fav}(z_2) + \frac{2}{9} D^{dis}(z_1)\overline{D}^{dis}(z_2)$$

$$N^C(\phi) = \frac{d\sigma(e^+e^- \to \pi\pi X)}{d\Omega dz_1 dz_2} = N^U(\phi) + N^L(\phi) \propto \frac{5}{9}[D^{fav}(z_1) + D^{dis}(z_1)][\overline{D}^{fav}(z_2) + \overline{D}^{dis}(z_2)] + \frac{4}{9} D^{dis}(z_1)\overline{D}^{dis}(z_2)$$

By a double ratio: $\frac{R^U_\alpha}{R^L_\alpha} := \frac{N^U_\alpha(\beta_\alpha)/\langle N^U_\alpha\rangle}{N^L_\alpha(\beta_\alpha)/\langle N^L_\alpha\rangle}, (\alpha = 0, 12)$

$$\frac{R^U_{12}}{R^L_{12}} = 1 + \cos(\phi_1 + \phi_2)\frac{\sin^2\theta}{1 + cos^2\theta}\left\{\frac{f\left(H_1^{\perp,fav}\overline{H}_2^{\perp,fav} + H_1^{\perp,dis}\overline{H}_2^{\perp,dis}\right)}{\left(D_1^{fav}\overline{D}_2^{fav} + D_1^{dis}\overline{D}_2^{dis}\right)} - \frac{f\left(H_1^{\perp,fav}\overline{H}_2^{\perp,dis}\right)}{\left(D_1^{fav}\overline{D}_2^{dis}\right)}\right\}$$

$\Downarrow$

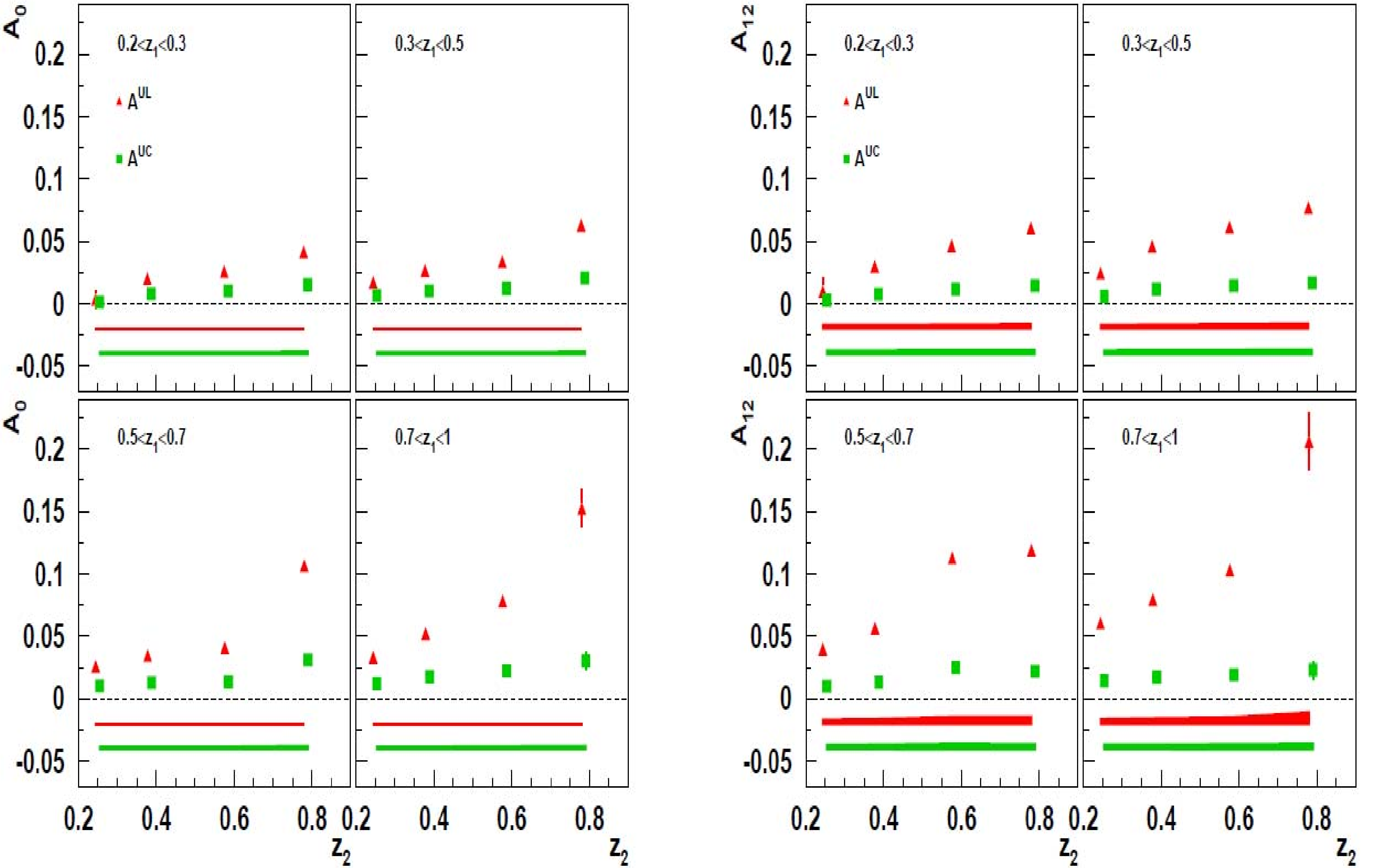
A^{UL}

Similarly, we also can get A^{UC} from the ratio R^U/R^C

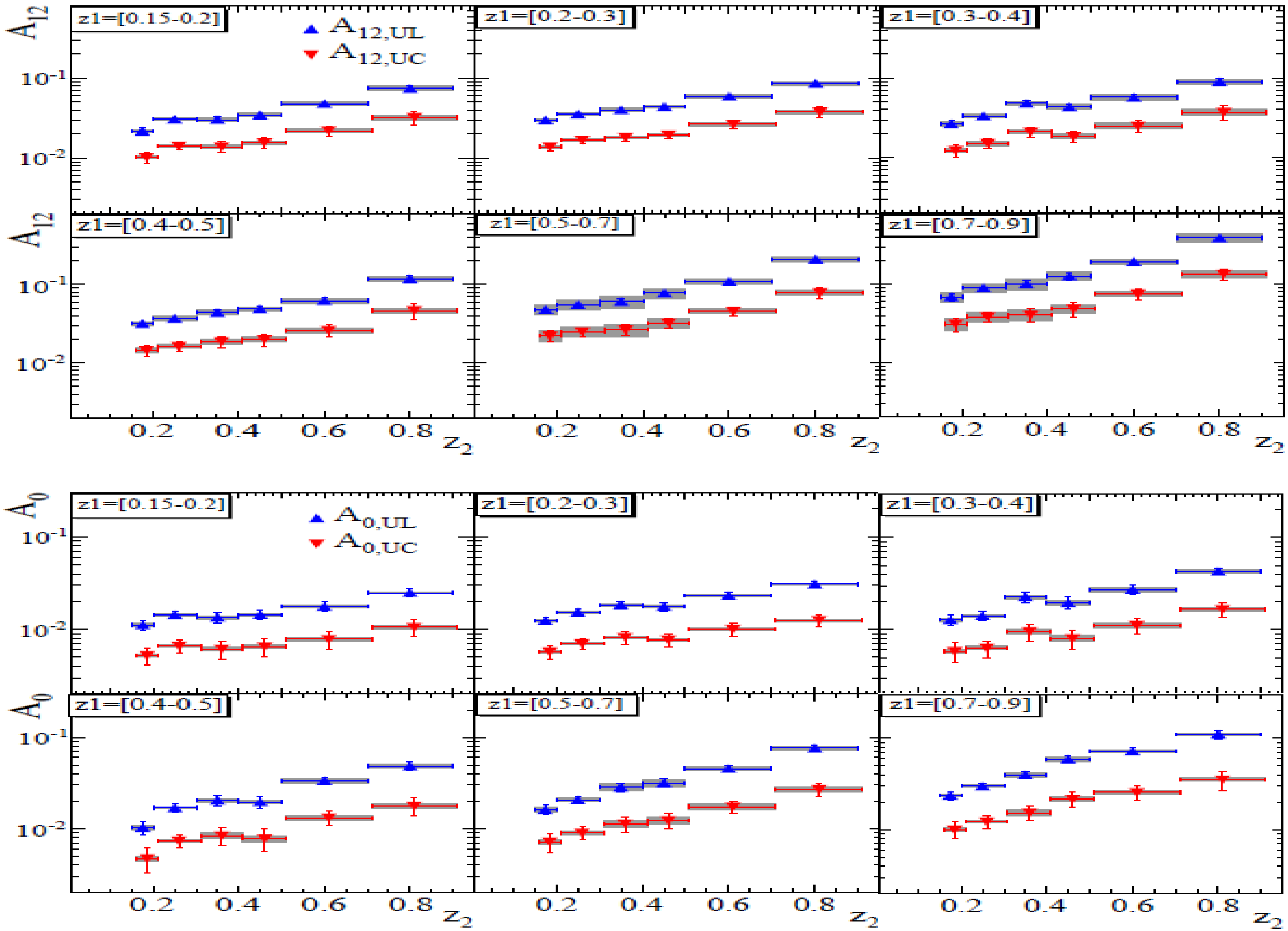
$$A^{UL} \sim \left\langle \frac{\sin^2\theta}{1+\cos^2\theta} \right\rangle \frac{\pi\langle k_{tC}^2\rangle}{4M^2} \left[\frac{H_1^{fav}\overline{H}_2^{fav} + H_1^{dis}\overline{H}_2^{dis}}{D_1^{fav}\overline{D}_2^{fav} + D_1^{dis}\overline{D}_2^{dis}} - \frac{H_1^{fav}\overline{H}_2^{dis} + H_1^{dis}\overline{H}_2^{fav}}{D_1^{fav}\overline{D}_2^{dis} + D_1^{dis}\overline{D}_2^{fav}} \right]$$

