

## **Experiments on Form Factors**

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Int. Workshop e+e- collisions from Phi to Psi Sept. 23-26, 2015, Hefei



## **Experiments on Form Factors**

How ? What ? Why ?





## **Experiments on Form Factors**

How?

What?

Why? did I accept to give this talk?





# Outline

- Introduction to Baryon Form Factors
- Measurements of Space-like Form Factors
  - Proton
  - Neutron
- Measurements of Time-like Form Factors
  - Proton
  - Neutron
  - Hyperons
- Outlook

3



- Electromagnetic Form Factors (EMFF) of hadrons are among the most basic quantities containing information about hadron internal structure.
- The electromagnetic structure of a hadron with spin S is given by 2S+1 Form Factors.
- Elastic Form Factors contain information the hadron ground state.



Electromagnetic Form Factors provide the most direct access to the spatial charge and magnetisation distributions.

EMFF of baryons are studied in:

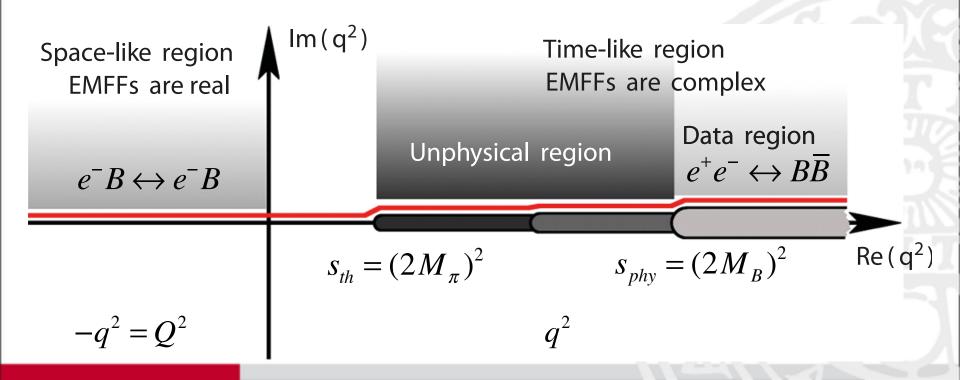
- elastic  $e^-B \rightarrow e^-B$  scattering; Space-Like FF
- baryon-antibaryon production/annihilation,  $e^+e^- \leftrightarrow B\overline{B}$ ; Time-Like FF

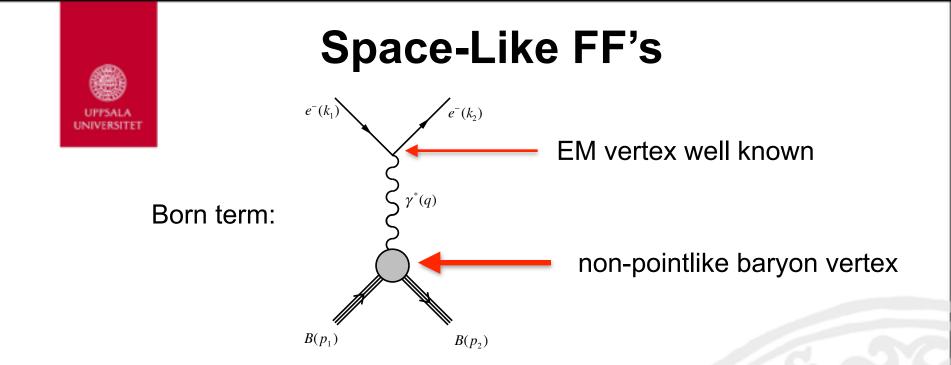


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- elastic  $e^-B \rightarrow e^-B$  scattering; Space-Like FF
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Baryon vertex matrix element:  $\Gamma^{\mu} = F_1^B(Q^2)\gamma^{\mu} + \frac{\kappa}{2M_B}F_2^B(Q^2)i\sigma^{\mu\nu}q_{\nu}$ 

 $F_1(Q^2)$  and  $F_2(Q^2)$  are related to non-helicity-flip and helicity-flip part of the hadronic current and are the Dirac and Pauli EMFF's, respectively.

Normalisation:  $F_1(0) = Z =$  baryon charge

 $F_2(0) = 1$ 



It is convenient to rewrite the Dirac and Pauli EEMFF's as

$$G_E = F_1 - \tau F_2$$
;  $\tau = \frac{Q^2}{4M_B^2}$   
 $G_M = F_1 + F_2$ 

These are the Sachs Form Factors with normalisation  $G_E(0) = Z$  and  $G_M(0) = 1 + \kappa = \mu_B$  = baryon magnetic moment.

The Sachs FF's correspond to the Fourier transformations of the charge and magnetic spatial distributions in the Breit frame ( $q = (0, \vec{q})$ ). (The situation is more complicated in reality.)

The Space-Like EEMFF is obtained from elastic electron scattering in terms of  $G_E^2$  and  $G_M^2$  (**Rosenbluth separation**) as:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E_e}{E_{beam}} \frac{1}{1+\tau} \left(G_E^2 + \frac{\tau}{\varepsilon}G_M^2\right);$$
  

$$\varepsilon = \frac{1}{1+2(1+\tau)\tan^2\theta_e/2} = \text{virtual photon polarisation}$$

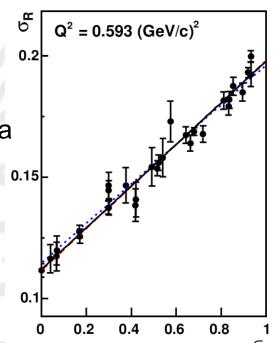


The linear dependence on  $\tau$  in  $\epsilon$  makes it possible to define a reduced cross section as

$$\sigma_{red} = \frac{\varepsilon(1+\tau)}{\tau} \frac{E_e}{E_{beam}} \frac{d\sigma}{d\Omega} / \left(\frac{d\sigma}{d\Omega}\right)_{Mott} = G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$

=>  $\sigma_{red}$  is expected to have linear dependence on  $\varepsilon$  at a given  $Q^2$  with a slope proportional to  $G_E^2$  and with the intercept  $G_M^2$ .

 $G_E^2$  and  $G_M^2$  are extracted from fits to experimental data by measuring the cross section at a given  $Q^2$  at different energies ( $\varepsilon$ ).





