$^{6}_{\Lambda\Lambda}$*He in Cluster EFT*

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- Discussion

Introduction

- Light double Λ hyper nuclei, ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, ${}^{11}_{\Lambda\Lambda}$Be, ...
- Λ-Λ interactions, H-dibaryon, Lattice QCD simulations
- Scattering length $a_{\Lambda\Lambda}$: from ^{12}C$(K^-, K^+\Lambda\Lambda X)$ data,

$$a_{\Lambda\Lambda} = -1.2 \pm 0.6\,\text{fm}\,,$$

[Gasparyan et al. PRC85(2012)], and from heavy ion collisions,

$$a_{\Lambda\Lambda} \leq -1.25\,\text{fm}\,,$$

[Ohnishi et al. NPA914(2013)377], the $\Lambda\Lambda$ model potentials

$$a_{\Lambda\Lambda} \simeq -0.27 \sim -3.8\,\text{fm}\,, \quad r_{\Lambda\Lambda} \simeq 0.34 \sim 15.0\,\text{fm}\,.$$

${}^{\;\;6}_{\Lambda\Lambda}$*He: References*

- Experiment:
  - Nagara event, $B_{\Lambda\Lambda} = 6.93 \pm 0.16$ MeV.

    [PRL87(2001)212502,NPA835(2010)207,PRC88(2013)014003]
- Theory:
  - Old works: e.g., Dalitz, Rajasekaran, NP50(1964)450, Tang, Herndon, Schmid, PL10(1964)450, and so on.
  - $\Lambda\Lambda$-$N\Sigma$ mixing due to $\Delta_M \sim 20$ MeV.
  - Three-body ΛNN force, Bodmer *et al.*, PRC29(1984)684.
  - Input for the exotic hyper-nuclear search

Cluster EFT

- Effective Field Theories
  - Model independent approach
  - Separation scale
  - Counting rules
  - Parameters should be fixed by experiments

