ΛΛ Interaction and Neutron stars

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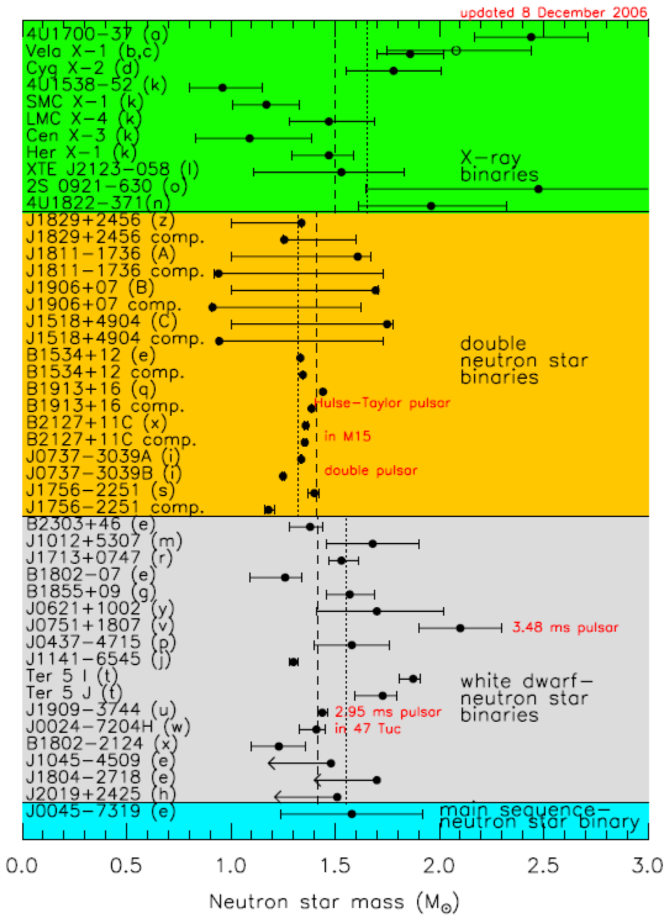
1. Hyperonization puzzle

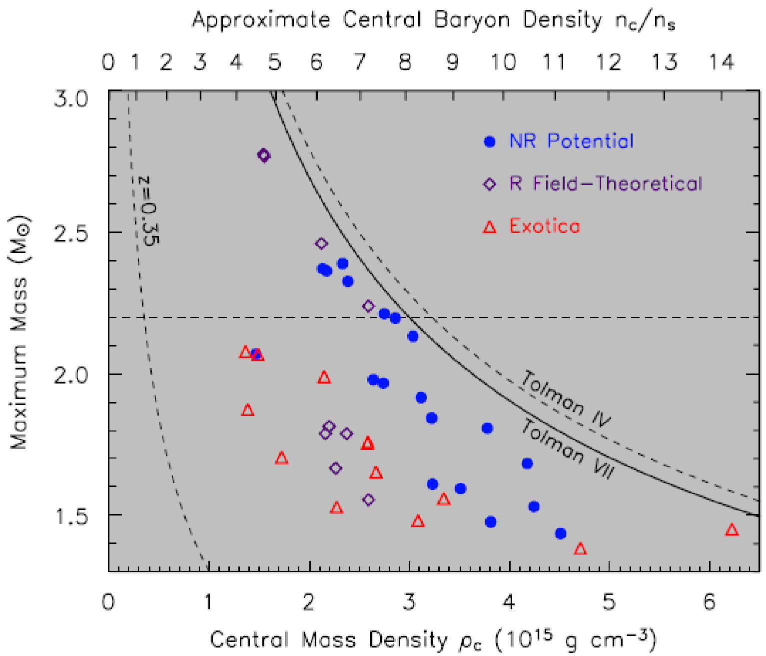
● Status of neutron star mass measurement