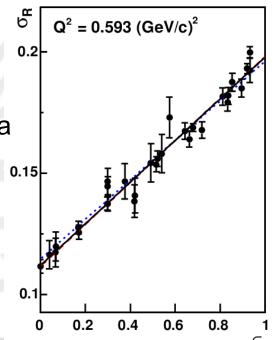
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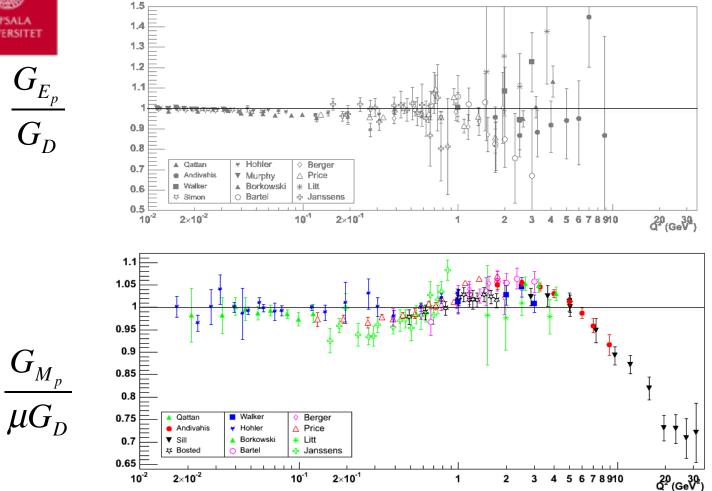
 $G_E^2$  difficult to measure at high  $Q^2$ 

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UP7SALA UNIVERSITET In the static limit is  $G_{M_p} = \mu_p G_{E_p}$ , and this is roughly consistent with data. Rosenbluth separation:

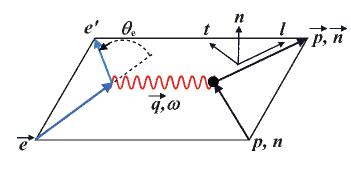


The  $q^2$  dependence of the proton EMFF is well characterised by a dipole behaviour:  $G_D = \left(1 + \frac{Q^2}{.71}\right)^{-2}$ 



An alternative to extract EEMFF is offered by polarisation measurements.

A longitudinally polarised electron transfers its polarisation in elastic scattering with non-zero  $P_l$  and  $P_t$  components:



$$\frac{G_E}{G_M} = -\frac{P_t}{P_l} \frac{(E_{beam} + E_e)}{2M_B} \tan \frac{\theta_e}{2}$$

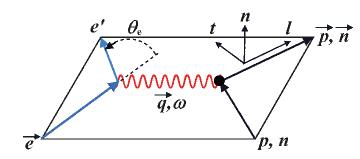




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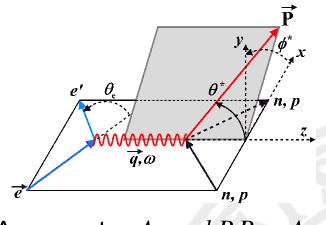
 $A = \frac{\sigma_{+} - \sigma_{-}}{\sigma_{+}} =$ 

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Polarised target:



Asymmetry: 
$$A_{exp} = hP_eP_{target}A$$

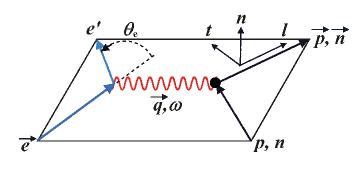
$$-\frac{2\sqrt{\tau(1+\tau)}\tan\frac{\theta_e}{2}}{G_E^2 + \frac{\tau}{\varepsilon}G_M^2} \left(\sin\theta^*\cos\phi^*G_E G_M + \sqrt{\tau(1+(1+\tau))}\tan^2\frac{\theta_e}{2}\cos\theta^*G_M^2\right)$$

$$\theta^* = \pi / 2, \phi^* = 0,180 \Longrightarrow A_y = -\frac{2\sqrt{\tau(1+\tau)}\tan\frac{\theta_e}{2}\frac{G_E}{G_M}}{\left(\frac{G_E}{G_M}\right)^2 + \frac{\tau}{\varepsilon}}$$



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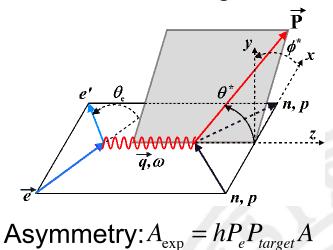


$$\frac{G_E}{G_M} = -\frac{P_t}{P_l} \frac{(E_{beam} + E_e)}{2M_B} \tan \frac{\theta_e}{2}$$

• Many systematical effects cancel when taking ratios.

• Higher sensitivity to  $G_E \otimes$ .

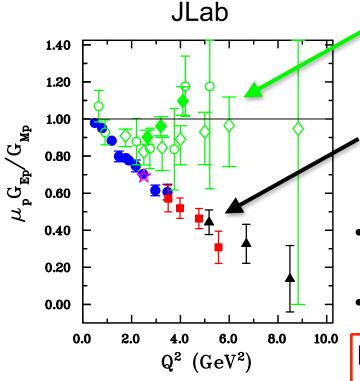
Polarised target:



$$A = \frac{\sigma_{+} - \sigma_{-}}{\sigma_{+} + \sigma_{-}} =$$

$$-\frac{2\sqrt{\tau(1+\tau)}\tan\frac{\theta_{e}}{2}}{G_{E}^{2} + \frac{\tau}{\varepsilon}G_{M}^{2}} \left(\sin\theta^{*}\cos\phi^{*}G_{E}G_{M} + \sqrt{\tau(1+(1+\tau))}\tan^{2}\frac{\theta_{e}}{2}\cos\theta^{*}G_{M}^{2}\right)$$

$$\theta^{*} = \pi/2, \phi^{*} = 0,180 \Rightarrow A_{y} = -\frac{2\sqrt{\tau(1+\tau)}\tan\frac{\theta_{e}}{2}\frac{G_{E}}{G_{M}}}{\left(\frac{G_{E}}{G_{M}}\right)^{2} + \frac{\tau}{\varepsilon}} \qquad 10$$



Rosenbluth separation

polarisation measurements

Why this difference?

- Effect from two-photon exchange? Polarisation expt's much less sensitive
- Radiation effects?

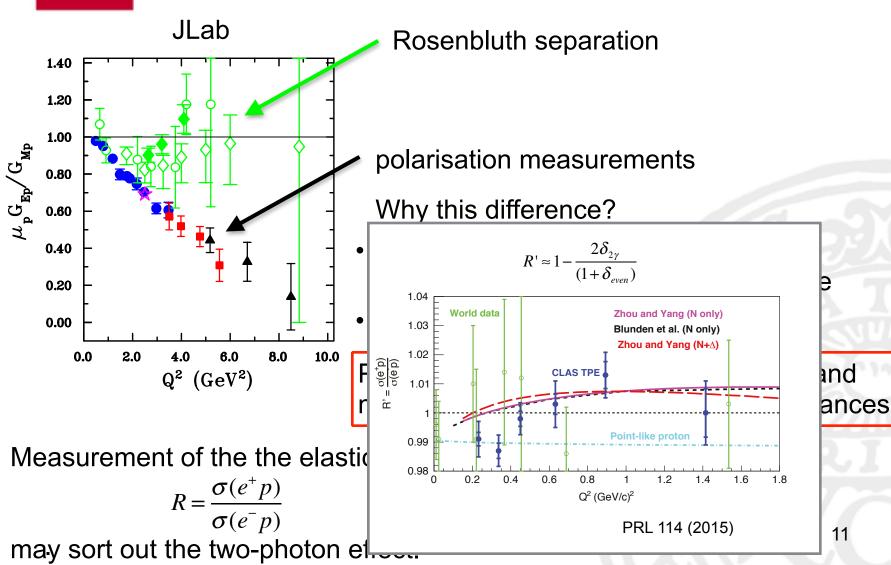
Points to a difference between charge and magnetisation distributions at short distances.

