Shapes and energy spectra of hypernuclei in the Skyrme Hartree-Fock method

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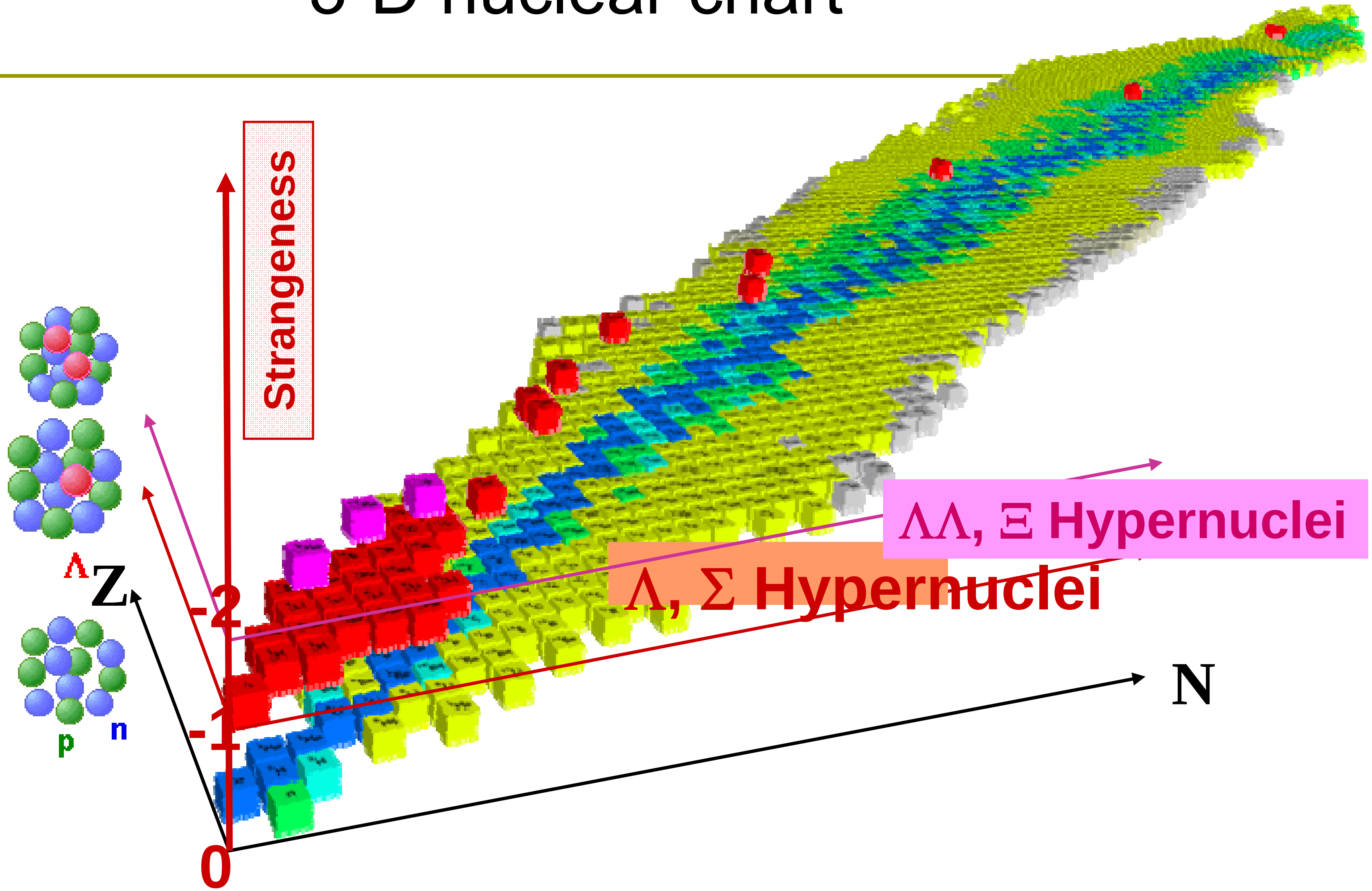
Outline

- Introduction
- Extended Skyrme Hartree-Fock
- Properties of hypernuclei
  - shapes of hypernuclei
  - enegery spetra
- Summary

3-D nuclear chart



Why to study hypernuclei?

Nucleon-nucleon interaction

Hyperon-nucleon interaction

Impurity of nuclear system

Multistrange system: Neutron star, …

Theoretical studies about hypernuclei

Energy spectrum

Decay properties

The effect of hyperon(s)

Shapes

....

Theoretical studies about the shapes of hypernuclei

Studies based on spherical symmetry:

1. Relativistic mean-field model (RMF)
2. Skyrme Hartree-Fock model (SHF)
3. Woods-Saxon potential + YN interaction
4. Few-body theory

Calculations considering deformation:

Deformed HF with nonrealistic YN interaction:

T. H. Ho and A.Volkov, Phys. Lett. B30, 303, 1969.

W. H. Bassichis, A. Gal, Phys. Rev. C1, 28, 1970.

J. Zofka, Czech, J. Phys. B30, 95, 1980.

Nilsson Model:

assume the same deformation for core and hypernuclei:

K. Hagino, Phys. Rev. C63, 044318, 2001

Considering deformation self-consistently:

Deformed SHF with Microscopic YN int.

X.-R. Zhou, H.-J. Schulze, H. Sagawa,et al., Phys. Rev. C76, 034312(2007)

Relativistic mean-field model (RMF):

Myaing Thi Win, H. Hagino, et al., Phys. Rev. C 78, 054311 (2008)

Triaxial SHF with Skyrme-like YN interaction:

Myaing Thi Win, H. Hagino, et al., Phys. Rev. C **83**, 014301 (2011)

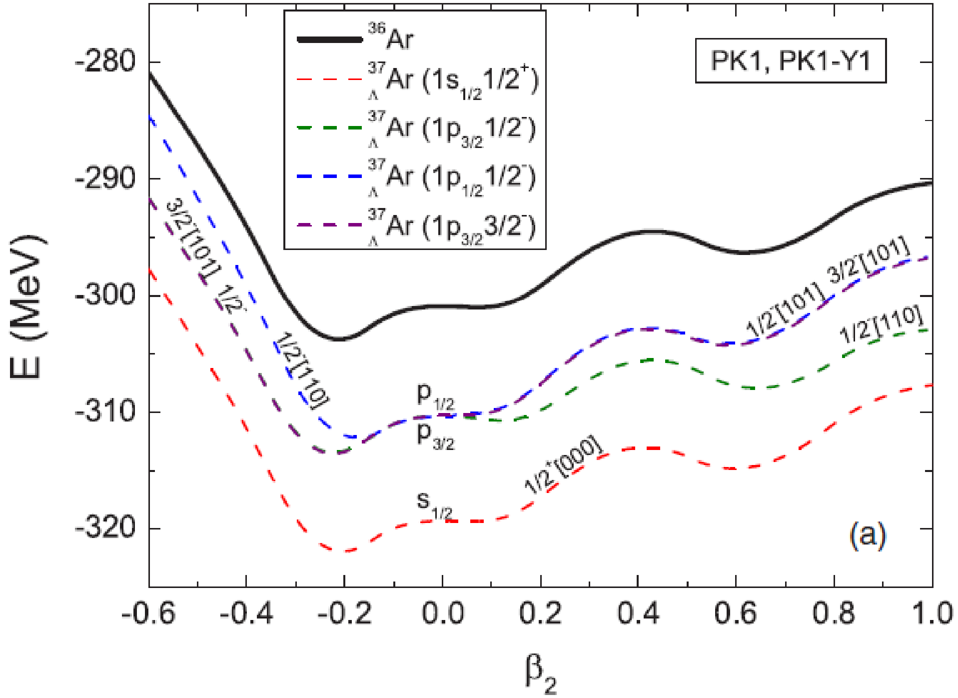
Antisymmetrized molecular dynamics (AMD):

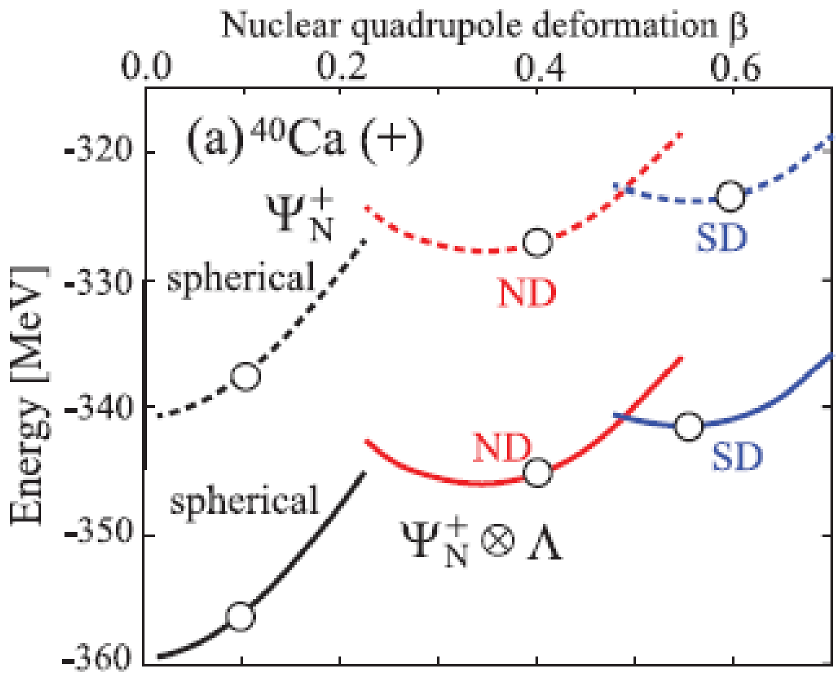
M. Isaka,, et al., Phys. Rev. C **83**, 044323 (2011)

Triaxial RMF:

Bing-Nan Lu (吕炳楠), S.-G.Zhou, Phys. Rev. C **84**, 014328 (2011)

Superdeformation in hypernuclei





Bing-Nan Lu, E. Hiyama, H. Sagawa, and S.-G. Zhou, Phys. Rev. C 89, 044307 (2014)

M. Isaka, K. Fukukawa, M. Kimura, E. Hiyama, H. Sagawa, and Y. Yamamoto, Phys. Rev. C 89, 024310 (2014)

Why to study deformations of hypernuclei

Many p-shell and sd-shell nuclei are deformed.

For example, experimentally, ^{10}B and ^{11}C have large quadrupole moments.

F. Ajzenberg-Selove, Nucl. Phys. **A490**, 1 (1988); **A506**, 1(1990).

Also, ^{8}Be is known to be strongly deformed due to its double-α structure.

