Parton distribution functions and fragmentation functions on lattice

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Based on works done with: Jian-Wei Qiu

appear soon

Collins asymmetry study at BEPCII

IHEP, P. R. China Nov. 1, 2013

Outline

- **Introduction**
  - Parton distribution functions / fragmentation functions
  - Difficulties in calculation
- **New idea for PDF**
  - Quasi-distribution: time independent operator
  - Conjecture: factorization
- **All order factorization**
- **A next-to-leading order example**
- **Summary and outlook**

Definition of PDF

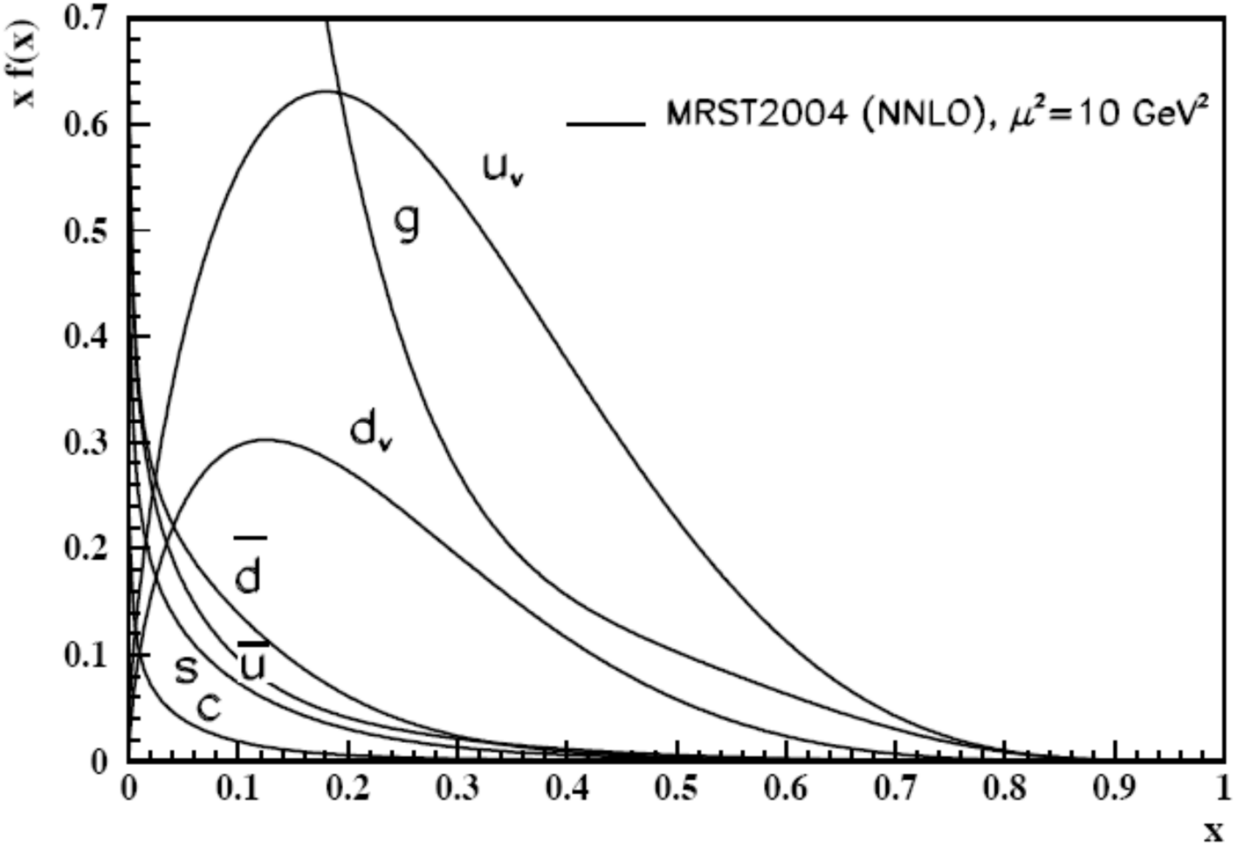
- **Lightcone quark distribution**

$$q(x,\mu^2) = \int \frac{d\xi_-}{4\pi} e^{-ix\xi_- P_+} \langle P|\overline{\psi}(\xi_-)\gamma_+ \exp\left\{-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right\}\psi(0)|P\rangle$$