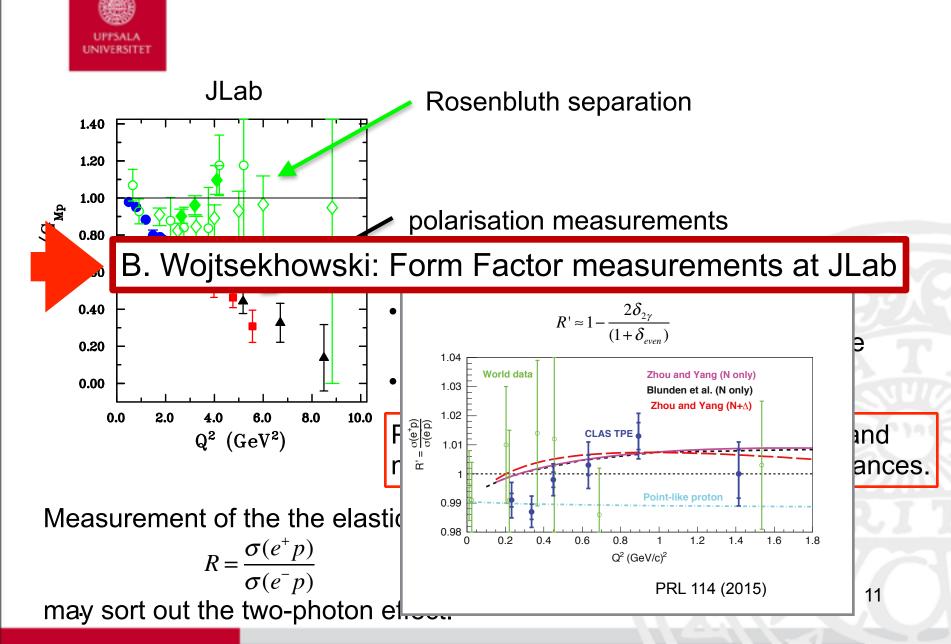
Measurement of the the elastic cross section ratio:

$$R = \frac{\sigma(e^{-}p)}{\sigma(e^{-}p)}$$
  
may sort out the two-photon effect

 $-(a^+-a)$ 





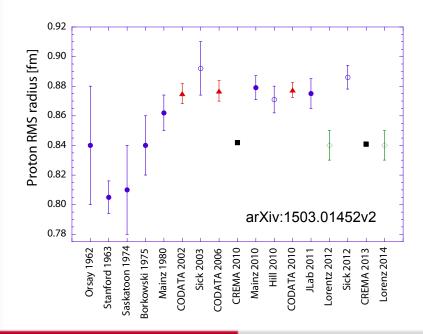




Non rel.: 
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left| \int_{V} \rho(\mathbf{r}) e^{i\mathbf{q}\mathbf{r}} d\mathbf{r} \right|$$
$$G_{E_p} = 1 - \frac{Q^2}{6} \left\langle r_{E_p}^2 \right\rangle + \frac{Q^4}{120} \left\langle r_{E_p}^4 \right\rangle \cdots$$

Low  $Q^2$ :  $\frac{dG_{E_p}}{dQ^2} \cong -\frac{1^2}{6} \langle r_{E_p}^2 \rangle \Big|_{Q^2=0}$ 

=> The proton RMS radius is obtained from the slope

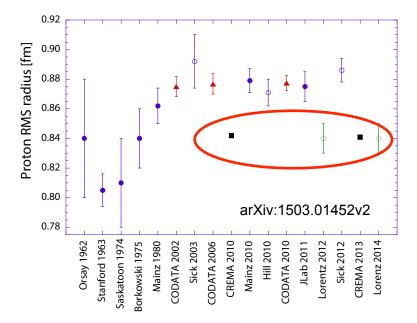




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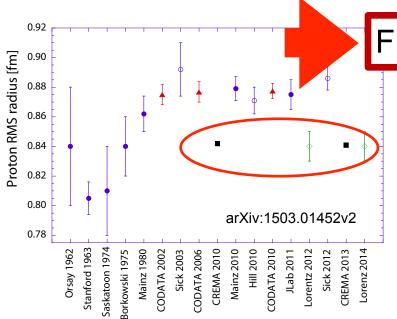
=>=> muonic hydrogen measurements at PSI give a 4% (7 $\sigma$ ) smaller proton RMS radius.



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=> The proton RMS radius is obtained from the slope



#### Friday morning session

=>=> muonic hydrogen measurements at PSI give a 4% (7 $\sigma$ ) smaller proton RMS radius.



## Neutron SL EEMFF

 $G_{En}$ : Earlier extracted from elastic ( $e^{-}$ , d) scattering. Now polarisation measurements.

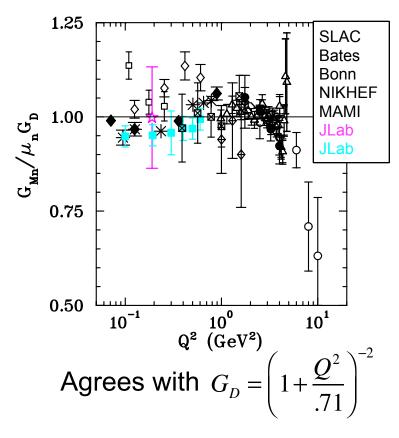
The slope as Q<sup>2</sup>  $\longrightarrow$  0 determines, in principle,  $\langle r_{E_n}^2 \rangle$  (*n*,*e*<sup>-</sup>), however, more precise (-.1161±.0022 fm<sup>2</sup>) (PDG)

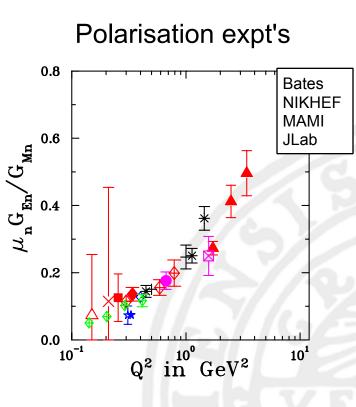
 $G_{Mn}$ : Earlier Rosenbluth separation from quasi-free ( $e^{-},n$ ) scattering on deuterons. Requires large corrections. Now polarisation measurements.

Polarisation measurements use recoil polarisation and double polarisation measurements deuteron and <sup>3</sup>He targets.



