

Recent developments in the theory of quarkonia production

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Heavy quarkonium

➤ Heavy quarkonium, a non-relativistic QCD system:

a) Constituent quarks are heavy quark-antiquark pair (J/ψ

$\psi', \chi_{cJ}, Y(nS), \chi_{bJ}(nP) \dots$)

b) Relative momentum between the quark pair is small

Charmonium: $v^2 \approx 0.3$

Bottomonium: $v^2 \approx 0.1$

c) A simple system: could be similar to a QED bound state, like hydrogen

➤ Multiple well-separated scales :

Quark mass:

M

Momentum:

Mv

Energy:

Mv^2

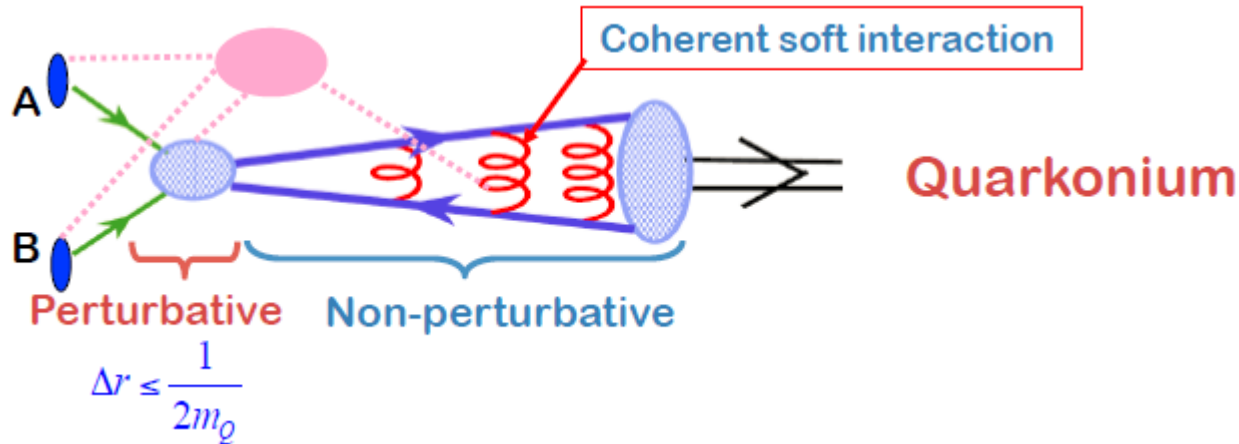


$$M \gg Mv \gg Mv^2 \sim \Lambda_{\text{QCD}}$$

➤ Production: a way to study hadronization and QGP

Factorization and hadronization models

- **Short distance and long distance parts. Hadronization followed by production of an off-shell heavy quark pair.**



- **Approximation: on-shell pair + hadronization**

Different assumptions/treatments on how the heavy quark pair becomes a heavy quarkonium: different factorization models

$$\sigma_{AB \rightarrow H+X} = \sum_n \int_n d\Gamma_{(Q\bar{Q})_n} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{(Q\bar{Q})_n}} \right] F_{(Q\bar{Q})_n \rightarrow H}(p_Q, p_{\bar{Q}}, P_H)$$

Historical review of quarkonium production

0. 1974 Discovery of J/ψ

E598 Collaboration (1974)
SLAC-SP-017 Collaboration (1974)

1. 1974 - CSM and CEM

Color Singlet Model: fine until 1994, ψ' surplus Einhorn, Ellis (1975), Chang (1980),
Berger, Jone (1981), ...

Color Evaporation Model: wrong for ratio Fritzsche (1977), Halzen (1977), ...

2. 1994 - NRQCD

➤ 1994-2004 NRQCD@LO Bodwin, Braaten, Lepage, 9407339, ...

✓ Self consistent, explain ψ' surplus

✗ Polarization puzzle, double charmonium, ...

➤ 2005-2014 NRQCD@NLO Zhang, Gao, Chao, 0506076, ...

✓ B-factories and hadron colliders, separately

➤ Very high p_T , low p_T region, plain NRQCD fails

It was proved that
both CSM and CEM
are special cases in
NRQCD framework.

Bodwin, Braaten, Lee, 0504014

3. 2014 -

High p_T : collinear factorization, SCET

Collins, Soper (1982)

Kang, Qiu, Sterman, 1109.1520

Fleming, Leibovich, Mehen, Rothstein 1207.2578

Kang, YQM, Qiu, Sterman, 1401.0923, ...

Low p_T : CGC+NRQCD

Kang, YQM, Venugopalan, 1309.7337

Qiu, Sun, Xiao, Yuan, 1310.2230

YQM, Venugopalan, 1408.4075

YQM, Venugopalan, Zhang, 1503.07772

Outline

I. NLO NRQCD: success and failure

II. Collinear factorization: large p_T

III. CGC+NRQCD: small p_T

χ_{cJ} @hadron colliders: a successful example

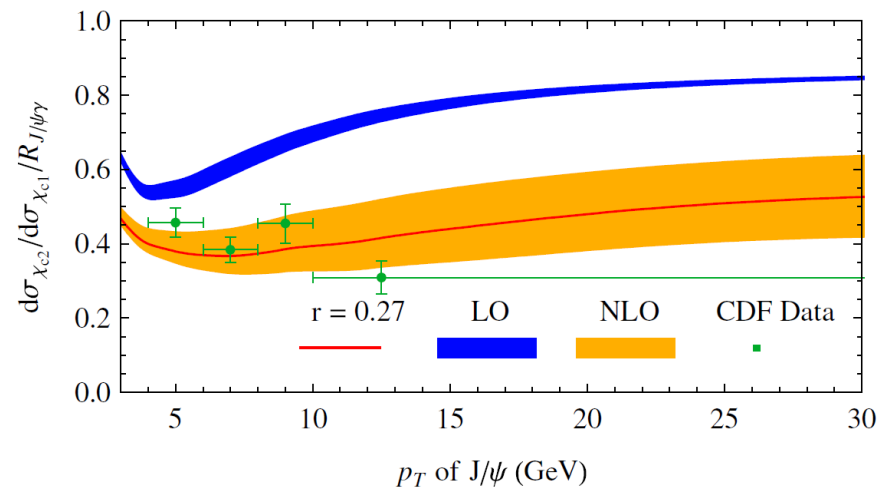
➤ NRQCD framework:

$$d\sigma_H = \sum_{\kappa} d\hat{\sigma}^{\kappa} \langle \mathcal{O}_{\kappa}^H \rangle$$

➤ χ_{cJ} production: $d\sigma_{\chi_{cJ}} \approx d\hat{\sigma}_{3P_J^{[1]}} \langle \mathcal{O}(3P_0^{[1]}) \rangle + (2J+1)d\hat{\sigma}_{3S_1^{[8]}} \langle \mathcal{O}(3S_1^{[8]}) \rangle$

- $d\hat{\sigma}$: can be calculated perturbatively
- $\langle \mathcal{O}(3P_0^{[1]}) \rangle$: can be determined by potential model
- $\langle \mathcal{O}(3S_1^{[8]}) \rangle$: a number, the only free parameter, fit Tevatron data $d\sigma_{\chi_{c2}}/d\sigma_{\chi_{c1}}$

