Resonances from the hadron hadron chiral interaction in the strange and charmed sectors

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Extracting the two Λ(1405) poles from πΣ photoproduction data

Analysis of lattice QCD data to extract the $D_{s0}(2317)$ and $D_{s1}(2460)$ resonances and evidence for their KD and KD* molecular structure

Meson interaction

Pseudoscalar-pseudoscalar interaction: channels

1) $\pi^+ \pi^-$
2) $\pi^0 \pi^0$
3) $K^+ K^-$
4) K^0 Kbar^0
5) $\eta\eta$

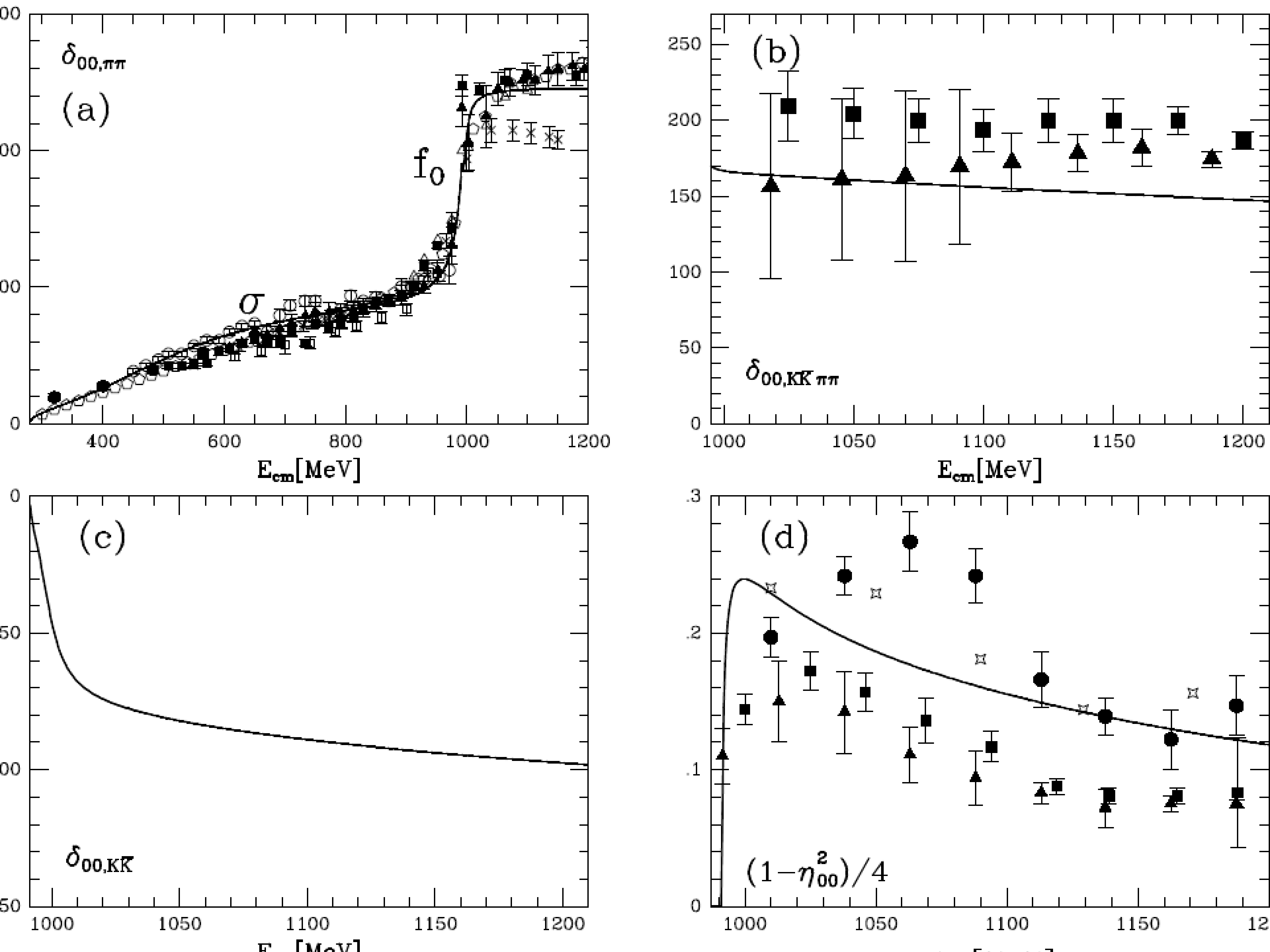
We use the chiral unitary approach: Bethe Salpeter equations in coupled channels

$$T=(1-VG)^{-1}V$$

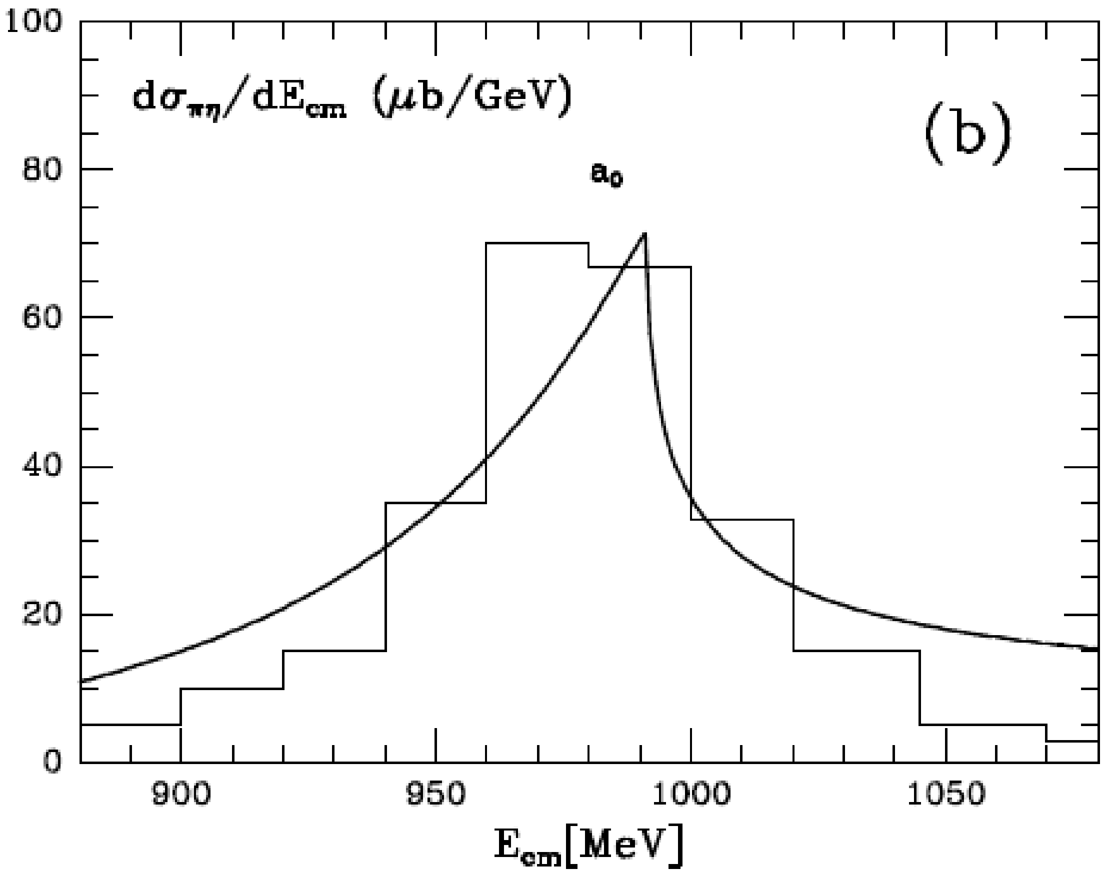
With V obtained from the chiral Lagrangians and G the loop function of two meson propagators .