## Neutron SL EEMFF





 $G_{En} \neq G_{Ep}$ 

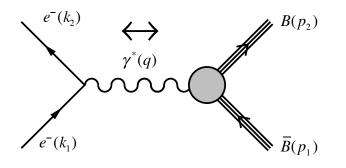
=> similarity between  $G_{Mn}$  and  $G_{Mp}$ 

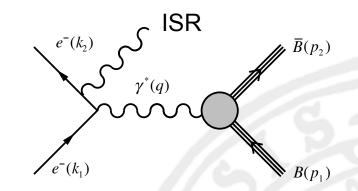
arXiv:1503.01452v2



**Time-Like FF's** 

 $e^+e^- \leftrightarrow \overline{B}B$ 

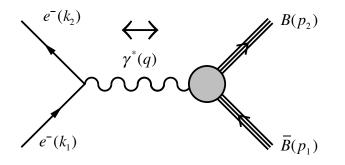


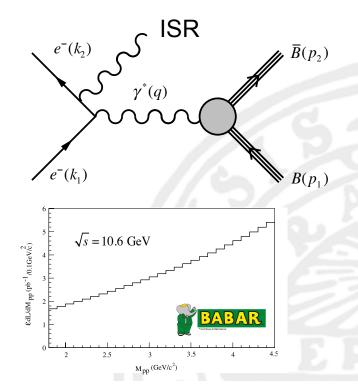




**Time-Like FF's** 

 $e^+e^- \leftrightarrow \overline{B}B$ 





Non-zero momentum of final state particles at threshold.



## **Time-Like Form Factors**

• Time-like FF's are complex:

 $\operatorname{Re}\left[G_{E}(q^{2})G_{M}^{*}(q^{2})\right] = \left|G_{E}(q^{2})\right| \left|G_{M}(q^{2})\right| \cos \Delta \phi$  $\operatorname{Im}\left[G_{E}(q^{2})G_{M}^{*}(q^{2})\right] = \left|G_{E}(q^{2})\right| \left|G_{M}(q^{2})\right| \sin \Delta \phi$ 

 $\Delta \phi$  = the relative phase between  $G_E$  and  $G_M$ .

- => Three observables determine the Time-Like Form Factors.
- The relative phase between  $G_E$  and  $G_M$  gives polarisation effects on the final state even when the initial state is unpolarised.



Time-Like Elastic FF can be extracted from the differential cross sections in analogy with the Rosenbluth technique.

$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2 \beta C}{4q^2} \left( \left| G_M \right|^2 \left( 1 + \cos^2 \theta \right) + \frac{1}{\tau} \left| G_E \right|^2 \sin^2 \theta \right);$$
  
$$\tau = \frac{q^2}{4m_B^2}, \ \beta = \sqrt{1 - 1/\tau}, \ C = \text{Coulomb factor} = \frac{y}{(1 - e^{-y})}, \ y = \pi \alpha / \beta$$

A measurement of the differential cross section at one energy is sufficient to extract the modulii of  $|G_E|$  and  $|G_M| \stackrel{\textcircled{o}}{=} .$ 

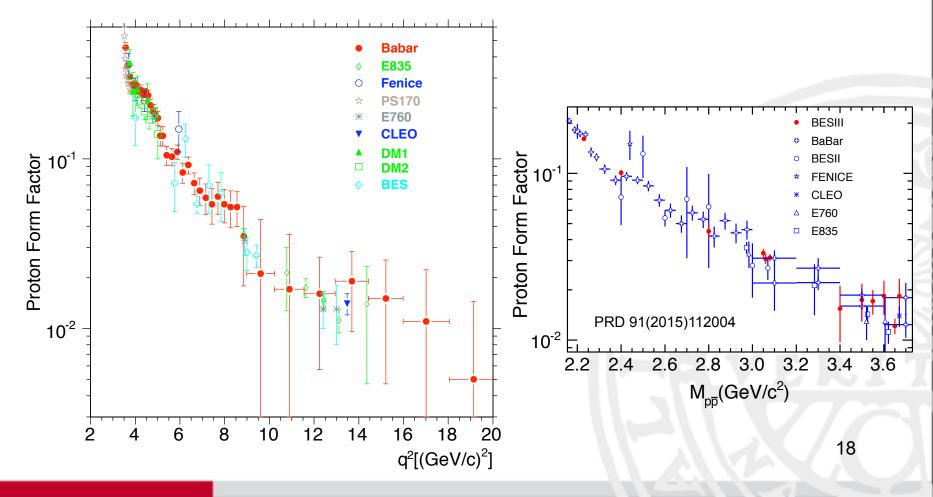
The total cross section gives access to effective form factor:

$$\sigma_{tot} = \frac{4\pi\alpha^2\beta C}{3q^2} \left[ \left| G_M \right|^2 + \frac{\left| G_E \right|^2}{2\tau} \right] \Leftrightarrow \left| G_{eff} \right| = \left( \frac{\sigma_{tot}}{4\pi\alpha^2\beta C / 3q^2} \right)^{\frac{1}{2}}$$

At threshold:  $\tau = 1$  and  $|G_E| = |G_M|$ , The Coulomb factor leads to a non-zero cross section for a charged final state.

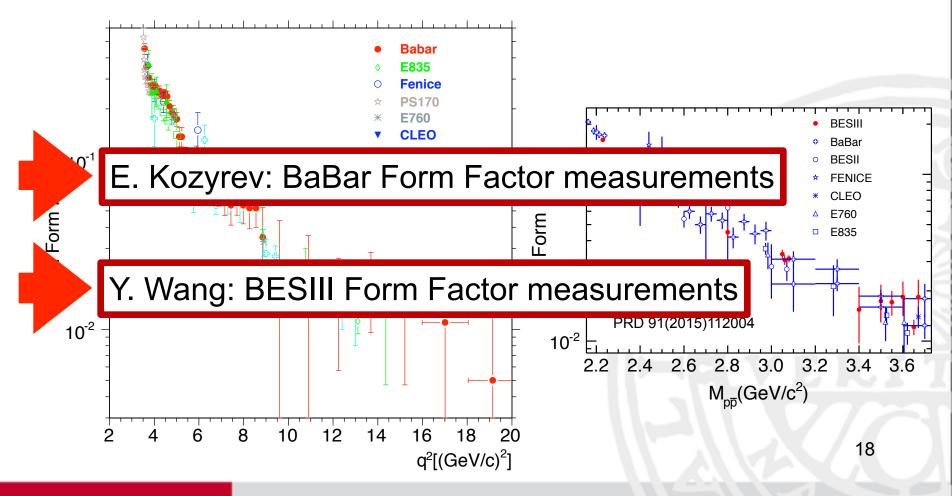


- Most experiments in the TL region only provide  $G_{eff}$ .
- The *q*<sup>2</sup> dependence is obtained from energy scan or by using initial state radiation (ISR):