Several models for deformed nuclei

Alpha-model

Projected shell model (PSM)

Deformed Skyrme Hartree-Fock (DSHF)

Relativistic mean-field model (RMF)

Antisymmetrized molecular dynamics (AMD)

……

Microscopic hyperon-nucleon interaction for deformed hypernuclei

Free YN interaction

BHF cal. for asymmetric matter

YN: Nijmegen soft-core hyperon-nucleon potential NSC89,NSC97a,NSC97f,ESC08

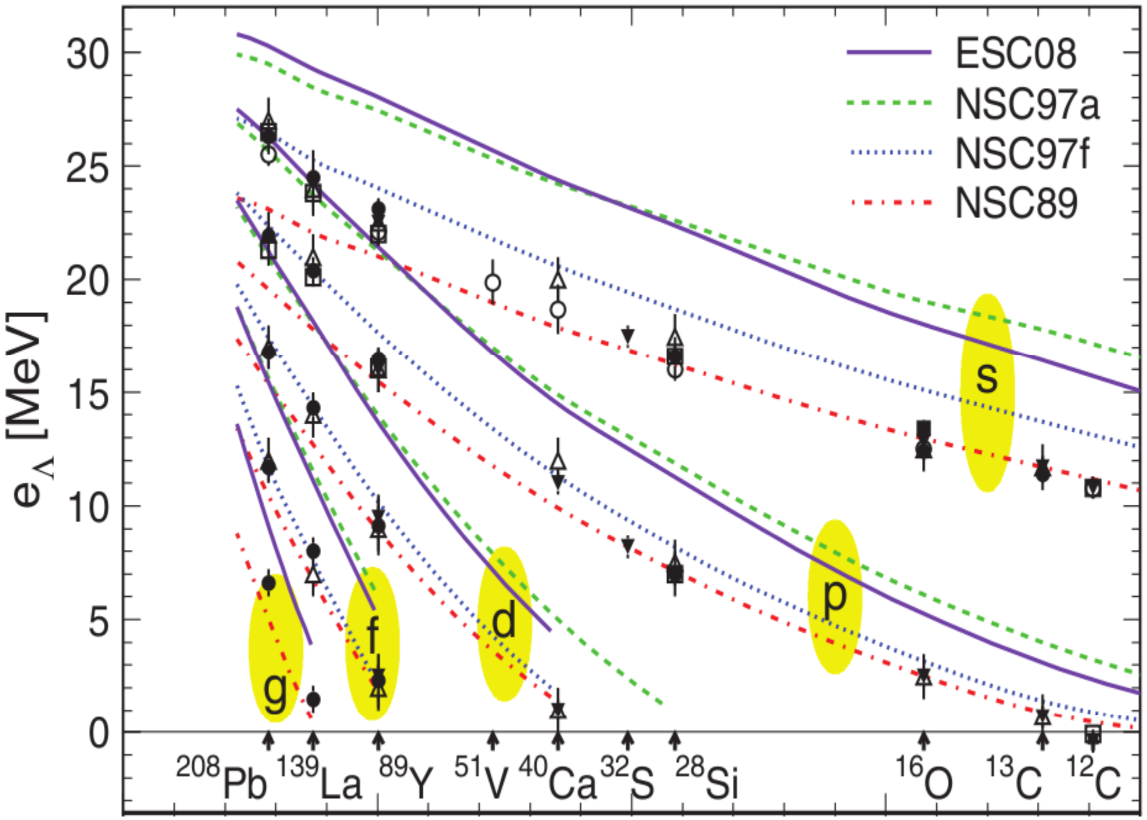
NN: Argonne v18 nucleon-nucleon interaction

Effective YN interaction

MF cal.

B_Y, Hypernuclear Structure

Comparison of different hyperon-nucleon potential



H.-J. Schulze PHYSICAL REVIEW C **88**, 024322 (2013)

Extended DSHF including hyperon-nucleon interaction

DSHF + YN interaction (NSC89):

Total energy of a hypernucleus in extended DSHF:

$$E = \int d^3 r \, \epsilon_{\mathrm{SHF}}(r)$$

where the energy density

$$\mathcal{E}_{\mathrm{SHF}} = \varepsilon_N[\rho_n, \rho_p, \tau_n, \tau_p, \boldsymbol{J}_n, \boldsymbol{J}_p] + \varepsilon_\Lambda[\rho_n, \rho_p, \rho_\Lambda, \tau_\Lambda]$$

Due to the YN force,

$$\epsilon_\Lambda = \frac{\tau_\Lambda}{2m_\Lambda} + \epsilon_{N\Lambda}(\rho_N, \rho_\Lambda) + \left(\frac{m_\Lambda}{m^*_\Lambda(\rho_n, \rho_p, \rho_\Lambda)} - 1\right)\frac{\tau_\Lambda - C\rho_\Lambda^{5/3}}{2m_\Lambda}$$

The energy density functional $\varepsilon_{N\Lambda}$ is obtained from a fit *to the* binding energy per baryon, $B/A(\rho_n, \rho_p, \rho)$, of asymmetric hypermatter, as generated by BHF calculations.

$$\varepsilon_{N\Lambda} = (\rho_n + \rho_p + \rho_\Lambda)\frac{B}{A}(\rho_n, \rho_p, \rho_\Lambda) - (\rho_n + \rho_p)\frac{B}{A}(\rho_n, \rho_p, 0) - \frac{C\rho_\Lambda^{5/3}}{2m_\Lambda}$$

Effective mass of hyperon

$$\frac{m_\Lambda^*}{m_\Lambda} = \left[1 + \frac{U_\Lambda\left(k_F^{(\Lambda)}\right) - U_\Lambda(0)}{k_F^{(\Lambda)2}/2m_\Lambda}\right]^{-1}$$

In practice we use the following parametrizations:

$$\varepsilon_{N\Lambda} \approx -\Big[368 - (1717 + 268\alpha - 920\alpha^2)\rho_N + (2932 - 776\alpha + 2483\alpha^2)\rho_N^2\Big]\rho_N\rho_\Lambda + \big(449 - 2470\rho_N + 5834\rho_N^2\big)\rho_N\rho_\Lambda^{5/3},$$

$$\frac{m_\Lambda^*}{m_\Lambda} \approx 1 - (1.58 + 0.12\alpha - 0.12\alpha^2 + 0.54y - 0.14y^2)\rho_N + (4.11 + 2.11\alpha + 2.88\alpha^2 + 0.35y + 1.17y^2)\rho_N^2 - (4.03 + 7.08\alpha + 5.18\alpha^2 - 0.93y + 3.27y^2)\rho_N^3,$$

Extended SHF equation

Minimizing the total energy of the hypernucleus, one arrives with extended SHF equation

$$\left[-\nabla\cdot\frac{1}{2m_q^*(r)}\nabla + V_q(r) - i\nabla W_q(r)\cdot(\nabla\times\boldsymbol{\sigma})\right]\phi_q^i(\boldsymbol{r}) = -e_q^i\,\phi_q^i(\boldsymbol{r})$$

with the modified mean field by hyperon:

$$V_N = V_N^{\mathrm{SHF}} + \frac{\partial\epsilon_{N\Lambda}}{\partial\rho_N} + \frac{\partial}{\partial\rho_N}\left(\frac{m_\Lambda}{m_\Lambda^*(\rho_N)}\right)\frac{\tau_\Lambda - C\rho_\Lambda^{5/3}}{2m_\Lambda}$$

$$V_\Lambda = \frac{\partial\epsilon_{N\Lambda}}{\partial\rho_\Lambda} - \left(\frac{m_\Lambda}{m_\Lambda^*(\rho_N)} - 1\right)\frac{5}{3}\frac{C\rho_\Lambda^{2/3}}{2m_\Lambda},$$

Pairing interaction

We take a density-dependent delta pairing

$$V_q(\boldsymbol{r}_1, \boldsymbol{r}_2) = V_q' \left[1 - \frac{\rho_N(r)}{\rho_0}\right] \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

For light nuclei,

Nucl. Phys. **A551**, 434 (1993)

