Chiral quark model study of 5-quark systems

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Outline

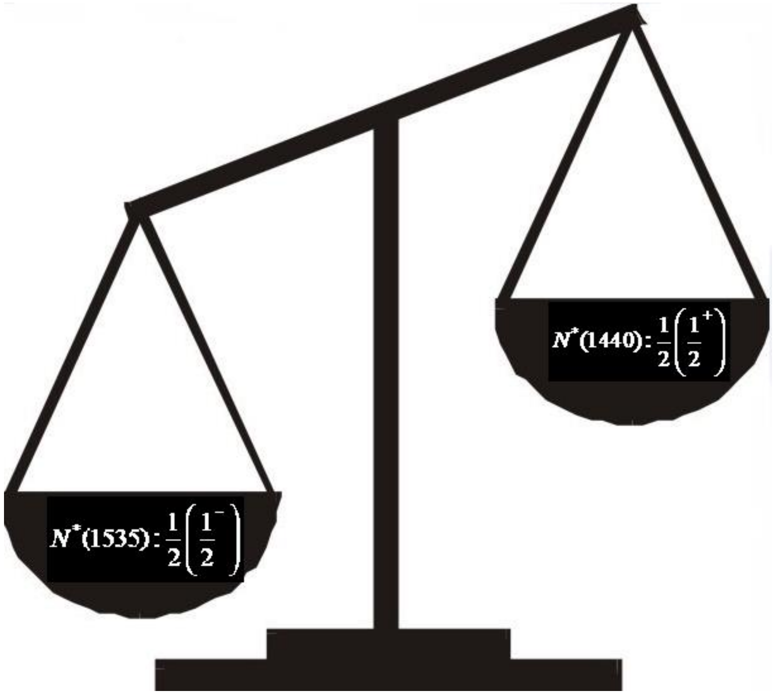
- Problems of 3-quark model
- Chiral quark model of 5-quark
- 5-quark configurations
- Gaussian basis function
- Baryon, meson spectra and 5-quark states
- Summary

Problems for the classical 3q model

- Mass order reverse

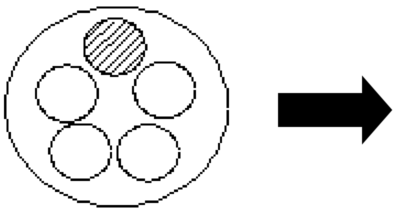
$$E_{SHO} = \left(2n + L + \frac{3}{2}\right)\hbar\omega$$

$$E_{n=0,L=1} < E_{n=1,L=0}$$



- $\bar{u}$ and $\bar{d}$ asymmetry

$$\bar{d} - \bar{u} \approx 0.12$$



12%

- Postive μ_s of proton

Hamiltonian of 5-quark

- General

$$H = \sum_{i=1}^{5}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{CM} + \sum_{j>i=1}^{5} V_{ij}^{C}\left(\vec{r}_{ij}\right)$$

- The central part of potential

1. OGE

$$V_{OGE}^{C}\left(\vec{r}_{ij}\right) = \alpha_s \frac{\lambda_i^c \cdot \lambda_j^c}{4}\left[\frac{1}{r_{ij}} - \frac{\pi}{2}\delta\left(r_{ij}\right)\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{3m_i m_j}\vec{\sigma}_i \cdot \vec{\sigma}_j\right)\right]; \qquad \alpha_s = \frac{\alpha_0}{\ln\left(\frac{\mu^2 + \mu_0^2}{\Lambda_0^2}\right)} \qquad \delta\left(r_{ij}\right) = \frac{e^{\frac{-r_{ij}^2}{\beta^2}}}{\left(\beta\sqrt{\pi}\right)^3}$$

2. Screened potential

$$V_{CON}^{C}\left(\vec{r}_{ij}\right) = -\left(\lambda_i^c \cdot \lambda_j^c\right)\left(a_c r_{ij}^2 - V_0\right)$$

3. Quark-quark interaction

$$V_{qq}^{C}\left(\vec{r}_{ij}\right)=V_{\pi}^{C}\left(\vec{r}_{ij}\right)+V_{\sigma}^{C}\left(\vec{r}_{ij}\right)+V_{\eta}^{C}\left(\vec{r}_{ij}\right)$$

each interaction being given by

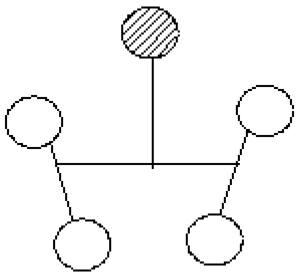
$$V_{\pi}^{C}\left(\vec{r}_{ij}\right)=\frac{g_{ch}^{2}}{4\pi}\frac{m_{\pi}^{2}}{12m_{i}m_{j}}\frac{\Lambda_{\pi}^{2}}{\Lambda_{\pi}^{2}-m_{\pi}^{2}}m_{\pi}\left[Y\left(m_{\pi}r_{ij}\right)-\frac{\Lambda_{\pi}^{3}}{m_{\pi}^{3}}Y\left(\Lambda_{\pi}r_{ij}\right)\right]\left(\vec{\sigma}_{i}\cdot\vec{\sigma}_{j}\right)\sum_{a=1}^{3}\left(\lambda_{i}^{a}\cdot\lambda_{j}^{a}\right)$$