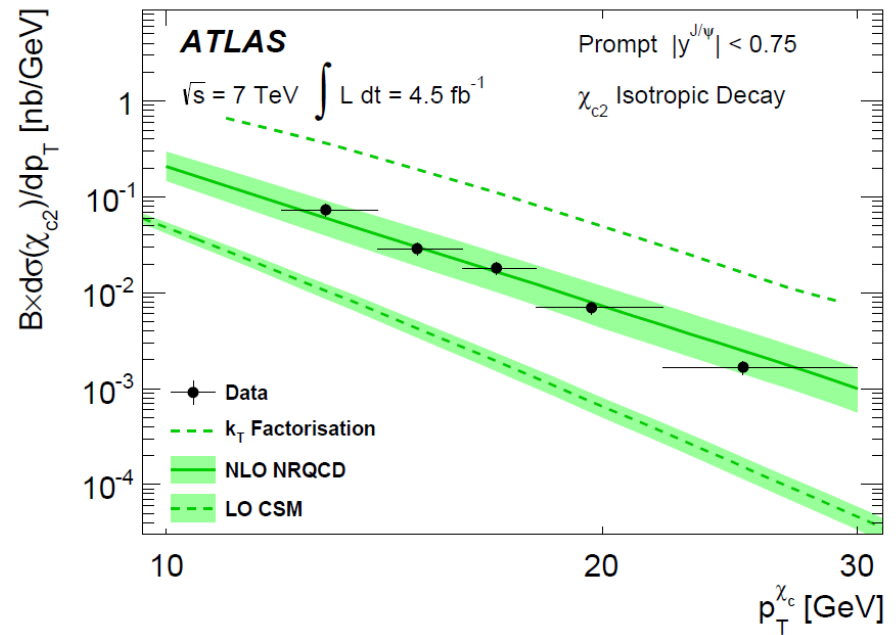
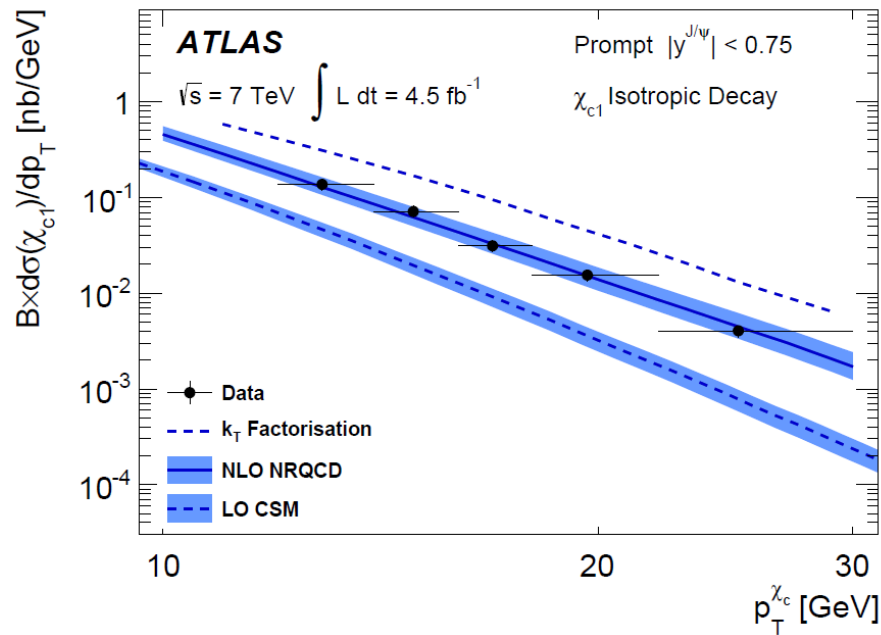
YQM, Wang, Chao, 1002.3987



χ_{cJ} @hadron colliders: predictions

➤ Comparison with new data

ATLAS, 1404.7035



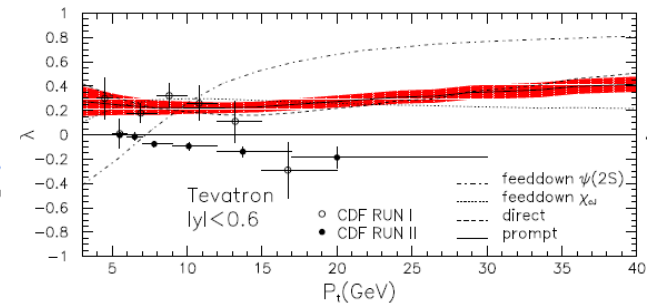
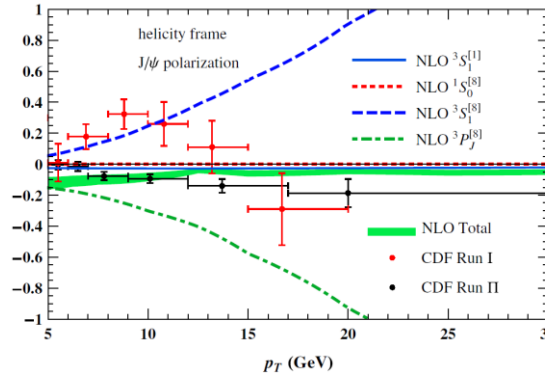
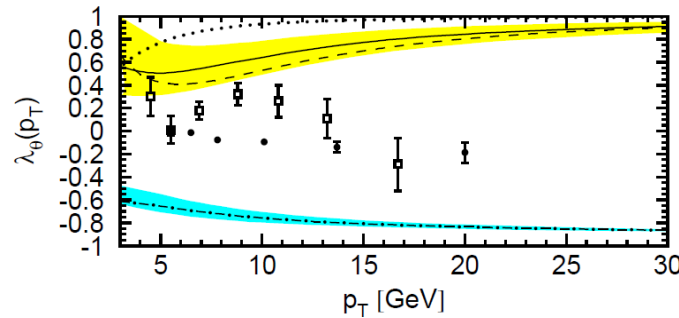
Perfect agreement!

J/ψ @hadron colliders

See Han Hao's talk
for more details

➤ Polarization@NLO NRQCD:

- Results depend on the treatment of the 3 CO LDMEs
- Our $^1S_0^{[8]}$ dominant mechanism can explain data