- **Boost invariant along z-direction**

- **Parton interpretation emerges in $A_+ = 0$ gauge.**

PDF by fitting data



- **Different fits give different distributions**

Lattice QCD

- **The main non-perturbative approach to solve QCD**

- **An intrinsically Euclidean approach**

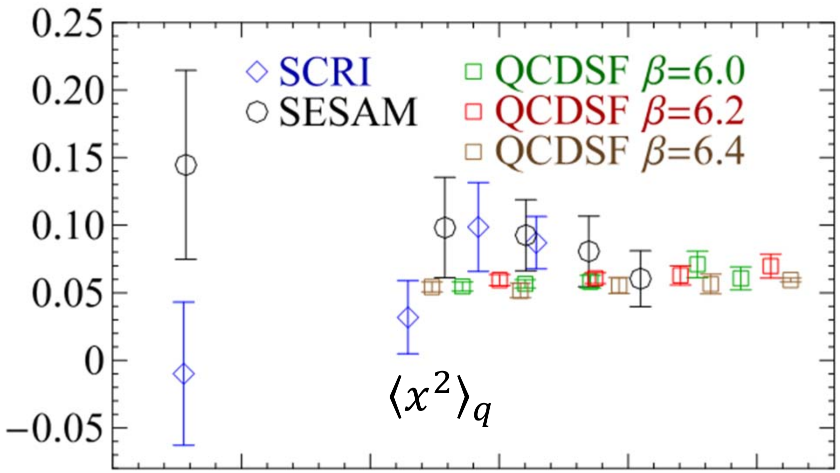
  - "time" is Euclidean $\tau = i\,t$
  - No direct implementation of physical time

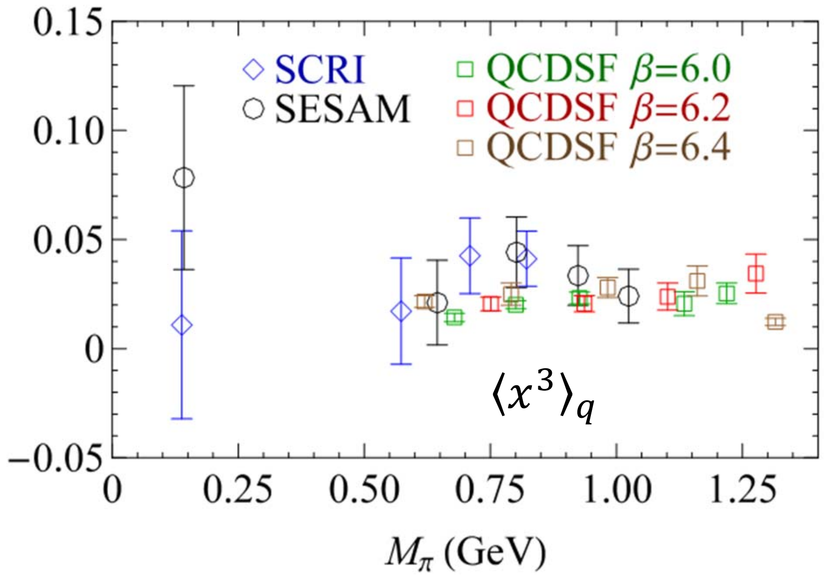
- **Cannot calculate operators like PDF, which is time dependent**

PDF from lattice

- **Only moments of distributions can be calculated: local operator**
  - $\langle x^n \rangle = \int_0^1 dx\, x^n q(x)$