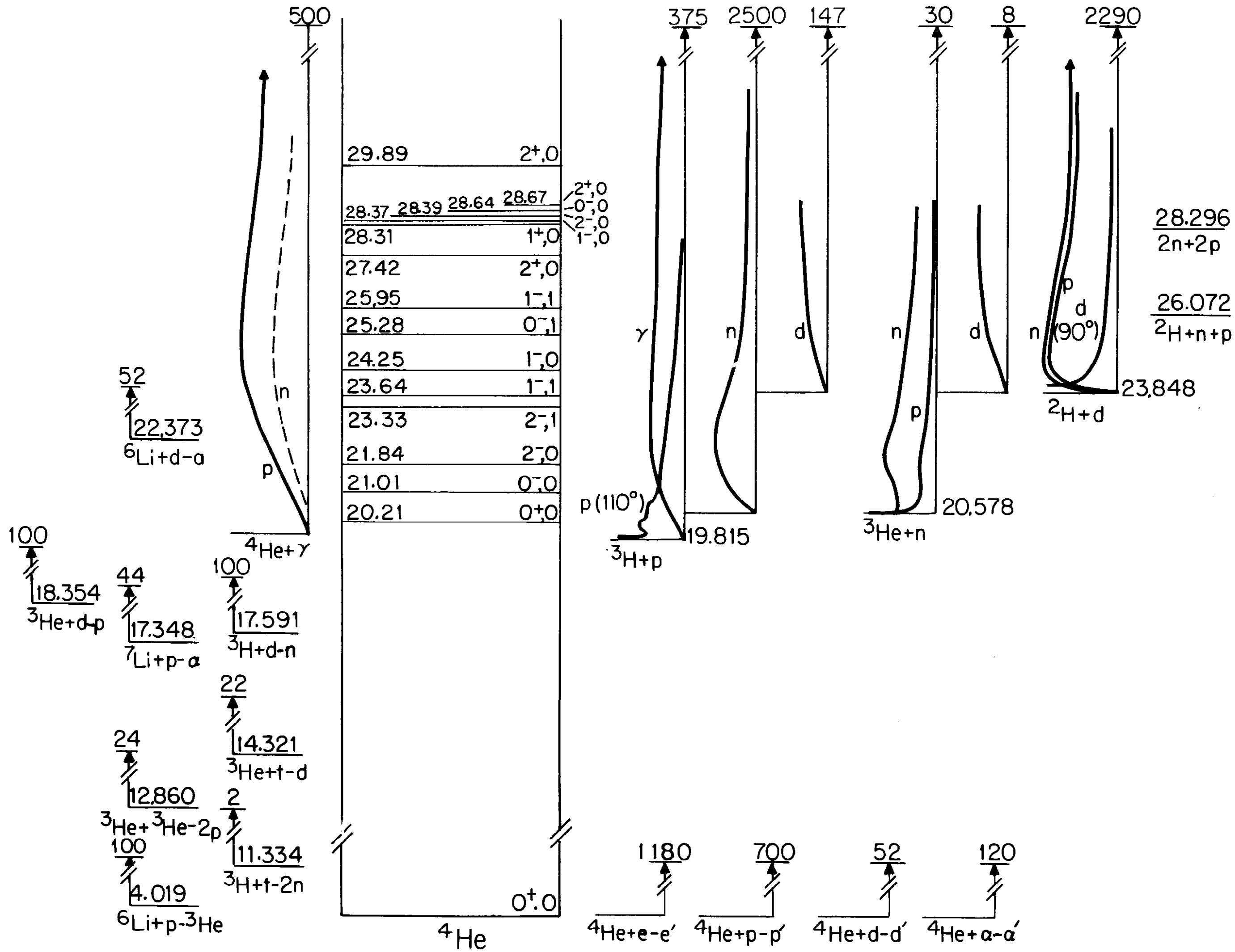
RG analysis and limit cycle

- RG analysis in EFTs

  The effective Lagrangian are expanded in terms of the number of derivatives in EFTs. However, the interaction can become singular, and thus RG analysis is a useful tool to determine the counting rules of the singular interactions.

- Limit cycle
  - "Limit cycle" suggested by Wilson
  - "Efimov states" in the unitary limit
  - "Determination equation" of the limit cycle obtained by Danilov

Alpha cluster in EFT



${}^{6}_{\Lambda\Lambda}$He in Cluster EFT

- ${}^{6}_{\Lambda\Lambda}$He in Cluster EFT
  - We choose the energy of breakup channel (p^3H or n^3He) of ^{4}He as a large energy scale, $B_H \simeq$ 20MeV
  - Thus the $\Lambda\Lambda$-$N\Sigma$ mixing becomes irrelevant because of $B_H \simeq \Delta_M$.
  - The typical energy scale is $B_\Lambda \simeq$ 3MeV from ${}^{5}_{\Lambda}$He.
  - ${}^{6}_{\Lambda\Lambda}$He as $\Lambda\Lambda\alpha$ cluster system in Cluster EFT at LO.

Specifications

- The large momentum scale, $\Lambda_H = \sqrt{2\mu B_H}$ ∼170MeV, whereas the typical scale, $\gamma_{\Lambda\alpha} = \sqrt{2\mu_{\Lambda\alpha} B_\Lambda}$ ≃73MeV. The expansion parameter $\gamma_{\Lambda\alpha}/\Lambda_H \sim 0.43$.
- We consider S-waves, spin singlet channel, and leading order terms only.
- We find a result sensitive to the momentum cutoff Λ_c, three-body interaction $g_1(\Lambda_c)$ is introduced, and it is described by four parameters, $\gamma_{\Lambda\alpha}$, $a_{\Lambda\Lambda}$, $g_1(\Lambda_c)$, Λ_c, at LO.

