A New Skyrme Force for Hypernuclei

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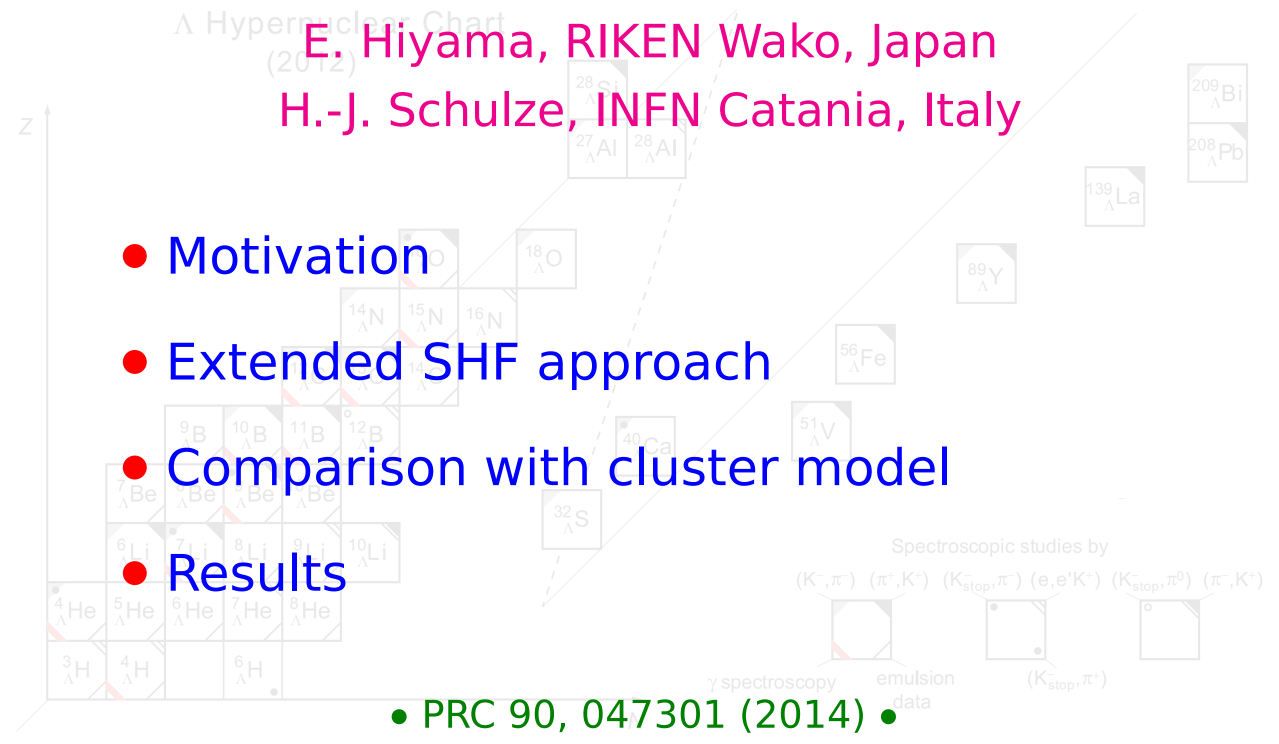
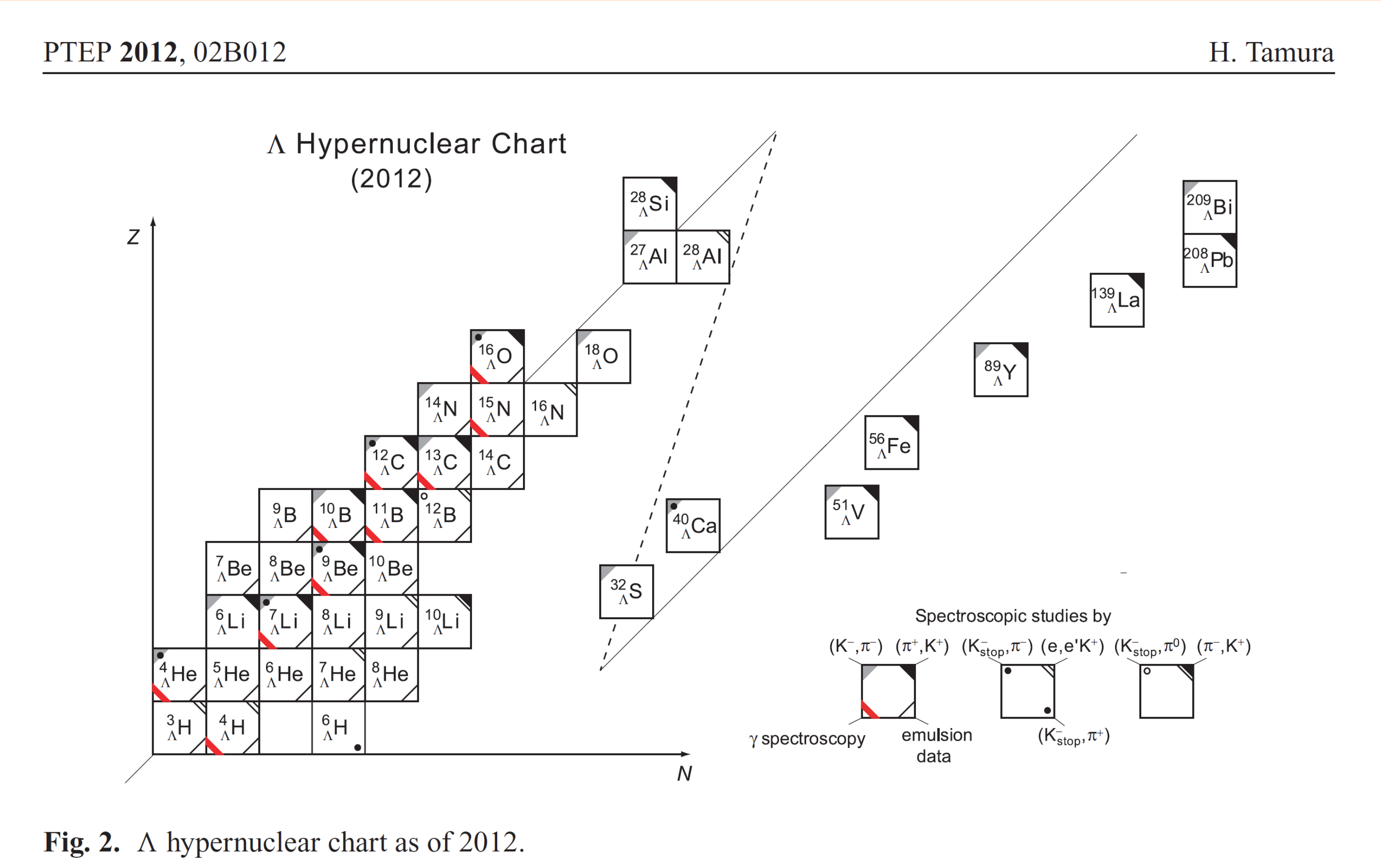