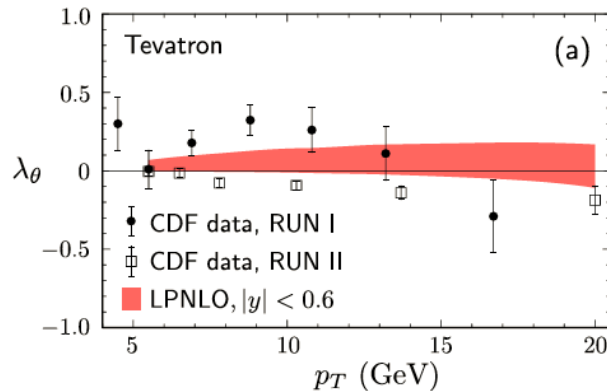


Butenschön, Kniehl, 1201.1872

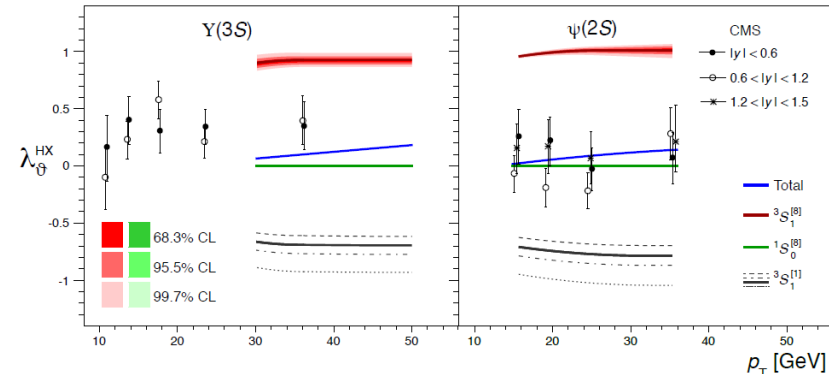
Chao, YQM, Shao, Wang, Zhang, 1201.2675

Gong, Wan, Wang, Zhang, 1205.6682

➤ $^1S_0^{[8]}$ dominant mechanism: agreed by new studies



Bodwin, Chung, Kim, Lee, 1403.3612

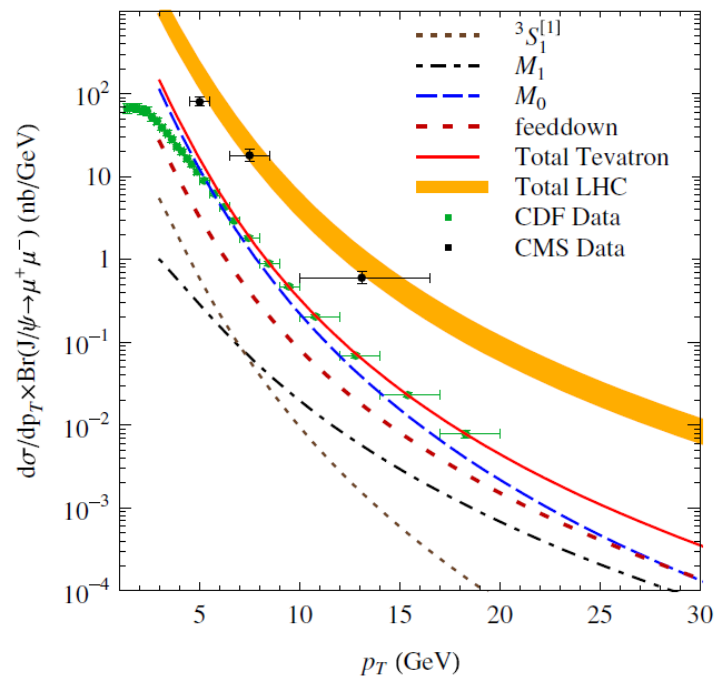
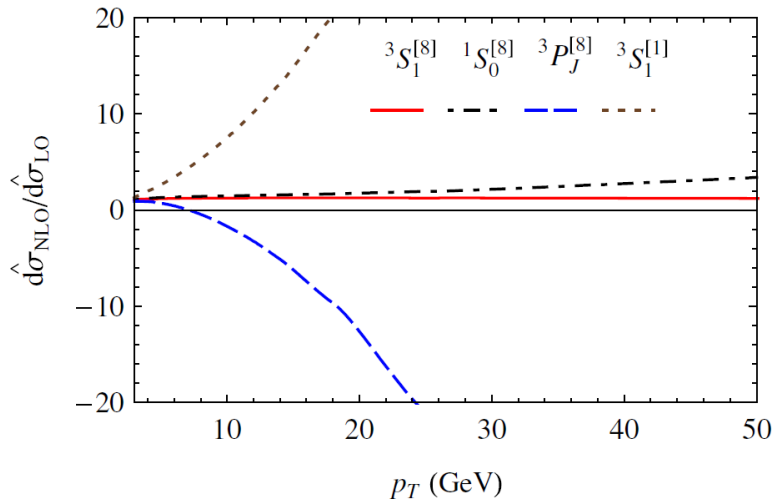


Faccioli, Knunz, Lourenco, Seixas, Wohri, 1403.3970

Problems with NLO NRQCD

- High p_T : large corrections for some channels
- Low p_T : different behavior from data

YQM, Wang, Chao, 1009.3655
Butenschön, Kniehl, 1009.5662



NLO NRQCD: summary

- **Most puzzles can be understood qualitatively**
 - Including J/ψ polarization puzzle, although under debating
- **It can not provide a good quantitative description for data at very high p_T region**
- **Can not describe low p_T region data**
- **Other methods are needed for these extreme regions**

Outline

I. NLO NRQCD: success and failure

II. Collinear factorization: large p_T

III. CGC+NRQCD: small p_T

Collinear factorization for high p_T production

- **When $p_T \gg m$, power expansion first: $1/p_T$**

Large $\log(p_T/m)$: can be resummed by solving evolution equation

- **Leading power: collinear factorization, single parton fragmentation**

Collins, Soper (1982)
Braaten, Yuan, 9303205
Nayak, Qiu, Sterman, 0509021

- **NLP: important for heavy quarkonium production**

Kang, Qiu, Sterman, 1109.1520

- **A rigorous collinear factorization method up to NLP**

Kang, YQM, Qiu, Sterman, 1401.0923
Kang, YQM, Qiu, Sterman, 1411.2456

Collinear factorization approach

➤ Ideas:

$$E \frac{d\sigma_{J/\psi}}{d^3P} : \left| \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \vdots \end{array} \right| \approx \left| \begin{array}{c} \text{Diagram 3} \log^n \left(\frac{P_T^2}{\mu_0^2} \right) \\ \text{Diagram 4} \mathcal{O} \left(\frac{1}{P_T^4} \right) \\ \vdots \end{array} \right| + \left| \begin{array}{c} \text{Diagram 5} \mu_0^2 \log^n \left(\frac{P_T^2}{\mu_0^2} \right) \\ \text{Diagram 6} \mathcal{O} \left(\frac{1}{P_T^6} \right) \\ \vdots \end{array} \right|$$

The diagram illustrates the collinear factorization approach for the cross-section $E \frac{d\sigma_{J/\psi}}{d^3P}$. It shows a series of diagrams on the left representing the full cross-section, which is then approximated by a sum of diagrams on the right. The diagrams on the right are separated by a horizontal dashed line, indicating different orders of collinear factorization. The top part of the right side shows diagrams with logarithmic enhancements $\log^n \left(\frac{P_T^2}{\mu_0^2} \right)$ and $\mu_0^2 \log^n \left(\frac{P_T^2}{\mu_0^2} \right)$, while the bottom part shows diagrams with power-law corrections $\mathcal{O} \left(\frac{1}{P_T^4} \right)$ and $\mathcal{O} \left(\frac{1}{P_T^6} \right)$.

➤ Factorization correct to all order

Qiu, Sterman (1991)

Kang, YQM, Qiu, Sterman, 1401.0923

Factorization formalism and evolution

➤ Factorization formalism:

Kang, YQM, Qiu, Sterman, 1401.0923

$$\begin{aligned}
 d\sigma_{A+B \rightarrow H+X}(p_T) &= \sum_f d\hat{\sigma}_{A+B \rightarrow f+X}(pf = p/z) \otimes D_{H/f}(z, m_Q) \\
 &+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B \rightarrow [Q\bar{Q}(\kappa)]+X}(p(1 \pm \zeta)/2z, p(1 \pm \zeta')/2z) \\
 &\quad \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q) \\
 &+ \mathcal{O}(m_Q^4/p_T^4)
 \end{aligned}$$

produce pair at $1/m_Q$

produce pair at $1/p_T$

$\kappa = v, a, t$ for spin, and 1, 8 for color.

➤ Independence of the factorization scale: $\frac{d}{d \ln(\mu)} \sigma_{A+B \rightarrow HX}(P_T) = 0$

➤ Evolution equations at NLP:

produce pair between $[1/m_Q, 1/p_T]$

$$\begin{aligned}
 \frac{d}{d \ln \mu^2} D_{H/f}(z, m_Q, \mu) &= \sum_j \frac{\alpha_s}{2\pi} \gamma_{f \rightarrow j}(z) \otimes D_{H/j}(z, m_Q, \mu) \\
 &+ \frac{1}{\mu^2} \sum_{[Q\bar{Q}(\kappa)]} \frac{\alpha_s^2}{(2\pi)^2} \Gamma_{f \rightarrow [Q\bar{Q}(\kappa)]}(z, \zeta, \zeta') \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu)
 \end{aligned}$$

$$\begin{aligned}
 \frac{d}{d \ln \mu^2} \mathcal{D}_{H/[Q\bar{Q}(c)]}(z, \zeta, \zeta', m_Q, \mu) &= \sum_{[Q\bar{Q}(\kappa)]} \frac{\alpha_s}{2\pi} K_{[Q\bar{Q}(c)] \rightarrow [Q\bar{Q}(\kappa)]}(z, \zeta, \zeta') \\
 &\quad \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu)
 \end{aligned}$$

Predictive power

➤ Calculation of short-distance hard parts in pQCD:

Power series in α_s , without large logarithms
LO is now available for all partonic channels

Kang, YQM, Qiu, Sterman, 1411.2456

➤ Calculation of evolution kernels in pQCD:

Power series in α_s , without large logarithms

Kang, YQM, Qiu, Sterman, 1401.0923

LO is now available for both mixing kernels and pair evolution kernels of all spin states of heavy quark pairs

➤ Universality of input fragmentation functions at μ_0 :


$$D_{H/f}(z, m_Q, \mu_0) \quad \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$$

- Complicated: different quarkonium states require different input distributions!
- Modeling: apply NRQCD to the input distributions at initial scale