$$A^{UC} \sim \left\langle \frac{\sin^2\theta}{1+\cos^2\theta} \right\rangle \frac{\pi\langle k_{tC}^2\rangle}{4M^2} \left[\frac{H_1^{fav}\overline{H}_2^{fav} + H_1^{dis}\overline{H}_2^{dis}}{D_1^{fav}\overline{D}_2^{fav} + D_1^{dis}\overline{D}_2^{dis}} - \frac{\left(H_1^{fav} + H_1^{dis}\right)\left(\overline{H}_2^{fav} + \overline{H}_2^{dis}\right)}{\left(D_1^{fav} + D_1^{dis}\right)\left(\overline{D}_2^{fav} + \overline{D}_2^{dis}\right)} \right]$$

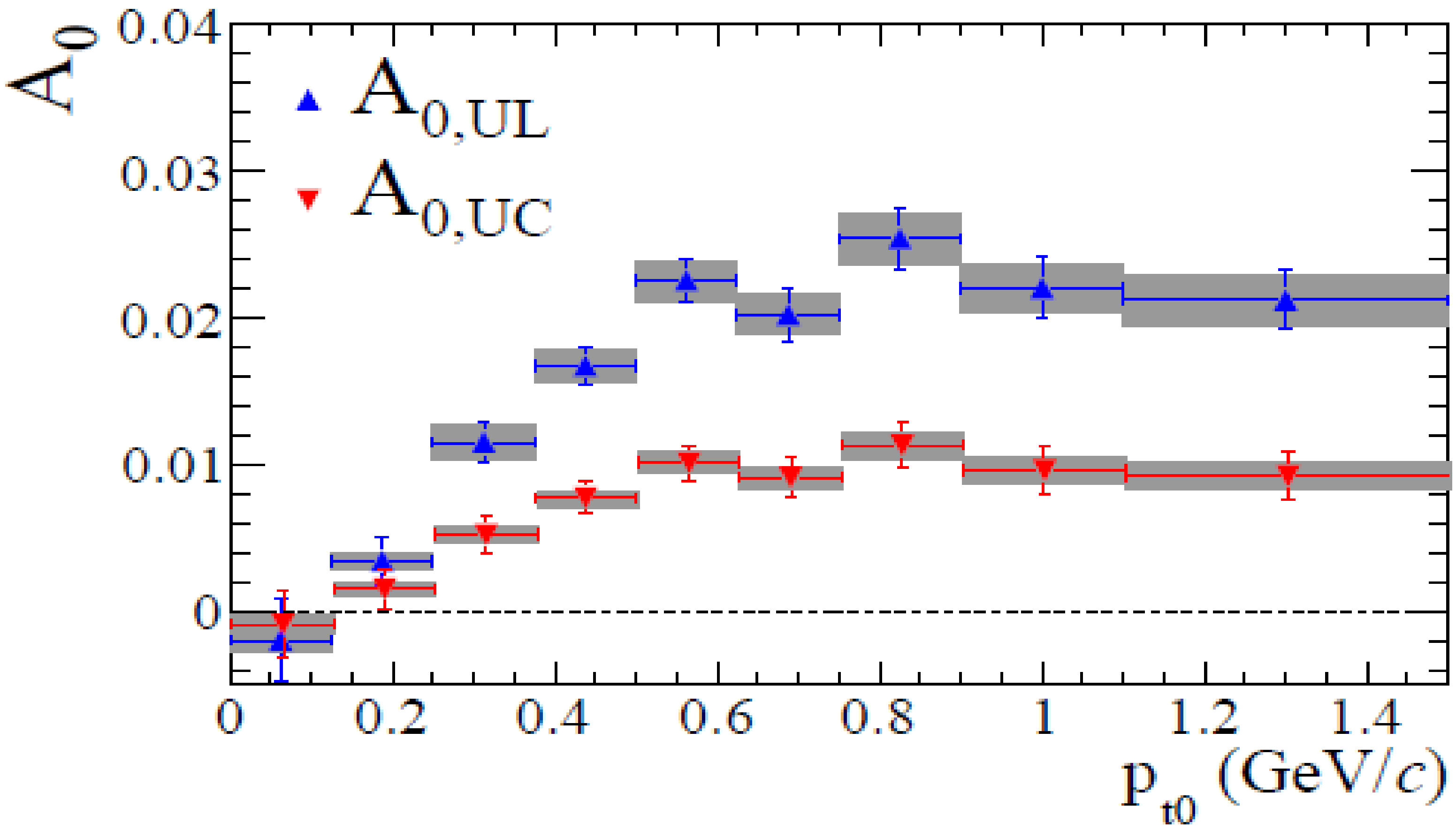
The data at BELLE (Phys. Rev. D 78, 032011 (2008))



The data at BABAR arXiv:1209.5278



The Pt distribution for Collins asymmetries at BABAR



It is an important test for the TMD factorization with the Collins function.