- Motivation
- Extended SHF approach
- Comparison with cluster model
- Results

• PRC 90, 047301 (2014) •

Lambda Hypernuclear Chart:



PTEP **2012**, 02B012 H. Tamura

Fig. 2. Λ hypernuclear chart as of 2012.

Lambda Hypernuclear Chart:

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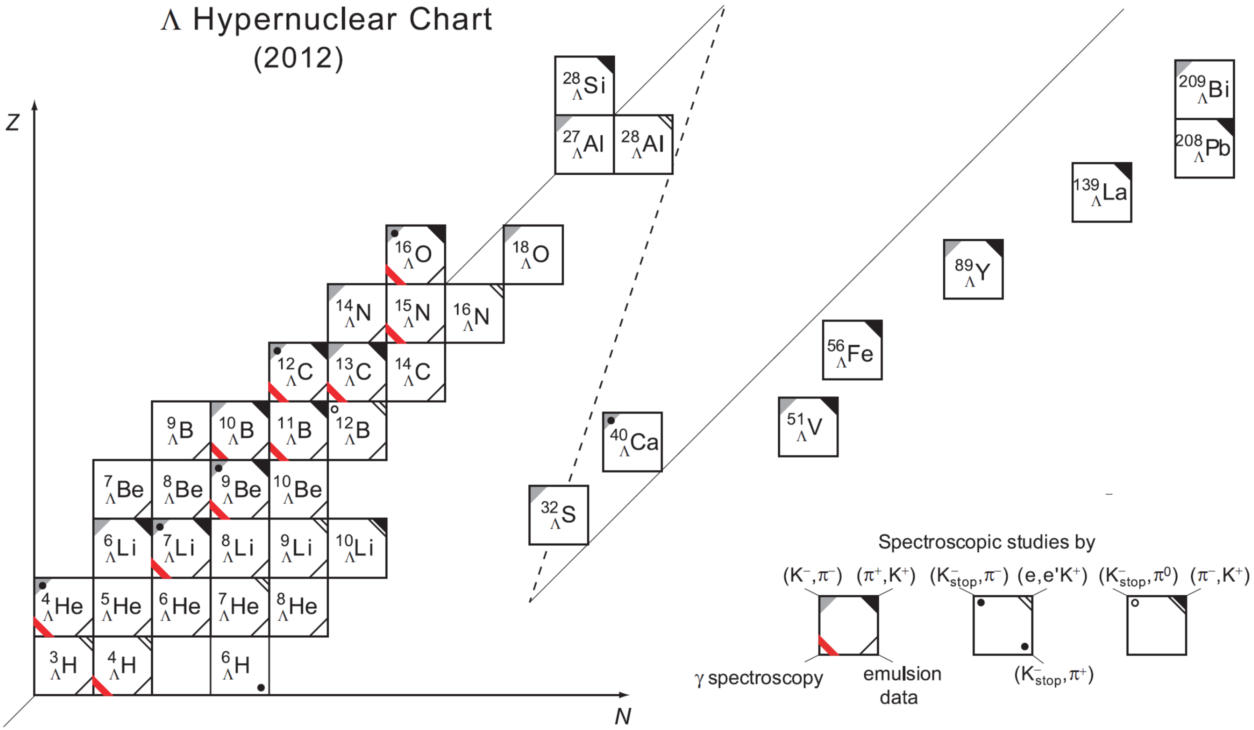
few-body (cluster) models <— —> mean-field (eff.int.) models