$$G_{jj}(s) = \int_0^{q_{max}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1+\omega_2}{\omega_1\omega_2[P^{02}-(\omega_1+\omega_2)^2+i\epsilon]}$$

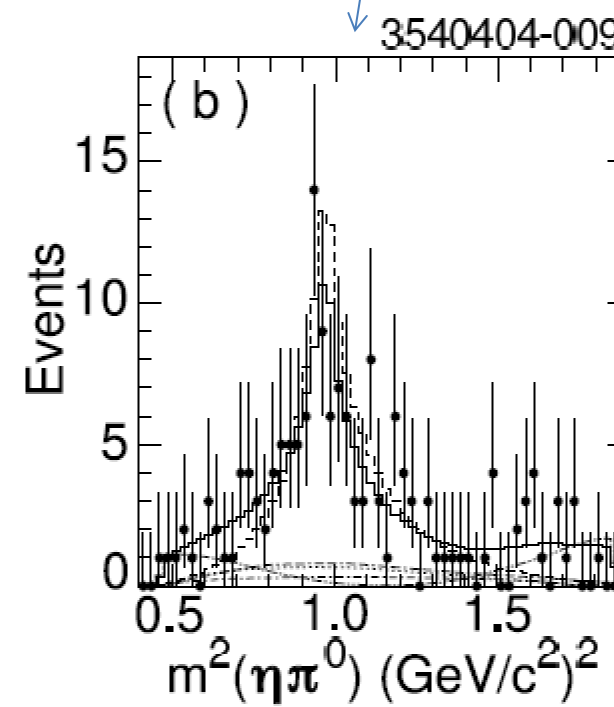
$$\begin{aligned}
&V_{11} = -\frac{1}{2f^2}s, && V_{12} = -\frac{1}{\sqrt{2}f^2}(s-m_\pi^2), && V_{13} = -\frac{1}{4f^2}s, \\
&V_{14} = -\frac{1}{4f^2}s, && V_{15} = -\frac{1}{3\sqrt{2}f^2}m_\pi^2, && V_{22} = -\frac{1}{2f^2}m_\pi^2, \\
&V_{23} = -\frac{1}{4\sqrt{2}f^2}s, && V_{24} = -\frac{1}{4\sqrt{2}f^2}s, && V_{25} = -\frac{1}{6f^2}m_\pi^2, \\
&V_{33} = -\frac{1}{2f^2}s, && V_{34} = -\frac{1}{4f^2}s, && V_{35} = -\frac{1}{12\sqrt{2}f^2}(9s-6m_\eta^2-2m_\pi^2), \\
&V_{44} = -\frac{1}{2f^2}s, && V_{45} = -\frac{1}{12\sqrt{2}f^2}(9s-6m_\eta^2-2m_\pi^2), && V_{55} = -\frac{1}{18f^2}(16m_K^2-7m_\pi^2),
\end{aligned} \tag{8}$$



$\delta_{00,\pi\pi}$
(a)
f_0
σ
400
600
800
1000
1200
E_{cm}[MeV]
(b)
$\delta_{00,K\bar{K}\pi\pi}$
250
200
150
100
50
0
1000
1050
1100
1150
1200
E_{cm}[MeV]
(c)
$\delta_{00,K\bar{K}}$
1000
1050
1100
1150
1200
(d)
$(1-\eta_{00}^2)/4$
.3
.2
.1
0
1000
1050
1100
1150



$D^0 \to K_S^0 \eta \pi^0$ **Decay**



Oset, Ramos NPA98

Coupled channels:

$$K^-p,\ \bar{K}^0n,\ \pi^0\Lambda,\ \pi^0\Sigma^0,\ \pi^+\Sigma^-,\ \pi^-\Sigma^+,\ \eta\Lambda,\ \eta\Sigma^0,\ K^0\Xi^0 \text{ and } K^+\Xi^-$$

$$T = [1 - VG]^{-1}V$$

$$G_i = i\int \frac{d^4q}{(2\pi)^4}\frac{M_i}{E_i(\vec{q}\,)} \times \frac{1}{k^0 + p^0 - q^0 - E_i(\vec{q}\,) + i\epsilon}\,\frac{1}{q^2 - m_i^2 + i\epsilon}$$

$$V_{ij}(\sqrt{s}) = -C_{ij}\frac{1}{4f^2}(2\sqrt{s} - M_i - M_j) \times \left(\frac{M_i + E_i}{2M_i}\right)^{1/2}\left(\frac{M_j + E_j}{2M_j}\right)^{1/2},$$

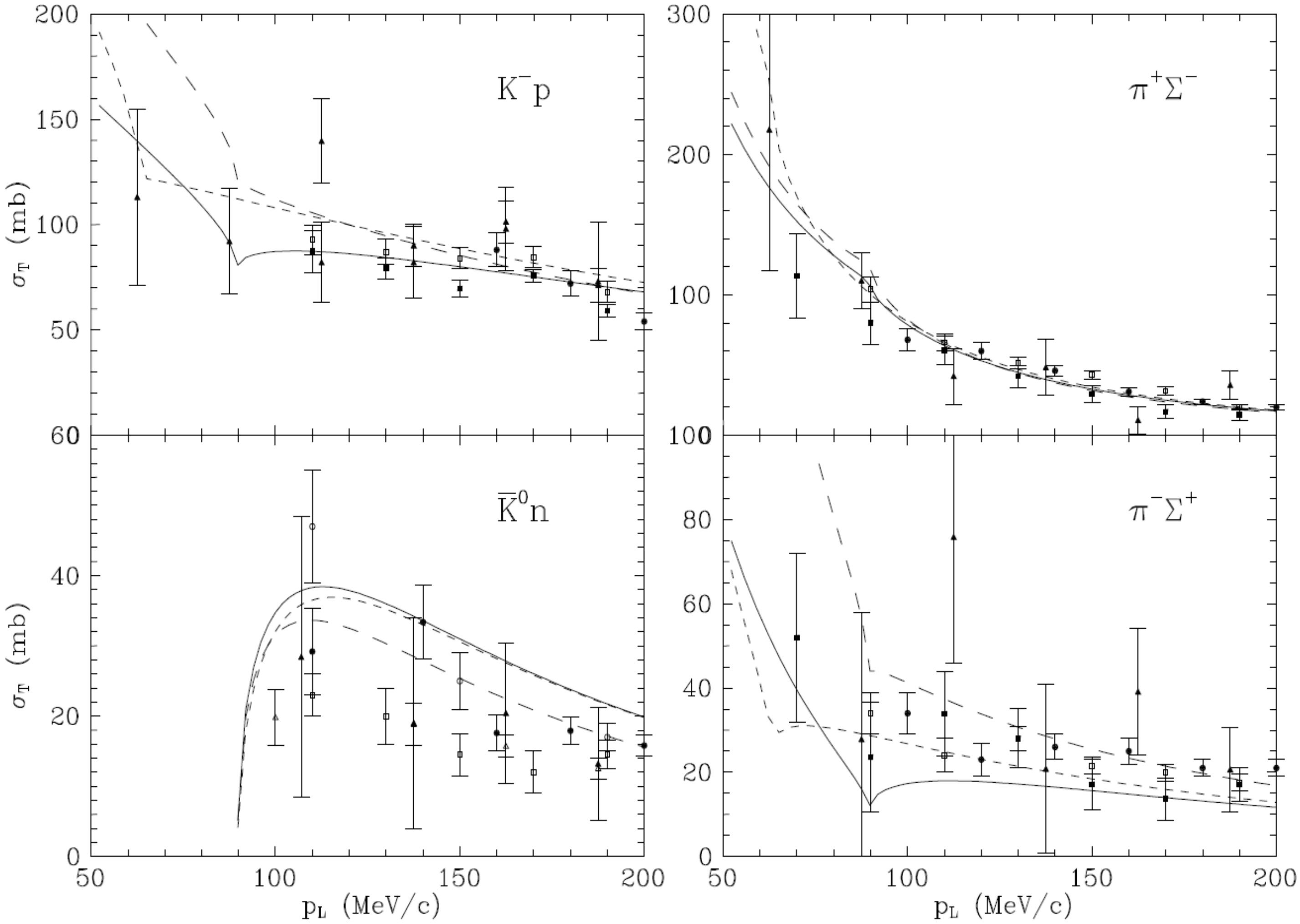
$$C_{ij} = \begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} \\ -\sqrt{\frac{3}{2}} & 4 \end{pmatrix}$$ for I=0