- **High moments: Yes, but hard**



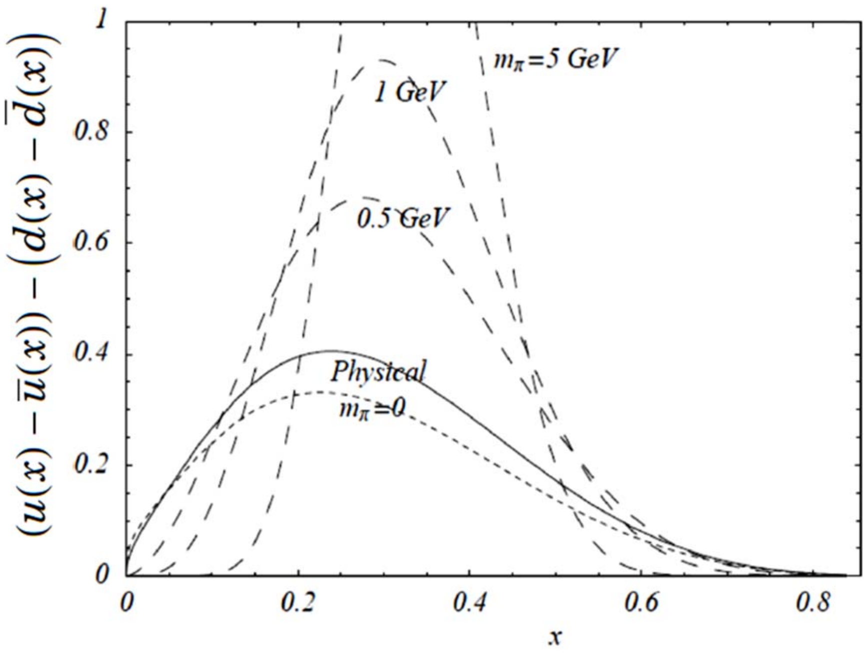


Dolgov et al., hep-lat/0201021 Gockeler et al., hep-ph/0410187

PDF from lattice

➢ **Make an ansatz of some smooth function form for the distribution and fix parameters by the $\langle x^n \rangle$**

- $x\, q(x) = a\, x^b (1-x)^c (1 + \epsilon \sqrt{x} + \gamma\, x)$



W. Dermold et al., Eur.Phys.J.direct C3 (2001) 1-15

- **Can not distinguish valence quark contribution from sea quark**

Ji's idea

Ji, 1305. 1539

- **Consider the space correlation with a large momentum P in the z-direction**

$$\tilde{q}(x,\mu^2,P_z) = \int \frac{dy}{4\pi} e^{iyP_z} \langle P|\overline{\psi}(y)\gamma_z \exp\left\{-ig\int_0^y dy' A_z(y')\right\}\psi(0)|P\rangle$$

- Quark fields separated along the z-direction
- The gauge-link along the z-direction
- The matrix element depends on the momentum P
- This distribution can be calculated using standard lattice method

- **No parton interpretation**

Large P_z limit

Ji, 1305. 1539

- **Taking the limit $P_z \to \infty$ and renormailzing all the UV divergence, one has the standard quark distribution**

  - Ji gave an argument using the standard OPE (is that a rigorous proof?)

  - The Altarelli-Parisi evolution was derived this way

- **What happens for finite P_z?**

Finite but large P_z

Ji, 1305. 1539

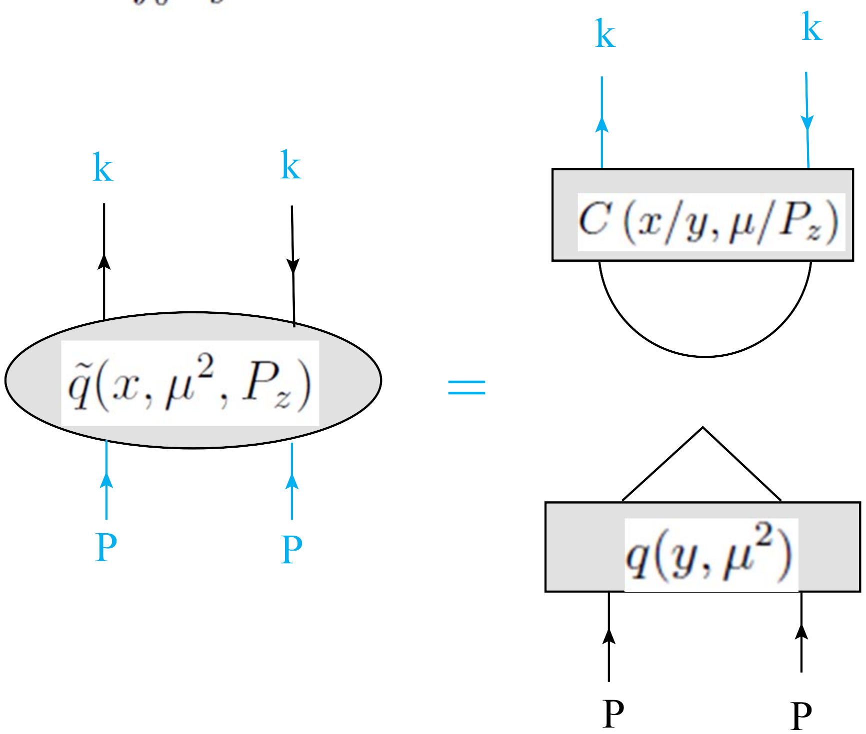
- It is potentially calculable in lattice QCD
- Since it differs from the standard PDF by simply an infinite P limit, it shall have the same IR physics
- It shall be related to the standard PDF by a matching condition