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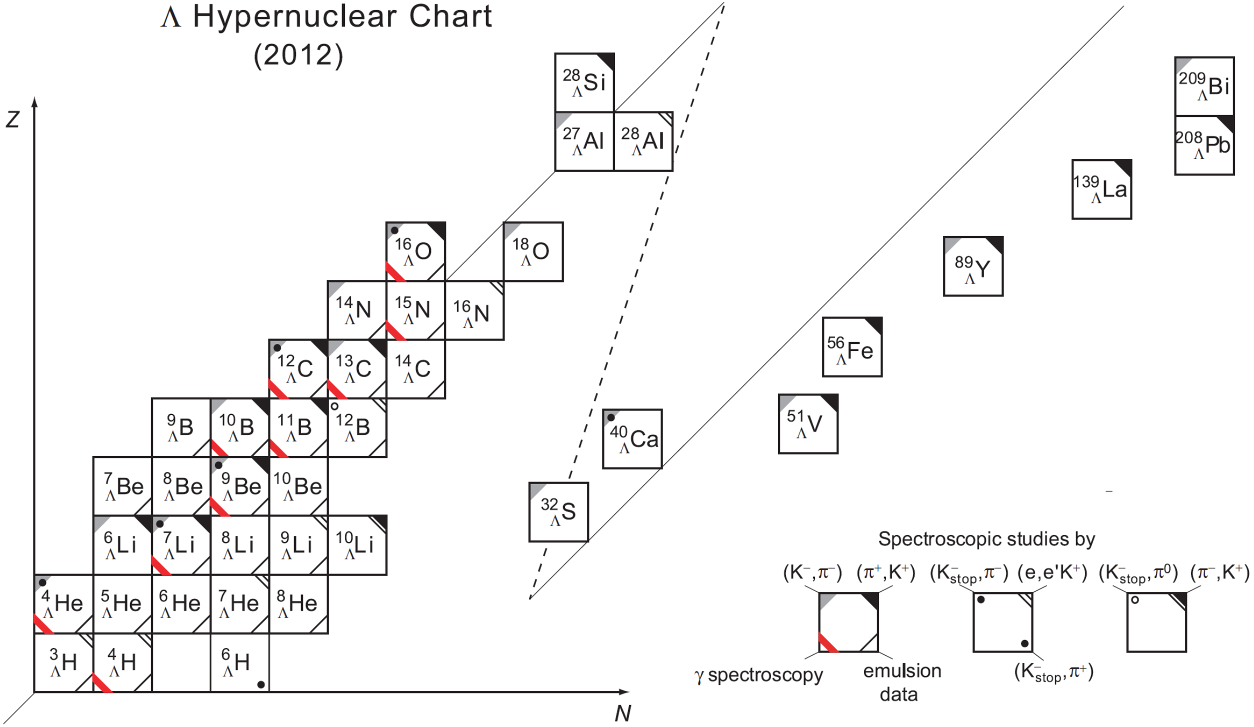

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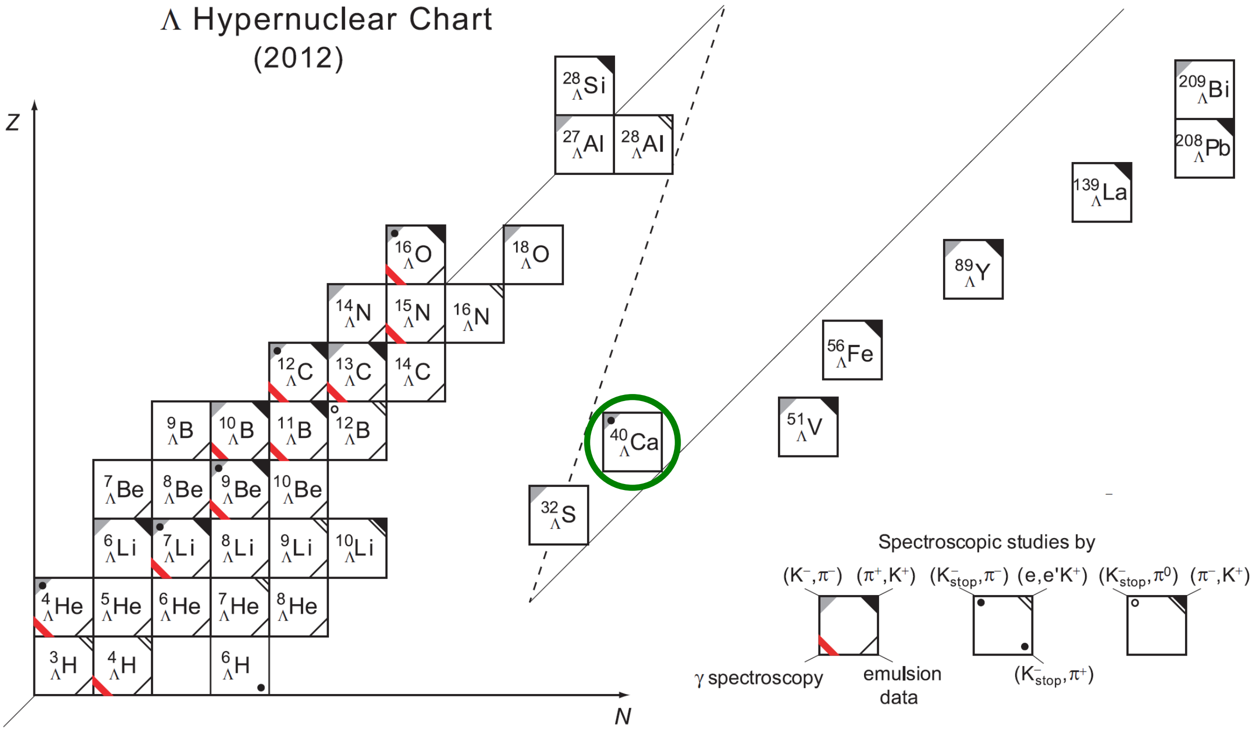