- We will abstract the Collins function by fitting these data points.
- We will choose CSS resummation formulism.

$$D_q(z_1, C_0/b)$$

$$\widetilde{Z}_{uu}(Q;b) = e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}(Q,b)} \Sigma_{i,j} \hat{C}_{qi}^{(e^+e^-)} \otimes D_{i/A}(z_1) \hat{C}_{qj}^{(e^+e^-)} \otimes D_{j/B}(z_2') ,$$

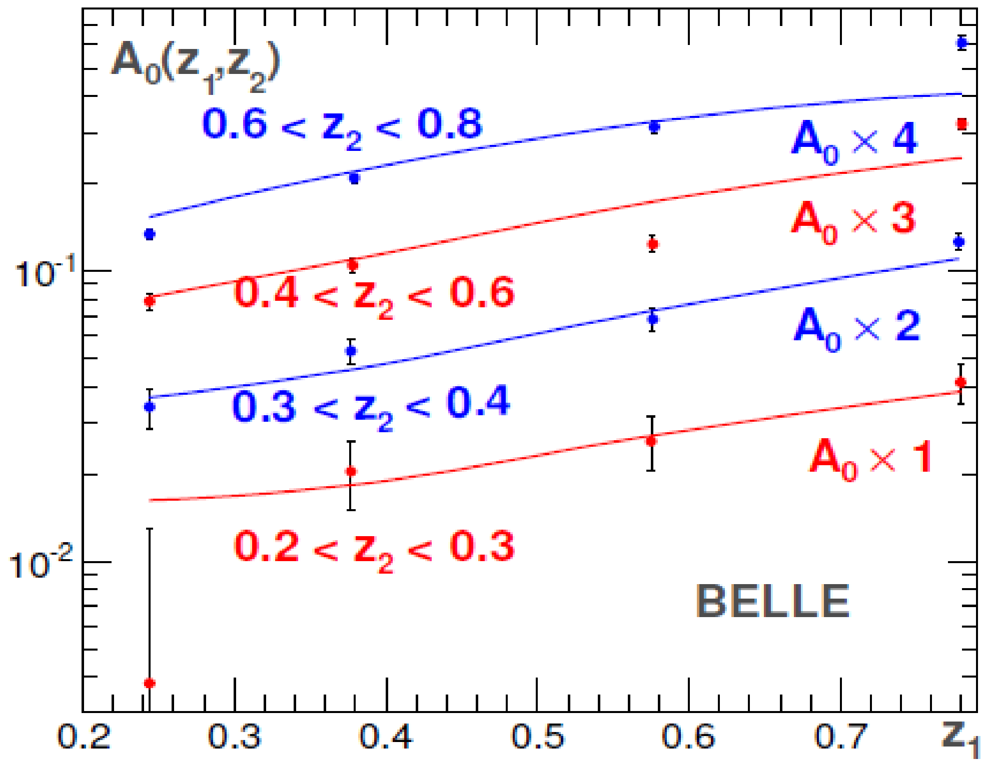
$$\widetilde{Z}_{\rm collins}^{\alpha\beta}(Q;b) = \left(\frac{-ib_\perp^\alpha}{2}\right)\left(\frac{-ib_\perp^\beta}{2}\right) e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}^T(Q,b)}$$

$$\times \Sigma_{i,j} \Delta\hat{C}_{qi}^{{\rm collins}(e^+e^-)} \otimes D_{i/A}^{(3)} \hat{C}_{qj}^{{\rm collins}(e^+e^-)} \otimes D_{j/B}^{(3)} ,$$

$$\hat{H}_{1q}(z_{h1}, C_0/b)$$

- And then, we can predict the Collins effect at BEPC.

abstracting Collins functions from BELLE data



P Sun and F Yuan
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our fitting in a trivial TMD factorization framework.

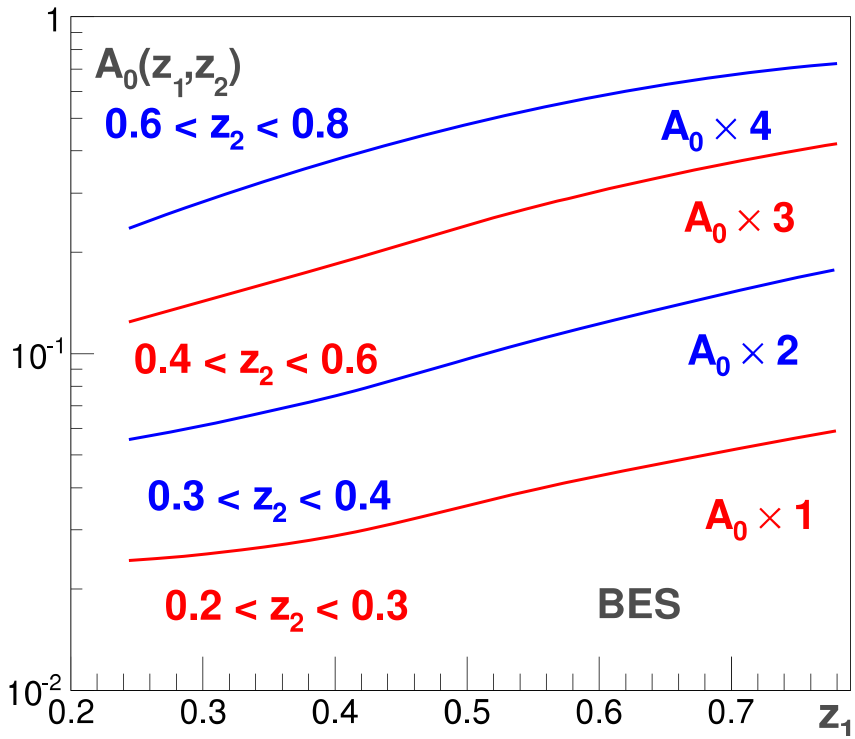
$H(z,b,Q_0=2.4\text{GeV})$

$$e^{-(g_h-g_c)b^2/z^2} N_q z^{\alpha_q}(1-z)^{\beta_q} \frac{(\alpha_q+\beta_q)^{\alpha_q+\beta_q}}{\alpha_q^{\alpha_q}\ \beta_q^{\beta_q}} D_q(z)$$

$$\chi^2/d.o.f = 1.22$$

flavor i	N_i	α_i	β_i	g_c (GeV^2)
u	0.34±0.006	3.9±0.71	0.85±0.29	0.013±0.002
d	-0.34±0.013	0.4±0.31	0.31±0.41	0.013±0.002

Test the evolution at BEPC



- $E_{c.m.}$=4.6GeV, di-pion in e^+e^- annihilation
- Because of energy evolution effect, It will be larger than that at BELLE by a factor 2