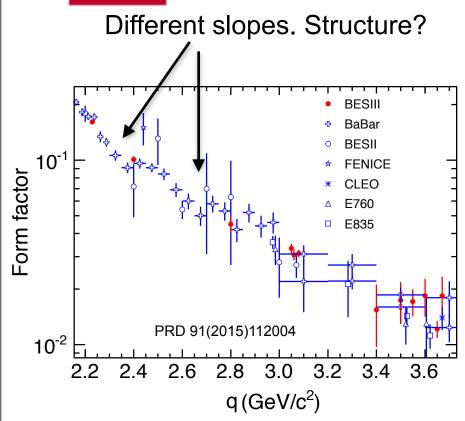


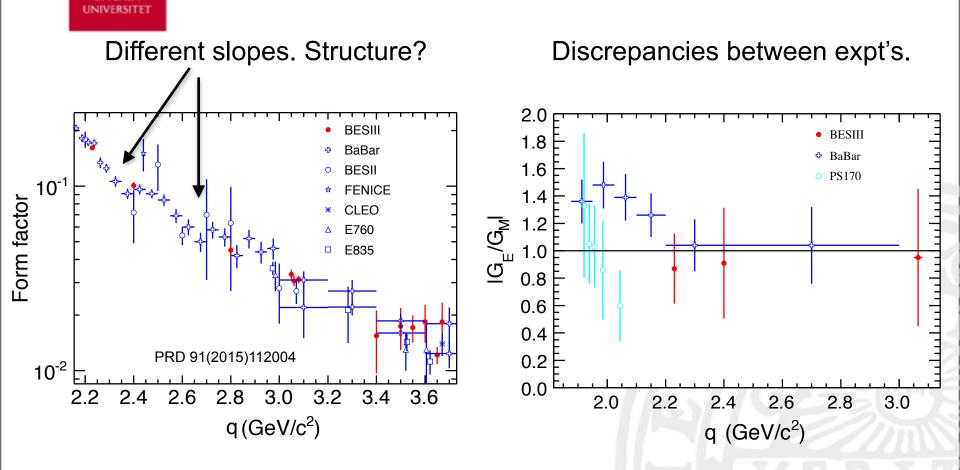


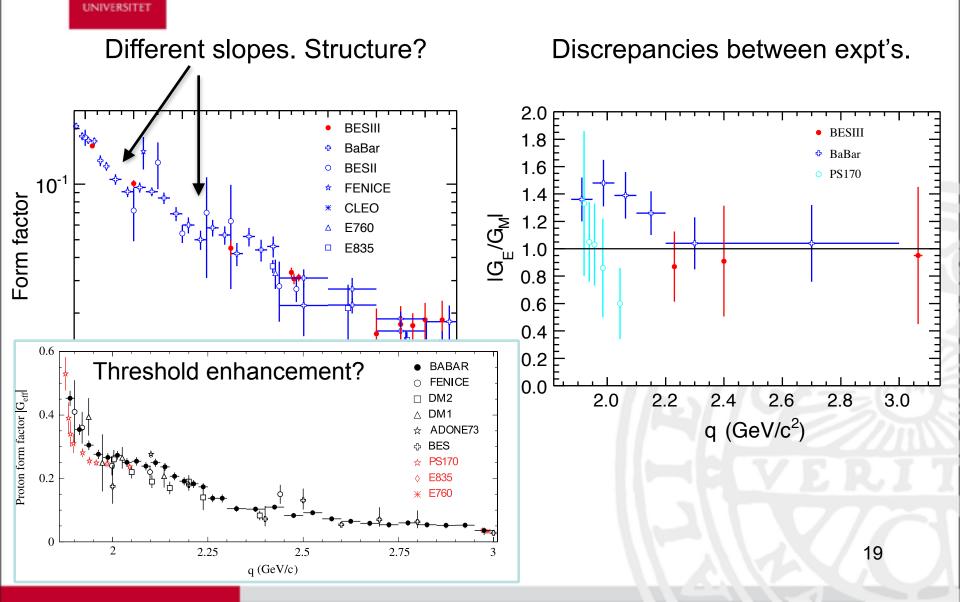
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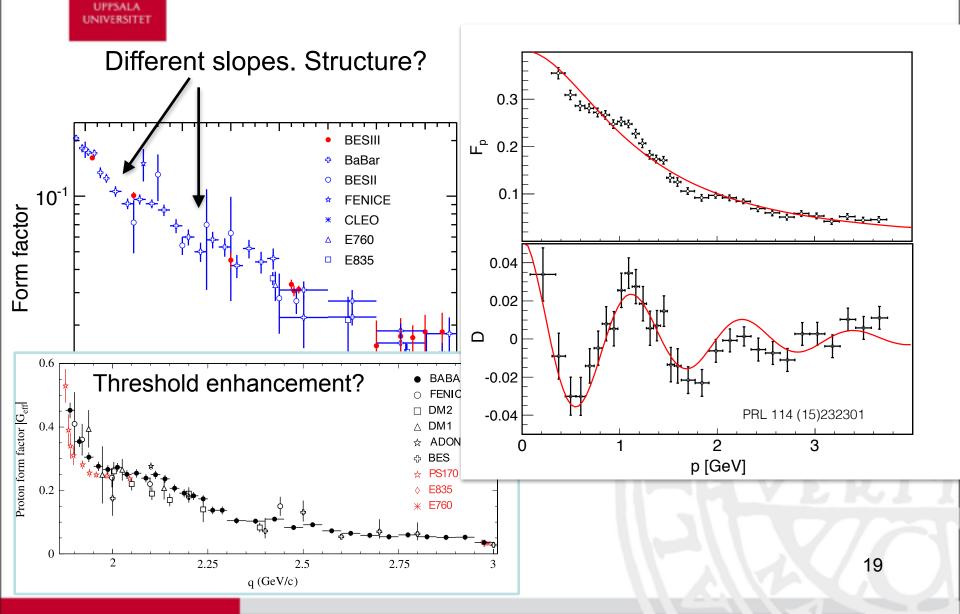














pQCD predicts that asymptotically

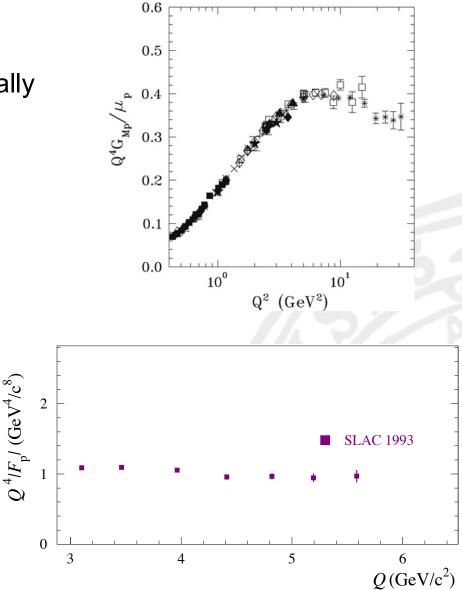
$$F_i^B(q^2) \rightarrow \left(\frac{1}{q^2}\right)^{i+1} \left(\ln\left(\frac{q^2}{\Lambda_{qcd}^2}\right)\right)^{\gamma}$$
$$i = 1, 2; \ \gamma = 2.148 \ (N_f = 2)$$
$$\Rightarrow G_E, G_M \propto \frac{1}{q^4} \text{ and } \frac{G_E}{G_m} \sim 1$$

PRL 31(73)1153 Nuov. Cim. L. 7(73)719

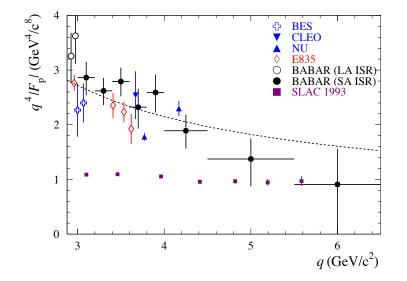
Analyticity:

$$G_{E,M}^{SL}\Big|_{q^2\to\infty} = G_{E,M}^{TL}\Big|_{q^2\to\infty}$$

pQCD scaling observed at  $Q > 3 \text{ GeV/c}^2$ ?