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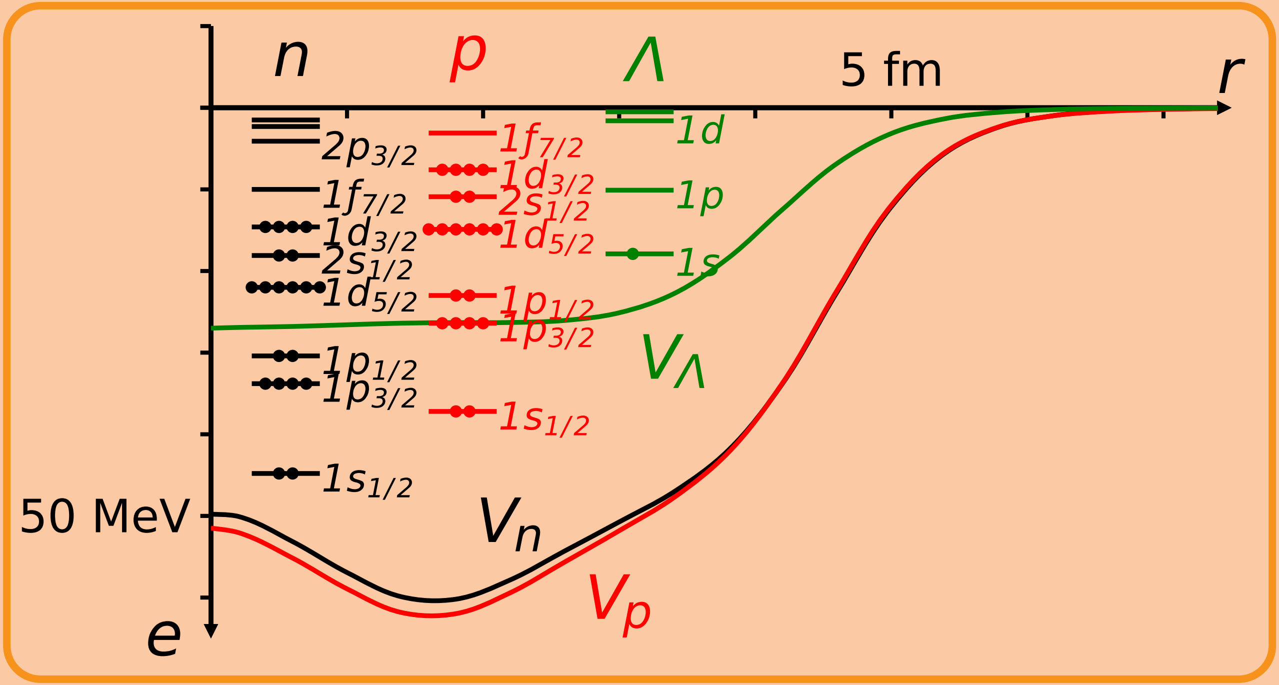
Hypernuclei: Typical Example: ${}^{40}_{\Lambda}\mathrm{Ca}$:



- Theoretical model:
  - Skyrme-Hartree-Fock (SHF) [Vautherin & Brink, PRC 5, 626 (1972)]
  - Standard *NN* force: SIII, SGII, SkI4, **SLy4**, . . .
  - Optimize *N*Λ Skyrme force

Extended SHF+BHF Model for Hypernuclei:

- Total energy of the hypernucleus:

$$E = \int \mathrm{d}^3 r \, \epsilon(r)$$

Energy density functional:

$$\epsilon = \epsilon_N[\tau_n, \tau_p, \rho_n, \rho_p, \boldsymbol{J}_n, \boldsymbol{J}_p] + \epsilon_\Lambda[\tau_\Lambda, \rho_\Lambda, \rho_N]$$

Local densities:

$$\rho_q = \sum_{i=1}^{N_q} |\phi_q^i|^2 \,, \quad \tau_q = \sum_{i=1}^{N_q} |\nabla \phi_q^i|^2 \,, \quad \boldsymbol{J}_q = \sum_{i=1}^{N_q} \phi_q^{i\,*} (\nabla \phi_q^i \times \boldsymbol{\sigma})/i$$

i: occupied states, N_q: number of particles $q = n, p, \Lambda$

- Nucleonic part: standard Skyrme force:

$$\begin{aligned}\epsilon_N \;=\;& \frac{1}{2m_N}\tau_N + \left[b_0\rho_N^2 - d_0(\rho_n^2+\rho_p^2)\right]/2 \\ &+\; b_1\rho_N\tau_N - d_1\left(\rho_n\tau_n+\rho_p\tau_p\right) + d_1\left(\boldsymbol{J}_n^2+\boldsymbol{J}_p^2\right)/2 \\ &-\left[b_2\rho_N\Delta\rho_N - d_2\left(\rho_n\Delta\rho_n+\rho_p\Delta\rho_p\right)\right]/2 \\ &+\left[b_3\rho_N^2 - d_3(\rho_n^2+\rho_p^2)\right]\rho_N^{\gamma}/3 \\ &-\; b_4\rho_N\nabla\cdot\boldsymbol{J}_N - d_4\left(\rho_n\nabla\cdot\boldsymbol{J}_n+\rho_p\nabla\cdot\boldsymbol{J}_p\right)+\epsilon_{\mathrm{Coul.}}\end{aligned}$$