Calculation

- Lagrangian

$$\mathcal{L} = \mathcal{L}_\Lambda + \mathcal{L}_\alpha + \mathcal{L}_s + \mathcal{L}_t + \mathcal{L}_{\Lambda t} \,,$$

$$\mathcal{L}_\Lambda = \mathcal{B}_\Lambda^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2m_\Lambda} \right] \mathcal{B}_\Lambda + \cdots \,,$$

$$\mathcal{L}_\alpha = \phi_\alpha^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2m_\alpha} \right] \phi_\alpha + \cdots \,,$$

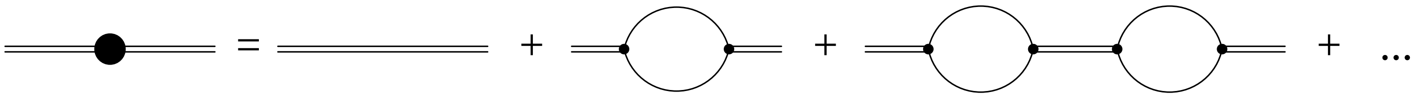
$$\mathcal{L}_s = \sigma_s s^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{4m_\Lambda} + \Delta_s \right] s - y_s \left[s^\dagger \left(\mathcal{B}_\Lambda^T P^{(^1S_0)} \mathcal{B}_\Lambda \right) + \text{H.c.} \right] + \cdots \,,$$

$$\mathcal{L}_t = \sigma_t t^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2 - \partial^2}{2(m_\alpha + m_\Lambda)} + \Delta_t \right] t - y_t \left[t^\dagger \mathcal{B}_\Lambda \phi_\alpha + \text{H.c.} \right] + \cdots \,,$$

$$\mathcal{L}_{\Lambda t} = -2m_\alpha y_t^2 \frac{g_1(\Lambda_c)}{\Lambda_c^2} \left(\mathcal{B}_\Lambda^T P^{(^1S_0)} t \right)^\dagger \left(\mathcal{B}_\Lambda^T P^{(^1S_0)} t \right) + \cdots \,,$$

Two-body part: $\Lambda\Lambda$ *in* $^1S_0$ *state*

- Dressed dibaryon propagator



- Renormalized dressed dibaryon propagator

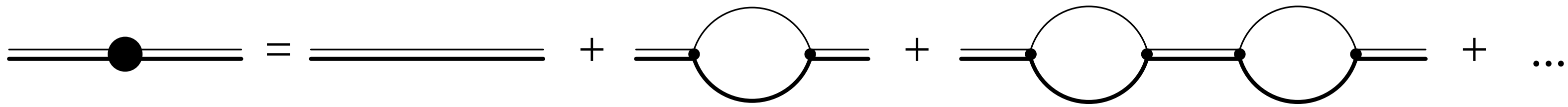
$$D_s(p_0,\vec{p}) = \frac{4\pi}{m_\Lambda y_s^2}\frac{1}{\frac{1}{a_{\Lambda\Lambda}} - \sqrt{-m_\Lambda p_0 + \frac{1}{4}\vec{p}^2 - i\epsilon}},$$

and

$$a_{\Lambda\Lambda} = -1.2 \pm 0.6 \text{ fm}.$$

Two-body part: $\Lambda\alpha$ ***in*** $^{5}_{\Lambda}$***He channel***

- Dressed $^{5}_{\Lambda}$He propagator



- Renormalized dressed $^{5}_{\Lambda}$He propagator

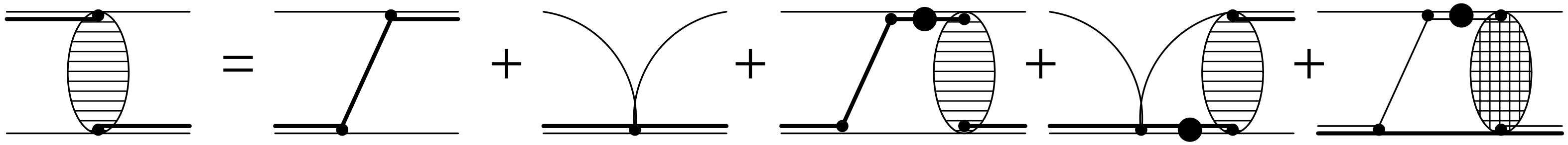
$$D_t(p_0,\vec{p}) = \frac{2\pi}{\mu_{\Lambda\alpha}y_t^2}\frac{1}{\gamma_{\Lambda\alpha} - \sqrt{-2\mu_{\Lambda\alpha}\left(p_0 - \frac{1}{2(m_\Lambda+m_\alpha)}\vec{p}^2\right)}}\,,$$