Channels Kbar N , πΣ,

Oset, Ramos NPA98

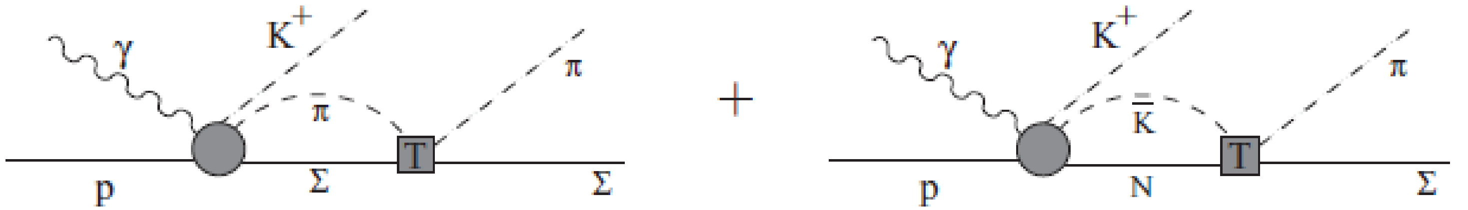
Coupled channels:

$$K^-p,\ \bar{K}^0n,\ \pi^0\Lambda,\ \pi^0\Sigma^0,\ \pi^+\Sigma^-,\ \pi^-\Sigma^+,\ \eta\Lambda,\ \eta\Sigma^0,\ K^0\Xi^0 \text{ and } K^+\Xi^-$$



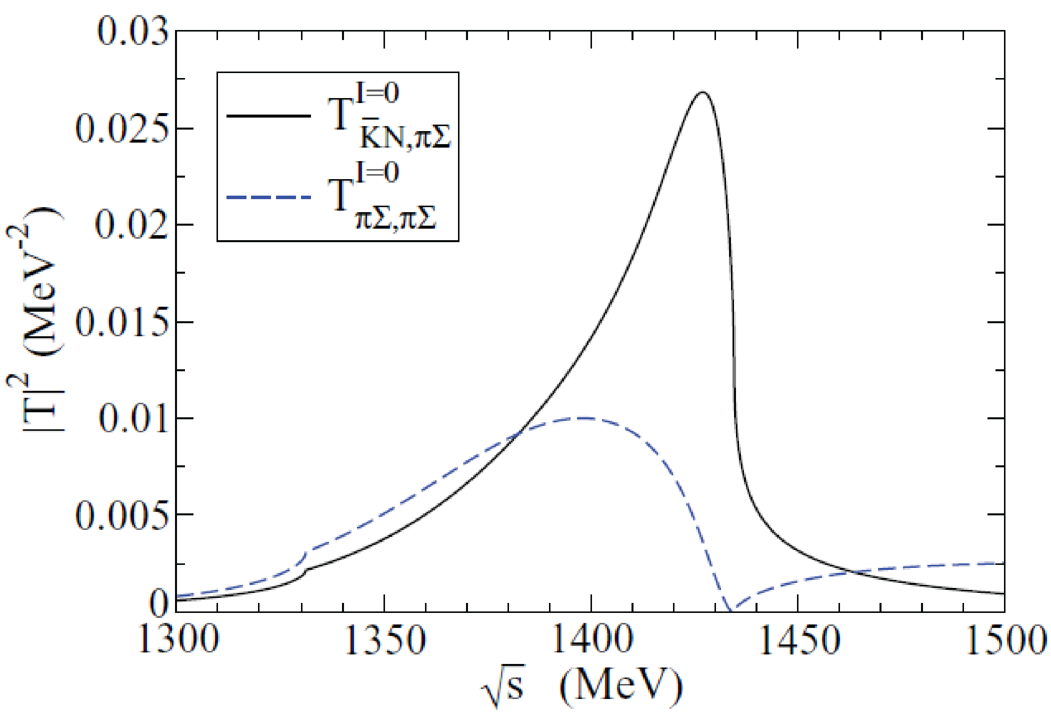
Description of the γ p -> K^+ $\pi\Sigma$ reaction