$$V_{\sigma}^{C}\left(\vec{r}_{ij}\right)=-\frac{g_{ch}^{2}}{4\pi}\frac{\Lambda_{\sigma}^{2}}{\Lambda_{\sigma}^{2}-m_{\sigma}^{2}}m_{\sigma}\left[Y\left(m_{\sigma}r_{ij}\right)-\frac{\Lambda_{\sigma}}{m_{\sigma}}Y\left(\Lambda_{\sigma}r_{ij}\right)\right]$$

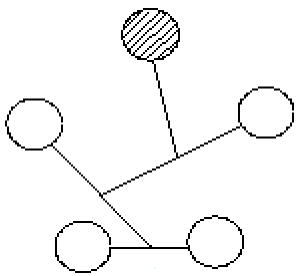
$$V_{\eta}^{C}\left(\vec{r}_{ij}\right)=\frac{g_{ch}^{2}}{4\pi}\frac{m_{\eta}^{2}}{12m_{i}m_{j}}\frac{\Lambda_{\eta}^{2}}{\Lambda_{\eta}^{2}-m_{\eta}^{2}}m_{\eta}\left[Y\left(m_{\eta}r_{ij}\right)-\frac{\Lambda_{\eta}^{3}}{m_{\eta}^{3}}Y\left(\Lambda_{\eta}r_{ij}\right)\right]\left(\vec{\sigma}_{i}\cdot\vec{\sigma}_{j}\right)\left[\cos\theta_{P}\left(\lambda_{i}^{8}\cdot\lambda_{j}^{8}\right)-\sin\theta_{P}\right]$$

$qqqq\bar{q}$ configurations

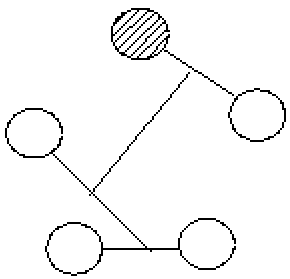
- Diquark type



- Ordered type



- Baryon-meson type



Wave function-GEM

- Example:

Two-body normalized Gaussian basis function,

$$\phi_{nlm}(\vec{r}) = \phi_{nl}^{G}(r) Y_{lm}(\hat{r})$$

$$= N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{r})$$

$$= \sqrt{\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}(2l+1)!!}} r^l e^{-\nu_n r^2} Y_{lm}(\hat{r})$$

Transform into the ISG form,

$$\phi_{nlm}(\vec{r}) = N_{nl} \lim_{\varepsilon \to 0} \left(\frac{l}{4\nu_n \varepsilon} \right)^l \sum_{j=0}^{\left|\frac{l-m}{2}\right|} A_{lm,j} \sum_{s=0}^{p} \sum_{t=0}^{q} \sum_{u=0}^{j} C_p^s C_q^t C_j^u e^{-\nu_n (\vec{r} - \varepsilon \vec{D})^2}$$

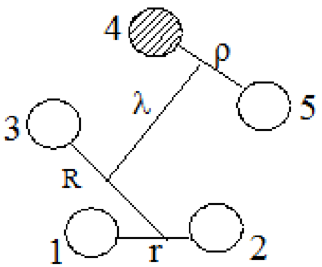
where,

$$A_{lm,j} = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} \frac{(l+m)!}{2^m} \frac{(-1)^j}{4^j j!(m+j)!(l-m-2j)!}$$