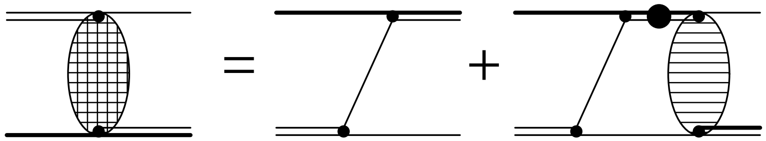
with

$$\gamma_{\gamma\alpha} = \sqrt{2\mu_{\Lambda\alpha}B_\Lambda} \simeq 73\ \text{MeV}\,.$$

Three-body part:

- S-wave Λ-${}^5_\Lambda$He scattering





$$
\begin{aligned}
a(p,k;E) &= K_{(a)}(p,k;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2} \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 \left[K_{(a)}(p,l;E) - \frac{g_1(\Lambda_c)}{\Lambda_c^2}\right] D_t^{LO}\left(E - \frac{1}{2m_\Lambda}l^2, \vec{l}\right) a(l,k;E) \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 K_{(b1)}(p,l;E) D_s^{LO}\left(E - \frac{1}{2m_\alpha}l^2, \vec{l}\right) b(l,k;E)\,, \\
b(p,k;E) &= K_{(b2)}(p,k;E) \\
&\quad - \frac{1}{2\pi^2}\int_0^{\Lambda_c} dl l^2 K_{(b2)}(p,l;E) D_t^{LO}\left(E - \frac{1}{2m_\Lambda}l^2, \vec{l}\right) a(l,k;E)\,,
\end{aligned}
$$

The determination equation

- Scale invariant of integral equations in asymptotic limit, where $a(p) \propto p^{-1-s}$.

$$
\begin{aligned}
1 \quad = \quad & \frac{1}{2\pi}\frac{m_\alpha}{\mu_{\Lambda\alpha}}\sqrt{\frac{\mu_{\Lambda(\Lambda\alpha)}}{\mu_{\Lambda\alpha}}}\int_0^\infty dx \ln\left(\frac{x^2+1+ax}{x^2+1-ax}\right)x^{s-1} \\
& + \frac{\sqrt{2}}{\pi^2}\frac{\sqrt{m_\Lambda \mu_{\Lambda(\Lambda\alpha)}\mu_{\alpha(\Lambda\Lambda)}}}{\mu_{\Lambda\alpha}^{3/2}}\int_0^\infty dx \ln\left(\frac{1+bx^2+x}{1+bx^2-x}\right)x^{s-1} \\
& \times \int_0^\infty dy \ln\left(\frac{b+y^2+y}{b+y^2-y}\right)y^{s-1},
\end{aligned}
$$