NLO is now available for all channels

YQM, Zhang, Qiu, 1311.7078

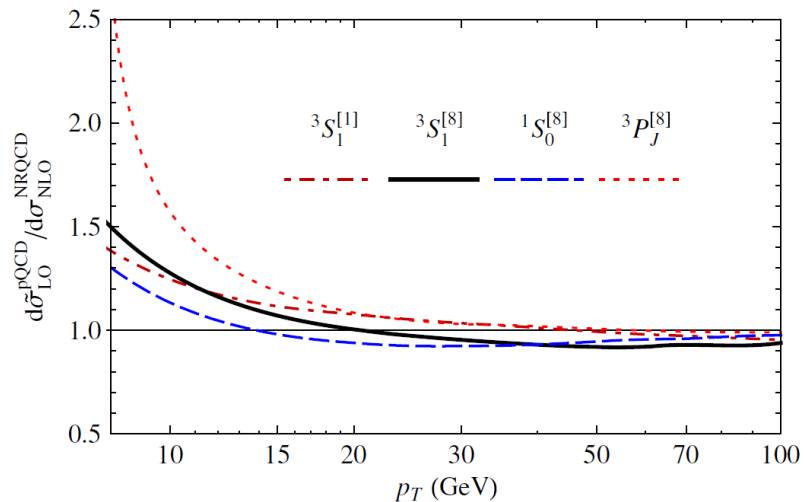
YQM, Zhang, Qiu, 1401.0524

YQM, Zhang, Qiu, 1501.04556

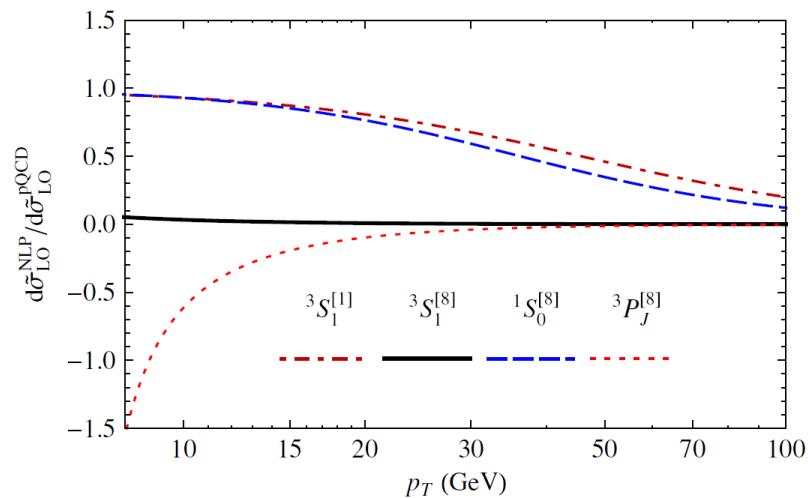
First application: reproducing NRQCD

YQM, Qiu, Sterman, Zhang, 1407.0383

➤ LP+NLP comparing with NLO NRQCD



LO analytical results reproduce NLO NRQCD calculations (numerical) !



- ◇ LP dominates: $^3S_1^{[8]}$ and $^3P_J^{[8]}$ channels
- ◇ NLP dominates: $^1S_0^{[8]}$ and $^3S_1^{[1]}$ channels

Large p_T : outlook

- **The collinear factorization framework is ready to use**
- **Solving the double parton evolution equations**
 - Resummation of $\log(p_T/m)$, better convergence
- **Calculating hard parts to NLO**
 - Before resummation, potentially can reproduce NNLO NRQCD
- **A full global analysis, based on collinear factorization formalism including NLP and evolution**

A lot of works to be done!

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Low p_T quarkonium production

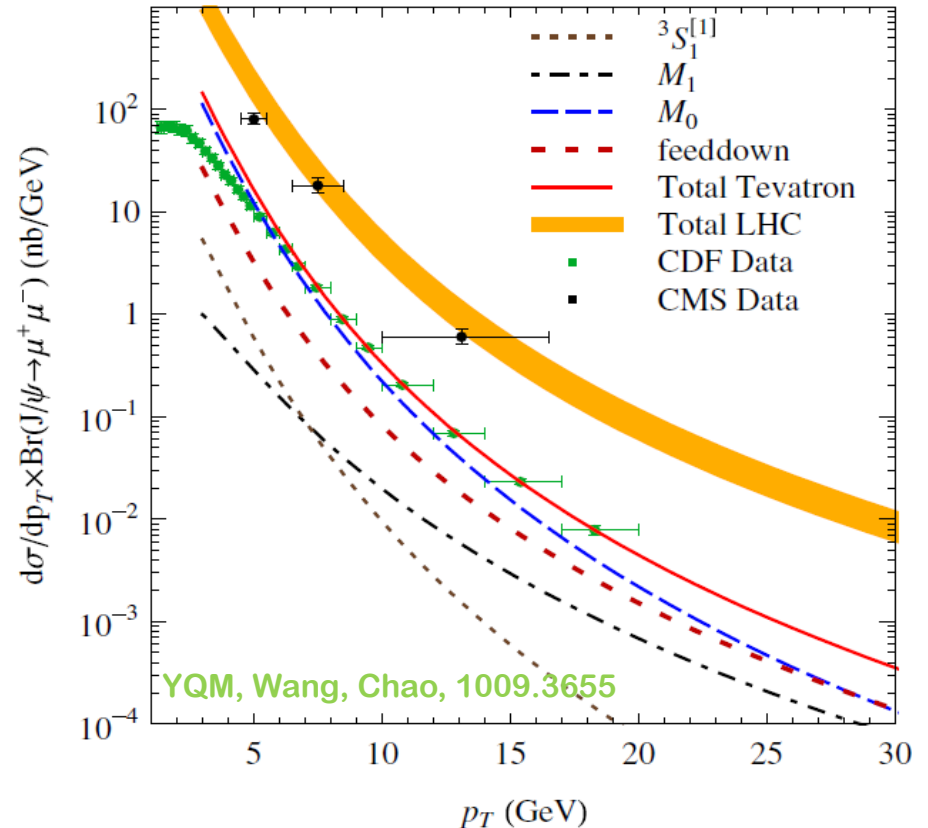
➤ Small p_T region

□ When $p_T \ll m_H$, fixed order gives

$$\frac{d\sigma}{dp_T} \propto \frac{1}{p_T}, \text{ while data goes to zero}$$

□ Far from understood

□ Dominate the total cross section



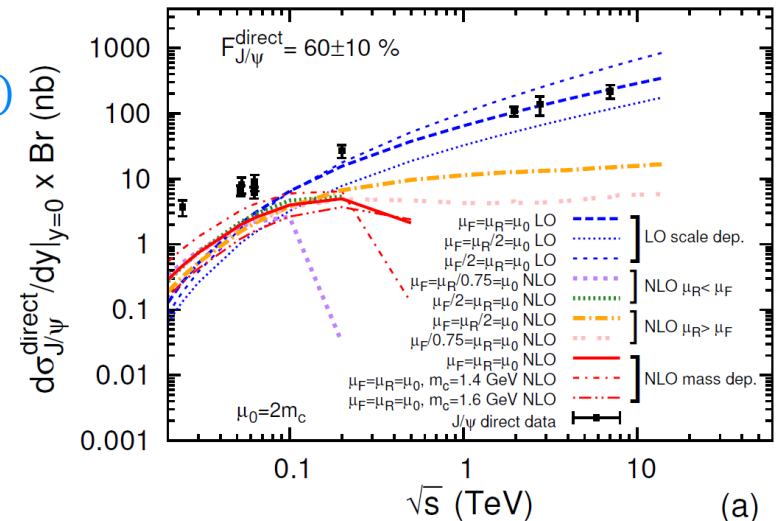
Small p_T v.s. small x

➤ Sudakov double logarithm Berger, Qiu, Wang, 0404158 Sun, Yuan, Yuan, 1210.3432

- ❑ Sudakov resummation of $\log^2(p_T/m_H)$ is needed at small p_T regime
- ❑ This resummation itself is still hard to explain the J/ψ data

➤ Why $\log^2(p_T/m_H)$ resummation is not enough?

- ❑ Total cross section is free of $\log(p_T/m_H)$
- ❑ Total cross section goes to negative at high energy
- ❑ Fixed order NRQCD fails to explain the data



Feng, Lansberg, Wang, 1504.00317

➤ Small- x effect can be important

- ❑ The only large logarithm is $\log(x)$

CGC effective field theory

McLerran, Venugopalan, 9309289

➤ Color Glass Condensate

- ◇ A tool to deal with small- x physics
- ◇ An effective field theory of QCD: separate $x < x_0$ configuration from $x > x_0$ configuration
- ◇ For small- x configuration: large saturation scale, **perturbatively calculable**
- ◇ For large- x configuration: $\Delta t^+ \sim \frac{1}{k^-} = \frac{2k^+}{k_{\perp}^2} \sim x$, life time of parton is long, determined before the collision, randomly distributed, **CGC average**
- ◇ JIMWLK evolution: guarantees the separation point x_0 independence

➤ NRQCD factorization:

Control the formation of quarkonium from $Q\bar{Q}$ -pair

$$d\sigma_H = \sum_{\kappa} d\hat{\sigma}^{\kappa} \langle \mathcal{O}_{\kappa}^H \rangle$$