$$V_q' = -410\,\mathrm{MeVfm}^3$$

Nucl. Phys. A722, c183, 2003

For medium-mass and heavy nuclei,

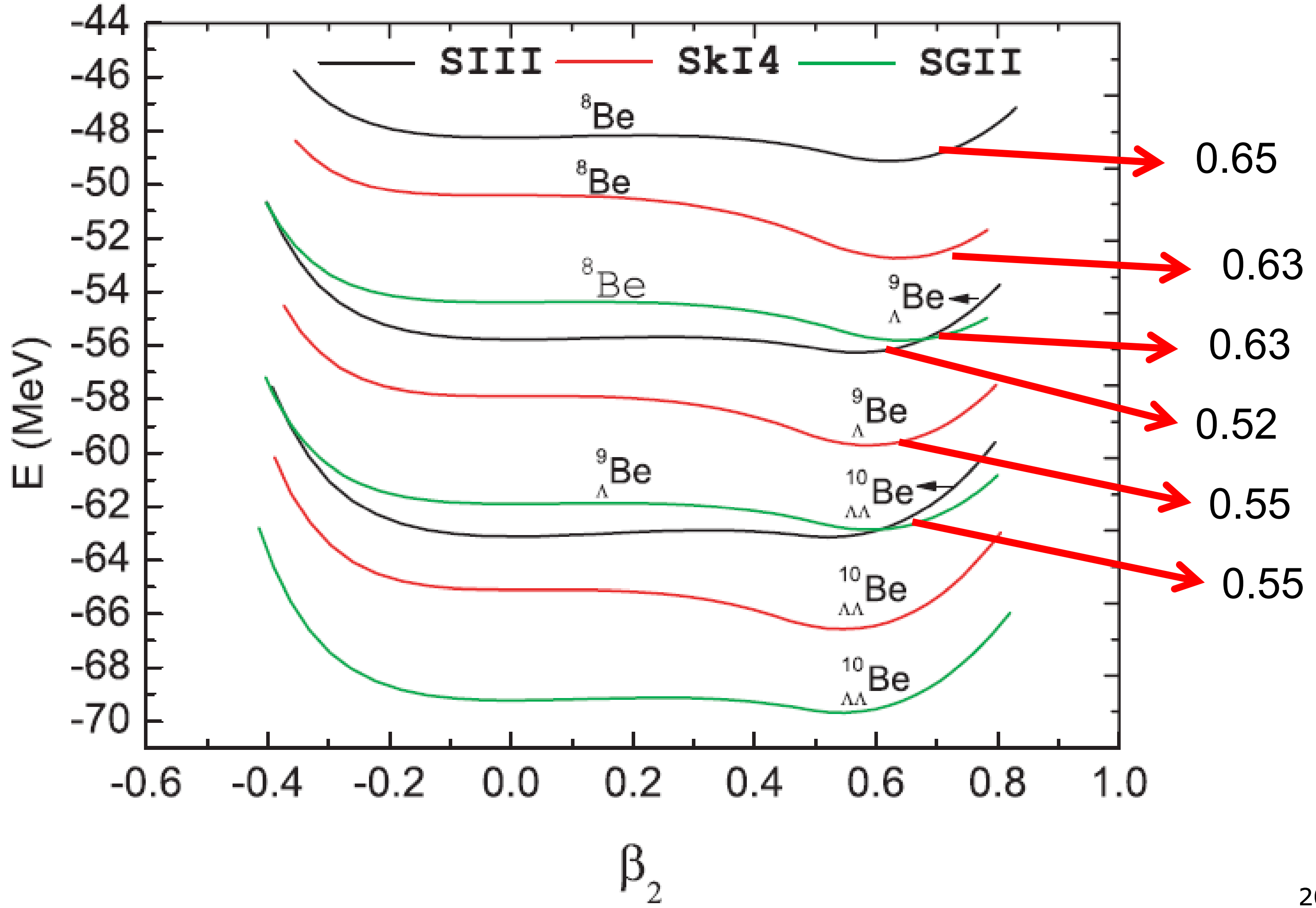
$$V_p' = -1146\,\mathrm{MeVfm}^3$$

$$V_n' = -999\,\mathrm{MeVfm}^3$$

Euro. Phys. J. A8, 59, 2000

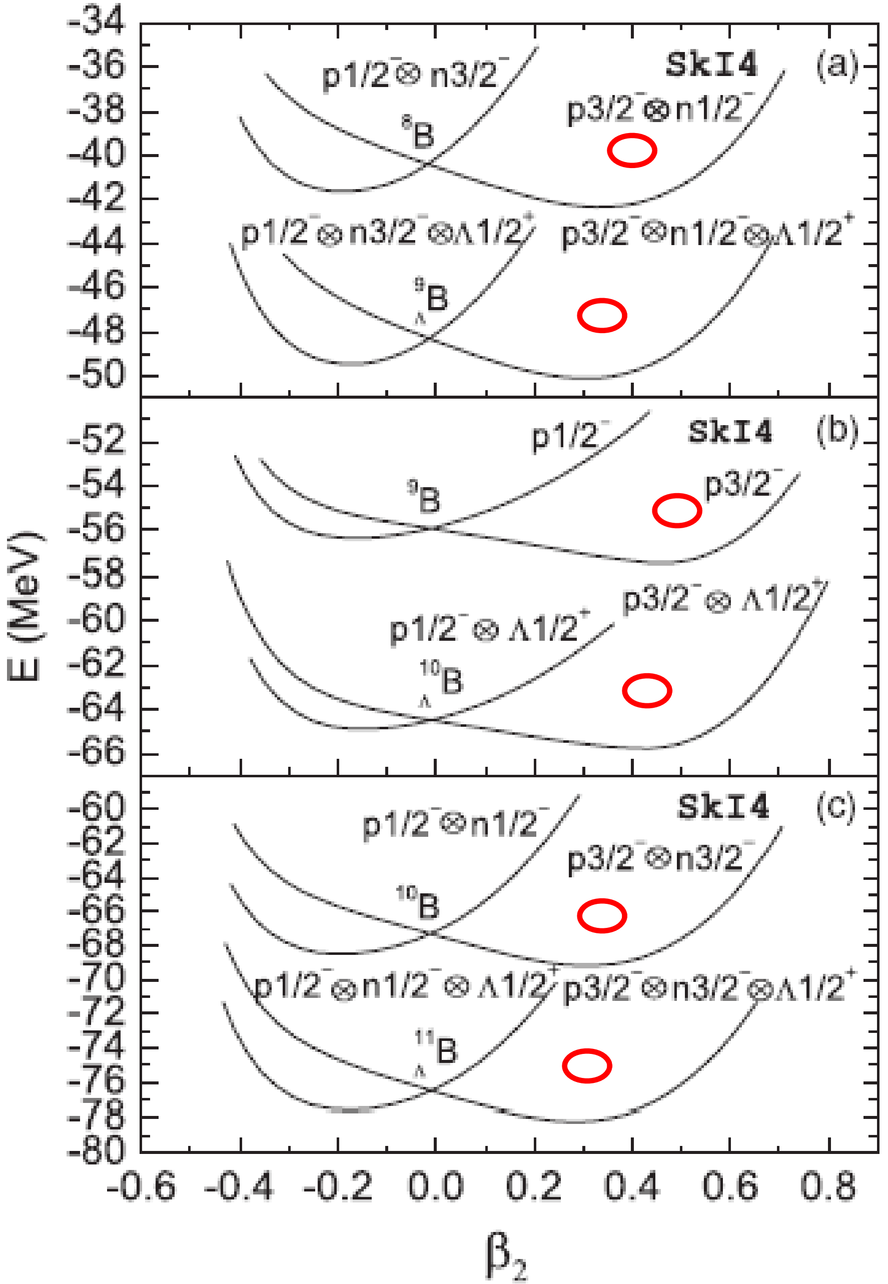
1. Shapes of Hypernuclei

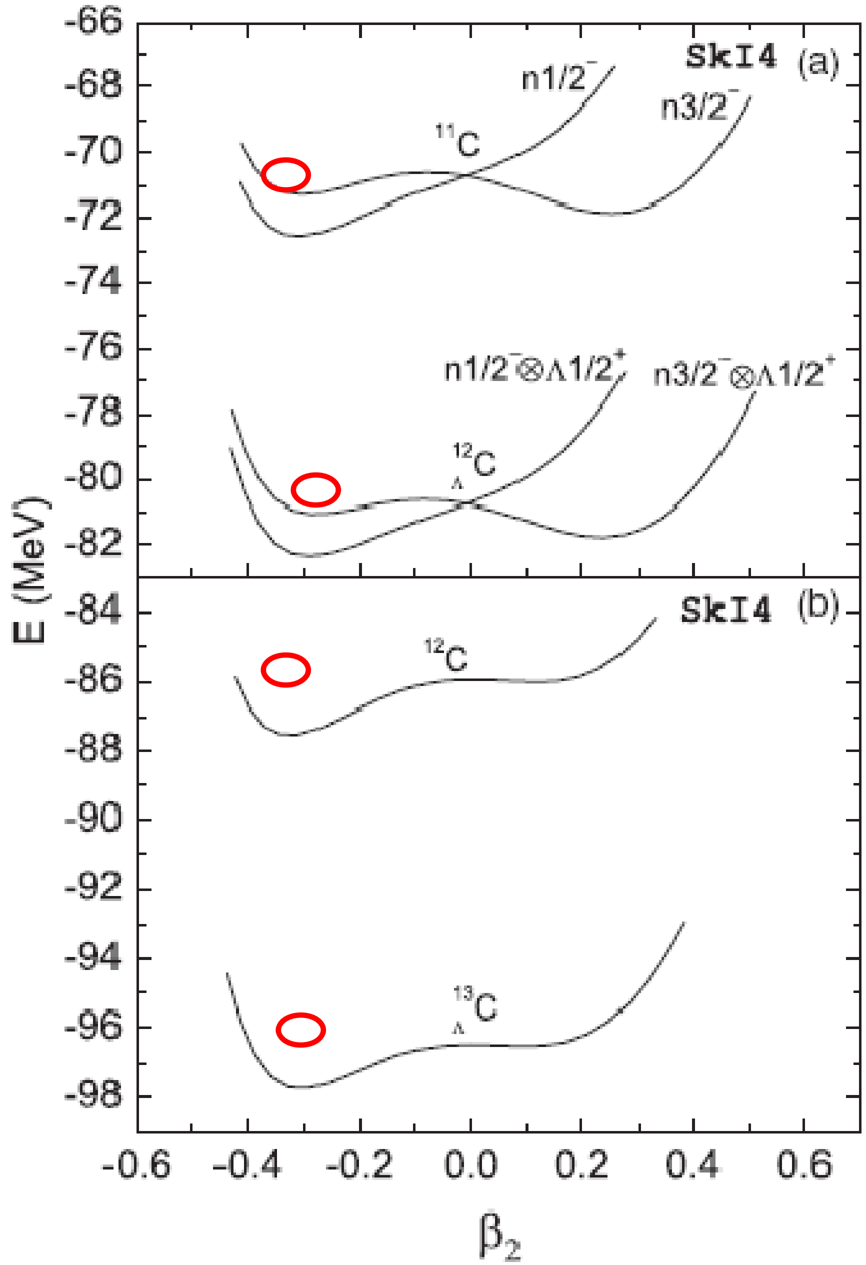
Binding energies vs deformations



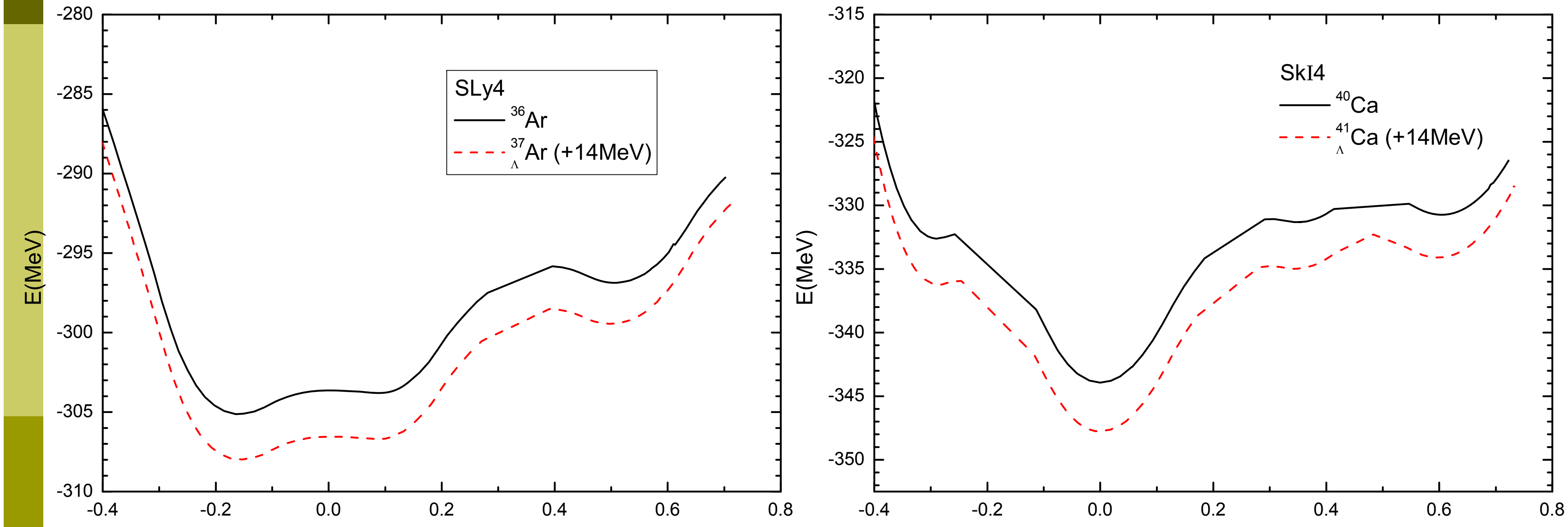
X.-R. Zhou, H.-J. Schulze, H. sagawa et.al, PRC76, 034312(2007)