$$p = l - m - 2j$$

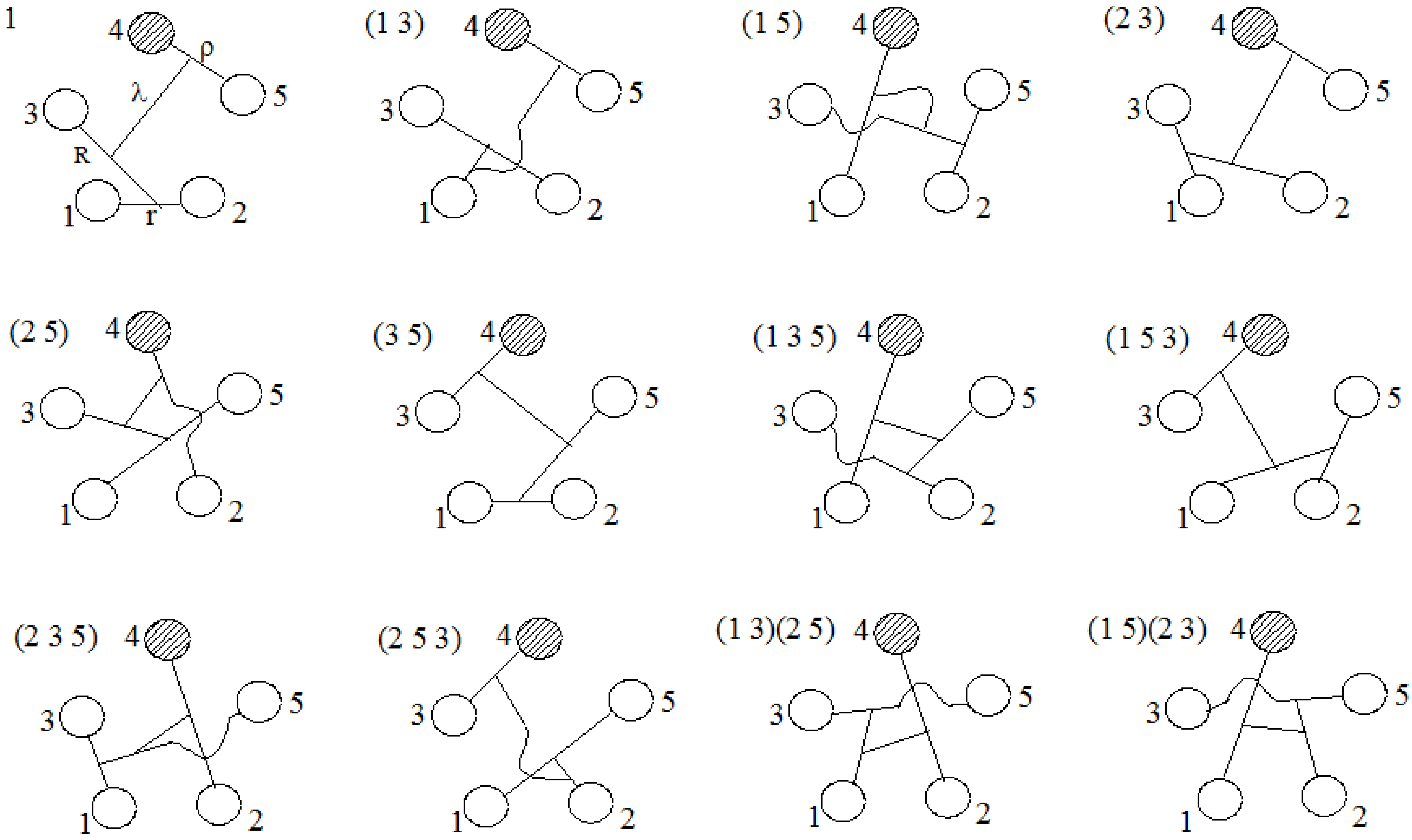
$$q = m + j$$

Wave function of the B-M configuration



$$\Psi = \sum_{g=1}^{N_{total}} C_g \mathrm{A} \phi_{n_1 l_1 m_1}(\vec{r}) \psi_{n_2 l_2 m_2}(\vec{R}) \omega_{n_3 l_3 m_3}(\vec{\rho}) \eta_{n_4 l_4 m_4}(\vec{\lambda}) \chi_i^{\sigma} \chi_j^{f} \chi_k^{c}$$

$$\begin{aligned} A = & 1-(1\,3)-(1\,5)-(2\,3)-(2\,5)-(3\,5) \\ & +(1\,3\,5)+(1\,5\,3)+(2\,3\,5)+(2\,5\,3)+(1\,3)(2\,5)+(1\,5)(2\,3) \end{aligned}$$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Bases of 5-quark

$I(J^P)$	$\frac{1}{2}(\frac{1}{2}^+);[\frac{1}{2}(\frac{3}{2}^+)]$	$\frac{1}{2}(\frac{5}{2}^+)$	$\frac{3}{2}(\frac{7}{2}^+)$
$N_{ijk}^{\sigma f c}$	45	25	4
$N_{n_1 n_2 n_3 n_4}^{x}$	625	625	625
N_{dim}	15000	8750	1250

LS Coupling

$$\left|J, M_J\right\rangle = \sum_{M_L, M_S} C^{J\,M_J}_{L\,M_L, S\,M_S} \left|L \quad M_L, S \quad M_S\right\rangle$$