◇ Via many channels, both CS and CO

➤ CGC: production of $c\bar{c}$ -pair

◇ Using CGC to calculate gluon distribution

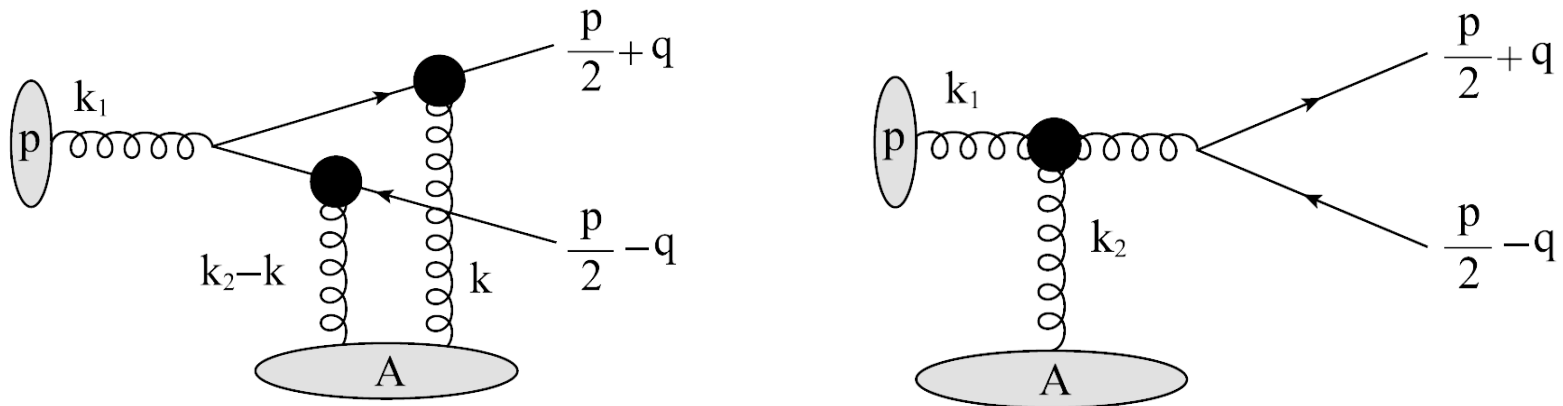
◇ Small x resummation is accounted by solving JIMWLK or BK evolution equations

➤ Scope of application:

- High energy p+A or p+p collision
- Quarkonium produced in forward rapidity region

LO and higher order

- **LO formula can only describe small p_T region data!**

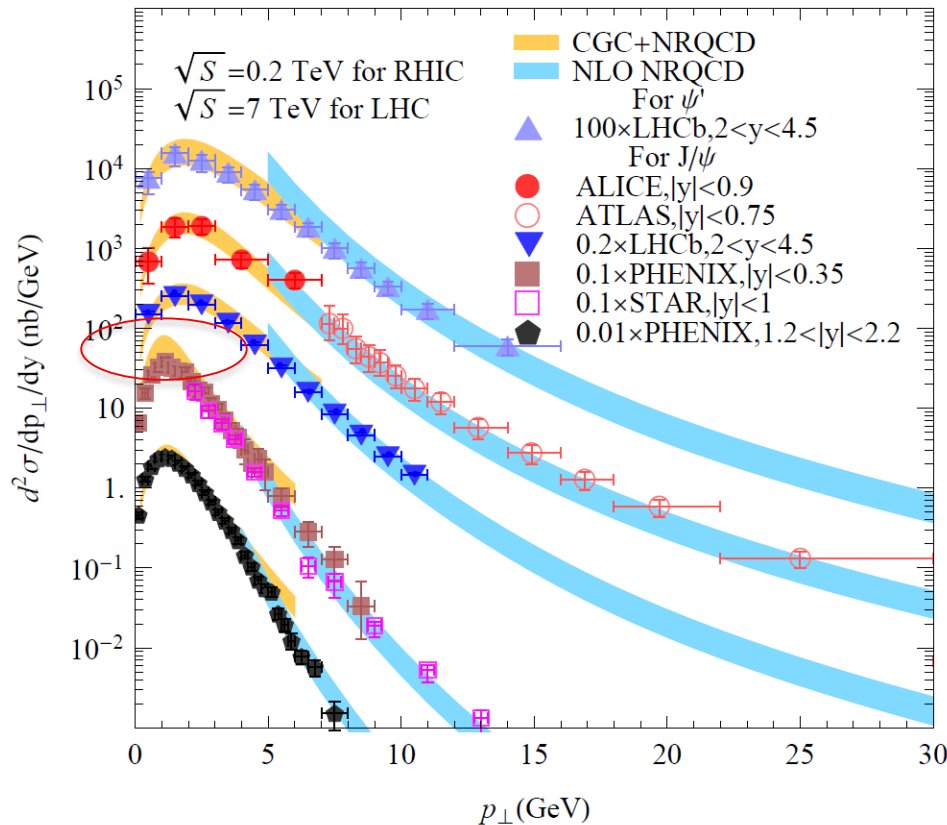


- ◇ **No final state radiation**
 - ◇ **Correct only if initial state radiation dominate (p_T can not be much larger than the saturation scale)**
- **NLO calculation is needed for CGC+NRQCD formula to give a consistent description of full p_T region**

J/ψ @ p+p: p_T dependence

YQM, Venugopalan, 1408.4075

➤ Agree with all small p_T data



- **RHIC data at central rapidity: agreement is not very good**
- **As expected: CGC+NRQCD is good for small x and forward rapidity**

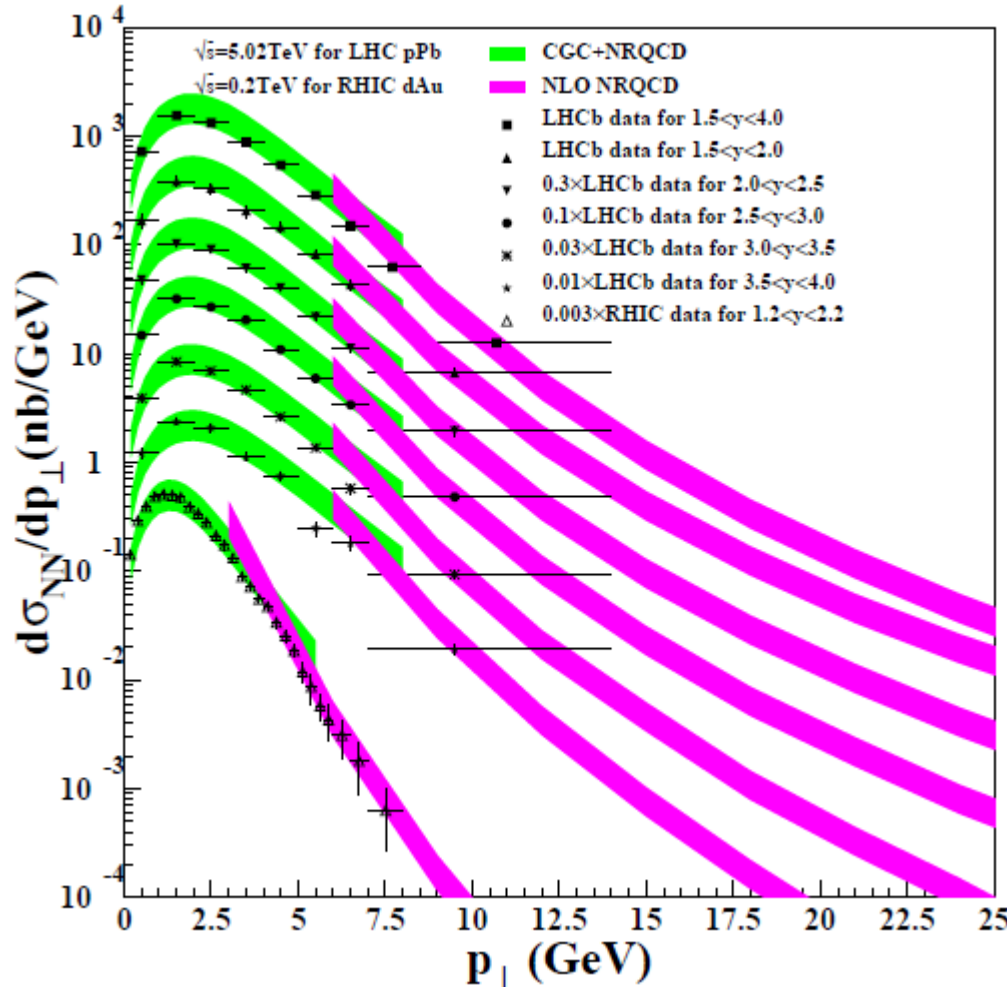
- ✓ Evolution of peaks agree!
- ✓ At moderate p_T region, smoothly matches with pQCD calculation: **NLO NRQCD**

J/ψ production at all p_T region can be described now!