Binding energies vs deformations



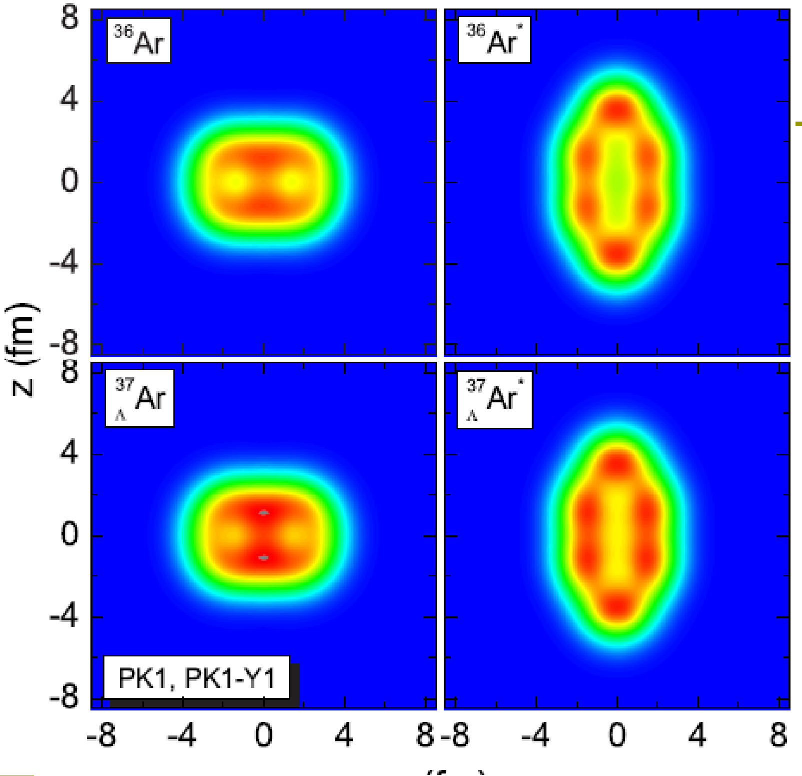


Superdeformation of hypernucei in SHF

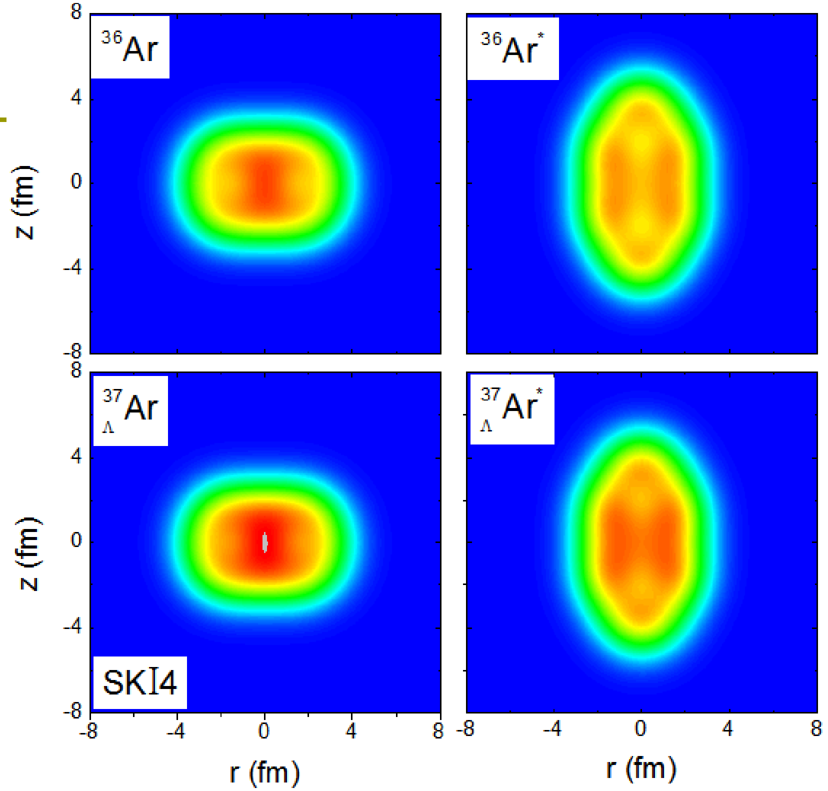


X.-R. Zhou, E. Hiyama, in preparation

Density distribution of superdeformed hypernuclei

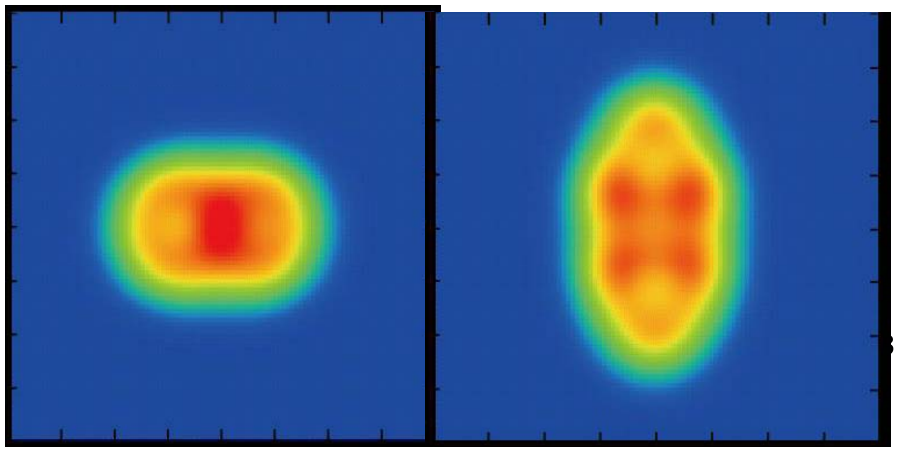


RMF



SHF

AMD



2. Energy spectra of hypernulcei

Limitations of extended SHF+BCS method:

- The SHF models can just give the single-particle energies and ground state of Λ hypernuclei in intrinsic frame of reference.
- The conservation of particle number is destroyed by BCS method.
- The study of the gamma spectra and electromagnetic transitions needs symmetry restoration.

Angular momentum and particle-number projection (AMP&PNP) are needed !

Projected SHF+BCS Model

The projected mean-field state

$$\left|\phi, JMK\right\rangle = P^{J}_{MK} P^{N} P^{Z} \left|\phi\right\rangle$$

The projection operator

$$P^{J}_{MK} = \frac{2J+1}{8\pi^2} \int d\Omega D^{J}_{MK}(\Omega) R(\Omega)$$

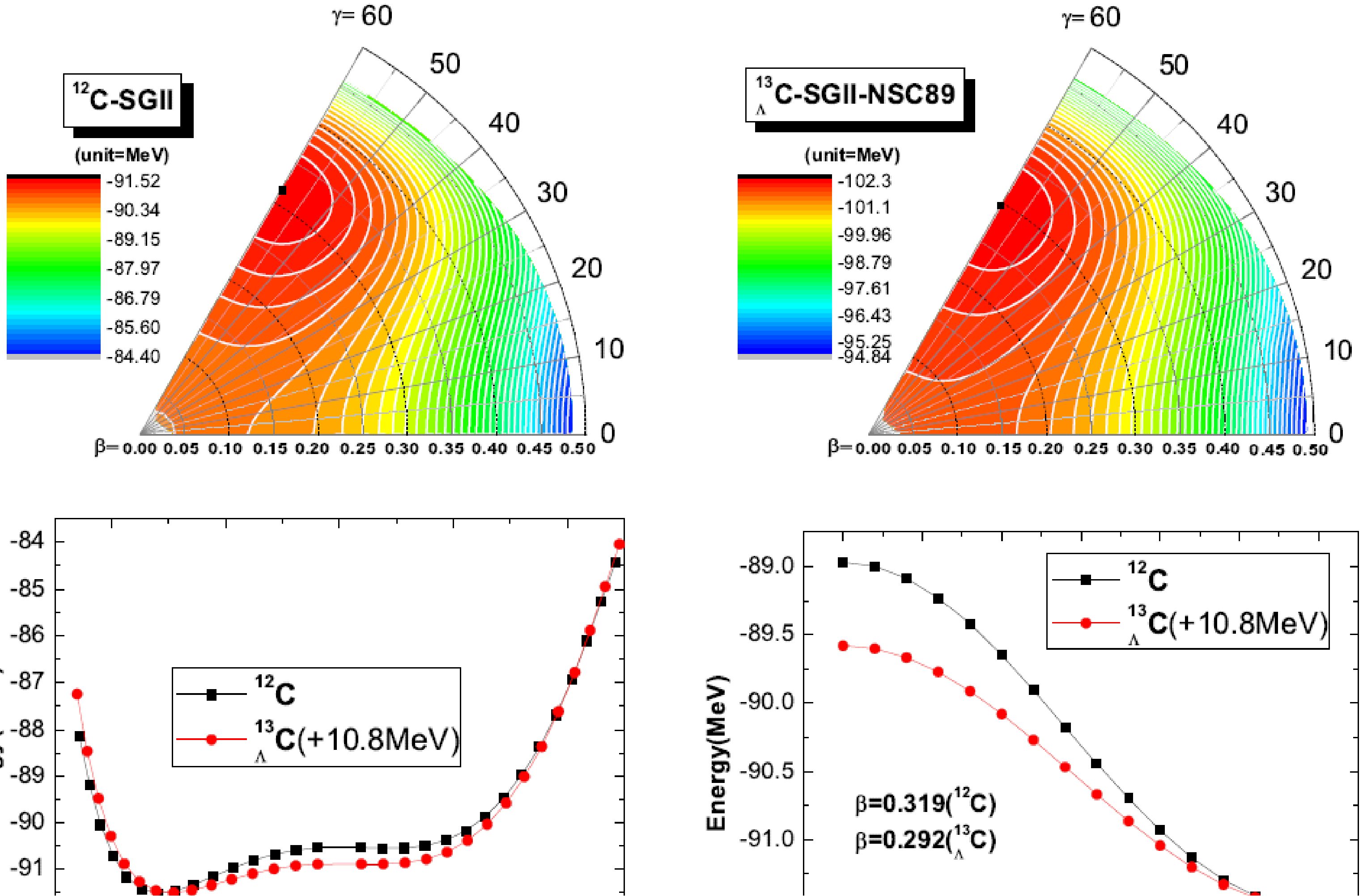
$$\hat{P}^{N} = \frac{1}{2\pi} \int e^{i\varphi(\hat{N}-N)} d\varphi.$$