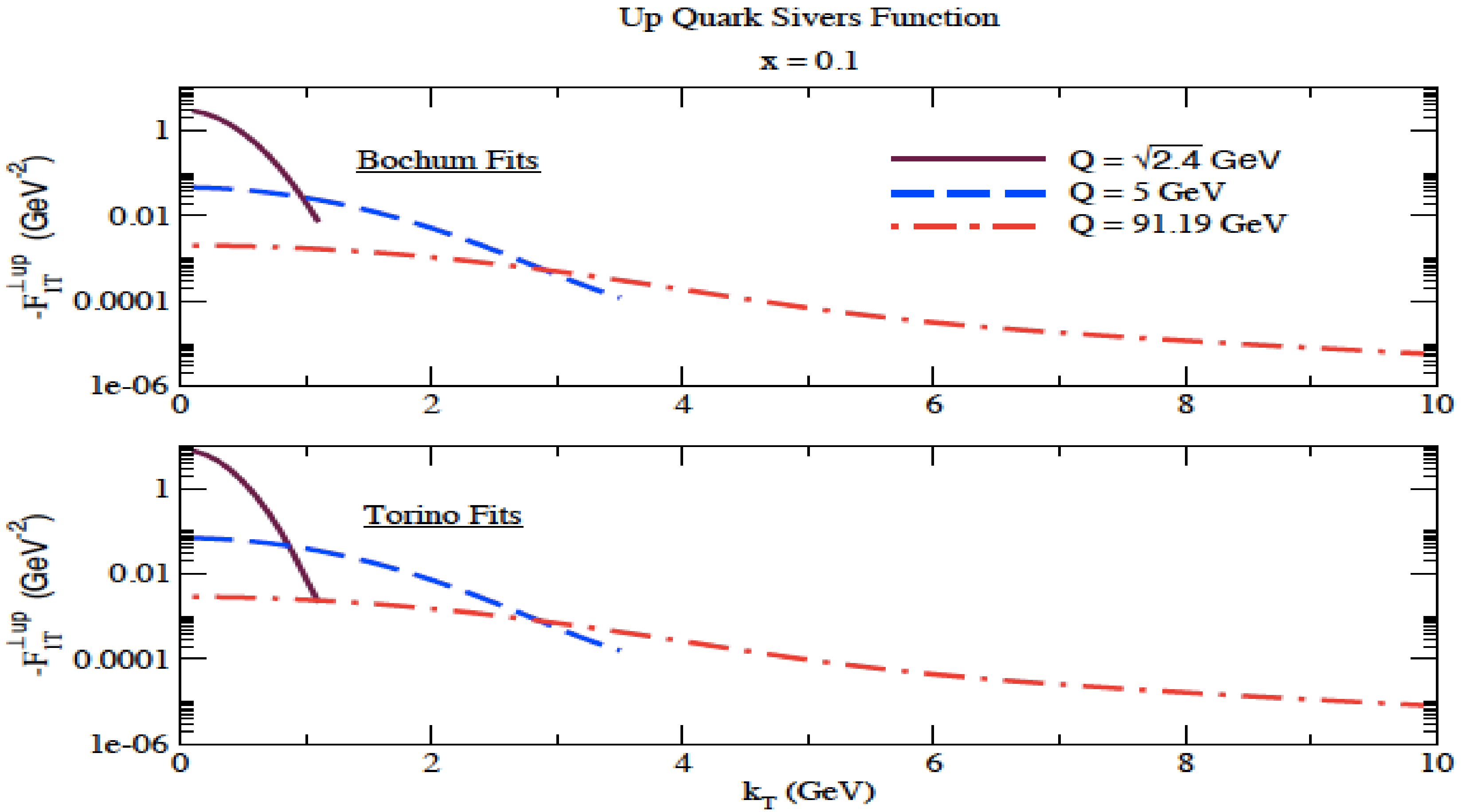
Summary

- We studied the Collins effect in the TMD factorization formulism.
- The TMD factorization will lead to a energy evolution effect.
- We can abstract the Collins function from the existing data at BELLE and BABAR.
- Then, we can predict the Collins asymmetry at BEPC.
- The experimental result at BEPC will supply an important test for TMD factorization.

Thank you very much!

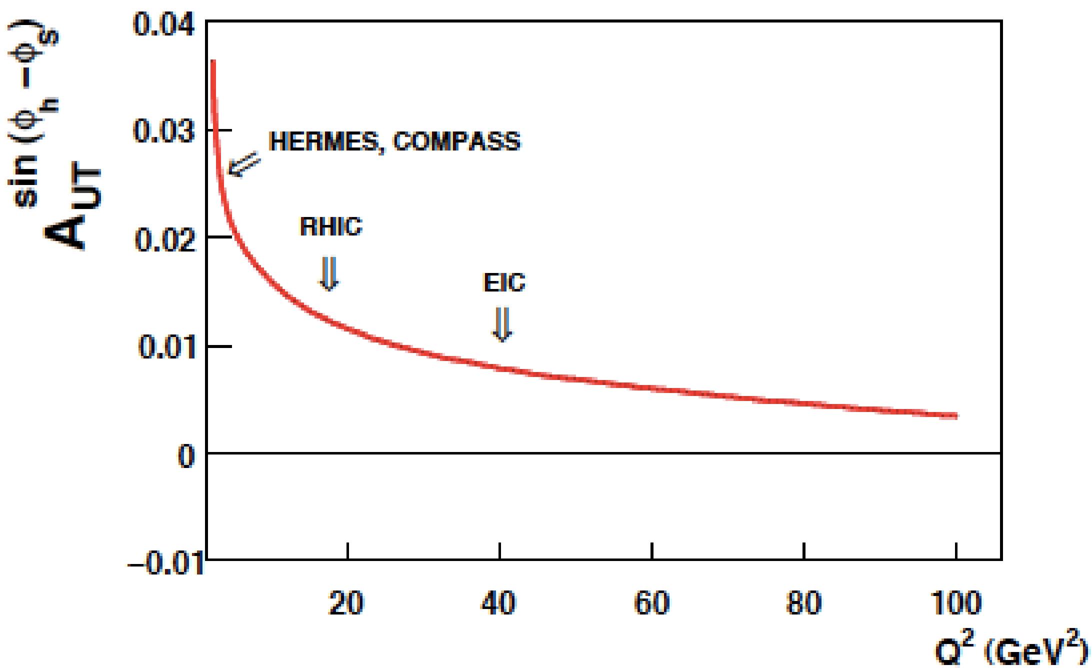
Energy Evolution in TMD factorization scheme

Aybat-Collins-Qiu-Rogers, 2011



Q^2-dependence

- Aybat-Prokudin-Rogers, 2011



Needs a cross check!

Collins scheme

$$W^{\mu\nu} =$$

$$\sum_f |\mathcal{H}_f(Q;\mu)^2|^{\mu\nu} \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T} \delta^{(2)}(\mathbf{k}_{1T} + \mathbf{q}_T - \mathbf{k}_{2T})$$

$$\times F_{f/p}(x, \mathbf{k}_{1T}; \mu; \zeta_F)\, D_{h/f}(z, z\mathbf{k}_{2T}; \mu; \zeta_D)$$

This version is much simpler than that of Ji Ma Yuan

$$\tilde{D}_{H/f}(z, \mathrm{b}_T; \mu; \zeta_D) = \tilde{D}^{\mathrm{unsub}}_{H/f}(z, \mathrm{b}_T; \mu; +\infty - y_h)\sqrt{\frac{\tilde{S}_{(0)}(\mathrm{b}_T; y_s, -\infty)}{\tilde{S}_{(0)}(\mathrm{b}_T; +\infty, -\infty)\tilde{S}_{(0)}(\mathrm{b}_T; +\infty, y_s)}} Z_D Z_2$$

$$\tilde{D}^{\mathrm{unsub}}_{H/f}(z, \mathrm{b}_T; \mu; y_A - y_h) = \sum_X \frac{1}{4N_{c,f}} \mathrm{Tr}_C \mathrm{Tr}_D \frac{1}{z} \int \frac{dw}{2\pi} e^{ik^+w^-} \langle 0|\gamma^+ W(w/2, \infty, n_A)\psi_f(w/2)|h, X\rangle$$

$$\times \langle h, X|\bar{\psi}_f(-w/2)W(-w/2, \infty, n_A)^\dagger|0\rangle_c$$

Ji Ma Yuan scheme, in SIDIS

Structure function is

$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp$$
$$\times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_T\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho)$$
$$\times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) \ ,$$

It depends on ρ

$$\hat{q}_h(z, P_{h\perp}, \mu, \hat{\zeta}/z, \rho) = \frac{1}{2z}\int \frac{d\xi^-}{2\pi}\frac{d^2\vec{b}}{(2\pi)^2} e^{-i(k^+\xi^- - \vec{k}_\perp\cdot\vec{b}_\perp)}$$
$$\times \sum_X \frac{1}{3}\sum_a \langle 0|\mathcal{L}_{\tilde{v}}(-\infty;0)\psi_{\beta a}(0)|P_h X\rangle \gamma^+_{\alpha\beta}$$
$$\times \langle P_h X|(\overline{\psi}_{\alpha a}(\xi^-, \vec{b})\mathcal{L}^\dagger_{\tilde{v}}(\xi^-, \vec{b}; -\infty)|0\rangle / S(b_\perp, \mu, \rho)$$

In Aybat-Collins-Qiu-Rogers framework

$$\tilde{K}\left[b, Q_0, g(Q_0)\right] = \left(\tilde{K}\left[b_*, C_1/b_*, g(C_1/b_*)\right] - \int_{C1/b_*}^{Q_0} \frac{d\mu}{\mu}\gamma_K(\mu) - g_K(b)\right)$$

And then

$$F(x,b;Q,Q) = F(x,b;Q_0,Q_0)\exp\Bigg\{\ln\frac{Q}{Q_0}\tilde{K}(b_*;\mu_b) + \int_{Q_0}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right]$$
$$+ \int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\ln\frac{Q}{Q_0}\gamma_K(g(\mu')) - g_K(b)\ln\frac{Q}{Q_0}\Bigg\}$$