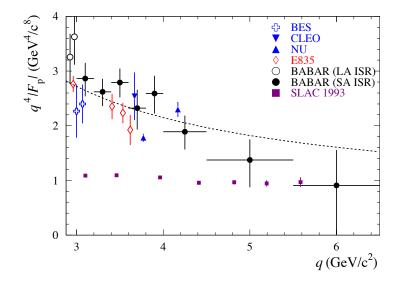




pQCD region reached at  $Q \approx 6$  Gev/c<sup>2</sup>?

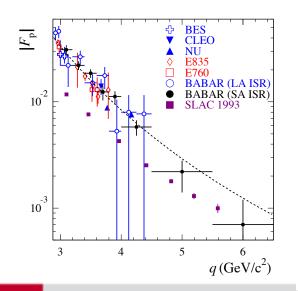
arXiv:1311.751v1



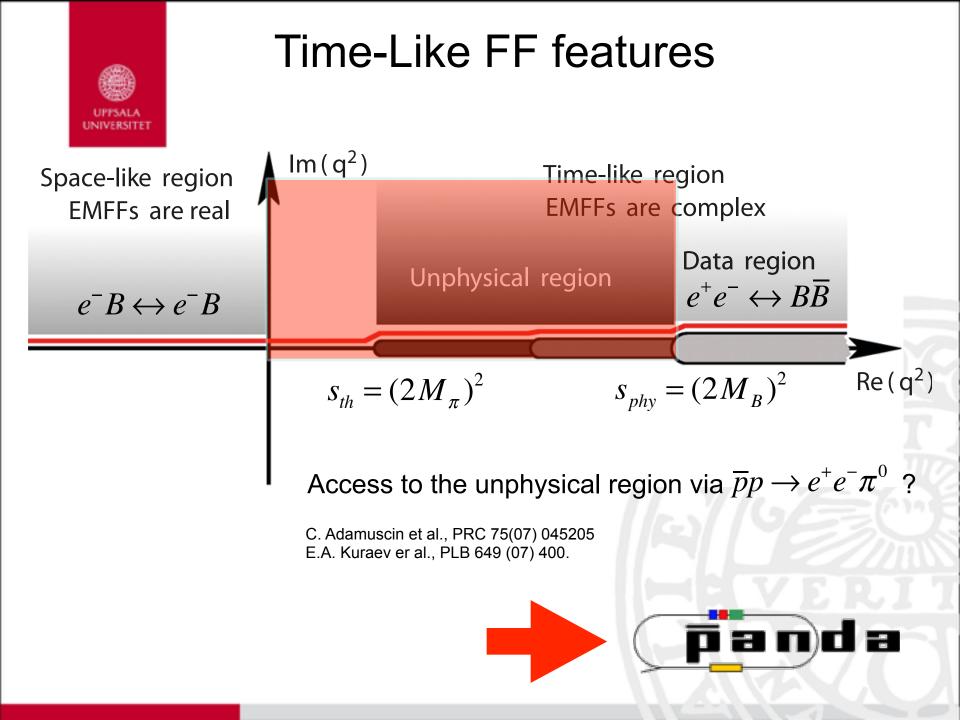


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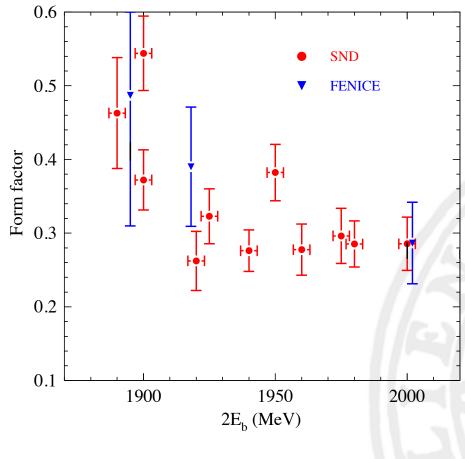


 $TL |F_p| = 2x SL |F_p| !?$ 





#### Scarse data on neutron TL FF:

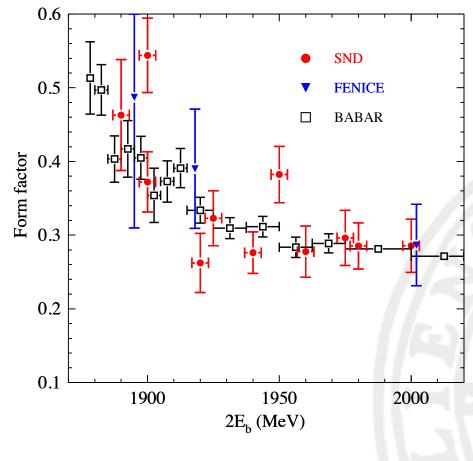


PRD 90 (14) 112007

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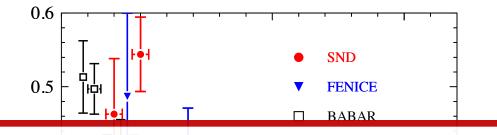
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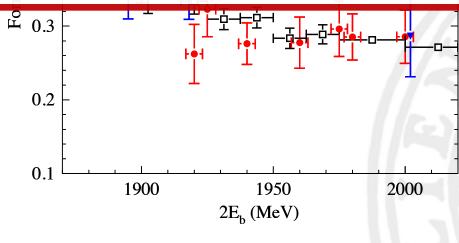
PRD 90 (14) 112007