Energies with angular momentum and E2 transitions

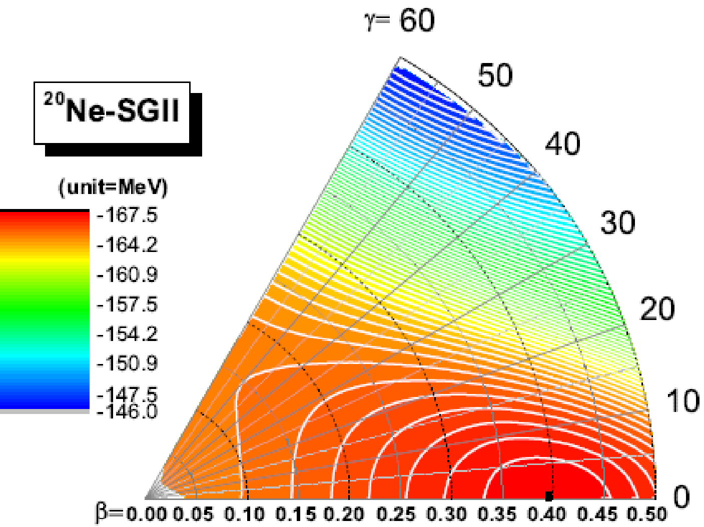
$$\mathrm{E}^{J}_{K} = \frac{\langle JMK, \phi | H | \phi, JMK \rangle}{\langle JMK, \phi | \phi, JMK \rangle}$$

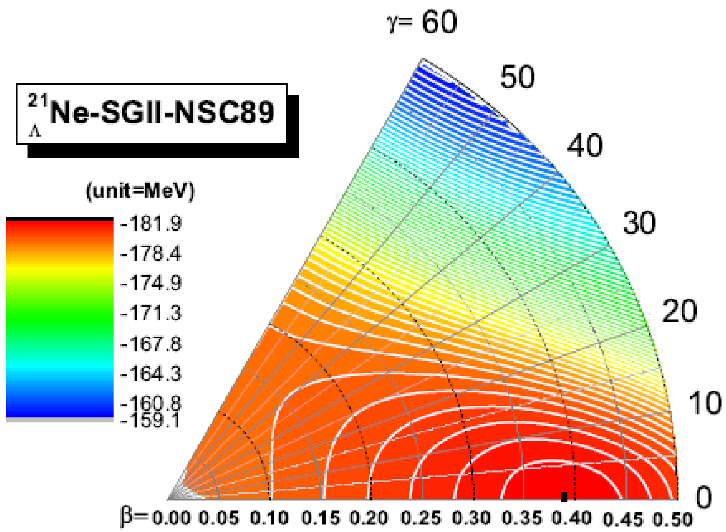
$$B(E\lambda, I_i \longrightarrow I_f) = \frac{1}{2I_i + 1} \left| \langle I_f K_f || \hat{Q}_\lambda | I_i K_i \rangle \right|^2$$

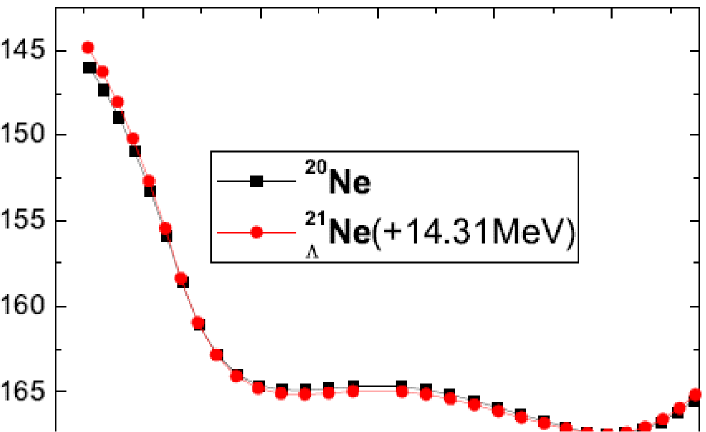
nergy potential surface of ^{12}C and $^{13}_{\Lambda}C$