L.Roca and EO, PRC2013

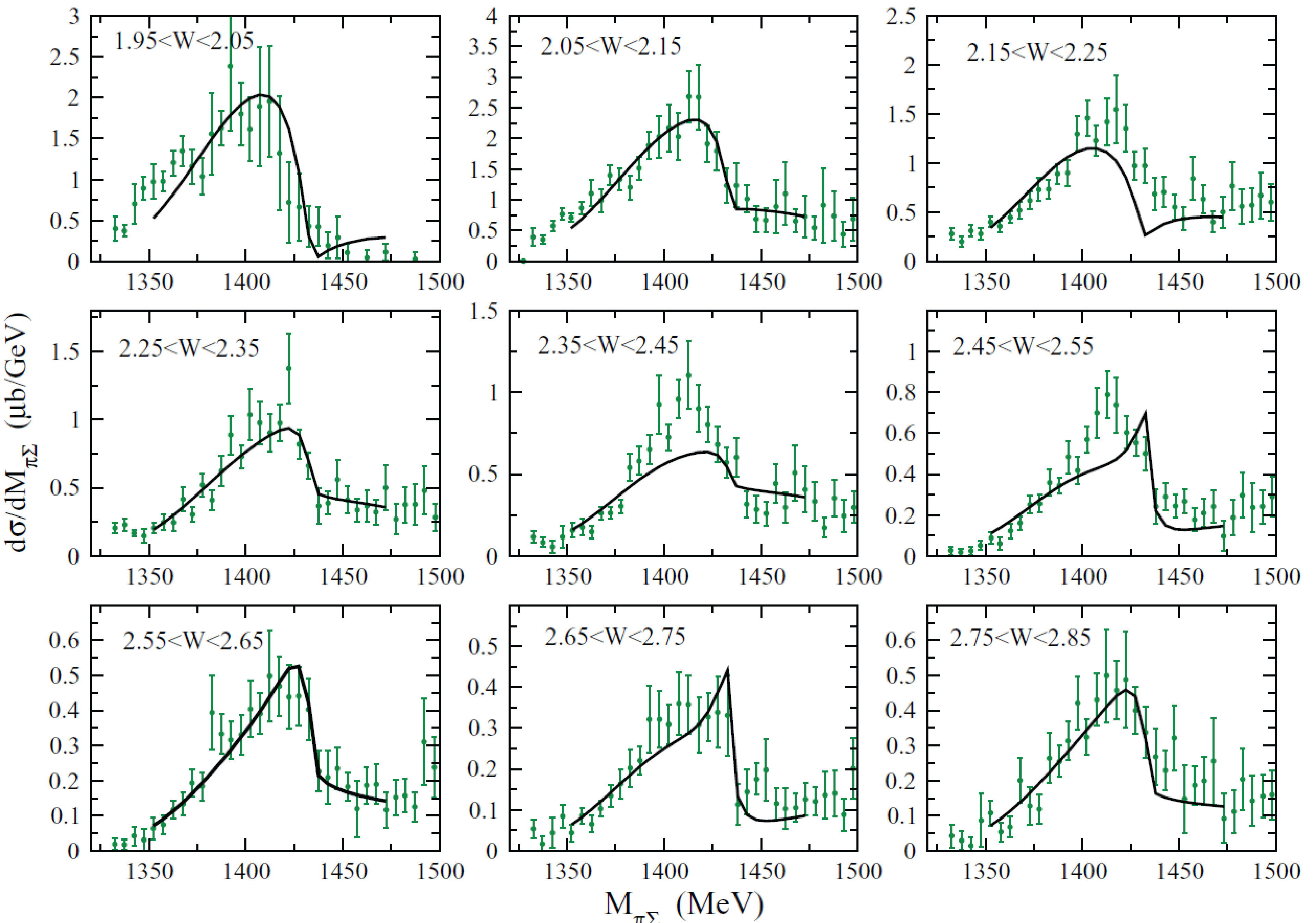


We look first at $\pi^0\Sigma^0$ which has I=0

$$t(W) = b(W) G_{\pi\Sigma} T^{I=0}_{\pi\Sigma,\pi\Sigma} + c(W) G_{\bar{K}N} T^{I=0}_{\bar{K}N,\pi\Sigma}$$



Results fitting the coeficients b(W) and C(W) to data of Moriya et. al on $\pi^0\Sigma^0$, PRC 2013



Improved fit

$$C_{ij} = \begin{pmatrix} 3\alpha_1 & -\sqrt{\frac{3}{2}}\alpha_2 \\ -\sqrt{\frac{3}{2}}\alpha_2 & 4\alpha_3 \end{pmatrix}$$

$$a_{KN} \rightarrow \alpha_4 a_{KN}\,, \; a_{\pi\Sigma} \rightarrow \alpha_5 a_{\pi\Sigma}$$

We allow the regulators a_i to vary a bit with respect to the original values

We respect unitarity in coupled channels . b(W) , c(W) are complex numbers for each energy, α_i are parameters of the potential

Why we improve over analysis of Moriya et al. ?

Moriya et al. $$t_I(m) = C_I(W) e^{i\Delta\phi_I} B_I(m)$$

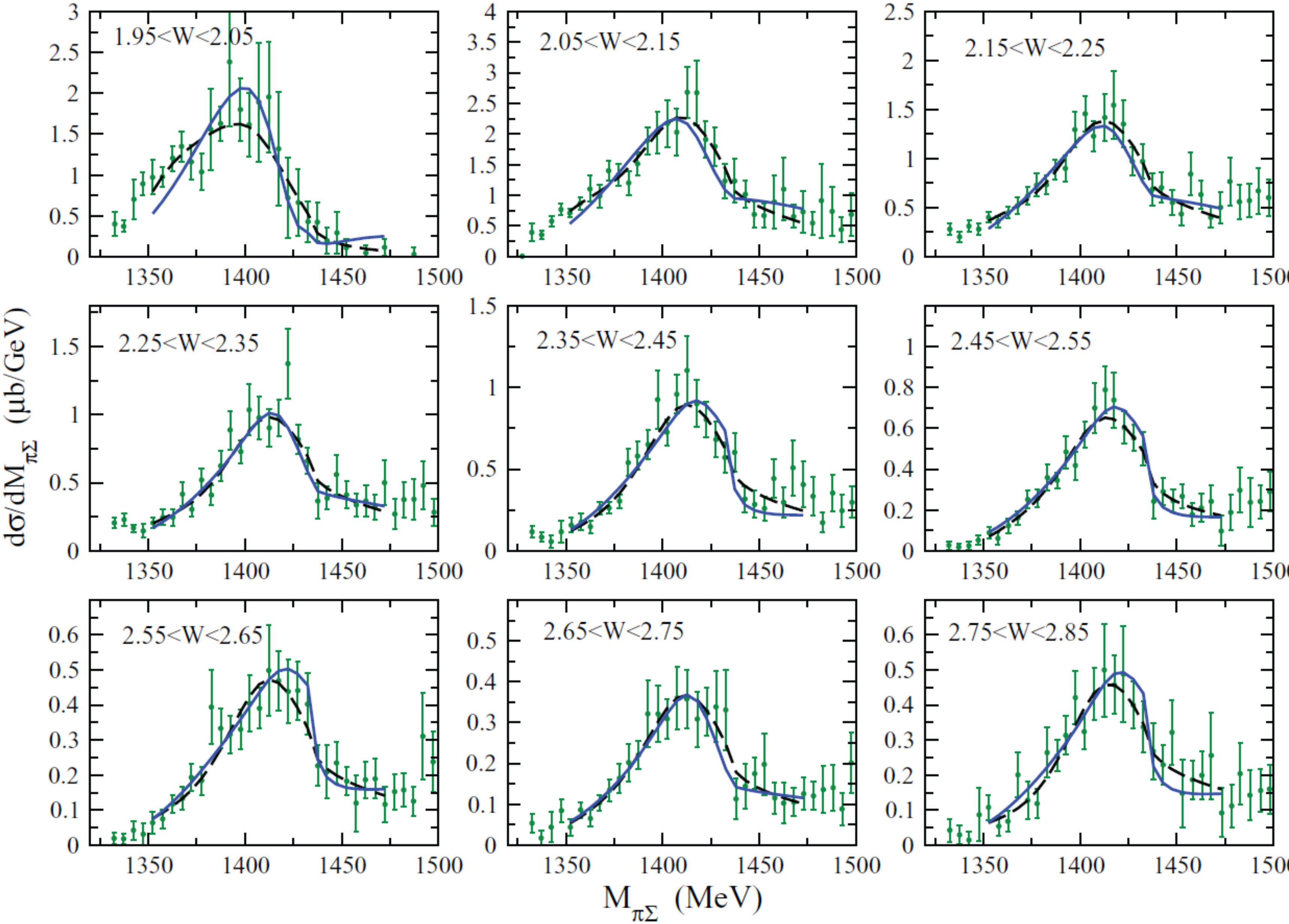
where $C_I(W)$ is a weight factor, $\Delta\phi_I$ a phase