J.M. Lattimer, M. Prakash / Physics Reports 442 (2007) 109–165



- Canonical range: (1.2-1.6) $M_\odot$
- Out of canonical range: large errors
- Exceptions: Large mass with small errors
  - * PSR J1614-2230: (1.97±0.04) $M_\odot$
  - * PSR J0348+0432: (2.01±0.04) $M_\odot$

● Theoretical uncertainties



J.M. Lattimer and M. Prakash, PRL 94 (2005) 1101

- Nucleon-NR potential: (1.4-2.4) $M_\odot$
- Nucleon-RMF: (1.5-2.8) $M_\odot$
- Nucleon & Hyperon-RMF: (1.4-1.8) $M_\odot$

- Hyperonization puzzle

Observation leaves narrow room for the existence of hyperons.

Hyperons should be kicked out?

How?

Other breakthrough?

2. Motif

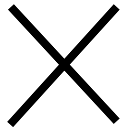
- What solutions possible
  - β-equilibrium is a wrong assumption
  - Modified gravity
  - Dark matter
  - Uncertainties in the hyperon interactions
- Hyperon interactions in RMF
  - Hyperon-Nucleon (YN): optical potential depth + quark counting
  - Hyperon-Hyperon (YY): SU(6) symmetry
- We try more realistic approach
  - YN and YY potential fitted to hyper-nuclear data
  - Skyrme force (NR potential): YN and YY as well as NN available

3. Skyrme force

- **NN interaction: more than 100 models**

$$
\begin{aligned}
v_{NN}(\boldsymbol{r}_{ij}) \;=\; & t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}t_1(1+x_1P_\sigma)[\boldsymbol{k}_{ij}^2\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij}^2] \\
& + t_2(1+x_2P_\sigma)\boldsymbol{k}_{ij}\cdot\delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij} + \frac{1}{6}t_3(1+x_3P_\sigma)\rho_N^\epsilon(\boldsymbol{R})\delta(\boldsymbol{r}_{ij}) \\
& + iW_0\boldsymbol{k}_{ij}\cdot\delta(\boldsymbol{r}_{ij})(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\times\boldsymbol{k}_{ij},
\end{aligned}
$$

=



  - Contact types with 10 parameters
  - Derivatives up to 2nd order plus density-dependent term

- **Models in the work**

NN model	ρ_0	B	S_v	L	K	m_N^*/m_N	$M_{\max}/M_\odot$
SLy4	0.160	16.0	32.0	45.9	230	0.694	2.06
SkI4	0.160	16.0	29.5	60.4	248	0.649	2.19
SGI	0.155	15.9	28.3	63.9	262	0.608	2.25

  - Distinct compression modulus
  - Satisfy mass-radius relation reasonably

- YN interaction: more than 10 models

$$v_{N\Lambda} = u_0(1+y_0P_\sigma)\delta(\boldsymbol{r}_{N\Lambda}) + \frac{1}{2}u_1\left[\boldsymbol{k}_{N\Lambda}^2\delta(\boldsymbol{r}_{N\Lambda}) + \delta(\boldsymbol{r}_{N\Lambda})\boldsymbol{k}_{N\Lambda}^2\right]$$
$$+u_2\boldsymbol{k}_{N\Lambda}\cdot\delta(\boldsymbol{r}_{N\Lambda})\boldsymbol{k}_{N\Lambda} + \frac{3}{8}u_3'(1+y_3P_\sigma)\rho_N^\gamma\left(\frac{\boldsymbol{r}_N+\boldsymbol{r}_\Lambda}{2}\right)\delta(\boldsymbol{r}_{N\Lambda}),$$

  - Contact types with 7 parameters
  - Derivatives up to 2nd order plus density-dependent term

- Models in the work

$N\Lambda$ model	γ	u_0	u_1	u_2	u_3'	y_0	y_3	$U_\Lambda^{\rm opt}$
HPΛ2	1	−399.946	83.426	11.455	2046.818	−0.486	−0.660	−31.23
OΛ2	1/3	−417.7593	1.5460	−3.2671	1102.2221	−0.3854	−0.5645	−28.27
YBZ6	1	−372.2	100.4	79.60	2000.	−0.107	0.	−29.73