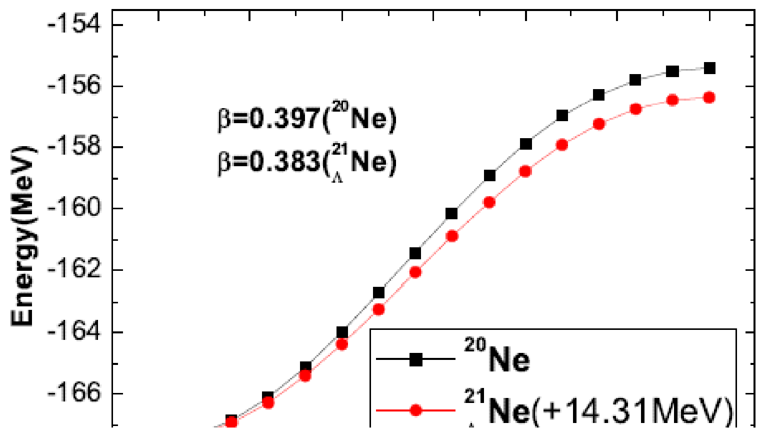


ergy potential surface of ^{20}Ne and $^{21}_{\Lambda}$Ne

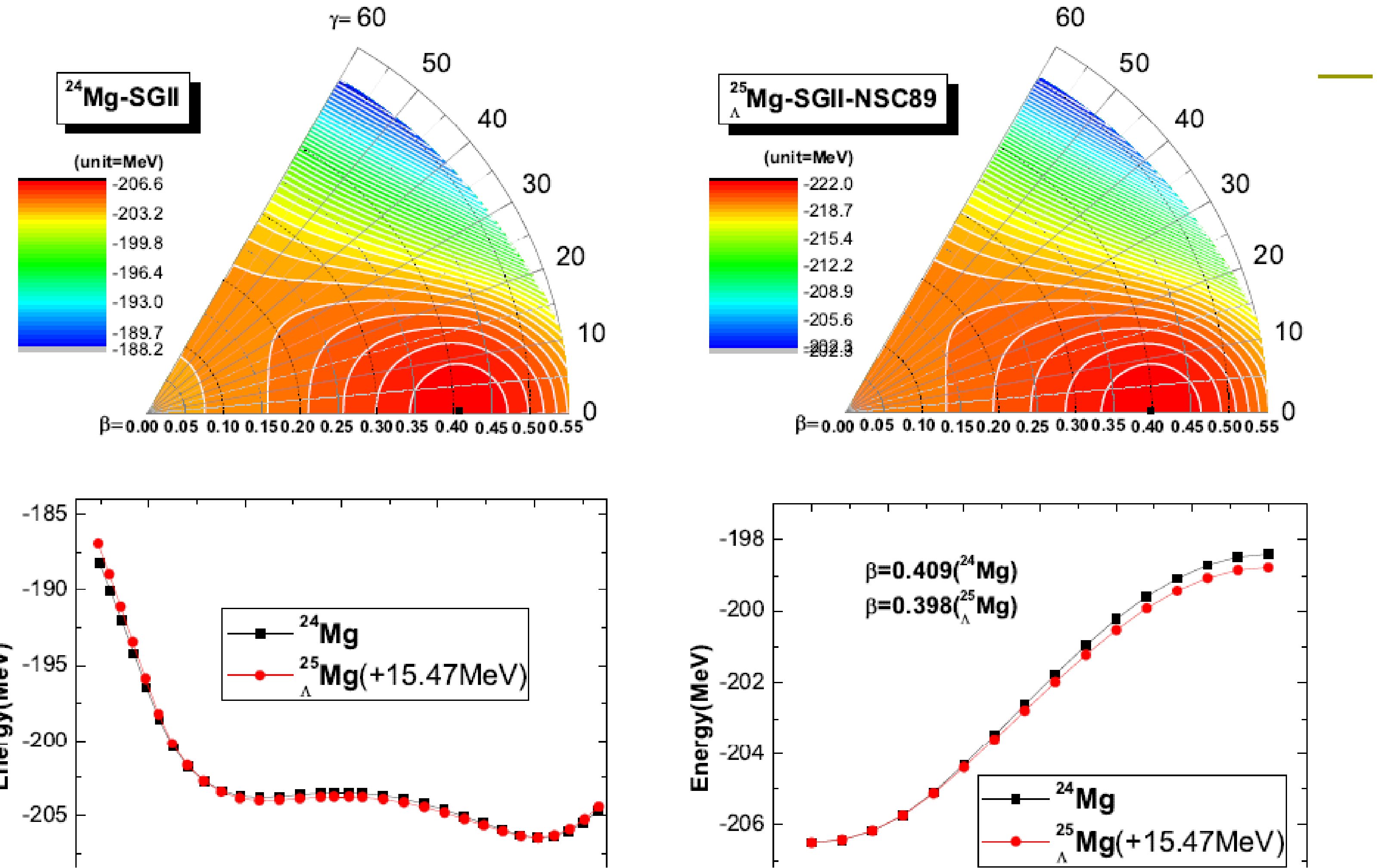




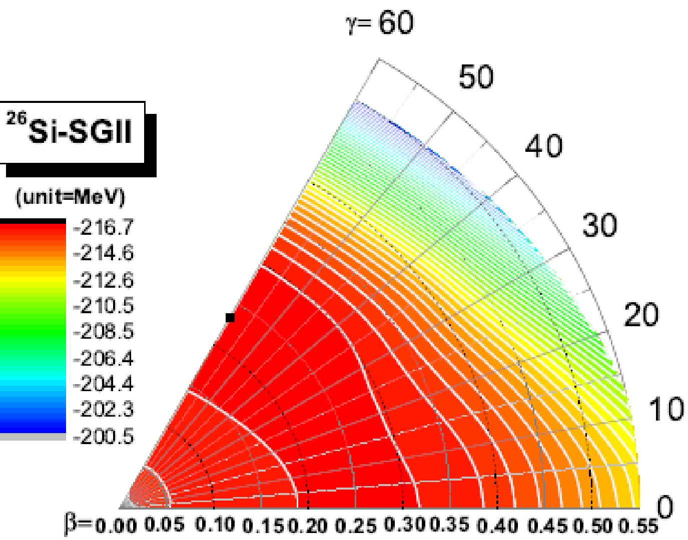


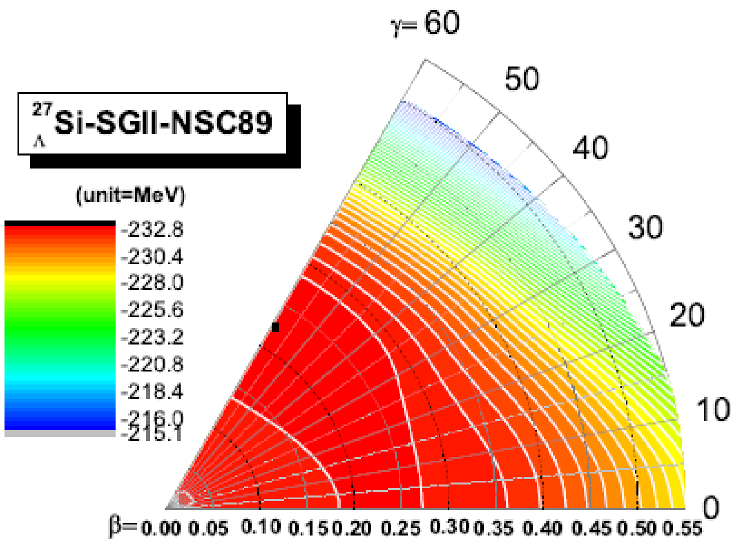


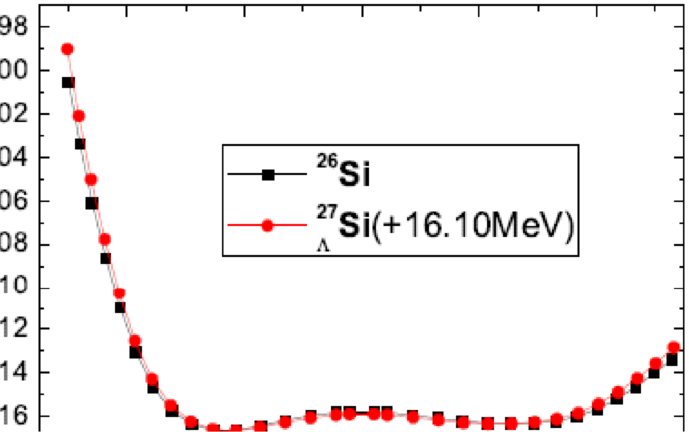
nergy potential surface of ^{24}Mg and $^{25}_{\Lambda}$Mg