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = \begin{cases} \sqrt{\frac{2}{3}}\left|1 \quad 1, \frac{1}{2} \quad -\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|1 \quad 0, \frac{1}{2} \quad \frac{1}{2}\right\rangle \\ \sqrt{\frac{1}{6}}\left|1 \quad 1, \frac{3}{2} \quad -\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|1 \quad 0, \frac{3}{2} \quad \frac{1}{2}\right\rangle + \sqrt{\frac{1}{2}}\left|1 \quad -1, \frac{3}{2} \quad \frac{3}{2}\right\rangle \end{cases} \Rightarrow \left|\frac{1}{2},\frac{1}{2}\right\rangle = \begin{cases} \left|1 \quad 0, \frac{1}{2} \quad \frac{1}{2}\right\rangle \\ \left|1 \quad -1, \frac{3}{2} \quad \frac{3}{2}\right\rangle \end{cases}$$

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = \begin{cases} \left|1 \quad 1, \frac{1}{2} \quad \frac{1}{2}\right\rangle \\ \sqrt{\frac{2}{5}}\left|1 \quad 1, \frac{3}{2} \quad \frac{1}{2}\right\rangle - \sqrt{\frac{3}{5}}\left|1 \quad 0, \frac{3}{2} \quad \frac{3}{2}\right\rangle \end{cases} \Rightarrow \left|\frac{3}{2},\frac{3}{2}\right\rangle = \begin{cases} \left|1 \quad 1, \frac{1}{2} \quad \frac{1}{2}\right\rangle \\ \left|1 \quad 0, \frac{3}{2} \quad \frac{3}{2}\right\rangle \end{cases}$$

Parameters of 5-quark

$m_{u,d}\left(MeV\right)$	$m_s\left(MeV\right)$	$a_c\left(MeV\ fm^{-2}\right)$	$V_0\left(MeV\right)$
313	540	59.5	16.3
α_0	β	$\mu_0\left(MeV\right)$	$\Lambda_0\left(fm^{-1}\right)$
3.1	0.485	340	0.106

Baryon and meson spectra

(3q)

$"N = 5 \quad r \in [0.3 - 1.7]"$

	N	Δ	Ω	Σ	Σ^*	Λ	Ξ	Ξ^*
i	929	1234	1629	1213	1382	1143	1356	1514
Exp.	939	1232	1672	1193	1384	1116	1318	1533

	π	η	ρ	ω
i	522	642	777	751
Exp.	139	547	770	782

The lowest energy of 5-quark systems

	$\frac{1}{2}(\frac{1}{2}^{+});[\frac{1}{2}(\frac{3}{2}^{+})]$	$\frac{1}{2}(\frac{5}{2}^{+})$	$\frac{3}{2}(\frac{7}{2}^{+})$
B+M	1451 $N+\pi$	1680 $N+\omega$	1985 $\Delta+\omega$
5q	1487	1637	1985

Summary

Discussion for $\frac{1}{2}(\frac{5}{2}^{+})$

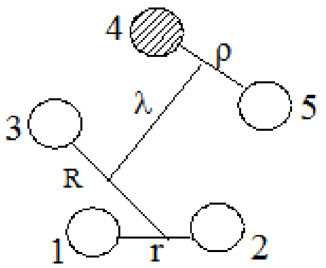
- Bound state

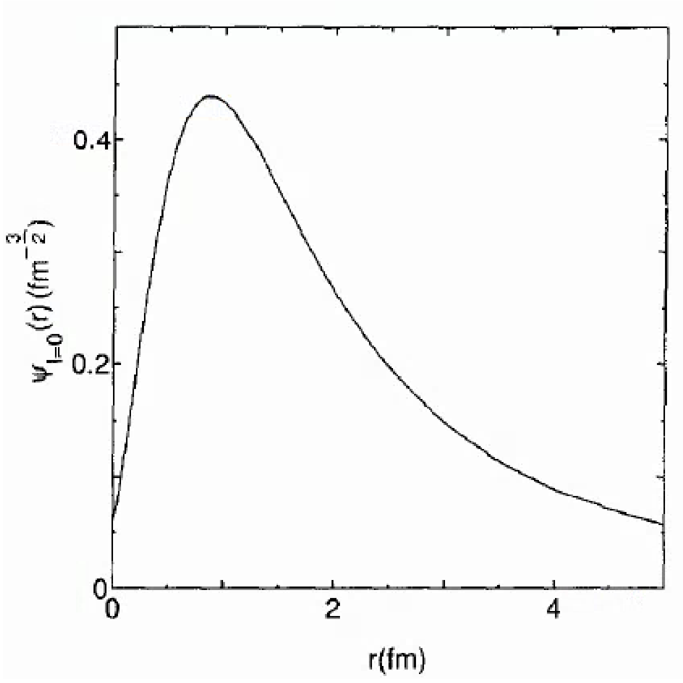
$$E_{5q} < E_{N+\omega}$$

- Molecular state

$$maybe \quad \lambda > 2$$

$$deuteron : \{n_{\max} = 30; r_{\max} = 30\,fm\}$$





Thanks!