$C_I(W)$ is real $\Delta\phi_I$ is energy independent

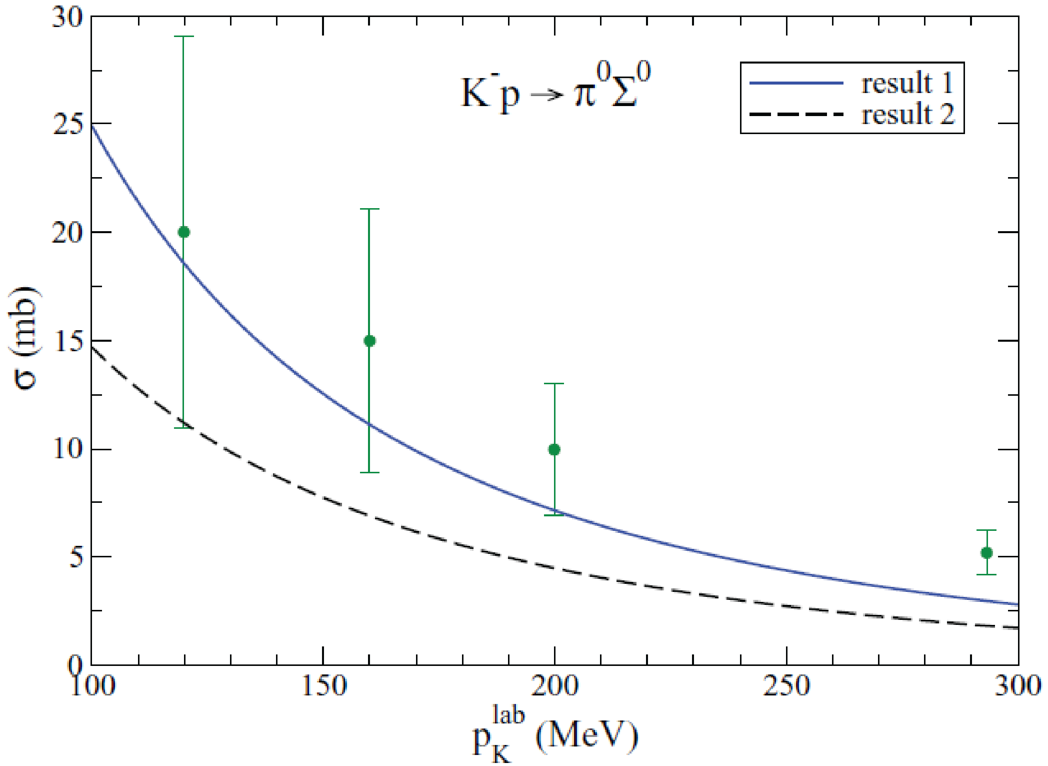
$B_I(m)$ a Breit-Wigner function

But amplitudes are not BW, and unitarity is lost when two BW amplitudes are summed

	α_1	α_2	α_3	α_4	α_5	$\Lambda(1405)$ poles (MeV)	
Solution 1	1.15	1.17	1.15	1.03	0.88	$1385-68i$	$1419-22i$
Solution 2	1.88	1.89	1.57	0.93	0.87	$1347-28i$	$1409-33i$

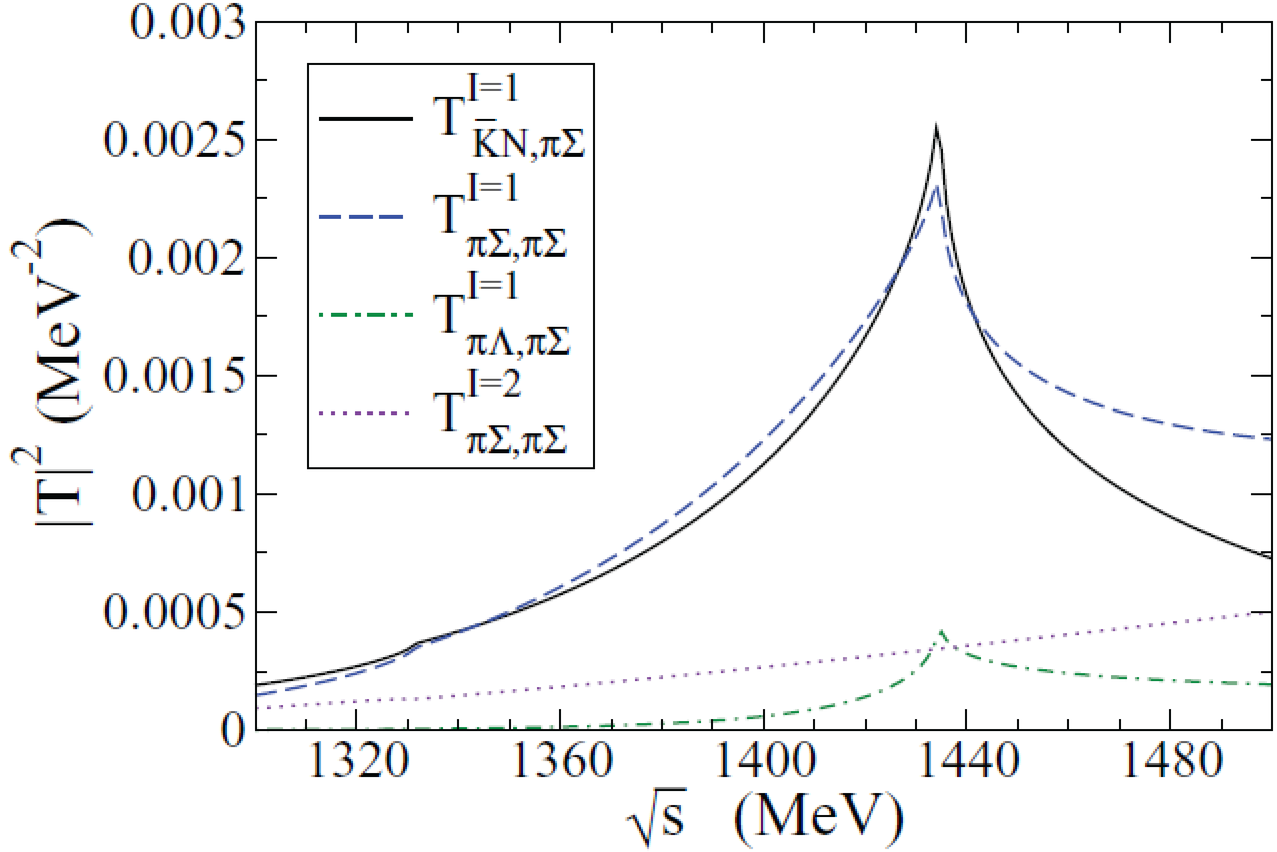


$\pi^0\Sigma^0$ are only in I=0



Solution 1 is favored

I=1 amplitudes in the standard approach to scattering data



Global fit to all photoproduction data

$$C^1_{ij} = \begin{pmatrix} 1 & -1 & -\sqrt{\frac{3}{2}} \\ -1 & 2 & 0 \\ -\sqrt{\frac{3}{2}} & 0 & 0 \end{pmatrix}$$