- SHF Schrödinger equation:

$$\left[-\nabla\cdot\frac{1}{2m_q^*(r)}\nabla + V_q(r) - i\nabla W_q(r)\cdot(\nabla\times\boldsymbol{\sigma})\right]\phi_q^i(r) \;=\; -e_q^i\phi_q^i(r)$$

- SHF mean fields:

$$V_N \;=\; V_N^{\mathrm{SHF}} + \frac{\partial\epsilon_\Lambda}{\partial\rho_N} \quad , \quad V_\Lambda = \frac{\partial\epsilon_\Lambda}{\partial\rho_\Lambda} \quad , \quad W_\Lambda = 0$$

Empirical Skyrme $N\Lambda$ Force:

M. Rayet, Ann.Phys. 102, 226 (1976); Nucl.Phys. A367, 381 (1981)

$$\begin{aligned}\epsilon_\Lambda = \frac{\tau_\Lambda}{2m_\Lambda} &+ a_0\rho_\Lambda\rho_N + a_3\rho_\Lambda\rho_N^{1+\alpha} \\ &+ a_1\left(\rho_\Lambda\tau_N + \rho_N\tau_\Lambda\right) - a_2\left(\rho_\Lambda\Delta\rho_N + \rho_N\Delta\rho_\Lambda\right)/2 \\ &- a_4\left(\rho_\Lambda\nabla\cdot\boldsymbol{J}_N + \rho_N\nabla\cdot\boldsymbol{J}_\Lambda\right) \\ &+ c_0\rho_\Lambda^2 + c_3\rho_\Lambda^2\rho_N^\gamma + c_1\rho_\Lambda\tau_\Lambda - c_2\rho_\Lambda\Delta\rho_\Lambda\end{aligned}$$

Parameters $a_0, \ldots, a_4, \alpha, c_0, \ldots, c_3, \gamma$

(Together with SLy4 NN force)

We use $a_0, a_1, a_2, a_3, \alpha = \gamma = 1$

Effective mass:

$$\frac{1}{2m_\Lambda^*} = \frac{1}{2m_\Lambda} + a_1\rho_N + c_1\rho_\Lambda$$

Fit to Hypernuclear Data:

- The SHF mean-field approach is not suited for light nuclei, but one can hope that for $B_\Lambda = E(^{A-1}Z) - E(^{A}_{\Lambda}Z)$ important cancellations regarding the *nuclear* core structure occur. We explore this idea pragmatically.

- Hypernuclear data set (19 + 16 data points):
  - $B_\Lambda^{(s)}$ for $^{5,6,8}_{\Lambda}\mathrm{He}$, $^{7,8,9}_{\Lambda}\mathrm{Li}$, $^{7,8,10}_{\Lambda}\mathrm{Be}$, $^{9,10,11,12}_{\Lambda}\mathrm{B}$, $^{12,13,14}_{\Lambda}\mathrm{C}$, $^{15,16}_{\Lambda}\mathrm{N}$

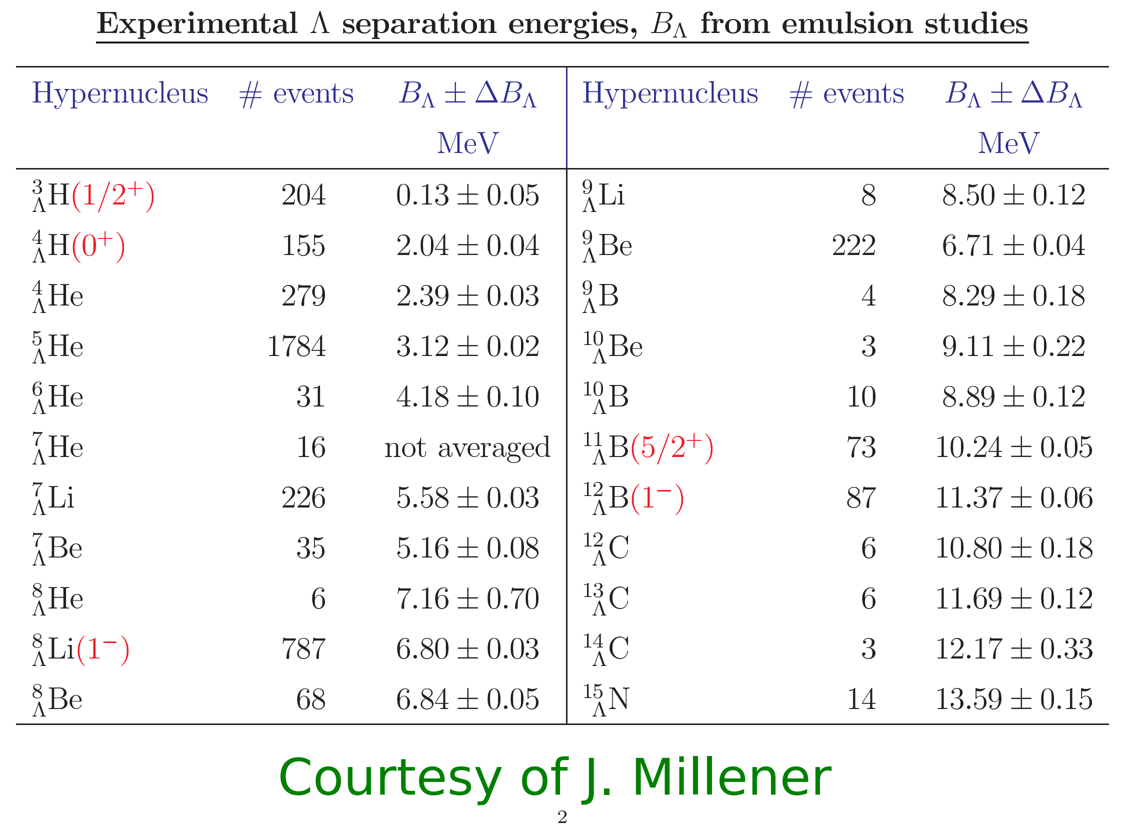
    Davis, NPA 754, 3 (2005)
  - $B_\Lambda(^{7}_{\Lambda}\mathrm{He}) = 5.68 \pm 0.03 \pm 0.25\ \mathrm{MeV}$