Conjecture: factorization

Ji, 1305. 1539

- **It is possible to have the following factorization (a little different from Ji's paper)**

$$\tilde{q}(x,\mu^2,P_z) = \int_0^1 \frac{dy}{y} C\left(x/y,\mu/P_z\right) q(y,\mu^2) + O(\Lambda^2_{\rm QCE}/P_z^2, M_N^2/P_z^2)$$



Key questions

- **Is there an all order factorization for finite but large P_z?**
- **How large of the P_z do we need?**

Power counting

Ma, Qiu, In preparation

➢ Scales:

- $\mu,\ P_z,\ \Lambda_{QCD}, M_N$

➢ Power counting

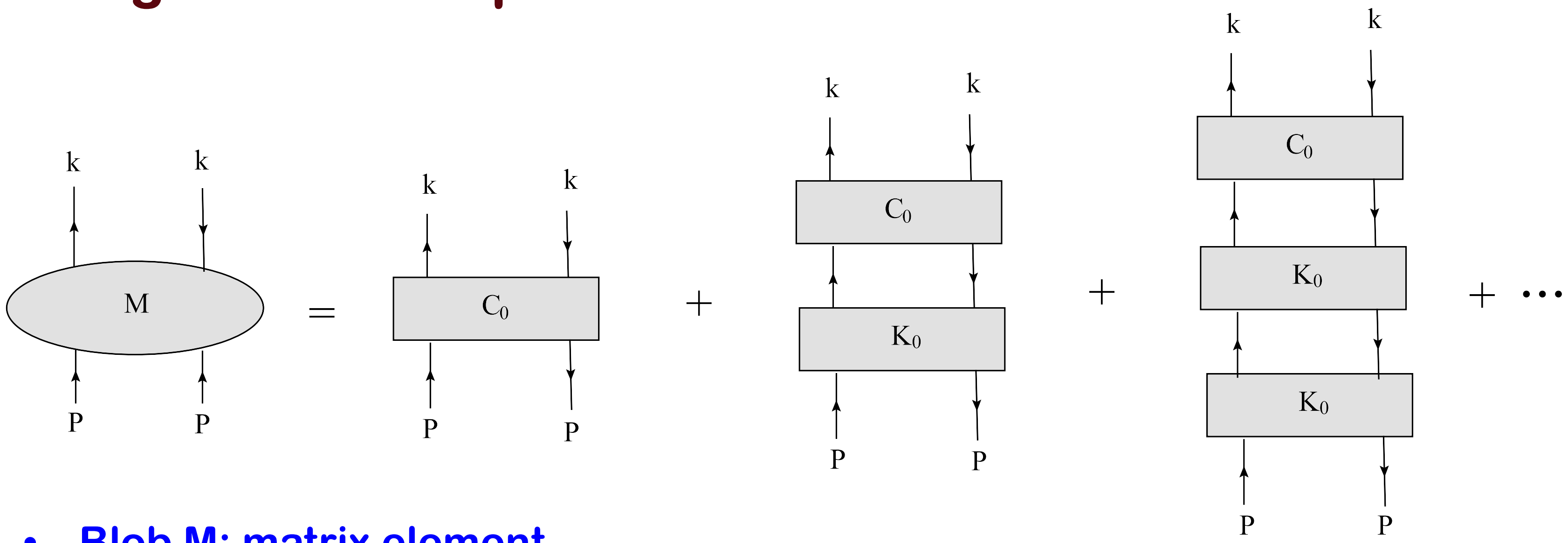
- $\mu,\ P_z \gg \Lambda_{QCD}, M_N$
- We will discuss only the leading power expansion in the above hierachy

➢ Factorization is nontrivial!