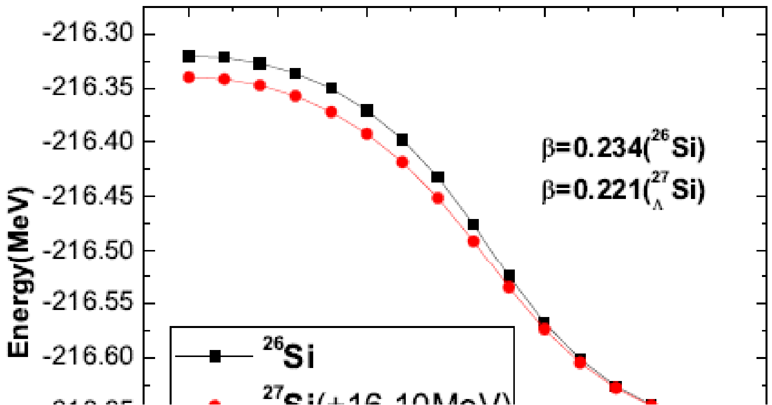


nergy potential surface of ^{26}Si and $^{27}_{\Lambda}$Si

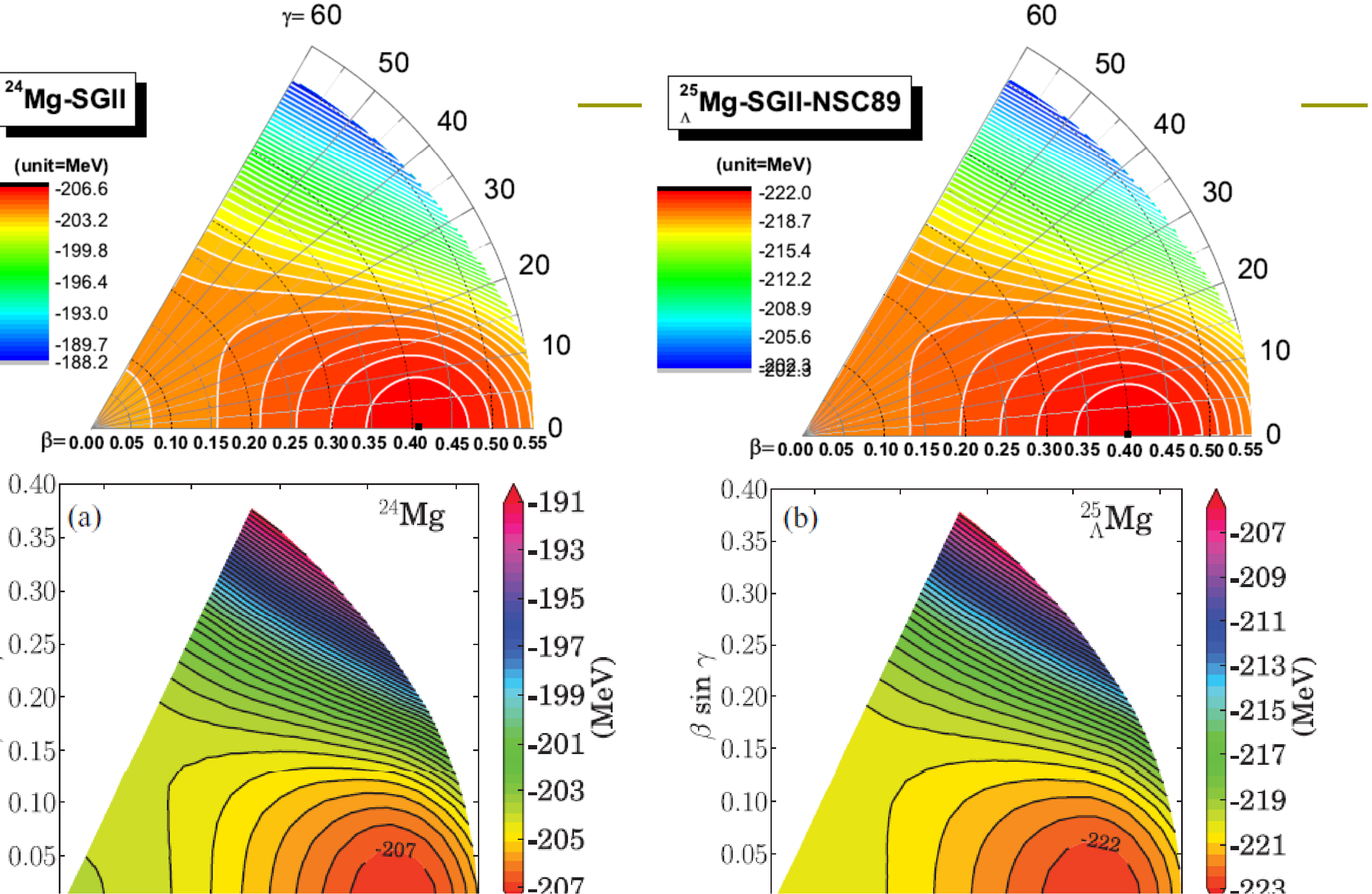




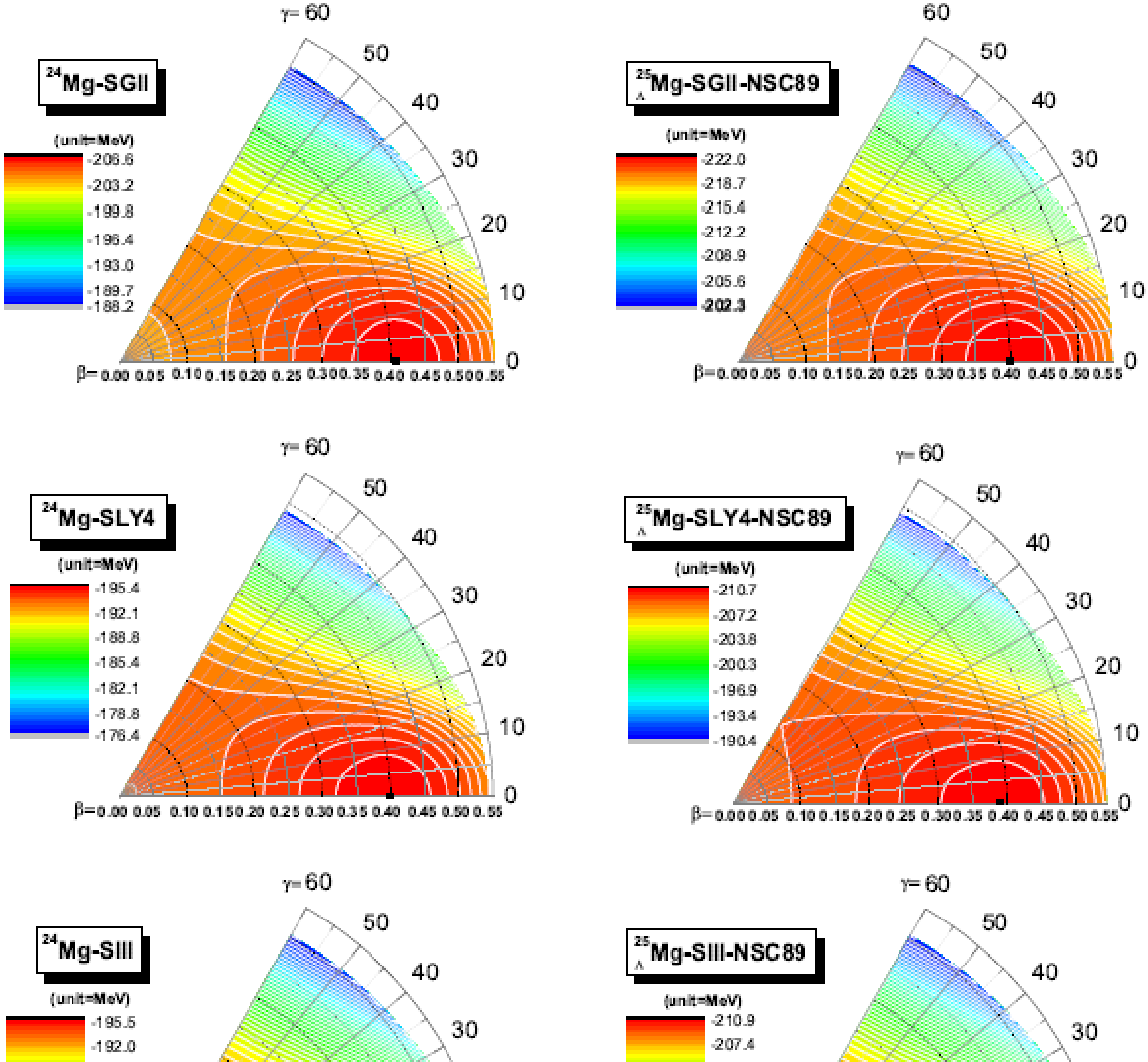




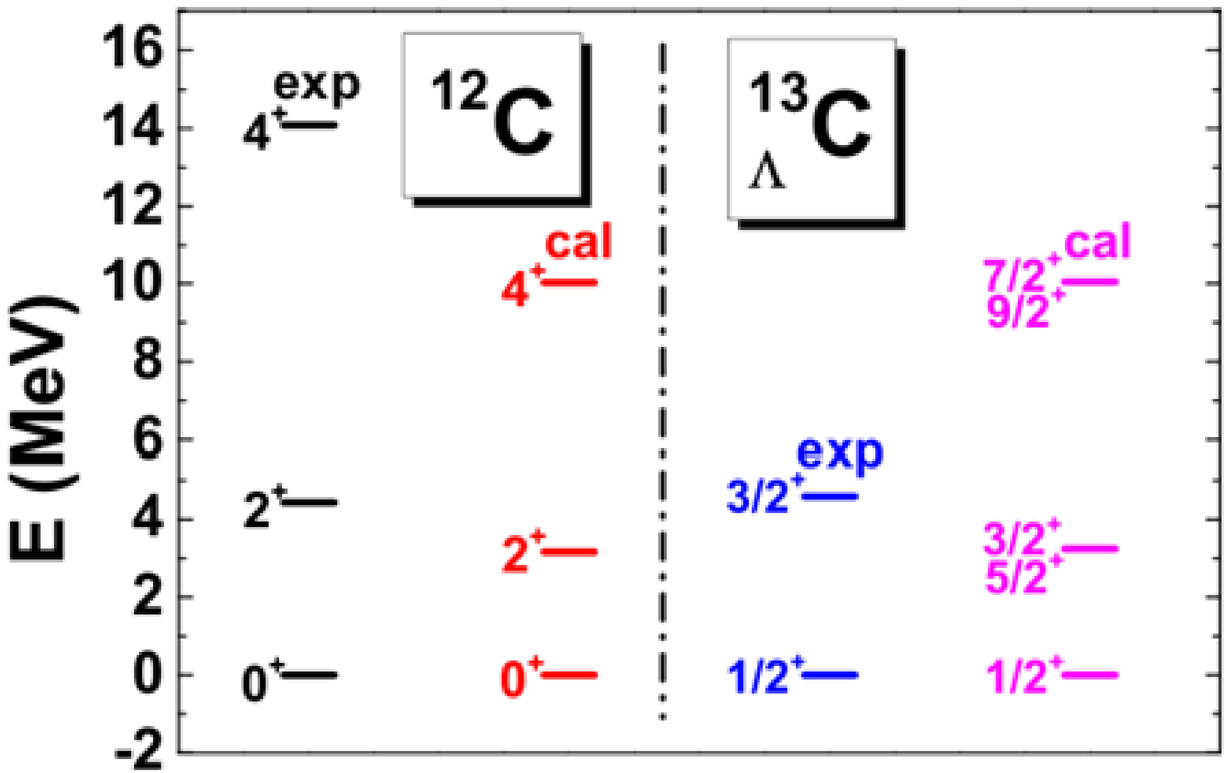
mparison of NSC89(upper) and Skyrme-type(lower) Interactions



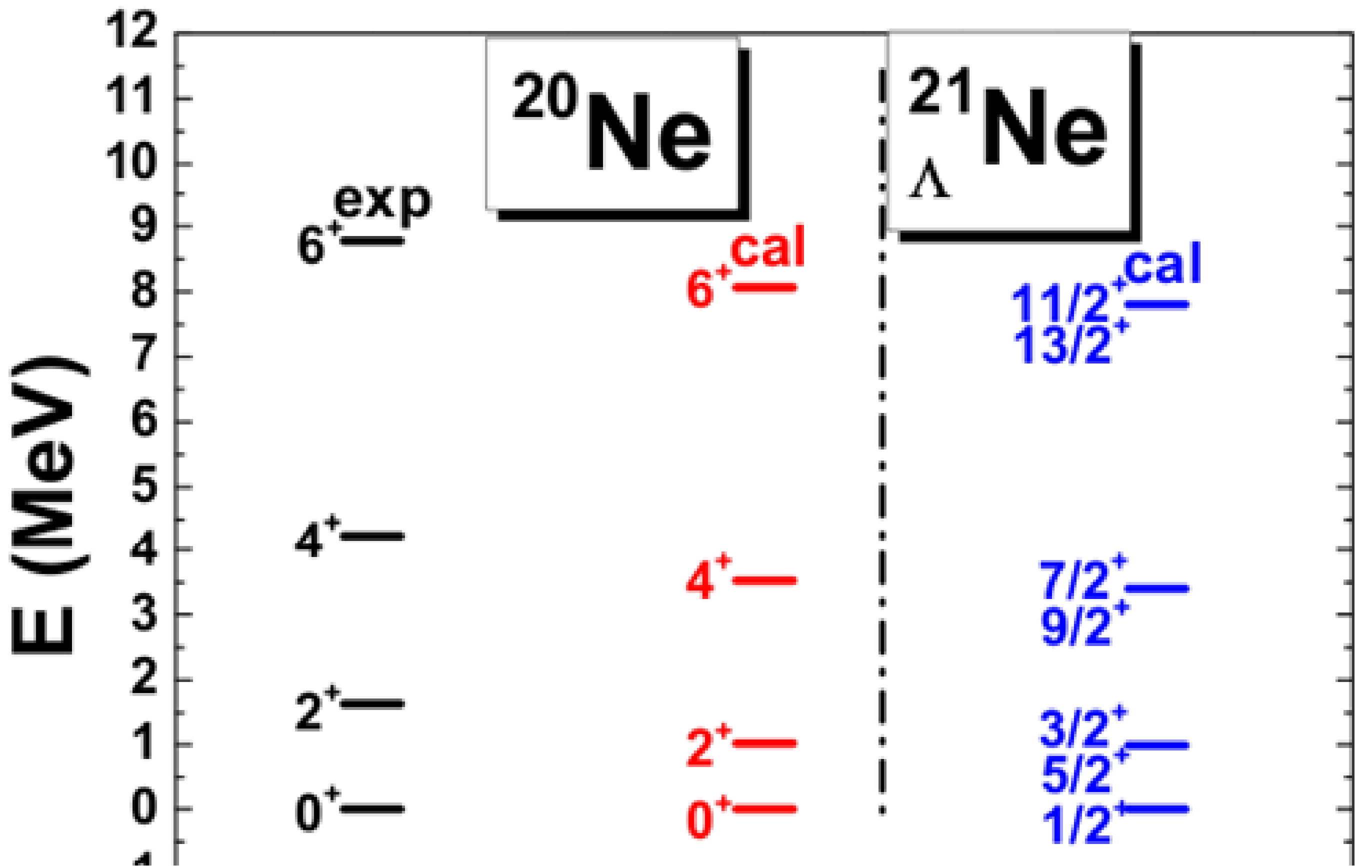
Comparison of different Skyrme parameters



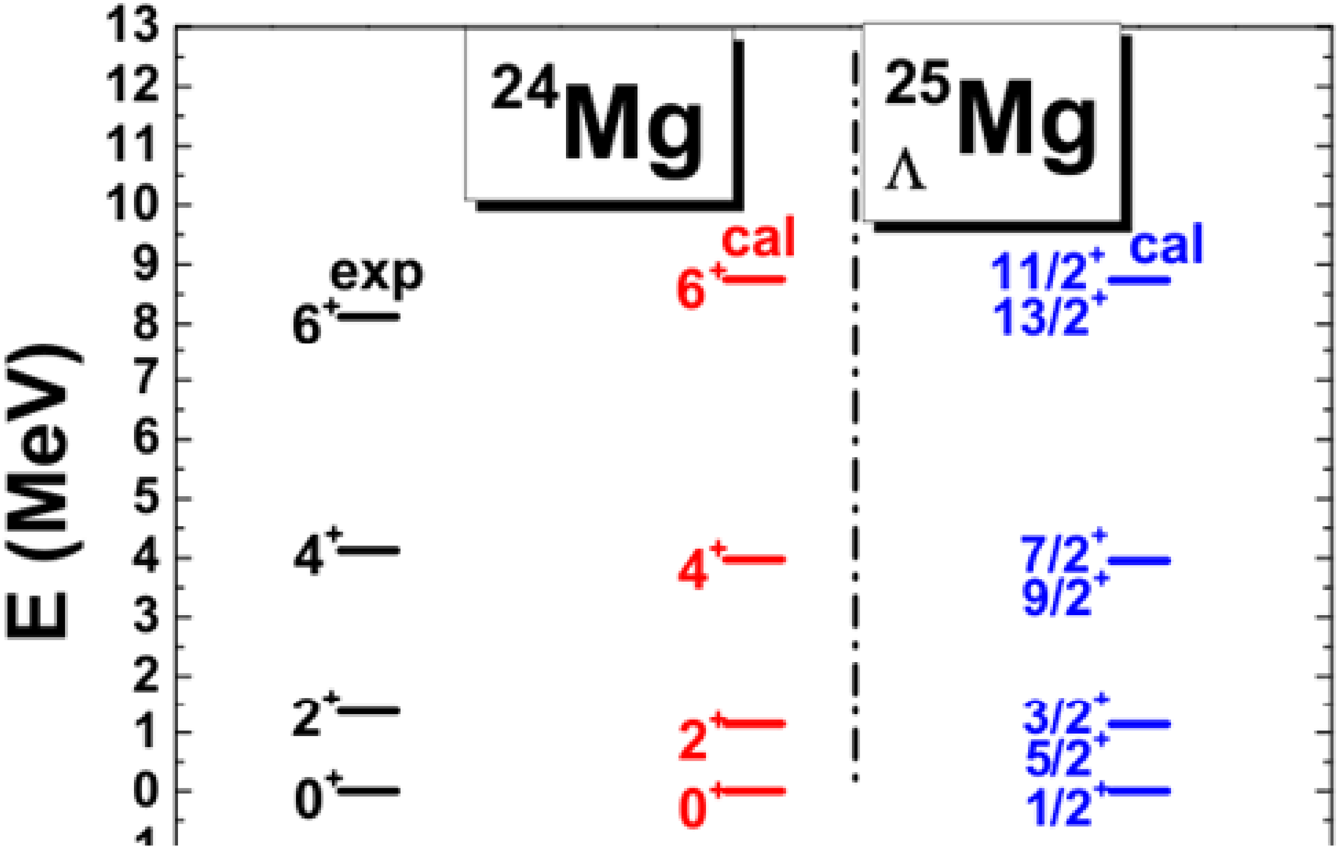
Energy spectra of ^{12}C and $^{13}_{\Lambda}C$