    JLAB Hall C, PRL 110, 012502 (2013)
  - $B_\Lambda^{(s,p,d,f,g)}$ for $^{28}_{\Lambda}\mathrm{Si}$, $^{89}_{\Lambda}\mathrm{Y}$, $^{139}_{\Lambda}\mathrm{La}$, $^{208}_{\Lambda}\mathrm{Pb}$

    Hashimoto & Tamura, PPNP 57, 564 (2006)

- Fit parameters a_0, a_1, a_2, a_3 → 'SLL4' $N\Lambda$ Force

Emulsion Data:



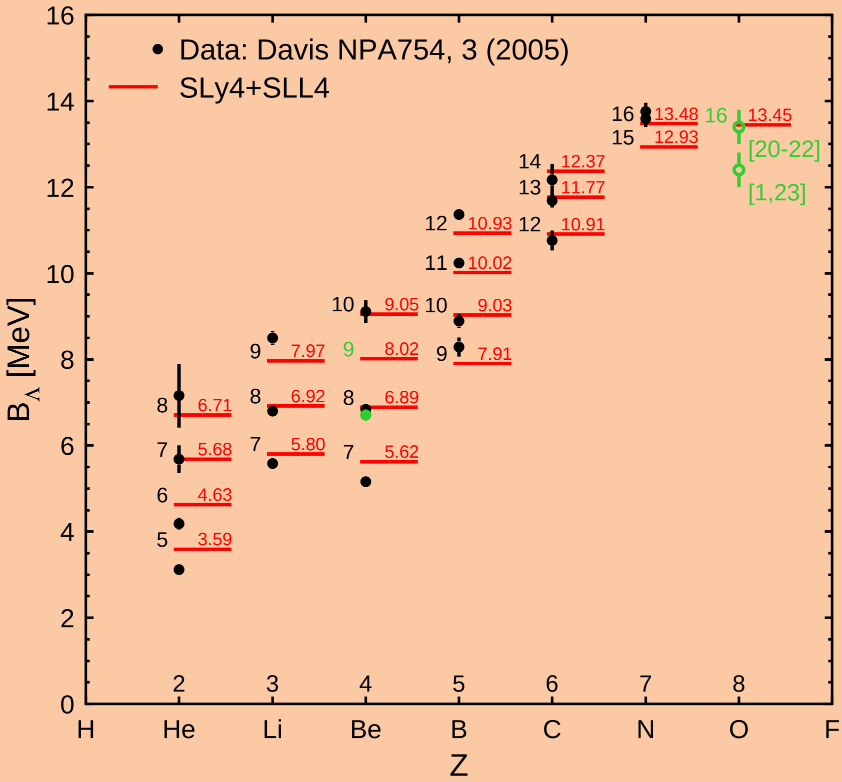
Experimental Λ separation energies, B_Λ from emulsion studies

Hypernucleus	# events	$B_\Lambda \pm \Delta B_\Lambda$ MeV	Hypernucleus	# events	$B_\Lambda \pm \Delta B_\Lambda$ MeV
${}^{3}_{\Lambda}$H(1/2$^+$)	204	0.13 ± 0.05	${}^{9}_{\Lambda}$Li	8	8.50 ± 0.12
${}^{4}_{\Lambda}$H(0$^+$)	155	2.04 ± 0.04	${}^{9}_{\Lambda}$Be	222	6.71 ± 0.04
${}^{4}_{\Lambda}$He	279	2.39 ± 0.03	${}^{9}_{\Lambda}$B	4	8.29 ± 0.18
${}^{5}_{\Lambda}$He	1784	3.12 ± 0.02	${}^{10}_{\Lambda}$Be	3	9.11 ± 0.22
${}^{6}_{\Lambda}$He	31	4.18 ± 0.10	${}^{10}_{\Lambda}$B	10	8.89 ± 0.12
${}^{7}_{\Lambda}$He	16	not averaged	${}^{11}_{\Lambda}$B(5/2$^+$)	73	10.24 ± 0.05
${}^{7}_{\Lambda}$Li	226	5.58 ± 0.03	${}^{12}_{\Lambda}$B(1$^-$)	87	11.37 ± 0.06
${}^{7}_{\Lambda}$Be	35	5.16 ± 0.08	${}^{12}_{\Lambda}$C	6	10.80 ± 0.18
${}^{8}_{\Lambda}$He	6	7.16 ± 0.70	${}^{13}_{\Lambda}$C	6	11.69 ± 0.12
${}^{8}_{\Lambda}$Li(1$^-$)	787	6.80 ± 0.03	${}^{14}_{\Lambda}$C	3	12.17 ± 0.33
${}^{8}_{\Lambda}$Be	68	6.84 ± 0.05	${}^{15}_{\Lambda}$N	14	13.59 ± 0.15