#### Scarse data on neutron TL FF:



A. Korol: Measurement of the timeline neutron and proton Form Factors at VEPP-2000

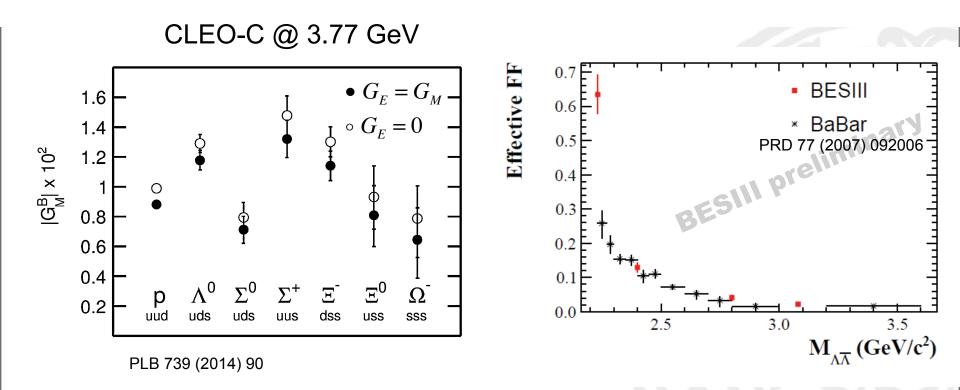


PRD 90 (14) 112007



## Hyperon Form Factors

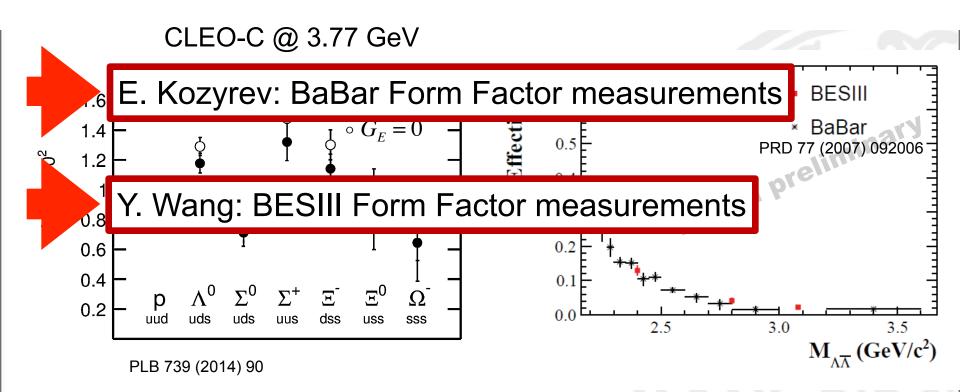
- What happens when adding strangeness to the baryons? SU(3) breaking?
- Only Time-Like Form Factors are experimentally accessible.
- e<sup>+</sup>e<sup>-</sup> -collisions are currently the best way to study hyperon structure.





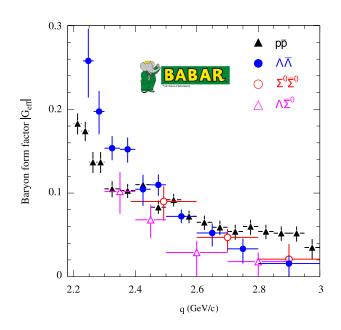
# Hyperon Form Factors

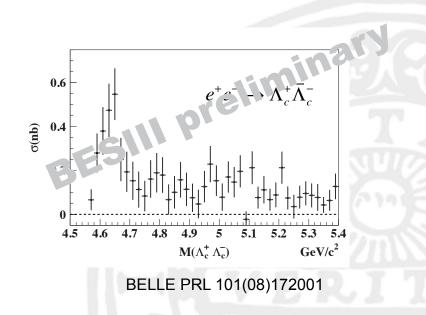
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Threshold enhancements?

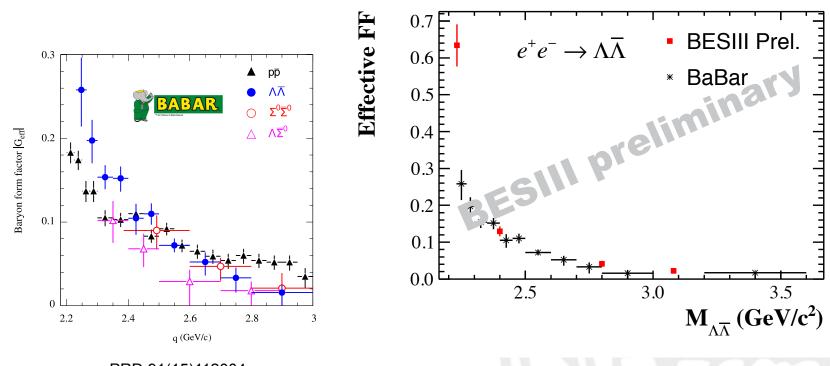




PRD 91(15)112004



Threshold enhancements?

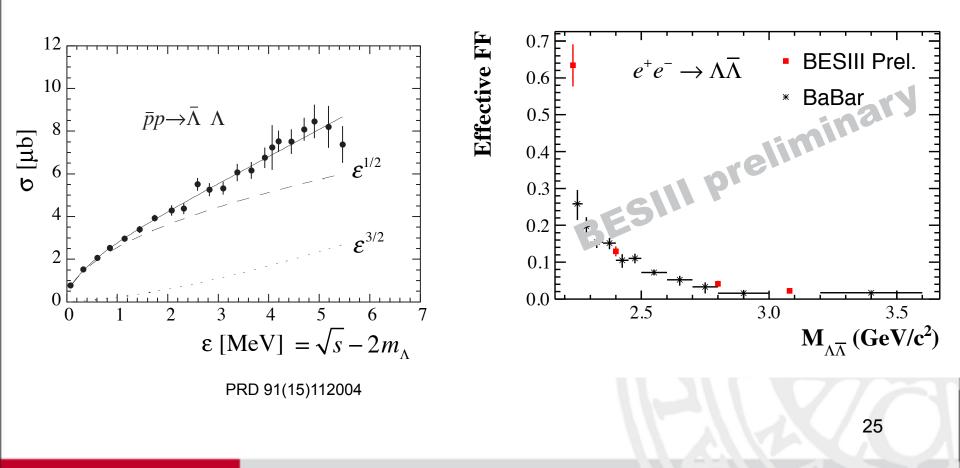


PRD 91(15)112004

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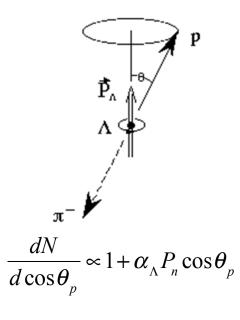
Threshold enhancements?

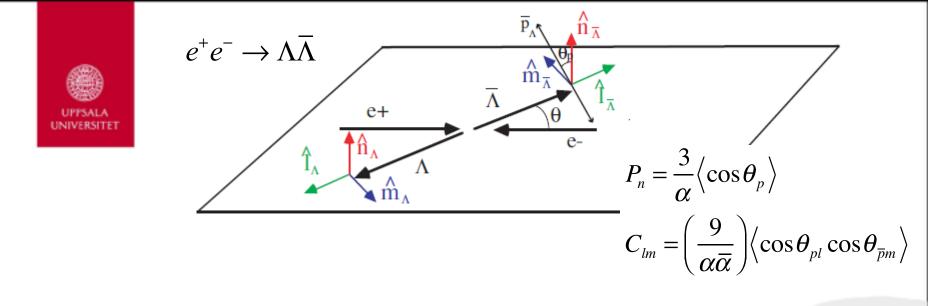




Need to measure polarisation to extract the phase between  $G_E$  and  $G_M$ .

Polarisation is available "for free" in hyperon TL FF's experiments because of their self-analysing weak decay.