Here $g_K(b)$ is $g_c \times b^2$

In Aybat-Collins-Qiu-Rogers framework

$$\tilde{K}\left[b, Q_0, g(Q_0)\right] = \left(\tilde{K}\left[b_*, C_1/b_*, g(C_1/b_*)\right] - \int_{C1/b_*}^{Q_0} \frac{d\mu}{\mu} \gamma_K(\mu) - g_K(b) \right)$$

And then

$$F(x, b; Q, Q) = F(x, b; Q_0, Q_0) \exp\left\{ \ln \frac{Q}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{Q_0}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right.$$
$$\left. + \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{Q}{Q_0} \gamma_K(g(\mu')) - g_K(b) \ln \frac{Q}{Q_0} \right\}$$

Here $g_K(b)$ is $g_c \times b^2$

Energy evolution

$$\widetilde{F}^{\alpha}_{\text{sivers}}(Q;b) = \widetilde{F}^{\alpha}_{\text{sivers}}(Q_0;b)e^{-\mathcal{S}_{Sud}(Q,Q_0,b)}$$

$$S(Q^2,Q_0^2,b) = -\ln\frac{Q}{Q_0}\tilde{K}\left[b,Q_0,g(Q_0)\right] + \int_{Q_0}^{Q}\left[\frac{d\mu}{\mu}\ln\left(\frac{Q}{\mu}\right)\gamma_K(\mu) - \gamma_F(g(\mu);1)\right]$$

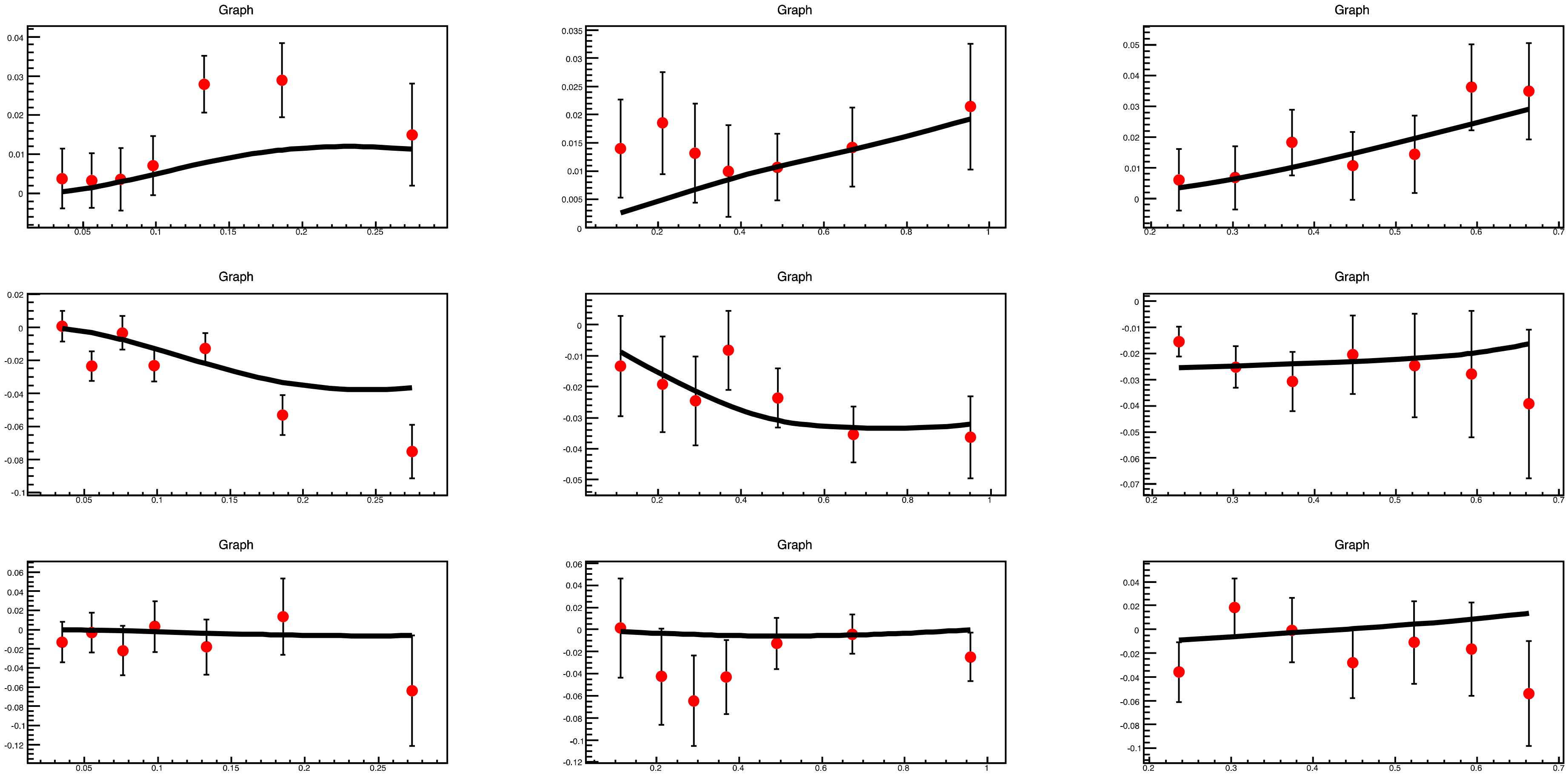
In our framework

$$\tilde{K}\left[b,Q_0,g(Q_0)\right] = -C_F\frac{\alpha_s[Q_0]}{\pi}\ln\left(\frac{Q_0^2b^2}{C_1^2}\right)$$