J/ψ @p+A: p_T dependence

YQM, Venugopalan, Zhang, 1503.07772

➤ Agree with all small p_T data, similar to p+p case

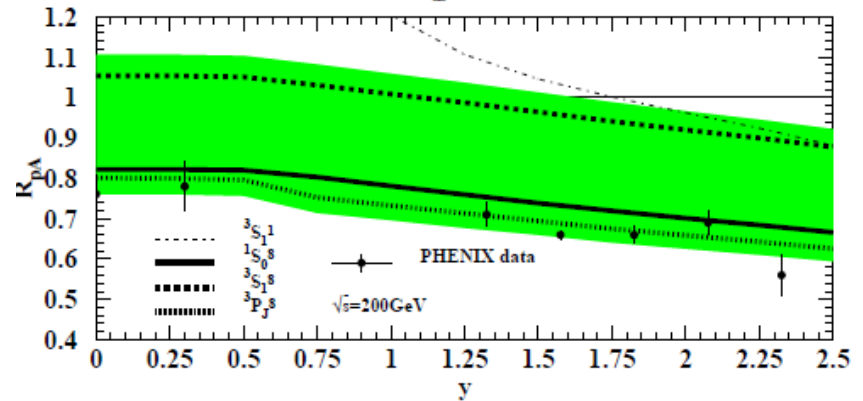
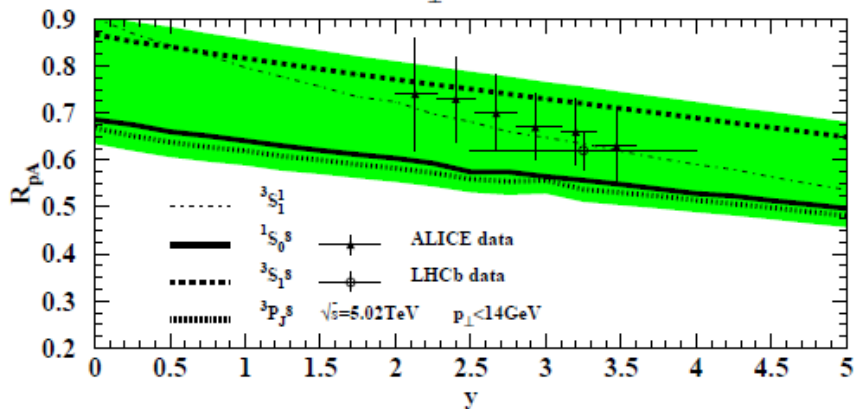
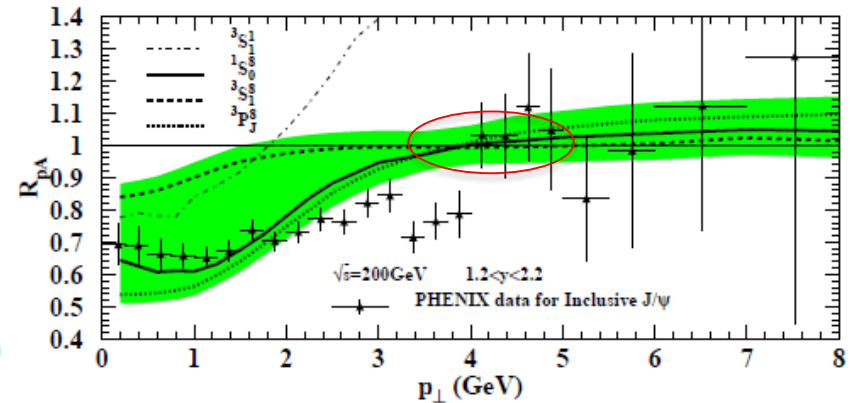
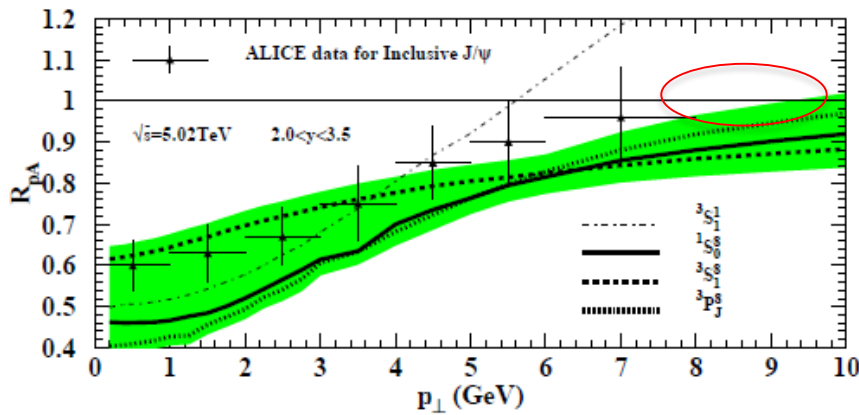


- ✓ Evolution of peaks agree!
- ✓ At moderate p_T region, smoothly matches with pQCD calculation: NLO NRQCD

J/ψ production at all p_T region can be described

R_{pA} : p_T and y dependence

➤ Agreement with data



✓ $R_{pA} \rightarrow 1$ at $p_T \approx 9\text{GeV}$ at LHC and $p_T \approx 4\text{GeV}$ at RHIC, both agree

CGC+NRQCD: outlook

- **Good description for J/ψ production at p+p and p+A collisions**
- **Apply for other quarkonium states is possible**
 - Plenty of data at LHC
- **NLO calculation in CGC framework is important and needed!!**

Thank you!

➤ CEM:

- ◇ A fixed fraction to become J/ψ if the invariant mass of $c\bar{c}$ -pair is below the D -meson threshold

$$\frac{d\sigma_{J/\psi}}{d^2\mathbf{p}_\perp dy} = F_{J/\psi} \int_{4m_c^2}^{4m_D^2} dM^2 \frac{d\sigma_{c\bar{c}}}{dM^2 d^2\mathbf{p}_\perp dy}$$