- The condition $\tilde{q}(x,\mu^2,P_z) \to q(x,\mu^2)$ in the limit $P_z \to \infty$ is not enough
- There may be terms like $\frac{\mu^2}{P_z^2}\ln\frac{\Lambda_{QCD}}{P_z^2}$, which is an IR divergence at finite P_z

Decomposition

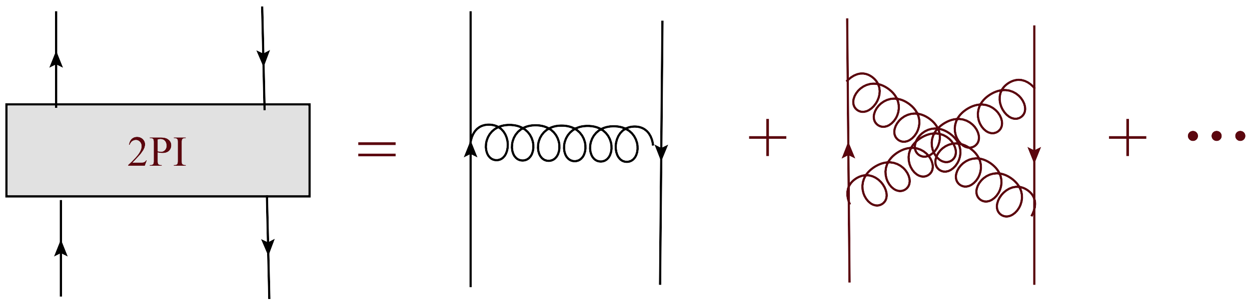
- **Replace the nucleon by a quark, generalized ladder diagrams decomposition** Mueller, PRD 1974



- **Blob M: matrix element**
- C_0, K_0: **2PI kernels**
- $p^2 \ll k^2$

2PI kernels

- **Diagrams**



- **Using physical gauge, 2PI diagrams are finite**

Ellis, Georgi, Machacek, Politzer, Ross, 1978, 1979

- **Landau rule: infrared divergenceare associated with physical, kinematically allowed subprocesses**
- **2PI diagrams: interference terms**

Proof of factorization

- **First factorize the last kernel ("x" finite, nonzero)**

$$\begin{aligned} M =& C_0 \sum_{i=0}^{\infty} K_0^i \\ =& C_0 \left[1 + \sum_{i=1}^{\infty} K_0^{i-1} \mathbb{P} K_0 + \sum_{i=1}^{\infty} K_0^{i-1} (1-\mathbb{P}) K_0 \right] \\ =& C_0 \left[1 + \sum_{i=1}^{\infty} K_0^{i-1} (1-\mathbb{P}) K_0 \right] + M \mathbb{P} K_0 \end{aligned}$$

  - $\mathbb{P}$: extracts the singular part of integration

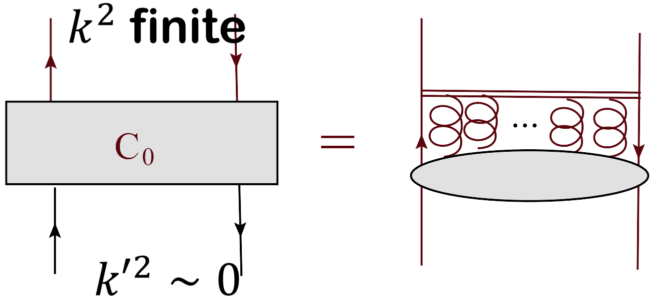
- **Using this technique recursively, we get the factorized form**

$$M = \left(C_0 \frac{1}{1-(1-\mathbb{P})K_0} \right) \left(\frac{1}{1-\mathbb{P}K_0} \right)$$