the order of the channels is $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$

$$|\pi^0\Sigma^0\rangle = \sqrt{\frac{2}{3}}|2\,0\rangle - \frac{1}{\sqrt{3}}|0\,0\rangle,$$

$$|\pi^+\Sigma^-\rangle = -\frac{1}{\sqrt{6}}|2\,0\rangle - \frac{1}{\sqrt{2}}|1\,0\rangle - \frac{1}{\sqrt{3}}|0\,0\rangle$$

$$|\pi^-\Sigma^+\rangle = -\frac{1}{\sqrt{6}}|2\,0\rangle + \frac{1}{\sqrt{2}}|1\,0\rangle - \frac{1}{\sqrt{3}}|0\,0\rangle$$

$$\begin{aligned} & t_{\gamma p\to K^+\pi^0\Sigma^0}(W) \\ & \quad = b_0(W) G^{I=0}_{\pi\Sigma} T^{I=0}_{\pi\Sigma,\pi\Sigma} + c_0(W) G^{I=0}_{\bar{K}N} T^{I=0}_{\bar{K}N,\pi\Sigma}, \\ & t_{\gamma p\to K^+\pi^\pm\Sigma^\mp}(W) \\ & \quad = b_0(W) G^{I=0}_{\pi\Sigma} T^{I=0}_{\pi\Sigma,\pi\Sigma} + c_0(W) G^{I=0}_{\bar{K}N} T^{I=0}_{\bar{K}N,\pi\Sigma} \\ & \qquad \pm\sqrt{\frac{3}{2}}\Big(b_1(W) G^{I=1}_{\pi\Sigma} T^{I=1}_{\pi\Sigma,\pi\Sigma} + c_1(W) G^{I=1}_{\bar{K}N} T^{I=1}_{\bar{K}N,\pi\Sigma} \\ & \qquad + d_1(W) G^{I=1}_{\pi\Lambda} T^{I=1}_{\pi\Lambda,\pi\Sigma}\Big), \end{aligned}$$

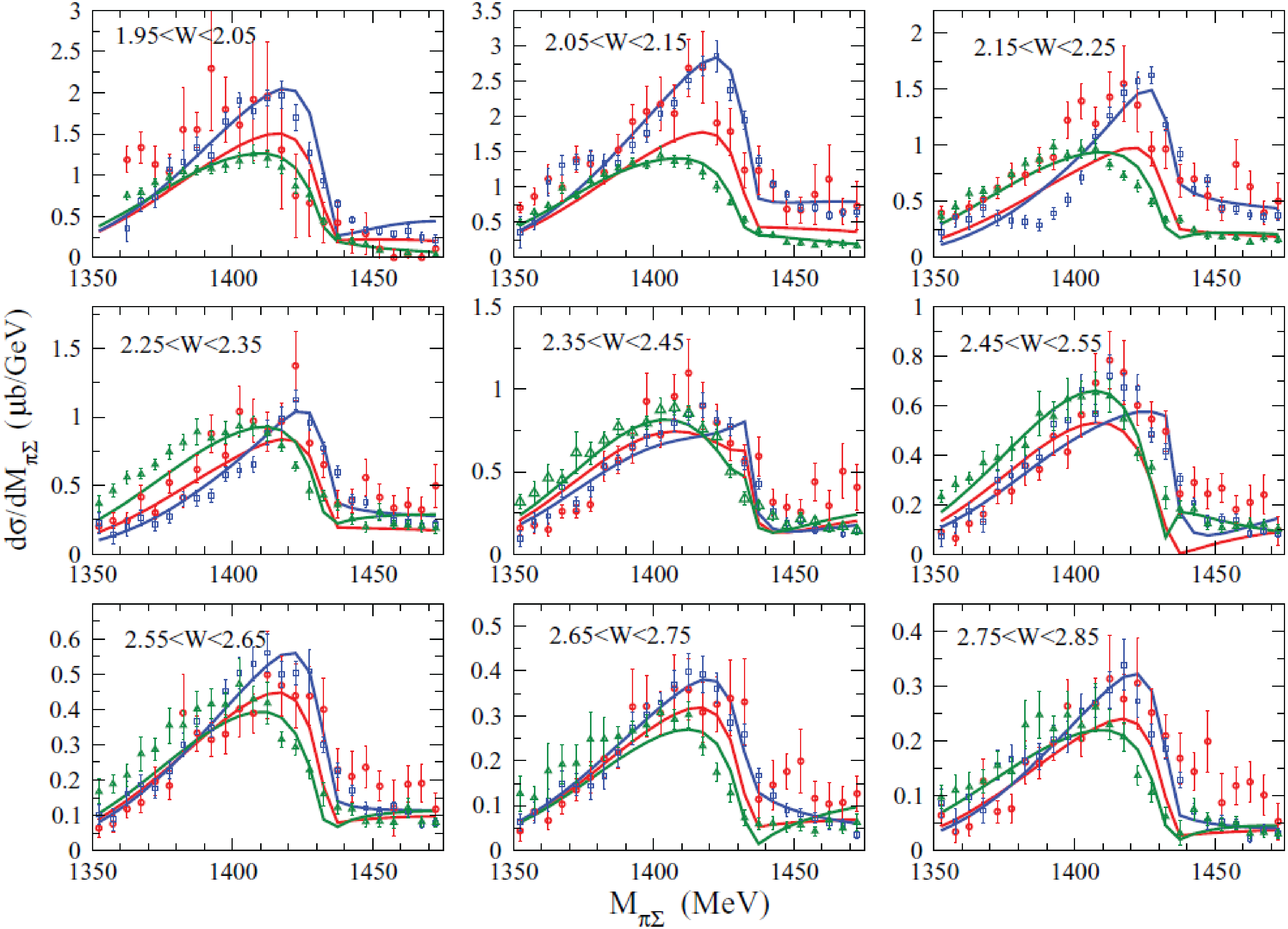


FIG. 2. (Color online) Fit to photoproduction data with fixed unitary amplitudes of $\alpha_i = 1$ and $\beta_i = 1$. Red: $\pi^0\Sigma^0$; blue: $\pi^-\Sigma^+$, green: $\pi^+\Sigma^-$. Experimental data are from Ref. [2].