Courtesy of J. Millener

Caution: In some cases only few events

Results: Light Single-Λ Hypernuclei:

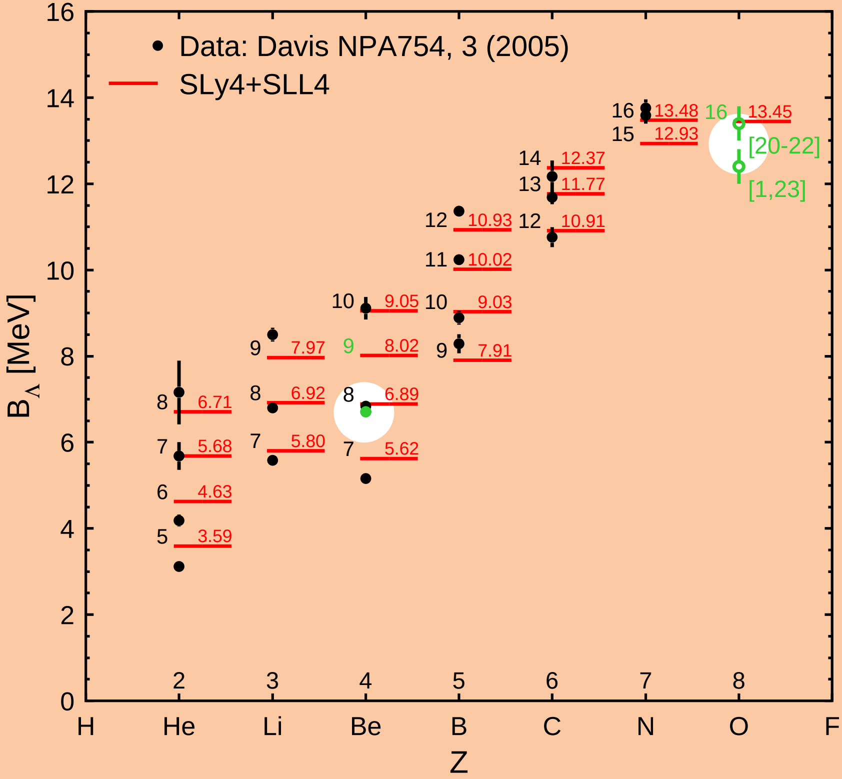


↪ Reasonable fit with $\langle \Delta B_\Lambda \rangle_{\mathrm{rms}} \approx 0.4$ MeV

Exceptions: overbound ${}^{16}_{\Lambda}$O ? and cluster nucleus ${}^{9}_{\Lambda}$Be :

large $\langle R_N^2 \rangle \rightarrow$ low $\rho_N \rightarrow$ low $V_\Lambda \rightarrow$ low B_Λ

Results: Light Single-Λ Hypernuclei:



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Exceptions: overbound ${}^{16}_{\Lambda}$O ? and cluster nucleus ${}^{9}_{\Lambda}$Be :