  - Divergences are included in the second term

Gauge choice

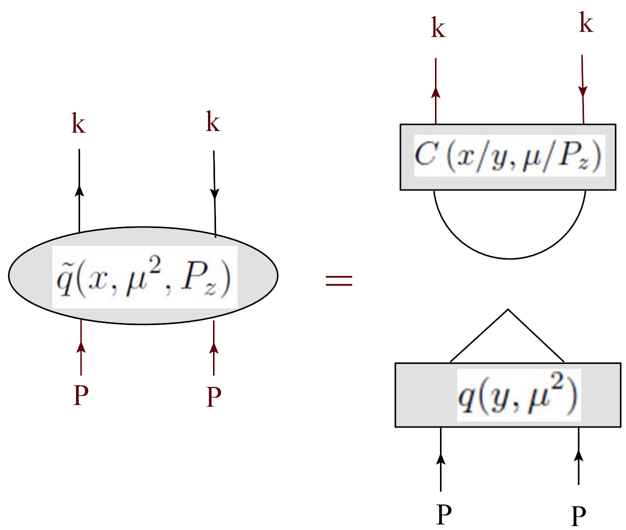
- **General axial gauge**



  - Double line: gauge link

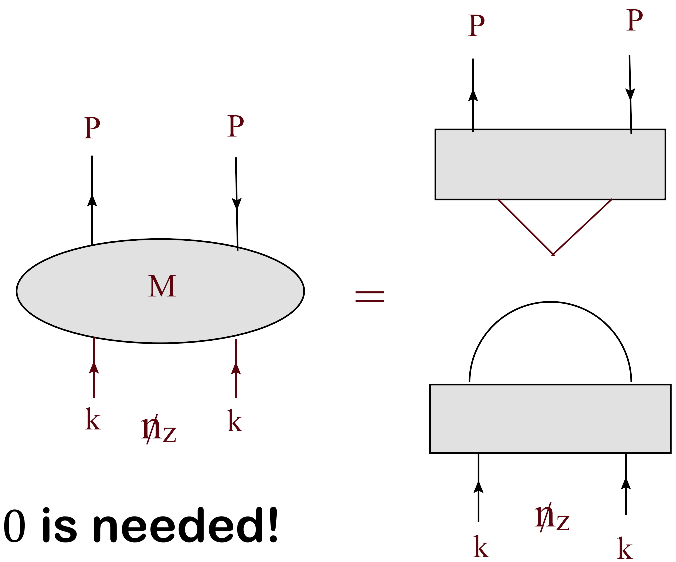
- **In the $A_z = 0$ gauge: $C_0 = 1$**

  - Renormalizetion is standard



Factorization for fragmentation function

- **Quasi fragmentation function: replace the "n" by "n_z" in ordinary fragmentation function**
- **Similar as the factorization for PDF, we all prove the factorization of quasi fragmentation function in terms of ordinary fragmentation function to all order in α_s**



$k^2 \gg P^2 \sim 0$ **is needed!**

- **Quasi Collins function?**
  Quasi Sivers function?
  Twist 3 PDF/ FF?

Matching

$$\tilde{q}(x,\mu^2,P_z)=\int_0^1\frac{dy}{y}C\left(x/y,\mu/P_z\right)q(y,\mu^2)+O(\Lambda^2_{\rm QCE}/P_z^2,M_N^2/P_z^2)$$

- **Leading order: $C^{(0)}$**

$$\tilde{q}^{(0)}(x,\mu^2,P_z)=\int_0^1\frac{dy}{y}C^{(0)}\left(x/y,\mu/P_z\right)q^{(0)}(y,\mu^2)$$

$$q^{(0)}(x,\mu^2)=\tilde{q}^{(0)}(x,\mu^2,P_z)=\delta(1-x)$$

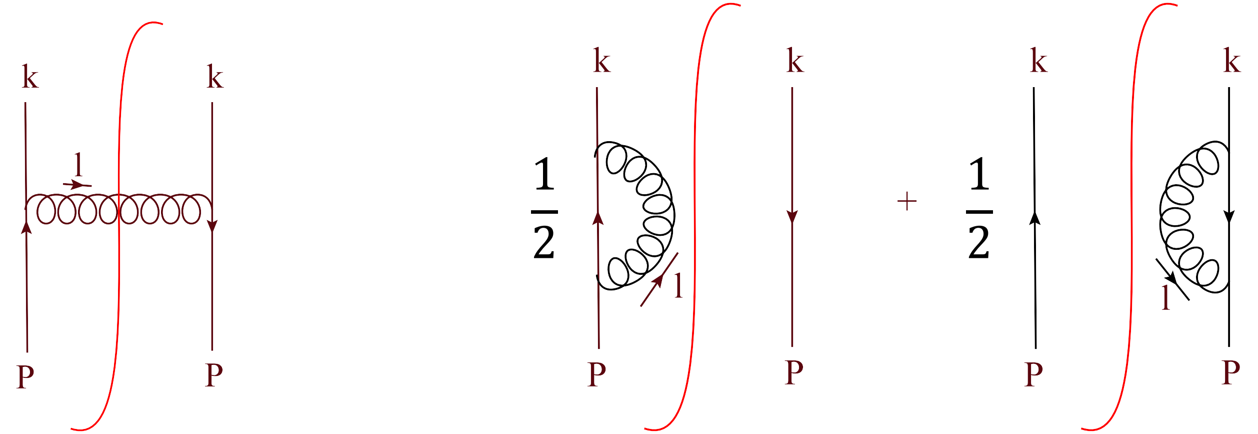
$$\Rightarrow \quad C^{(0)}\left(x/y,\mu/P_z\right)=\delta(1-x/y)$$