$$C_{ij}^0 = \begin{pmatrix} 3\alpha_{11}^0 & -\sqrt{\frac{3}{2}}\alpha_{12}^0 \\ -\sqrt{\frac{3}{2}}\alpha_{12}^0 & 4\alpha_{22}^0 \end{pmatrix}$$

for isospin $I = 0$ and

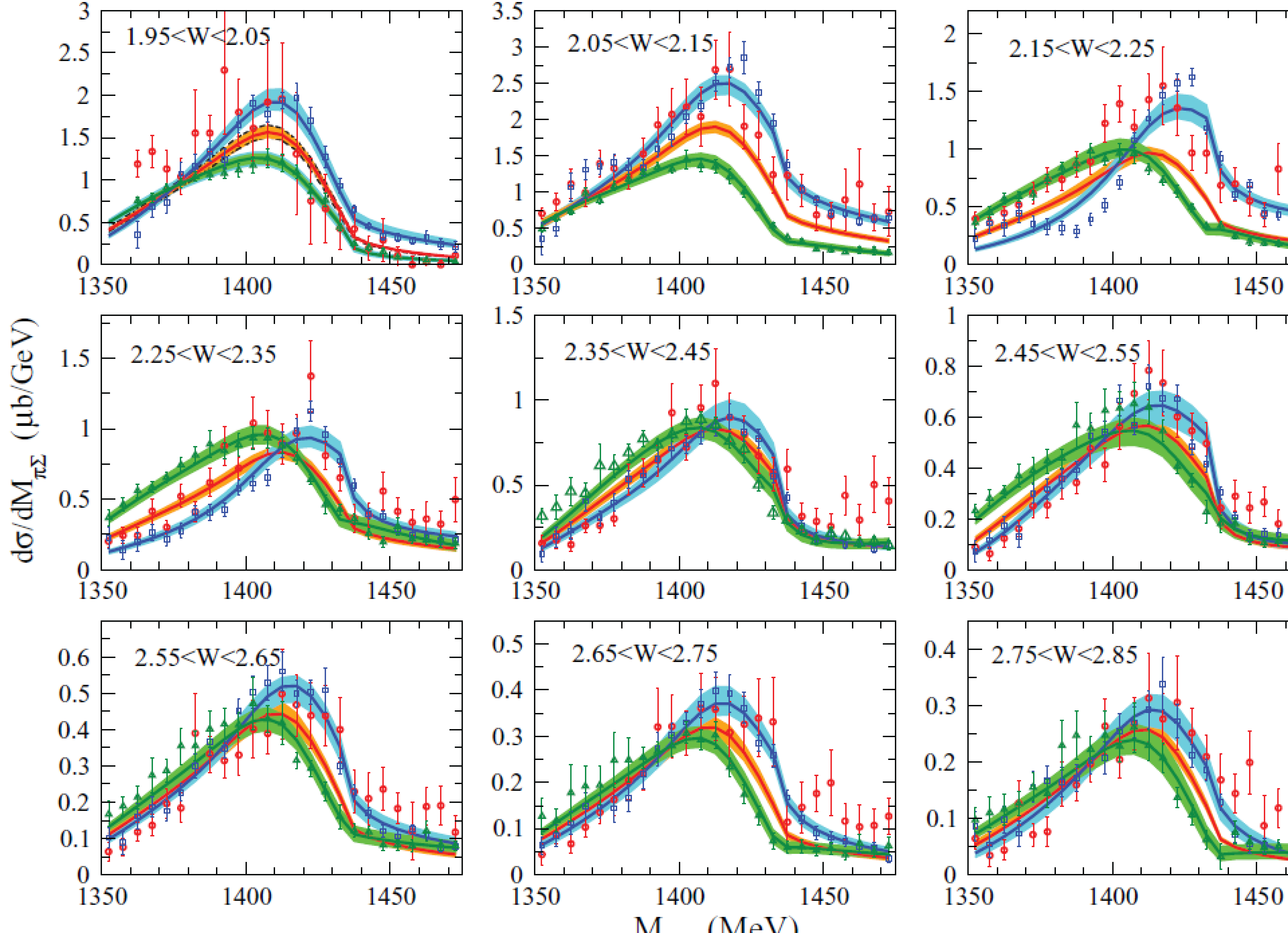
$$C_{ij}^1 = \begin{pmatrix} \alpha_{11}^1 & -\alpha_{12}^1 & -\sqrt{\frac{3}{2}}\alpha_{13}^1 \\ -\alpha_{12}^1 & 2\alpha_{22}^1 & 0 \\ -\sqrt{\frac{3}{2}}\alpha_{13}^1 & 0 & 0 \end{pmatrix}$$

for isospin $I = 1$.

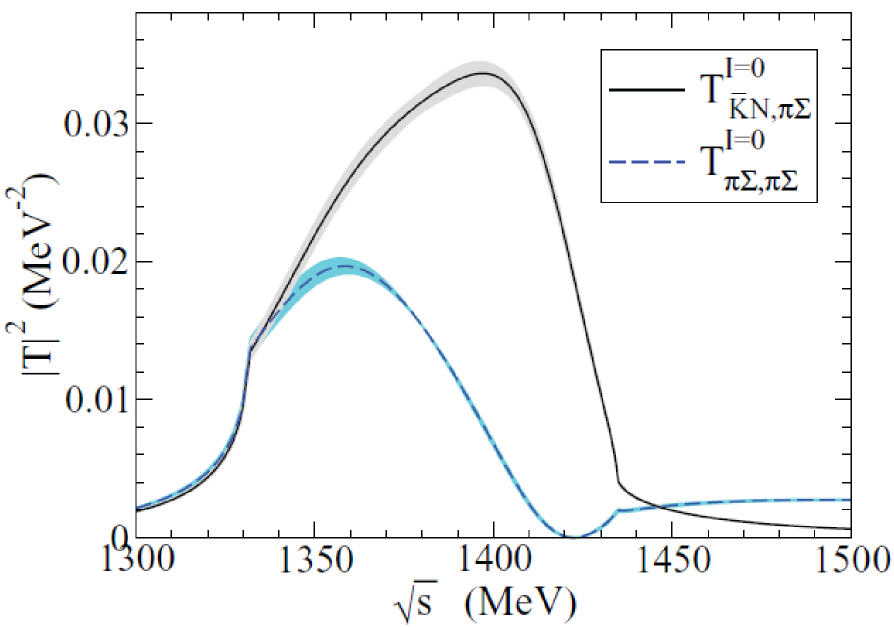
βi are the coefficients multiplying the standard subtraction contants

α_{11}^0	α_{12}^0	α_{22}^0	α_{11}^1	α_{12}^1	α_{13}^1	α_{22}^1	β_1	β_2	β_3
1.037	1.466	1.668	0.85	0.93	1.056	0.77	1.187	0.722	1.119

	$I = 0$		$I = 1$
Poles	$1352 - 48i$	$1419 - 29i$	–
$\lvert g_{\bar{K}N}\rvert$	2.71	3.06	–
$\lvert g_{\pi\Sigma}\rvert$	2.96	1.96	–