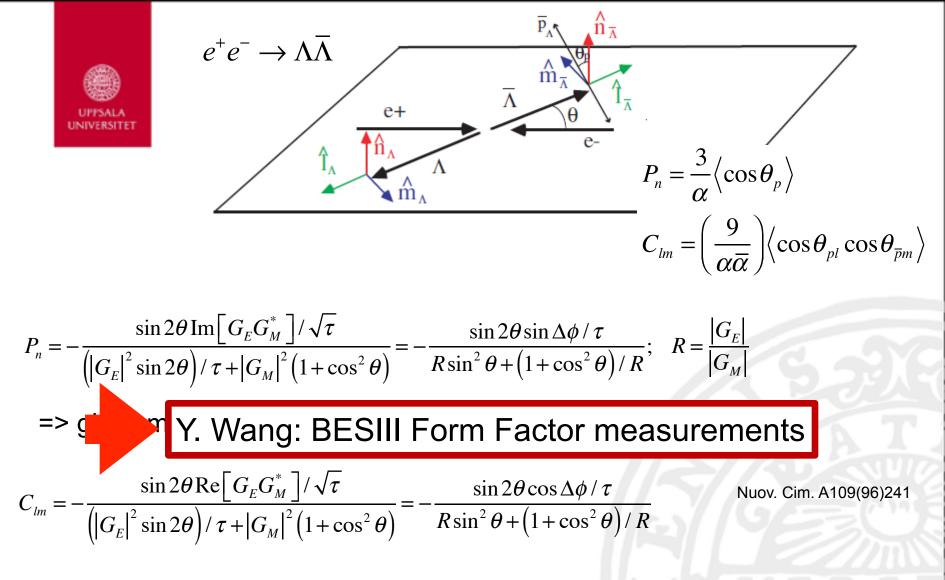
$$P_n = -\frac{\sin 2\theta \operatorname{Im} \left[ G_E G_M^* \right] / \sqrt{\tau}}{\left( \left| G_E \right|^2 \sin 2\theta \right) / \tau + \left| G_M \right|^2 \left( 1 + \cos^2 \theta \right)} = -\frac{\sin 2\theta \sin \Delta\phi / \tau}{R \sin^2 \theta + \left( 1 + \cos^2 \theta \right) / R}; \quad R = \frac{|G_E|}{|G_M|}$$

=> gives modulus of the phase  $\phi$ 

$$C_{lm} = -\frac{\sin 2\theta \operatorname{Re}\left[G_{E}G_{M}^{*}\right]/\sqrt{\tau}}{\left(\left|G_{E}\right|^{2}\sin 2\theta\right)/\tau + \left|G_{M}\right|^{2}\left(1+\cos^{2}\theta\right)} = -\frac{\sin 2\theta \cos \Delta\phi/\tau}{R\sin^{2}\theta + \left(1+\cos^{2}\theta\right)/R}$$
 Nuov. Cim. A109(96)241

=> gives the sign of the phase  $\phi$ 

A complete determination of the  $\Lambda$  Time-Like Form Factor using  $e^+e^- \rightarrow \Lambda \overline{\Lambda}$  is possible!

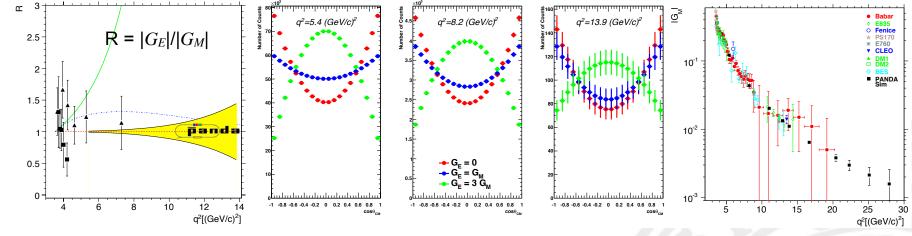


=> gives the sign of the phase  $\phi$ 

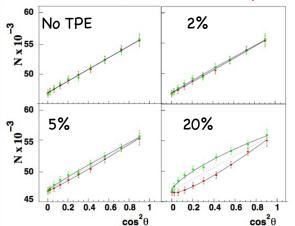
A complete determination of the  $\Lambda$  Time-Like Form Factor using  $e^+e^- \rightarrow \Lambda \overline{\Lambda}$  is possible!



High luminosity mode:  $2x10^{32}$  cm<sup>-2</sup>s<sup>-1</sup>  $\rightarrow$  100 fb<sup>-1</sup> in  $\approx$  100 days



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forward lepton backward lepton

- Individual determination of  $|G_E|$ ,  $|G_M|$  up to  $q^2 \approx 14$  (GeV/c)<sup>2</sup>
- $|G_M|$  up to  $q^2 \approx 28 \text{ (GeV/c)}^2$
- Sensitivity to odd cos-terms down to ≈ 5 %
   => two photon exchange can be investigated.

Long range: Polarised target to measure the phase.



# **Conclusions and Outlook**

(non-exhaustive list)

A lot has been learned on baryon elastic EM Form Factors and there is much more to come.

- SL: Polarisation measurements are superior to Rosenbluth separation in determining Form Factors.
  - New precise data at very low  $Q^2$  will shed light on the proton radius puzzle.
  - The importance of two-photon contributions will be clarified.
  - Neutron FF's at higher  $q^2$  .
- TL: New precise data at very low  $q^2$  will shed light on threshold enhancements.
  - First precise determination of  $|G_E|$  and  $|G_M|$  for the proton and the  $\Lambda$ .
  - First measurement of the phase between of  $|G_E|$  and  $|G_M|$  is within reach via  $e^+e^- \rightarrow \Lambda \overline{\Lambda}$ .
- SL+TL: Measure FF's at higher momentum transfers to investigate analyticity and the onset of pQCD. Flavour decomposition. 29



# **Conclusions and Outlook**

(non-exhaustive list)

A lot has been learned on baryon elastic EM Form Factors and there is much more to come.

SL: - Polarisation measurements are superior to Resenbluth separation in deterring Factors.

- New precise drift ery low  $Q^2$  with shed light on the proton radius puzzle.
- The importance of two-photon contributions will be clarified.
- Neutron FF's at higher  $q^2$  .

TL: - New precise data at very low  $q^2$  will shed light on threshold enhancements. P(G, q) = 0 for the problem of the standard for the st

reach via  $e^+e^- \rightarrow \Lambda\overline{\Lambda}$ .

SL+TL: Measure FF's at higher momentum transfers to investigate analyticity and the onset of pQCD. Flavour decomposition. 29



# Backup slide





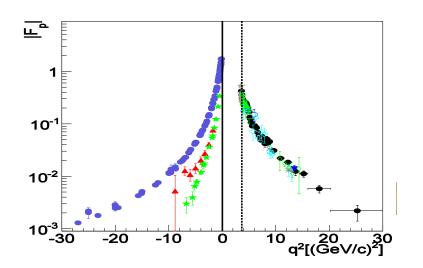






## Wishes

An unified description of baryon EM Form Factors.



Density distributions.