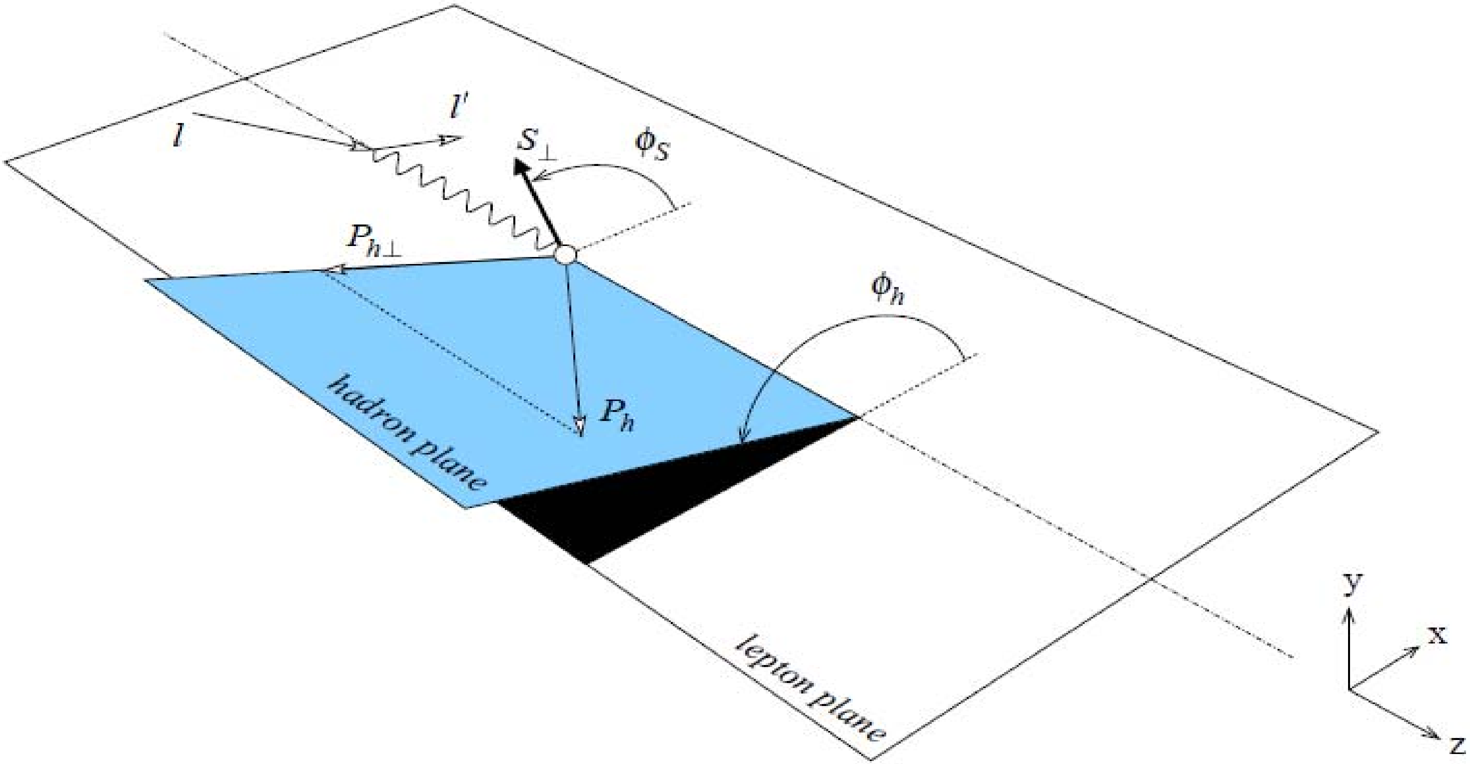
At the leading order of α_s

$$\mathcal{S}_{Sud} = 2C_F\int_{Q_0}^{Q}\frac{d\bar{\mu}}{\bar{\mu}}\frac{\alpha_s(\bar{\mu})}{\pi}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2b^2}{c_0^2} - \frac{3}{2}\right]$$

Collins asymmetries in SIDIS



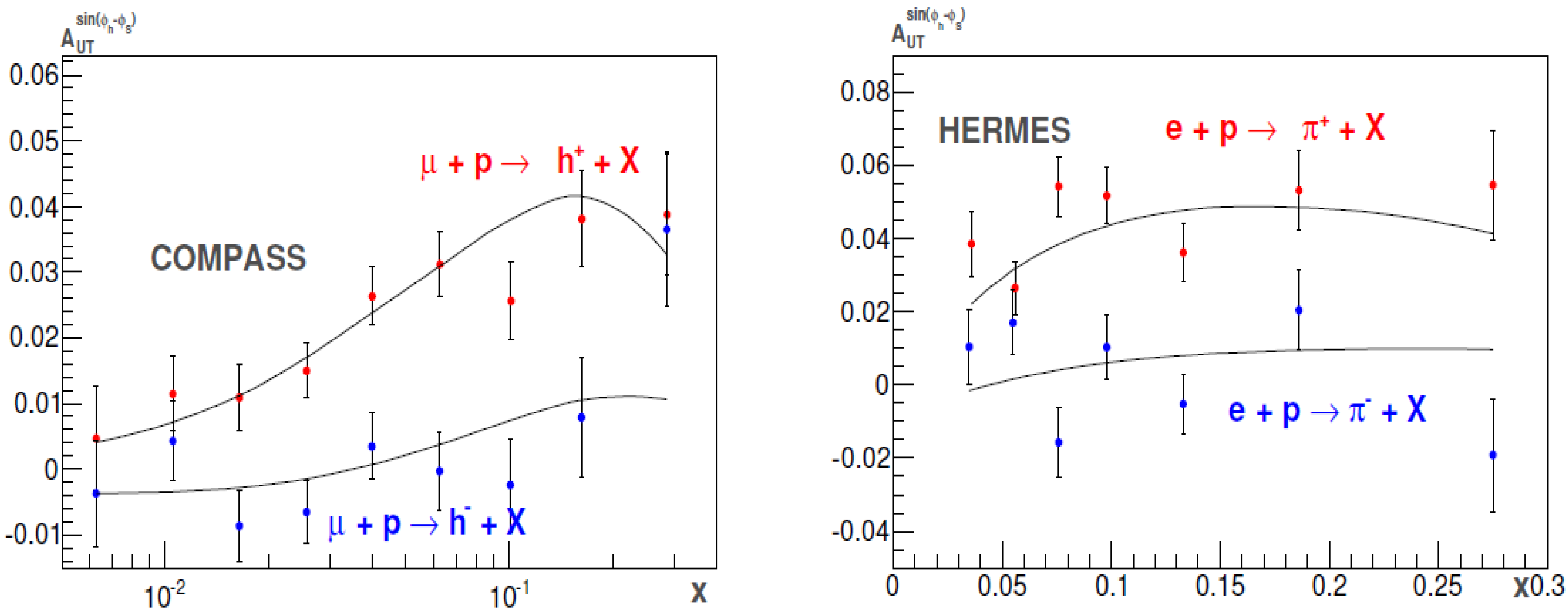
Sivers asymmetries in SIDIS



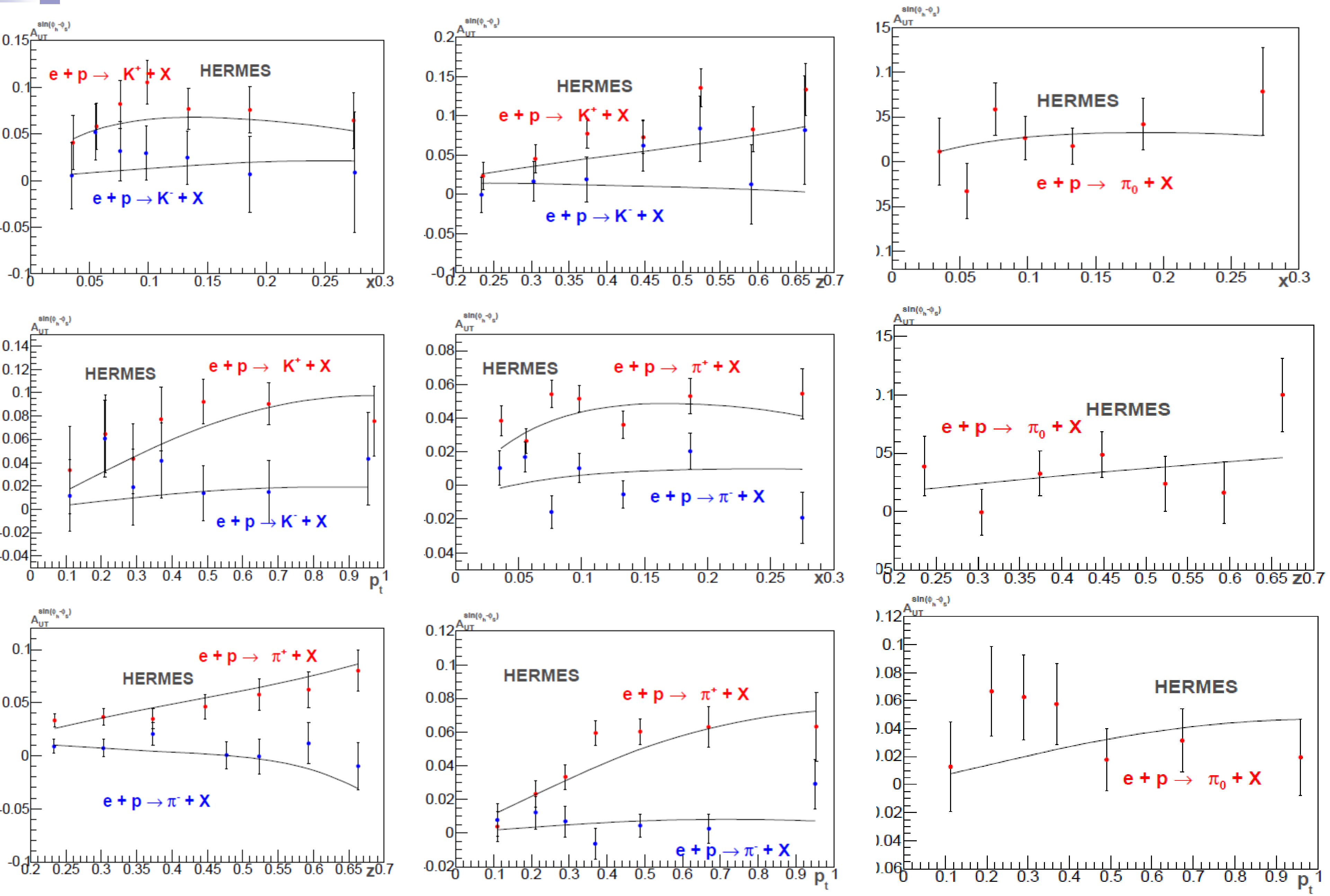
$$A_{UT}^{\sin(\phi_h-\phi_S)} = 2\,\frac{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow - d\sigma^\downarrow]\, \sin(\phi_h-\phi_S)}{\int d\phi_S\, d\phi_h\, [d\sigma^\uparrow + d\sigma^\downarrow]}$$