- **Next-to-leading order: $C^{(1)}$**

$$\tilde{q}^{(1)}(x,\mu^2,P_z)=\int_0^1\frac{dy}{y}C^{(1)}\left(x/y,\mu/P_z\right)q^{(0)}(y,\mu^2)+\int_0^1\frac{dy}{y}C^{(0)}\left(x/y,\mu/P_z\right)q^{(1)}(y,\mu^2)$$
$$=C^{(1)}\left(x,\mu/P_z\right)+q^{(1)}(x,\mu^2)$$

One loop example: quasi PDF

- **Diagrams: $A_z = 0$ gauge**



- **Polarization: double pole!!!**

$$\tilde{d}^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n_z^\beta + n_z^\alpha l^\beta}{l_z} - \frac{n_z^2 l^\alpha l^\beta}{l_z^2}$$

Integrand

- **Real**

$$\tilde{R} = C_F \frac{\alpha_s}{2\pi} \int \frac{2 d^{D-2} l_\perp}{(2\pi)^{D-3} l_\perp^2} \int dl_z \delta\left(1 - x - \frac{l_z}{P_z}\right) \times \left[\frac{1 - l_z/P_z}{l_z^2} \frac{l_\perp^2}{2\sqrt{l_\perp^2}} + \left(1 - \frac{l_z}{P_z} + \frac{1-\epsilon}{2}\frac{l_z^2}{P_z^2}\right) \frac{\sqrt{l_\perp^2 + l_z^2} + l_z}{l_z \sqrt{l_\perp^2 + l_z^2}}\right]$$

  - **Collinear divergent if** $l_z > 0$

- **Virtual**

$$\tilde{V} = -C_F \frac{\alpha_s}{2\pi} \delta(1-x) \int \frac{2 d^{D-2} l_\perp}{(2\pi)^{D-3} l_\perp^2} \int dl_z \left\{ \frac{l_\perp^2}{2 l_z^2 \sqrt{l_\perp^2 + l_z^2}} + \frac{l_\perp^2}{2 P_z l_z \sqrt{l_\perp^2 + (l_z - P_z)^2}} \times \left(1 - \frac{l_z}{P_z} + \frac{1-\epsilon}{2}\frac{l_z^2}{P_z^2}\right) \left[\frac{l_z}{\sqrt{l_\perp^2 + l_z^2}} - \frac{l_z - P_z}{\sqrt{l_\perp^2 + (l_z - P_z)^2}}\right] \right\}$$

  - **Collinear divergent if** $P_z > l_z > 0$

Sum over real and virtual

- **Final**

$$y = 1 - x \qquad \Lambda_y = \sqrt{\mu^2/P_z^2 + y^2} - |y|$$

$$\begin{aligned}\tilde{R} + \tilde{V} =& C_F \frac{\alpha_s}{2\pi} \int \frac{2d^{D-2}l_\perp}{(2\pi)^{D-3}l_\perp^2} \left[\frac{1+x^2-\epsilon(1-x)^2}{(1-x)_+} + \frac{3+\epsilon}{2}\delta(1-x)\right] \\ &+ C_F \frac{\alpha_s}{2\pi} \left\{ \left(\frac{\Lambda_y}{y^2}\right)_N - \left(\frac{\Lambda_y}{y}\right)_A - [1+(1-y)^2] \left(\frac{\ln\left(1+\frac{\Lambda_y}{2y}\right)}{y}\right)_N \right\} \\ &+ C_F \frac{\alpha_s}{2\pi} \left\{ \Lambda_1 \left[2 + \ln\left(1+\frac{2}{\Lambda_1}\right)\right] + \ln\left(1+\frac{\Lambda_1}{2}\right) \ln\left(\frac{\Lambda_1^2}{4+2\Lambda_1}\right) + 2\mathrm{Li}_2\left(-\frac{\Lambda_1}{2}\right) \right\}\end{aligned}$$