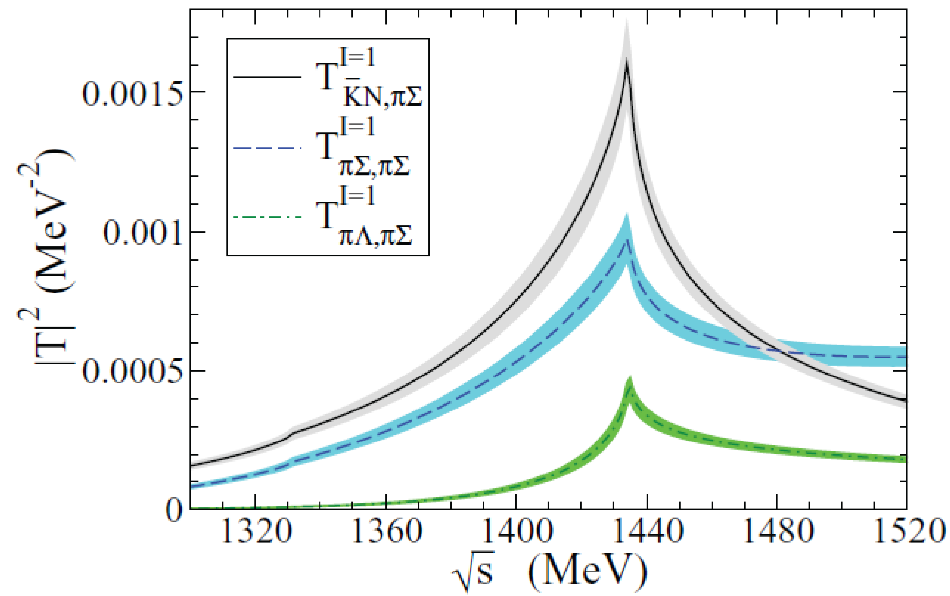


1.95<W<2.05
2.05<W<2.15
2.15<W<2.25
2.25<W<2.35
2.35<W<2.45
2.45<W<2.55
2.55<W<2.65
2.65<W<2.75
2.75<W<2.85
dσ/dM_πΣ (μb/GeV)
M_πΣ (MeV)



No poles for I=1 are found, but amplitudes ressemble much the shape of the $a_0(980)$ “resonance”.

The I=1 amplitude is only 1/4 of that of I=0, and essential to describe the photoproduction data.



Claim of I=1 state from analysis of $k^- p \to \Lambda \pi^+ \pi^-$

J. -J. Wu, S. Dulat and B. S. Zou, Phys. Rev. C **81** (2010) 045210.

P. Gao, J. -J. Wu and B. S. Zou, Phys. Rev. C **81**, 055203 (2010).

The $D_{s0}(2317)$ and $D_{s1}(2460)$ as KD and KD* molecules from lattice QCD

How to identify the nature of a resonance?

Imagine Meson-Baryon scattering in one channel and an interacting potential V, energy independent.

T= V/(1-VG) = 1/(V^{-1} –G)

$$G(s) = \int \frac{d^3\vec{q}}{(2\pi)^3} f(\vec{q}) \frac{\omega_P + \omega_B}{2\omega_P \omega_B} \frac{2M_B}{P^{02} - (\omega_P + \omega_B)^2 + i\varepsilon}, \tag{5}$$

Let T have a pole below threshold (bound state) at E_R

Close to a pole T= g^2/ (E-E_R), g^2 = lim (E-E_R) T = lim (E-E_R)/ (V^{-1} –G) = 1/ - ∂G/∂E

- g^2 ∂G/∂E = 1

Generalization to coupled channels
(Gamermann, Nieves, Ruiz Arriola, EO, PRD2010)

$$\sum_i g_i^2 \frac{dG_{ii}}{dE} \Big|_{E=E_\alpha} = -1$$

Each term in the sum represents the probability to have each channel in the wave function

Coupled chanels

$T=(1-VG)^{-1}V$, Take two channels and assume $V_{22}=0$ for simplicity.
Assume V energy independent.

$$T_{11} = \frac{v_{11} + v_{12}^2 G_2}{1 - (v_{11} + v_{12}^2 G_2) G_1}$$

We can eliminate channel 2 by means of V_{eff}, but now V_{eff} is energy dependent

in one channel with $V_{\rm eff}$, we will have

$$T_{\rm eff} = \frac{V_{\rm eff}}{1 - V_{\rm eff} G_1}. \qquad V_{\rm eff} = v_{11} + v_{12}^2 G_2$$

$$-\sum_{i,j} g_i g_j \left[\frac{\partial G_i^{II}(E)}{\partial E} \delta_{ij} + G_i^{II}(E) \frac{\partial V_{ij}(E)}{\partial E} G_j^{II}(E) \right]_{E \to E_R} = 1$$

T. Hyodo, Int J Mod Phys A 2013

Probability of meson-baryon

Probability of genuine component or missing channels

Reanalysis of lattice QCD spectra leading to the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$

A. Martínez Torres,[1] A. Ramos,[2] E. Oset,[3] and S. Prelovsek[4] arXiv 1412.1706

TABLE I. Energy levels found in Ref. [1] S. Prelosvsek et al 2014

	KD interpolator	KD^* interpolator
E_1 (MeV)	2086 (34)	2232 (33)
E_2 (MeV)	2218 (33)	2349 (34)
E_3 (MeV)	2419 (36)	2528 (53)

$$\tilde{T} = \frac{1}{V^{-1} - \tilde{G}}$$

$$V = \alpha + \beta(s - s_0),$$

$$\tilde{G} = G + \lim_{q_{\max}\to\infty}\left[\frac{1}{L^3}\sum_{q_i}^{q_{\max}} I(\vec{q}_i) - \int_{q<q_{\max}} \frac{d^3q}{(2\pi)^3} I(\vec{q})\right]$$

$$I(\vec{q}) = \frac{\omega_1(\vec{q}) + \omega_2(\vec{q})}{2\omega_1(\vec{q})\omega_2(\vec{q})\left[P^2 - (\omega_1(\vec{q}) + \omega_2(\vec{q}))^2 + i\epsilon\right]}$$

$$T = \frac{1}{V^{-1} - G}$$

$$B(KD) = 31 \pm 17 \text{ MeV}, \qquad B(KD^*) = 32 \pm 20 \text{ MeV}$$

$$a = -1.4 \pm 0.6 \text{ fm}, \quad r_0 = -0.1 \pm 0.2 \text{ fm for } KD,$$

$$a = -1.2 \pm 0.5 \text{ fm}, \quad r_0 = -0.5 \pm 0.5 \text{ fm for } KD^*$$

$$P(KD) = 72 \pm 12\ \%, \text{ for the } D^*_{s0}(2317)$$

$$P(KD^*) = 63 \pm 16\ \%, \text{ for the } D_{s1}(2460)$$

Luescher formula: In the box $\tilde{T} = \dfrac{1}{V^{-1} - \tilde{G}}$

If we have an eigenenergy of the box → $V^{-1}(E) - \tilde{G}(E) = 0$

In the continuum $T = \dfrac{1}{V^{-1} - G}$

$$T(E) = \left(V^{-1}(E) - G(E)\right)^{-1} = \left(\tilde{G}(E) - G(E)\right)^{-1}$$

Luescher provides phase shifts in the continuum only for the eigenenergies of the box
The previous method gives phase shifts at all energies and bound states if they exist.

Conclusions:

The data on π Σ photoproduction demonstrate the existence of two poles for the Λ(1405) one around 1420 MeV, the other one around 1355-1385 MeV. There is a strong cusp (virtual state) for I=1 around the Kbar p threshold.

The reanalysis of the lattice QCD data on KD and KD* scattering concludes that there are bound states of KD and KD*, which can be associated to the $D_{s0}(2317)$ and $D_{s1}(2460)$ resonances.

The application of a generalization of the Weinberg compositeness condition to the lattice results allows us to say that these resonances are mostly molecules of KD and KD*, with about 30% weight for other components.