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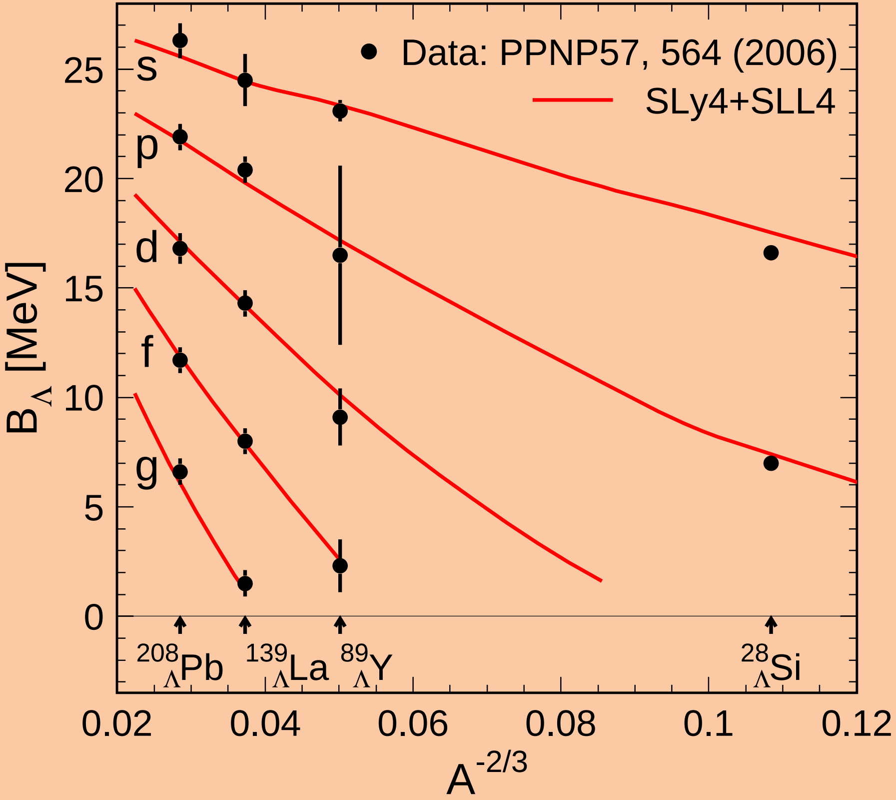
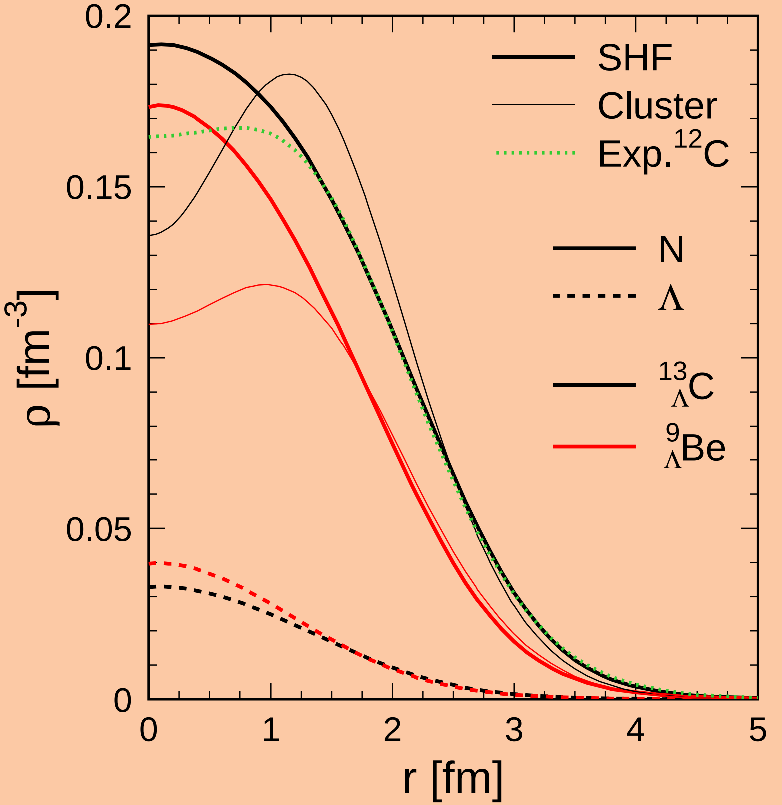
Results: Heavy Single-Λ Hypernuclei:



- Exp. data too low by ~ 0.6 MeV ? (J. Millener, H. Tamura): Emulsion $^{12}_{\Lambda}$C used for normalization is not accurate

Results: Density Distributions:

Compare SHF and cluster model, PTEP 013D01 (2014), results:



- Cluster structure causes larger radius and lower central nucleon density
- Exp. data (^{12}C) lie between cluster and SHF predictions

Atom. Data Nucl. Data 36, 495 (1987)

Results: $N\Lambda$ Skyrme Parameters:

Force		a_0	a_1	a_2	a_3	α	Δ_{B_Λ}[MeV]
SLL4		-316.0	23.25	13.88	650.0	1	0.40
SLL4’		-326.0	20.50	20.75	705.0	1	0.35
RAY13	[1]	-280.7	0	-16.25	750.00	1	1.02
YBZ6	[2]	-352.3	45.00	27.70	750.00	1	0.75
SKSH2	[3]	-290.0	0.35	10.68	693.75	1	0.66
YMR	[4]	-1056.2	26.25	35.00	1054.20	1/8	0.47
HPΛ2	[5]	-302.8	23.72	29.85	514.25	1	1.00

SLL4’ : $\Delta B_\Lambda\big|_{\mathrm{PPNP57}}$ corrected by 0.6 MeV

[1] M. Rayet, Ann. Phys. 102, 226 (1976); Nucl. Phys. A367, 381 (1981)

[2] Y. Yamamoto, H. Bandō, J. Žofka, Prog. Theor. Phys. 80, 757 (1988)

[3] F. Fernández, T. López-Arias, C. Prieto, Z. Phys. A334, 349 (1989)

[4] Y. Yamamoto, T. Motoba, T. Rijken, Prog. Theor. Phys. Suppl. 185, 72 (2010)

[5] N. Guleria, S. K. Dhiman, R. Shyam, Nucl. Phys. A886, 71 (2012)

Summary:

- Optimized Skyrme parameters for currently known light and heavy hypernuclei: $\Delta B_\Lambda \approx 0.4$ MeV
- Experimental open problems: ${}^{12}_{\Lambda}$C, ${}^{16}_{\Lambda}$O

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Outlook:

- More sophisticated Skyrme model: Pairing, Deformation, ...
- HFB etc. for dripline (hyper)nuclei
- Extension to double Λ hypernuclei
- Extend comparison with cluster model