HPΛ2, OΛ2: N. Guleria, S.K. Dhiman, R. Shyam, Int. J. Mod. Phys. E 23 (2014) 1450026,
YBZ6: Y. Yamamoto, H. Bando, Z. Zofka, PTP 80 (1988) 757

  - Reasonable depth of optical potential (~ -30 MeV)

- YY interaction: about 5 models

$$v_{\Lambda\Lambda}(\boldsymbol{r}_{ij}) = \lambda_0\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}\lambda_1\left[\boldsymbol{k}_{ij}^2\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij}^2\right] + \lambda_2\boldsymbol{k}_{ij}\cdot\delta(\boldsymbol{r}_{ij})\boldsymbol{k}_{ij} + \lambda_3\rho_N^{\alpha}(\boldsymbol{R})\delta(\boldsymbol{r}_{ij}),$$

  - Contact types with 5 parameters
  - Derivatives up to 2nd order plus density-dependent term

- Models in the work

ΛΛ model	λ_0	λ_1	λ_2	λ_3	α
SΛΛ1	−312.6	57.5	0	0	–
SΛΛ2	−437.7	240.7	0	0	–
SΛΛ3	−831.8	922.9	0	0	–
SΛΛ1′	−37.9	14.1	0	0	–
SΛΛ3′	−156.4	347.2	0	0	–

SΛΛ1,2,3: D.E. Lanskoy, PRC 58 (1998) 3351,
SΛΛ1′,3′ : F. Minato, S. Chiba, NPA 856 (2011) 55

Nuclei	$B_{\Lambda\Lambda}$(SΛΛ1)	$B_{\Lambda\Lambda}$(SΛΛ2)	$B_{\Lambda\Lambda}$(SΛΛ3)	$B_{\Lambda\Lambda}$(Exp.)
$^{6}_{\Lambda\Lambda}$He	11.88	9.25	7.60	6.91 ± 0.16 [19]
$^{10}_{\Lambda\Lambda}$Be	19.78	18.34	15.19	14.94 ± 0.13 [20]
$^{11}_{\Lambda\Lambda}$Be	20.55	19.26	16.27	20.49 ± 1.15 [19]
$^{12}_{\Lambda\Lambda}$Be	21.10	19.97	17.18	22.23 ± 1.15 [19]
$^{13}_{\Lambda\Lambda}$B	21.21	20.26	17.76	23.30 ± 0.70 [19]

  - Each model is partially good to reproduce empirical data

4. Our results

- Energy density: matrix element of Hamiltonian

$$\mathcal{H} = \mathcal{H}_N + \mathcal{H}_\Lambda + \mathcal{H}_e + \mathcal{H}_\mu$$

$$\begin{aligned}\mathcal{H}_N &= \sum_{i=n,p} \frac{\hbar^2}{2m_N}\tau_i + \rho_N(\tau_n+\tau_p)\left[\frac{t_1}{4}\left(1+\frac{x_1}{2}\right)+\frac{t_2}{4}\left(1+\frac{x_2}{2}\right)\right] \\ &+ \sum_{i=n,p}\tau_i\rho_i\left[-\frac{t_1}{4}\left(1+\frac{x_1}{2}\right)+\frac{t_2}{4}\left(1+\frac{x_2}{2}\right)\right] \\ &+\frac{t_0}{2}\left[\left(1+\frac{x_0}{2}\right)\rho_N^2-\left(\frac{1}{2}+x_0\right)(\rho_n^2+\rho_p^2)\right] \\ &+\frac{t_3}{12}\left[\left(1+\frac{x_3}{2}\right)\rho_N^2-\left(\frac{1}{2}+x_3\right)(\rho_n^2+\rho_p^2)\right]\rho_N^\epsilon,\end{aligned}$$