The Sivers effect is proportional to $\sin(\phi_h-\phi_S)$

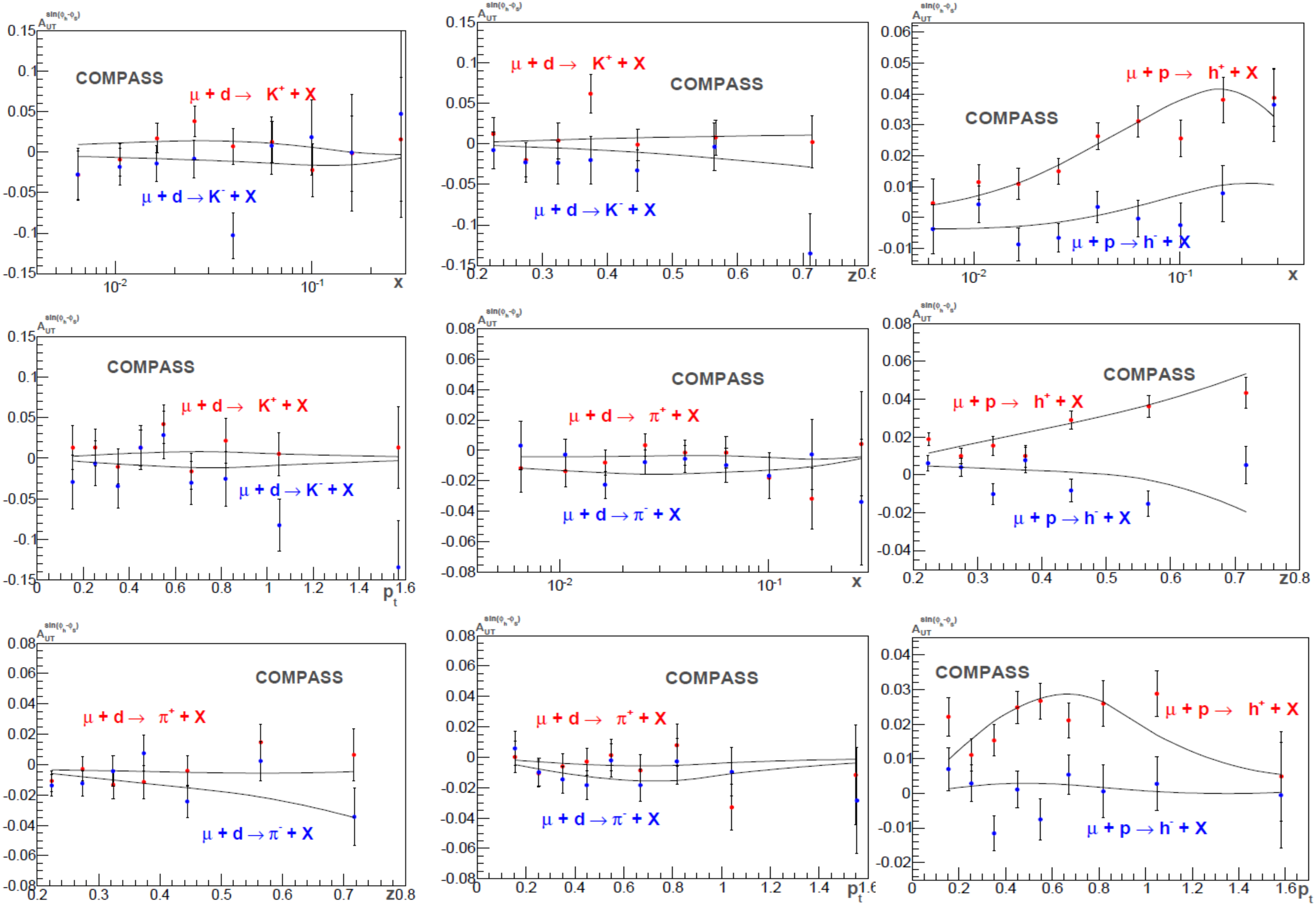
Fit to Sivers asymmetries



- With the evolution effects taken into account.
- Not so large Q difference



$A_{UT}^{\sin(\phi_h-\phi_s)}$
HERMES
e + p → K⁺ + X
e + p → K⁻ + X
x
z
p_t
e + p → π_0 + X
e + p → π^+ + X
e + p → π^- + X



$A_{UT}^{\sin(\phi_h-\phi_S)}$
COMPASS
μ + d → K⁺ + X
μ + d → K⁻ + X
x
COMPASS
μ + d → K⁺ + X
μ + d → K⁻ + X
z
COMPASS
μ + p → h⁺ + X
μ + p → h⁻ + X
x
COMPASS
μ + d → K⁺ + X
μ + d → K⁻ + X
p_t
COMPASS
μ + d → π⁺ + X
μ + d → π⁻ + X
x
COMPASS
μ + p → h⁺ + X
μ + p → h⁻ + X
z
COMPASS
μ + d → π⁺ + X
μ + d → π⁻ + X
z
COMPASS
μ + d → π⁺ + X
μ + d → π⁻ + X
p_t
COMPASS
μ + p → h⁺ + X
μ + p → h⁻ + X
p_t

$$\widetilde{F}_{UU}(Q;b) = e^{-\mathcal{S}_{sud}(Q,Q_0,b)}\widetilde{F}_{UU}(Q_0;b) \ ,$$
$$\widetilde{F}^{\alpha}_{\rm sivers}(Q;b) = e^{-\mathcal{S}_{sud}(Q,Q_0,b)}\widetilde{F}^{\alpha}_{\rm sivers}(Q_0;b)$$