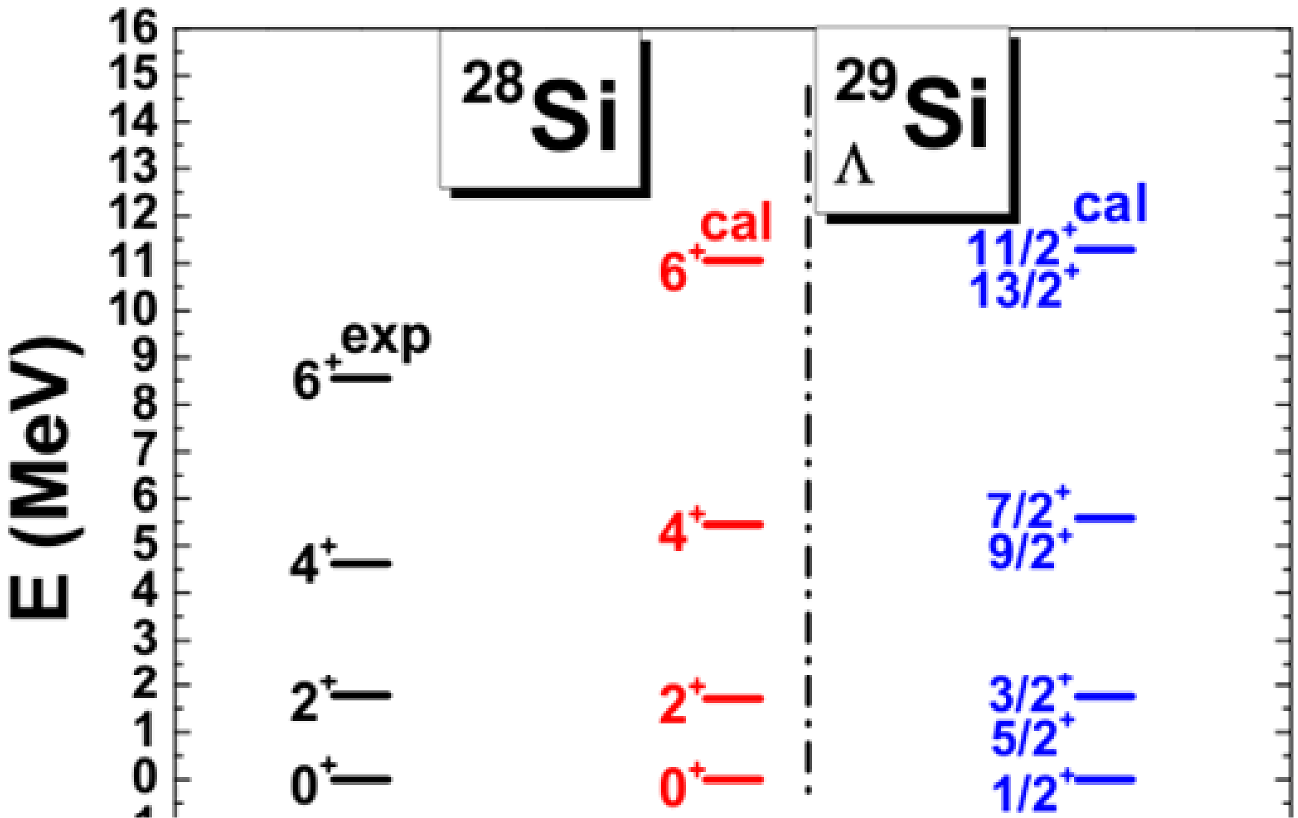
Energy spectra of ^{20}Ne and $^{21}_{\Lambda}$Ne



Energy spectra of ^{24}Mg and $^{25}_{\Lambda}$Mg



Energy spectrum of ^{28}Si and $^{29}_{\Lambda}Si$



E2 transition probabilities

$B(E2)$	^{12}C		$^{13}_{\Lambda}$C	^{20}Ne		$^{21}_{\Lambda}$Ne	^{24}Mg		$^{25}_{\Lambda}$Mg	^{28}Si		$^{29}_{\Lambda}$Si
$I_i \to I_f$	exp	cal	cal	exp	cal	cal	exp	cal	cal	exp	cal	cal
$^+(\frac{3^+/2\to1^+/2}{5^+/2\to1^+/2})$	4.65(26)	6.42	$\frac{5.61}{5.69}$	20.3(10)	19.49	$\frac{17.72}{17.72}$	21.5(10)	24.30	$\frac{22.91}{23.14}$	13.2(5)	14.23	$\frac{13.89}{13.61}$
$^+(\frac{7^+/2\to3^+/2}{9^+/2\to5^+/2})$	—	10.11	$\frac{8.23}{8.84}$	22.0(2)	28.39	$\frac{23.09}{25.73}$	39.0(4)	35.45	$\frac{30.53}{33.94}$	16.4(18)	21.46	$\frac{18.60}{20.88}$
$^+(\frac{11^+/2\to7^+/2}{13^+/2\to9^+/2})$	—	—	—	20.0(3)	32.52	$\frac{28.45}{29.46}$	—	41.15	$\frac{37.52}{39.22}$	10.6(10)	25.75	$\frac{24.09}{24.86}$

xp B(E2) values come from http://www.nndc.bnl.gov

evaluate the change of size the following quantity is introduced .

$$_B = \frac{B(E2, I_{ic}^+ \to I_{fc}^+)_H}{B(E2, I_i^+ \to I_f^+)}$$

TABLE III. Calculated Γ_B values.

Γ_B	^{12}C	^{20}Ne	^{24}Mg	^{28}Si
$2^+ \to 0^+$	0.885	0.903	0.957	0.964
$4^+ \to 2^+$	0.898	0.913	0.955	0.973
$6^+ \to 4^+$	–	0.908	0.956	0.970

Summary

1. The DSHF was extended to hypernuclei by including a microscopically derived hyperon-nucleon interaction.

2. The calculated core nuclei and the corresponding hypernuclei have similar deformations with the same sign when the core nuclei are well deformed. The superdeformation of hypernuclei is found in SHF.

3. The projected SHF+BCS model gives reasonable results of energy spectra and E2 transition rates for well-deformed sd-shell nuclei and hypernuclei .

Cooperators

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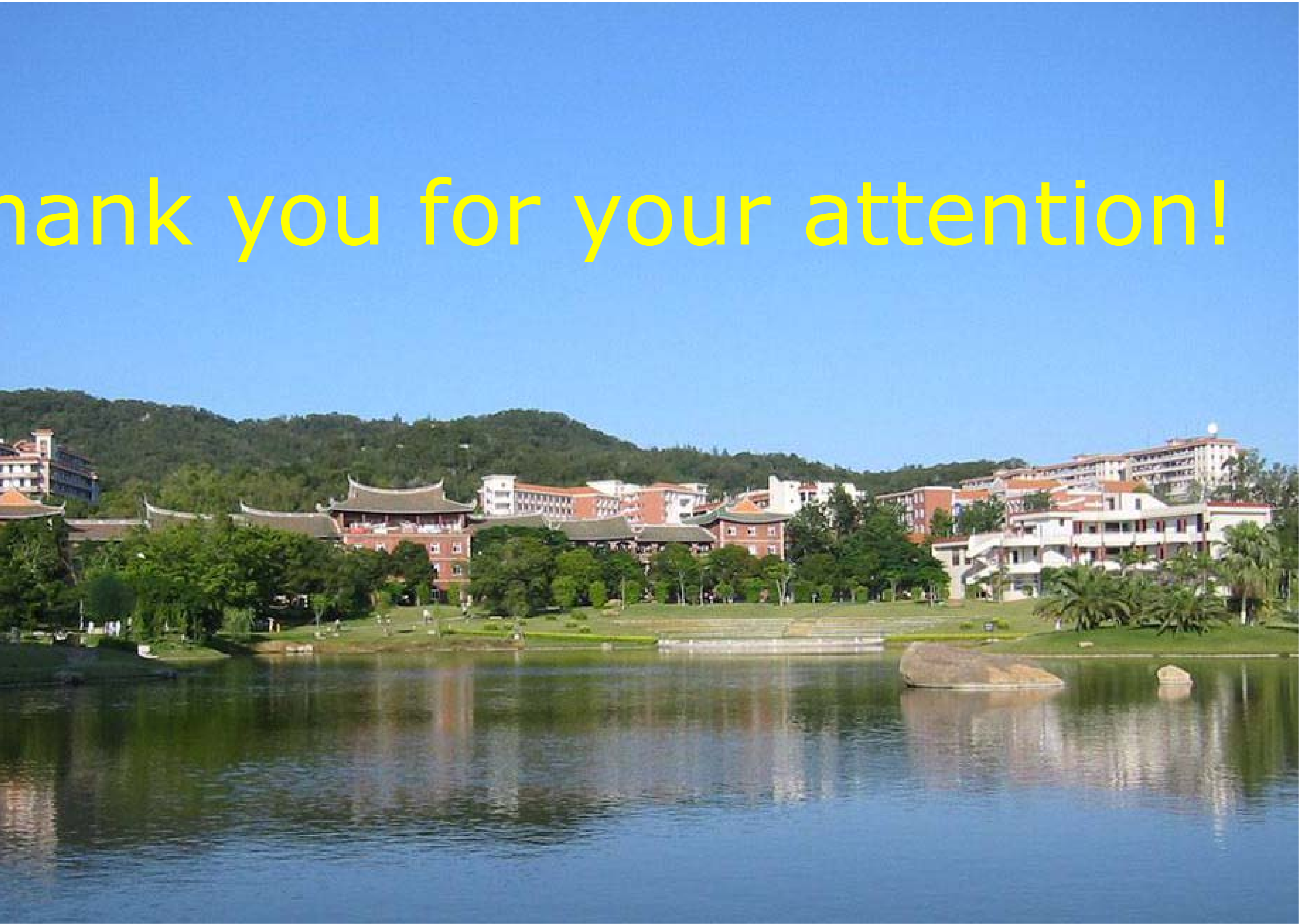
hank you for your attention!