$$\begin{aligned}\mathcal{H}_\Lambda &= \frac{\hbar^2}{2m_\Lambda}\tau_\Lambda + u_0\left(1+\frac{1}{2}y_0\right)\rho_N\rho_\Lambda+\frac{1}{4}(u_1+u_2)(\tau_\Lambda\rho_N+\tau_N\rho_\Lambda) \\ &+\frac{3}{8}u_3'\left(1+\frac{1}{2}y_3\right)\rho_N^{\gamma+1}\rho_\Lambda+\mathcal{H}_{\Lambda\Lambda},\end{aligned}$$

$$\mathcal{H}_{\Lambda\Lambda} = \frac{\lambda_0}{4}\rho_\Lambda^2+\frac{1}{8}(\lambda_1+3\lambda_2)\rho_\Lambda\tau_\Lambda+\frac{\lambda_3}{4}\rho_\Lambda^2\rho_N^\alpha,$$

- Functions of densities of nucleon and Λ

- Pressure: thermodynamic relation

$$P = \rho \frac{\partial}{\partial \rho} \left(\frac{\mathcal{E}}{\rho} \right)$$

- Conditions for neutron star matter
  - Baryon number conservation
  - Charge neutrality
  - β–equilibrium

⇓

- Density of each particle determined self-consistently
- Calculate energy density
- Calculate pressure
- Equation of state: pressure vs energy density

● Maximum mass

	SLy4 2.06		SkI4 2.19			SGI 2.25		
	HPΛ2	OΛ2	HPΛ2	OΛ2	YBZ6	HPΛ2	OΛ2	YBZ6
noΛΛ	1.51	1.08	1.52	1.19	1.80	1.52	1.22	1.79
SΛΛ1	1.40	1.00	1.41	1.12	1.70	1.42	1.16	1.69
SΛΛ2	1.58	1.28	1.57	1.30	1.79	1.57	1.31	1.77
SΛΛ3	1.85	1.57	1.87	1.62	2.03	1.88	1.65	2.04
SΛΛ1′	1.51	1.08	1.51	1.18	1.79	1.51	1.21	1.78
SΛΛ3′	1.76	1.43	1.76	1.47	1.97	1.77	1.49	1.96
FRF	1.61	1.22	1.60	1.25	1.86	1.59	1.26	1.84

FRF: E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, Y. Yamamoto, PRC 66 (2002) 024007