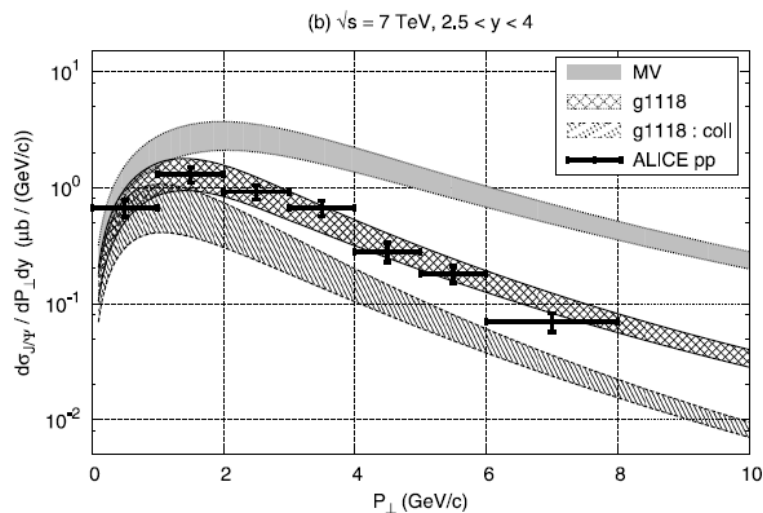
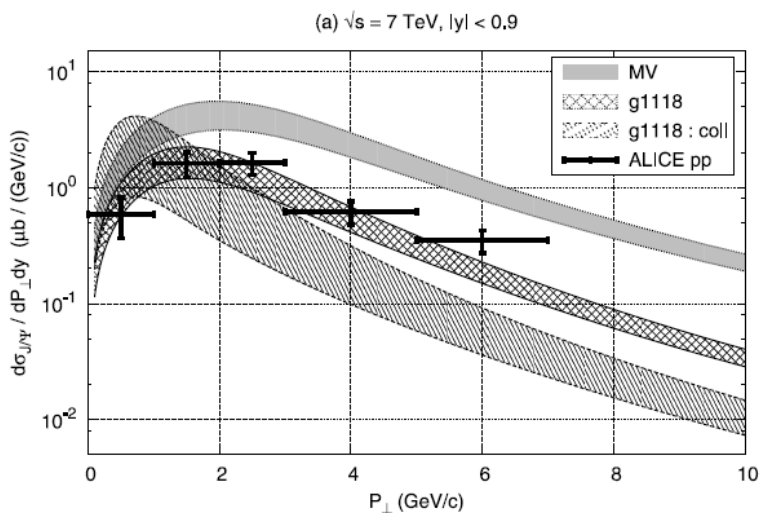
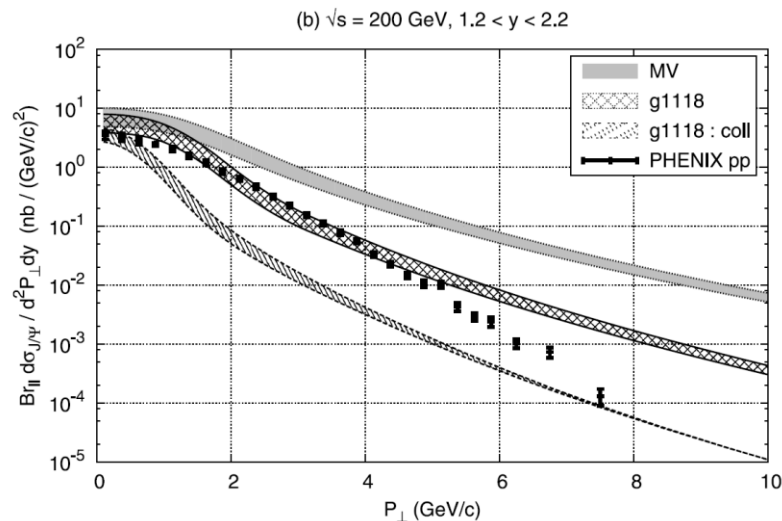
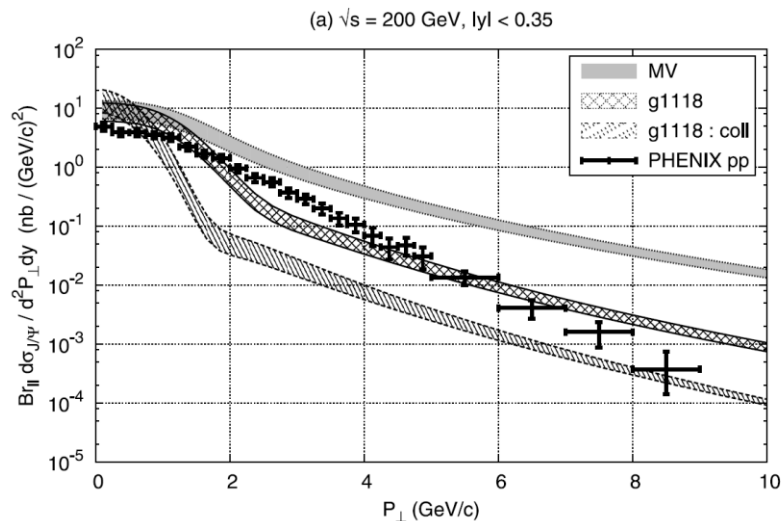
➤ CGC: production of $c\bar{c}$ -pair

- ◇ Using CGC to calculate gluon distribution
- ◇ Small x resummation is accounted by solving JIMWLK or BK evolution equations

CGC+CEM: p+p

Bad agreement:

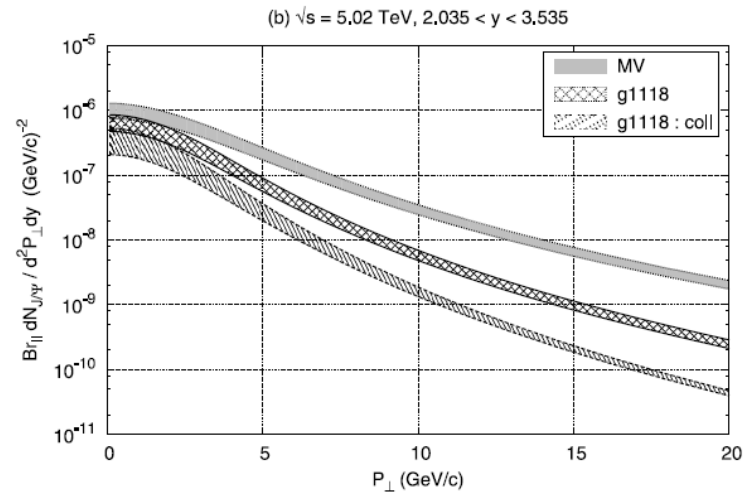
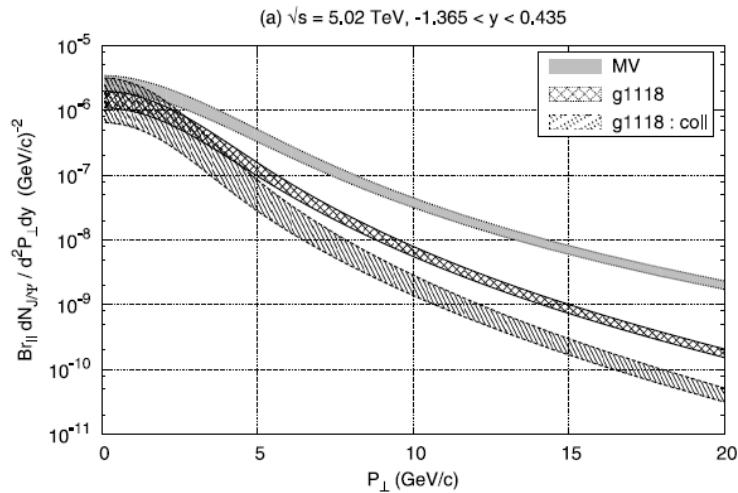
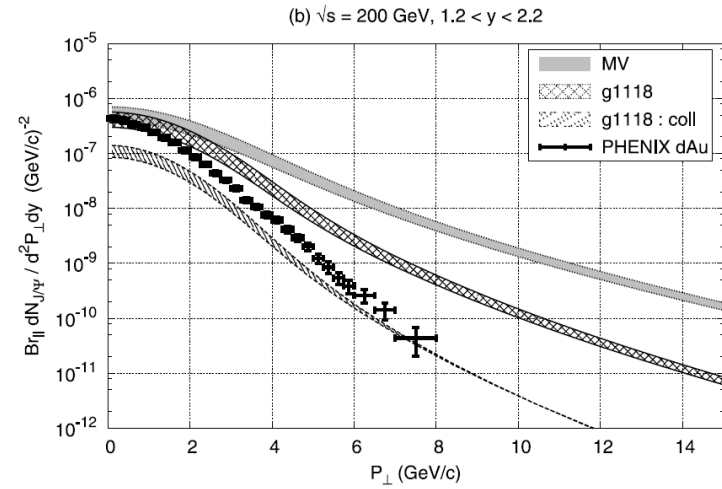
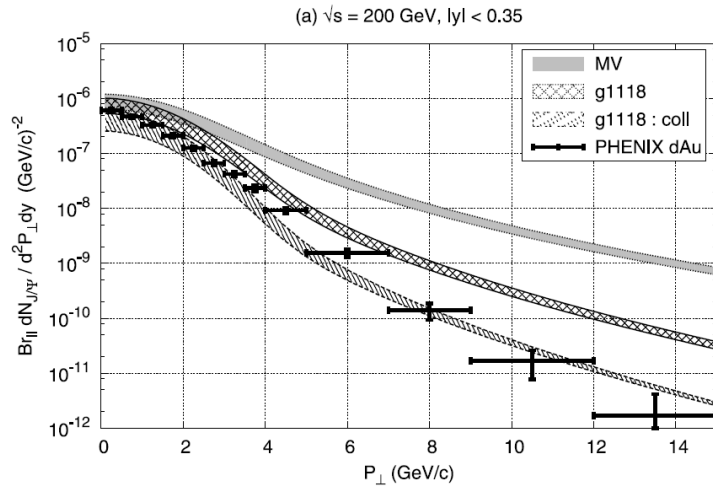
Fujii, Watanabe, 1304.2221