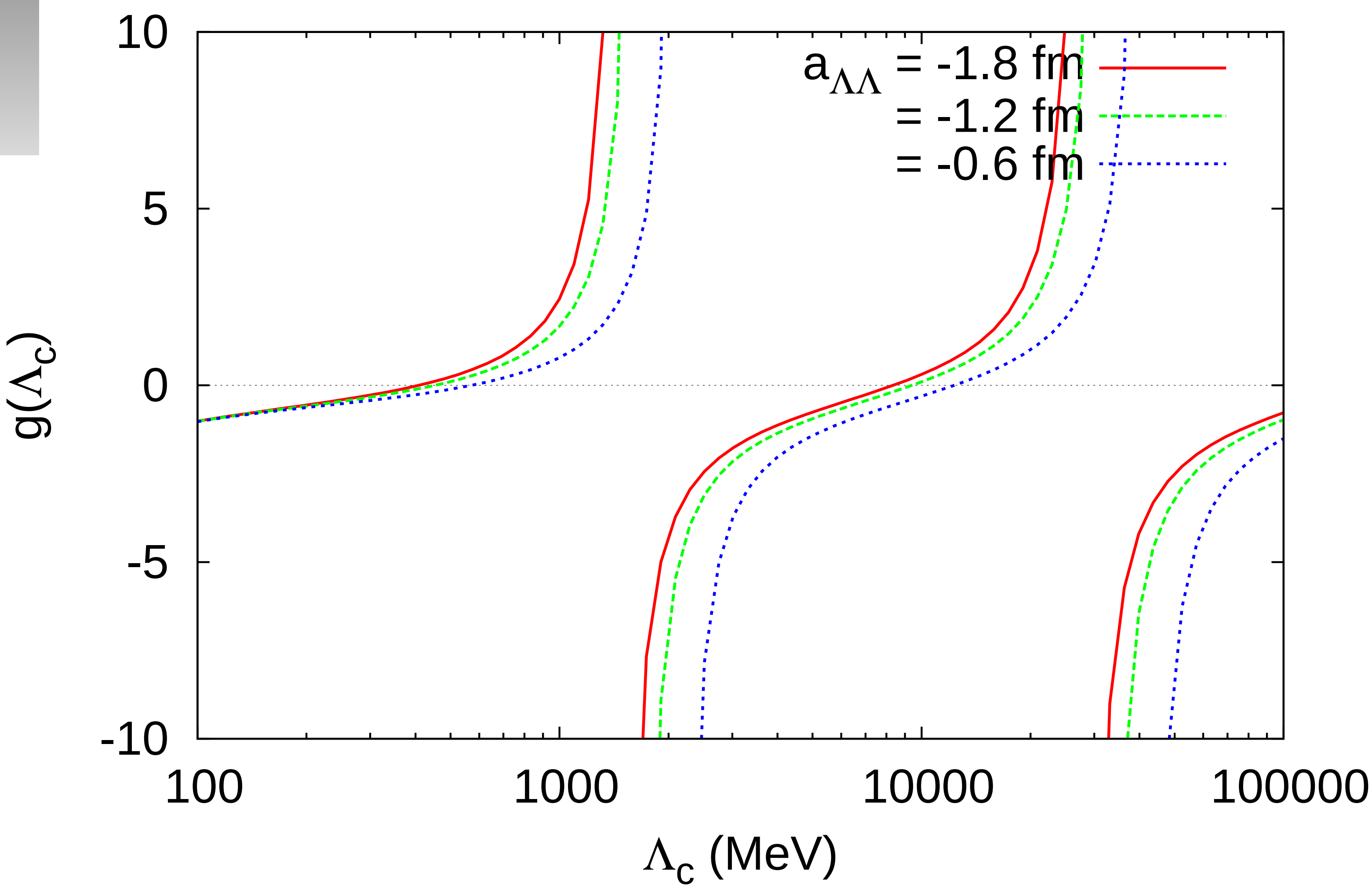
where $a = 2\mu_{\Lambda\alpha}/m_\alpha$ and $b = m_\Lambda/(2\mu_{\Lambda\alpha})$. The integrals above are known as Mellin transformation.

- We have an imaginary solution of s as $s = \pm i s_0$ where

$$s_0 = 1.04968\cdots.$$

Numerical results:

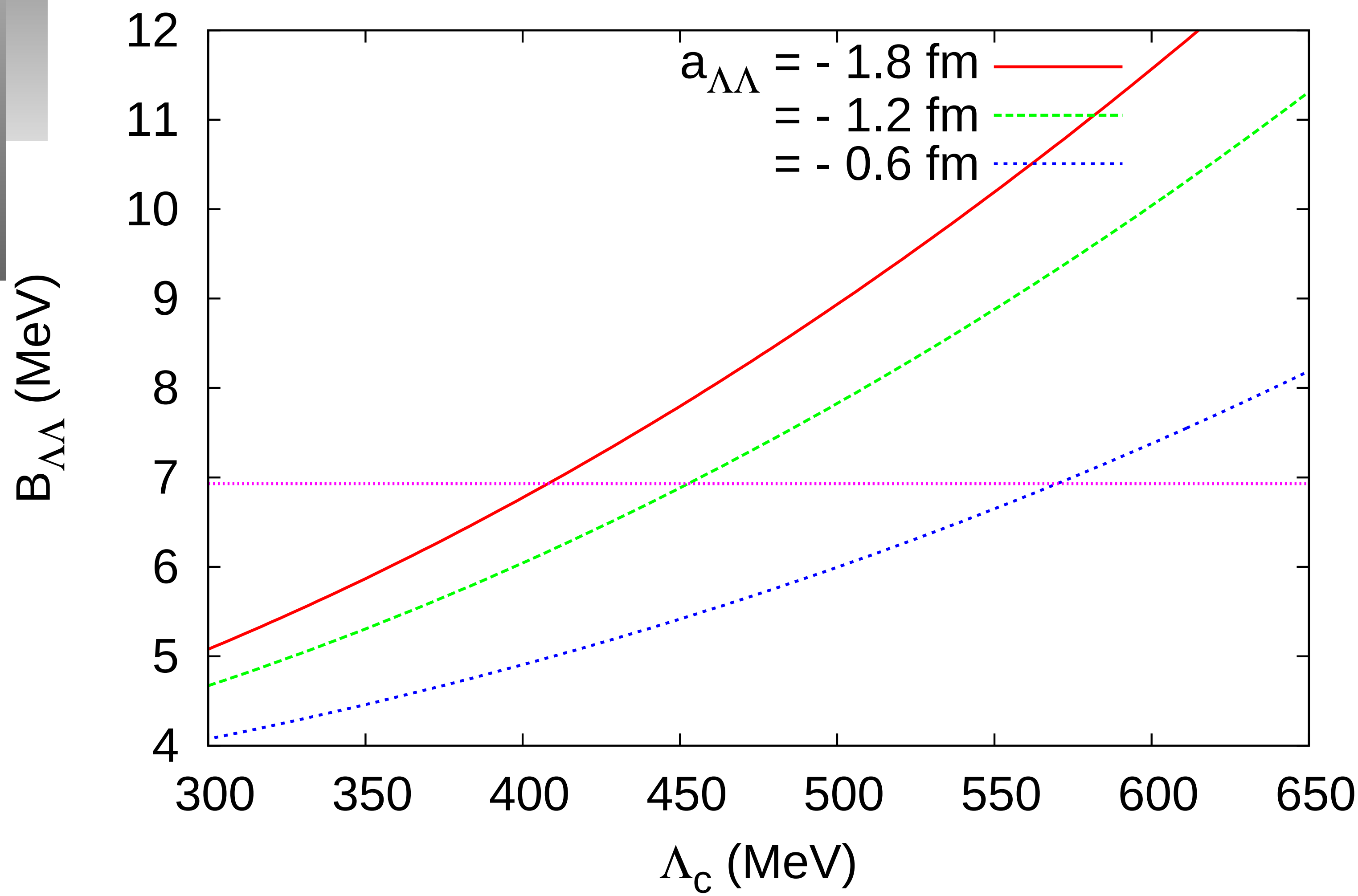
- With $g_1(\Lambda_c)$ (Input: $B_{\Lambda\Lambda}$ =6.93MeV)



$$\Lambda_n = \Lambda_0 \exp(n\pi/s_0)\,, \quad s_0 \simeq 1.05\,.$$

Numerical results:

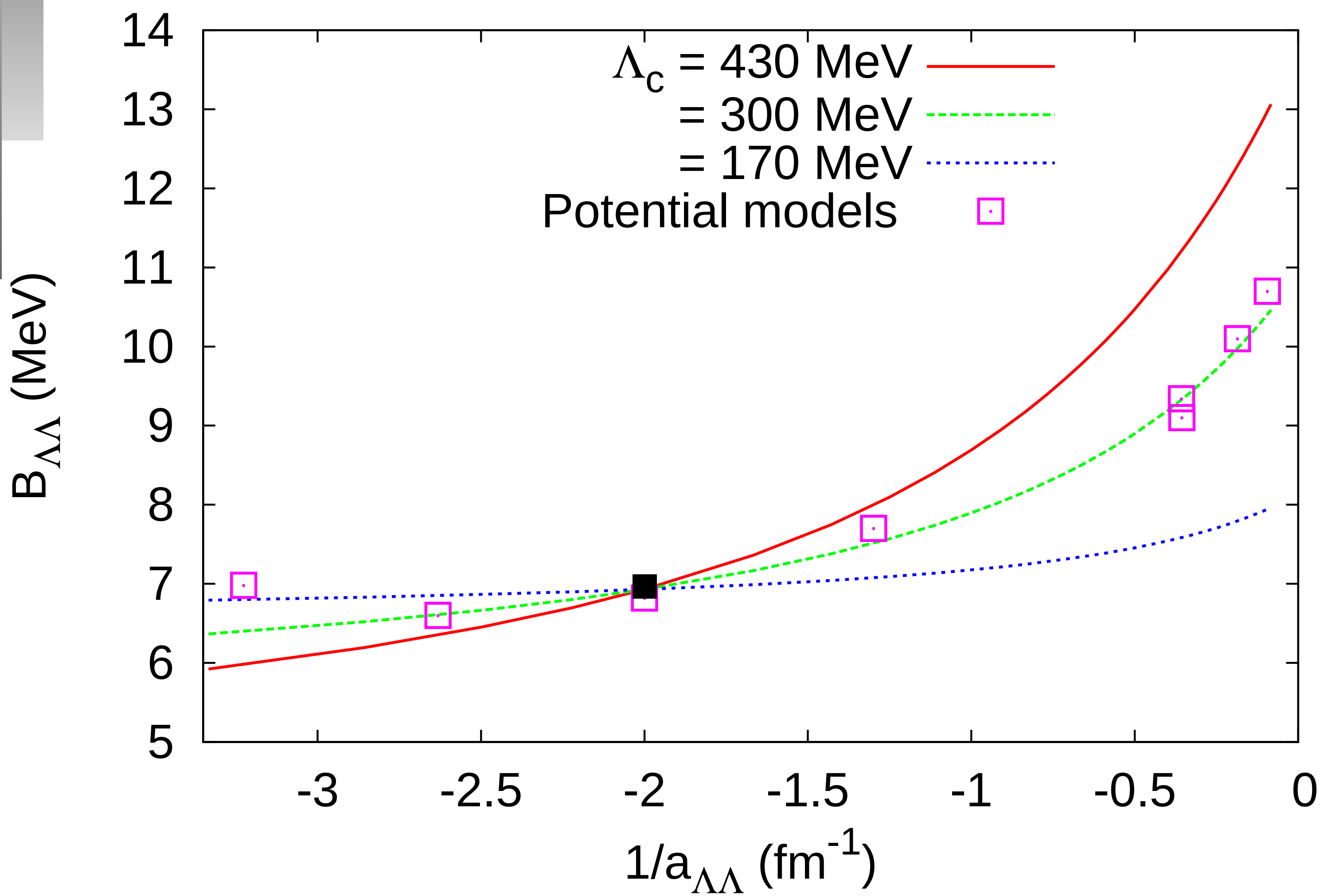
- Without $g_1(\Lambda_c)$



- $r_c = \Lambda_c^{-1} \simeq 0.35$ to 0.5 fm

Numerical results:

- With $g_1(\Lambda_c)$ (Input: $B_{\Lambda\Lambda}$ =6.93MeV, $a_{\Lambda\Lambda} = -0.5$fm)



Results and discussion

- ${}^{6}_{\Lambda\Lambda}$He, as $\Lambda\Lambda\alpha$ system, in Cluster EFT at LO is studied.
- Our result shows the limit cycle, sensitive to the cutoff, and the three-body contact interaction should be introduced at LO.
- The determination equation works well. Its imaginary solution implies present of a bound state. Thus it could be a useful tool to search for an exotic state in other three-body systems.
- The results of the $B_{\Lambda\Lambda}$-$a_{\Lambda\Lambda}$ correlation from the model potentials are well reproduced in this work.