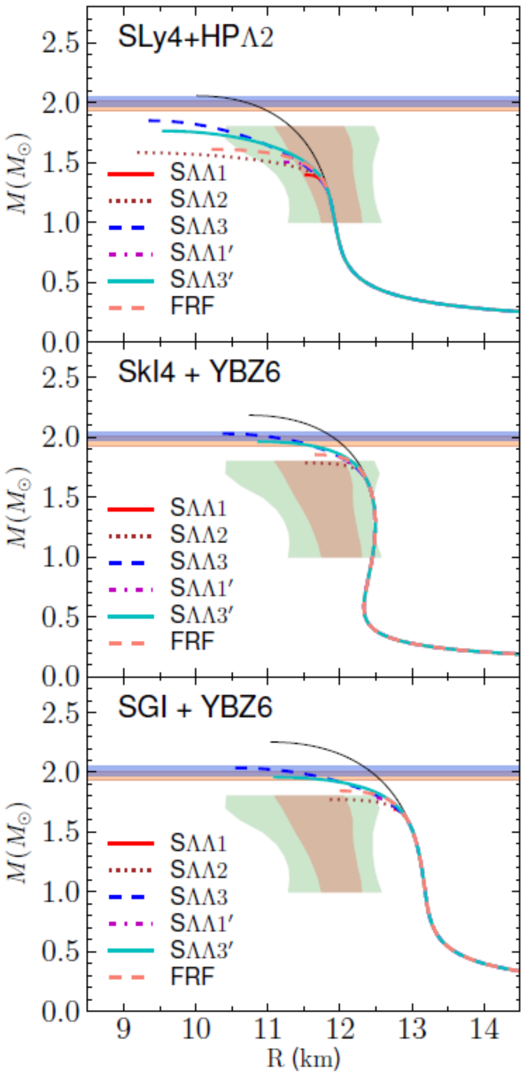
- Stiffness of interaction

  * NN: SGI > SkI4 > SLy4

  * YN: YBZ6 > HPΛ2 > OΛ2

  * YY: SΛΛ3 > SΛΛ3′ > FRF > SΛΛ2 > SΛΛ1′ > SΛΛ1

● Mass-radius relation



- Horizontal region: mass range of PSR J1614-2230 and PSR J0348+0432
- Green and brown region: mass-radius range suggested by A.W. Stein, J.M. Lattimer, ApJ 722 (2010) 33

- Hyperonization puzzle can be solved

  * SLy4+HPΛ2: SΛΛ3

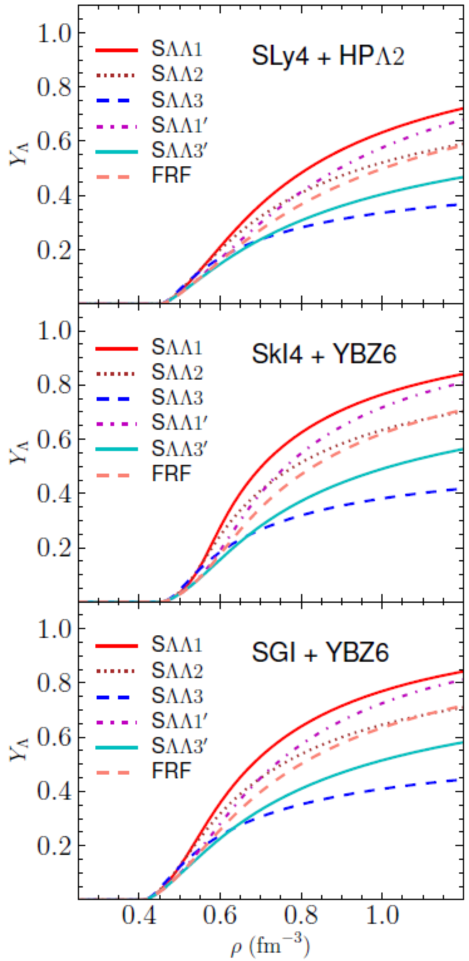
  * SkI4+YBZ6: SΛΛ3, SΛΛ3′, FRF

  * SGI+YBZ6: SΛΛ3, SΛΛ3′, FRF

- Λ population

  - Density where Λ starts to be created
  - Mass of the star with central density equal to critical density

	SLy4		SkI4			SGI		
	HPΛ2	OΛ2	HPΛ2	OΛ2	YBZ6	HPΛ2	OΛ2	YBZ6
$\rho_{\rm crit}$	0.453	0.380	0.374	0.340	0.455	0.352	0.325	0.412
$M/M_\odot$	1.17	0.90	1.19	1.04	1.51	1.21	1.08	1.47



  - Significant dependence on the model
  - The stiffer, the lesser
  - Neutron star cooling is sensitive to the internal composition: test the hyperonization puzzle with cooling curve

5. Summary

- Focus: the role of YN and YY interaction
- Skyrme force models: reproduce empirical data well
- YN and YY interactions: can be as important as NN
- Hyperon: can be compatible with observation
- Required improvements
  - $\Lambda\Lambda$ interactions: still poorly constrained
  - More and better statistics of experimental data essential

$\Lambda\Lambda$ model	λ_0	λ_1	λ_2	λ_3	α
S$\Lambda\Lambda$1	−312.6	57.5	0	0	–
S$\Lambda\Lambda$2	−437.7	240.7	0	0	–
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S$\Lambda\Lambda$1′	−37.9	14.1	0	0	–
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Thank you.