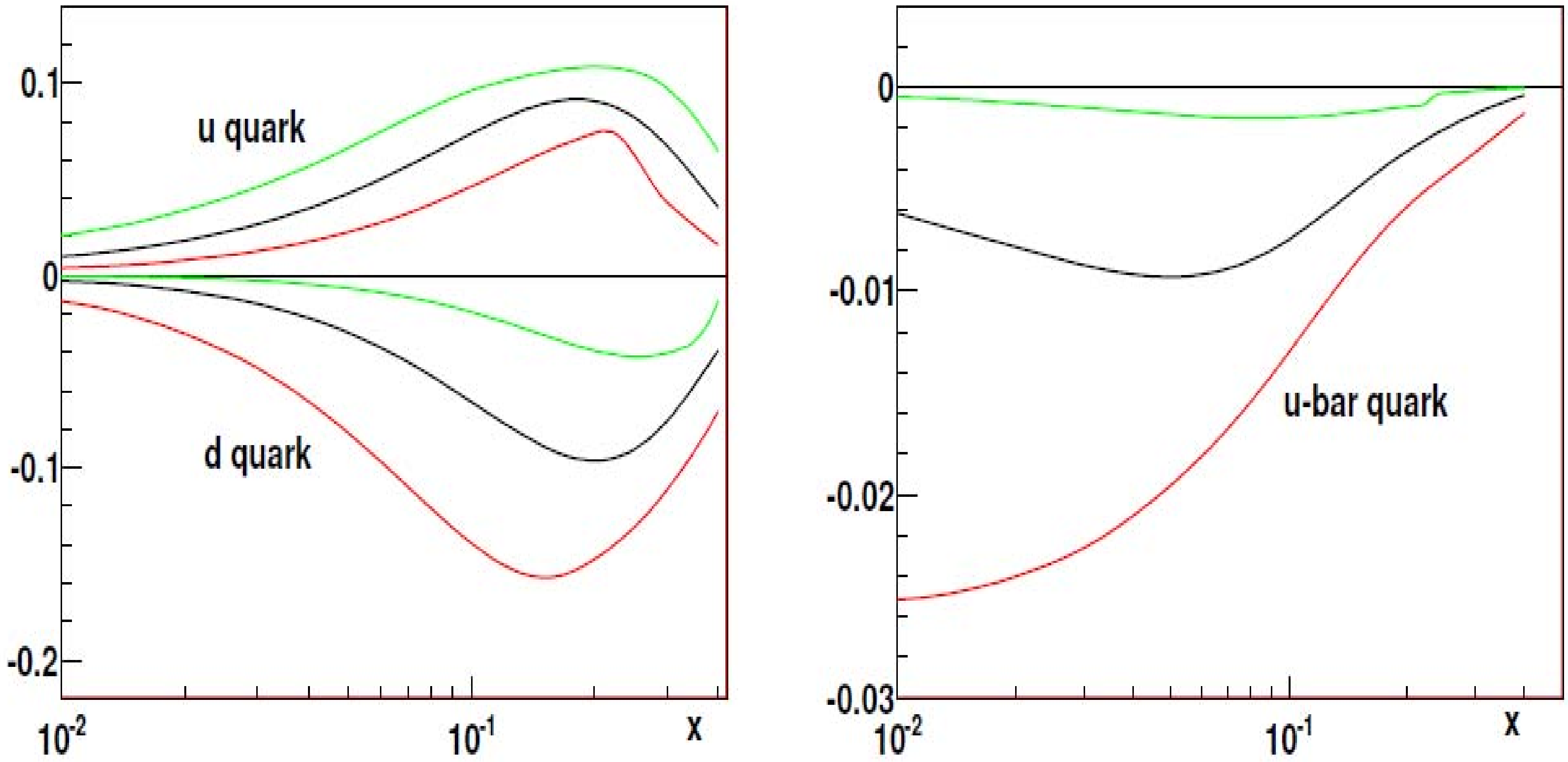
$$\widetilde{F}_{UU}(Q_0,b) = \sum_q e_q^2\, f_q(x_B,\mu=Q_0)\ D_q(z_h,\mu=Q_0)e^{-g_0 b^2 - g_h b^2/z_h^2} \ ,$$
$$\widetilde{F}^{\alpha}_{\rm sivers}(Q_0,b) = \frac{ib_\perp^\alpha M}{2}\sum_q e_q^2\, f_{1T}^{\perp(1)q}(x)\ D_q(z,\mu=Q_0)e^{-(g_0-g_s)b^2 - g_h b^2/z_h^2}$$

$$f_{1T}^{\perp(1)u}(x,\mu=Q_0) = N_u x^{\alpha_u}(1-x)^\beta \frac{(\alpha_u+\beta)^{\alpha_u+\beta}}{\alpha_u^{\alpha_u}\ \beta^\beta} f_u(x,\mu=Q_0) \ ,$$
$$f_{1T}^{\perp(1)d}(x,\mu=Q_0) = N_d x^{\alpha_d}(1-x)^\beta \frac{(\alpha_d+\beta)^{\alpha_d+\beta}}{\alpha_d^{\alpha_d}\ \beta^\beta} f_d(x,\mu=Q_0) \ ,$$
$$f_{1T}^{\perp(1)(\bar{u},\bar{d},s)}(x,\mu=Q_0) = N_{(\bar{u},\bar{d},s)} x^{\alpha_s}(1-x)^\beta \frac{(\alpha_s+\beta)^{\alpha_s+\beta}}{\alpha_s^{\alpha_s}\ \beta^\beta} f_{(\bar{u},\bar{d},s)}(x,\mu=Q_0)$$

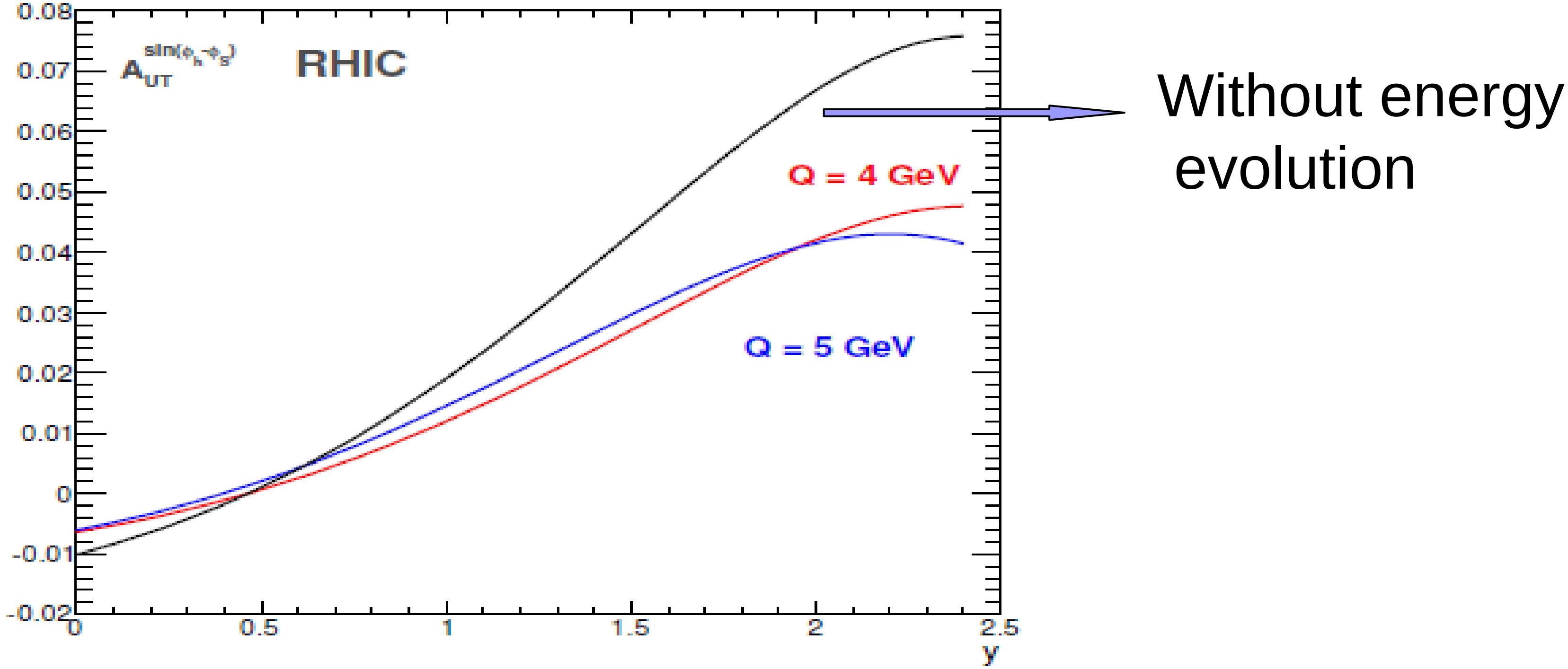
$$N_u = 0.13 \pm 0.023, \quad \alpha_u = 0.81 \pm 0.16, \quad \beta = 4.0 \pm 1.2 \ ,$$
$$N_d = -0.27 \pm 0.12, \quad \alpha_d = 1.41 \pm 0.28 \ ,$$
$$N_s = -0.07 \pm 0.06, \quad \alpha_s = 0.58 \pm 0.39 \ ,$$
$$N_{\bar{u}} = -0.07 \pm 0.05 \ ,$$
$$N_{\bar{d}} = -0.19 \pm 0.12 \ ,$$
$$g_s = 0.062 \pm 0.0053 \ .$$

$$\chi^2/d.o,f \approx 1.1$$

Uncertainties in the Sivers functions: moments



Predictions at RHIC



- About a factor of 2 reduction, as compared to previous order of magnitude difference