- **"N" and "A" are regularized functions**
- **Collinear divergence: same as that for ordinary PDF**
- **Finite terms:** $O(\frac{\alpha_s}{2\pi}\frac{\mu^2}{P_z^2})$**, good convergence!**

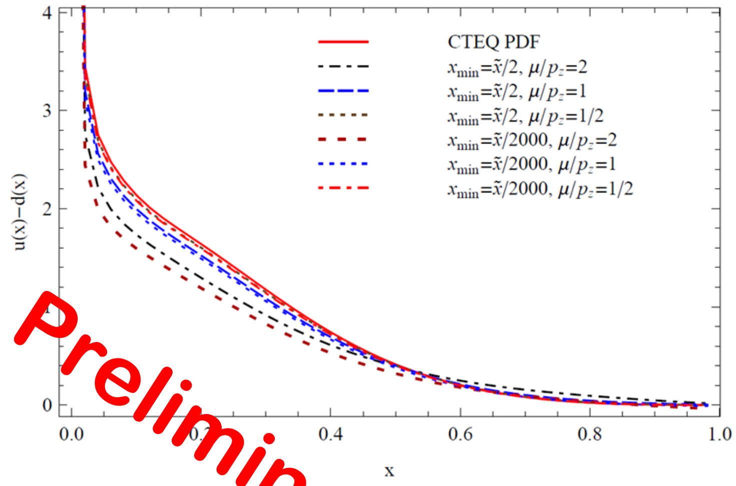
Preliminary numerical calculation

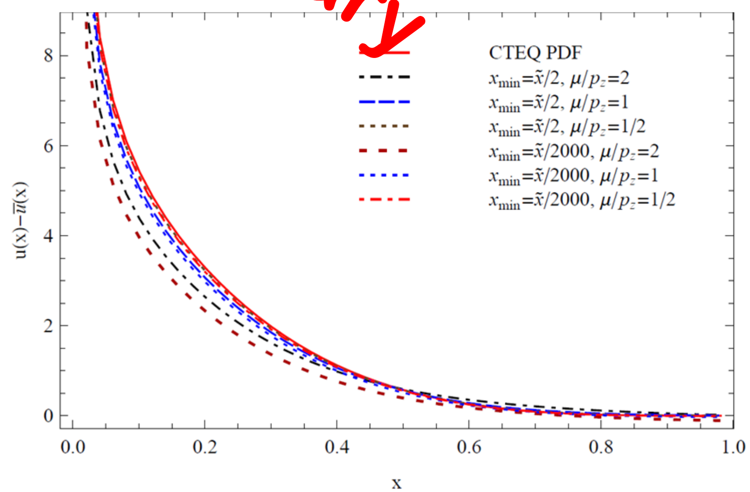
- **Final: use CTEQ6M as input**

$$\tilde{q}(x,\mu^2,P_z) = \int_0^1 \frac{dy}{y} C\left(x/y, \mu/P_z\right) q(y,\mu^2) + O(\Lambda^2_{\mathrm{QCE}}/P_z^2, M_N^2/P_z^2)$$

- **In principle: we need $P_z \gg \mu$, but $P_z = \mu$ is already good enough for these cases!**

- **Renormalization scheme dependent**





Comparison with Ji et al.'s work

Ji et al. 1310.7471

- **Ji et al. also worked out the one-loop calculation, and claimed the factorization at one-loop level. We have not yet figured out whether our results are consistent with each other.**

Summary

- Parton distribution functions, fragmentation function are important, but hard to calculate from first principle.
- Recent new idea: ordinary distributions can be obtained by quasi distributions by taking $P_z \to \infty$.
- Quasi distributions can be calculate only for finite P_z. Relation between ordinary distribution and quasi distribution is needed (factorization).
- We prove the factorization to all order in α_s: for convergent distribution, e.g. valence quark, .
- Explicit one-loop calculation confirm the factorization.
- Preliminary numerical results….

Thank you!