CGC+CEM: p+A

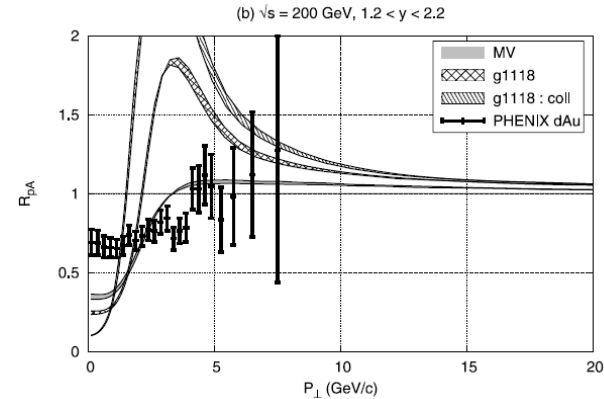
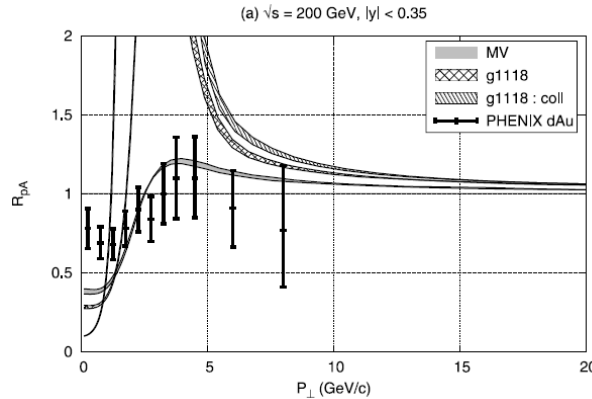
Bad agreement:

Fujii, Watanabe, 1304.2221



CGC+CEM: R_{pA}

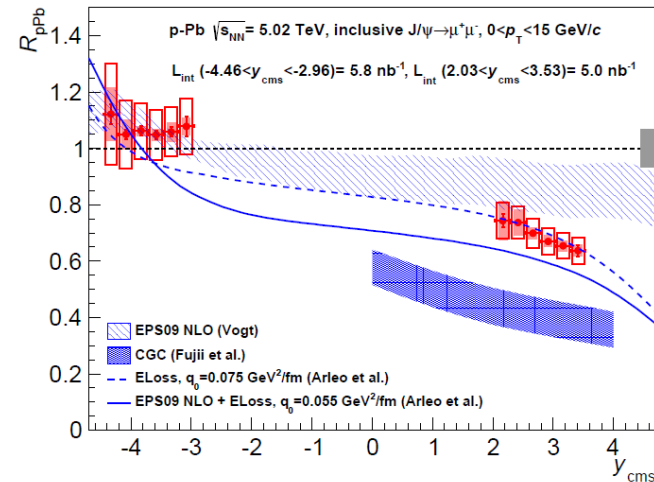
➤ RHIC:



Fujii, Watanabe, 1304.2221

➤ LHC:

- ◇ Disagree with data
- ◇ Rule out the CGC method???



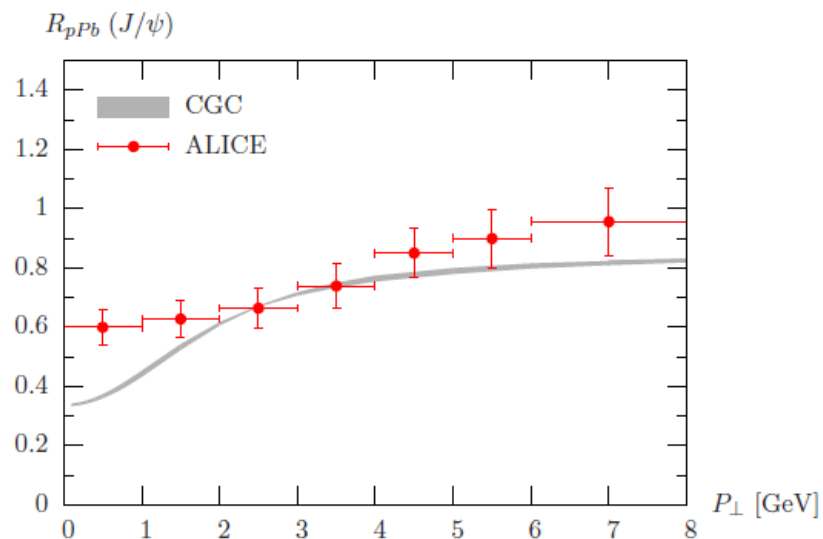
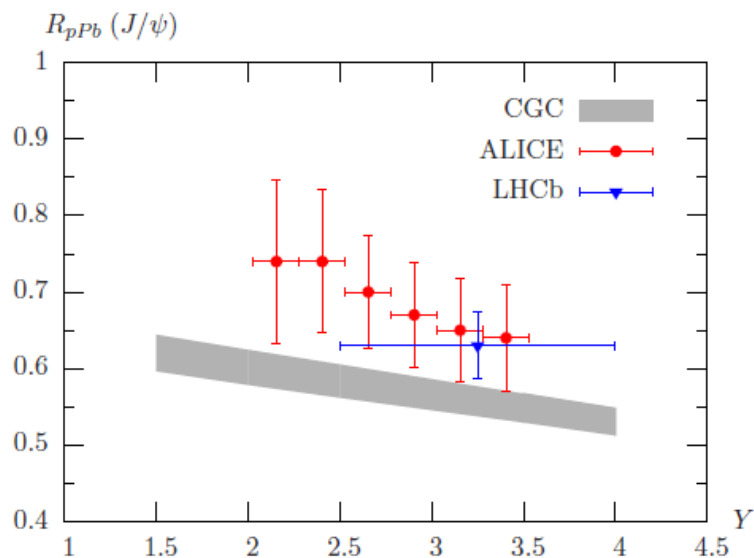
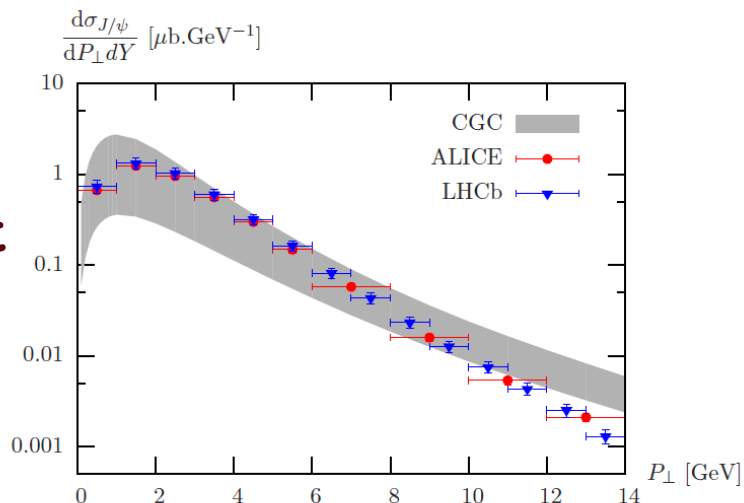
ALICE, 1308.6726

are also shown. Within our uncertainties, both the model based on shadowing only and the coherent energy loss approach are able to describe the data, while the CGC-based prediction overestimates the observed suppression. None of these models include a suppression related to the break-up of the $c\bar{c}$ pair.

CGC+CEM: improved

Ducloue, Lappi, Mantysaari, 1503.02789

- Using the collinear “hybrid” frame work
- Introduce impact-parameter-dependent initial condition
- Marginally describe data



Parameters for p+p

➤ An approximation for quadrupole

YQM, Venugopalan, 1408.4075

$$Q_{x_{\perp}x'_{\perp}y_{\perp}y'_{\perp}} \approx D_{x_{\perp}-x'_{\perp}} D_{y'_{\perp}-y_{\perp}} - D_{x_{\perp}-y'_{\perp}} D_{x'_{\perp}-y_{\perp}} + D_{x_{\perp}-y_{\perp}} D_{x'_{\perp}-y'_{\perp}} \\ + \frac{1}{2} (D_{x_{\perp}-y'_{\perp}} D_{x'_{\perp}-y_{\perp}} - D_{x_{\perp}-y_{\perp}} D_{x'_{\perp}-y'_{\perp}}) \\ \times (D_{x'_{\perp}-y_{\perp}} - D_{y'_{\perp}-y_{\perp}} + D_{y'_{\perp}-x_{\perp}} - D_{x'_{\perp}-x_{\perp}})$$

- ✓ Self-consistent: exact when any two adjacent positions coincide
- ✓ Checked: a good approximation to the quadrupole

➤ Dipole distributions:

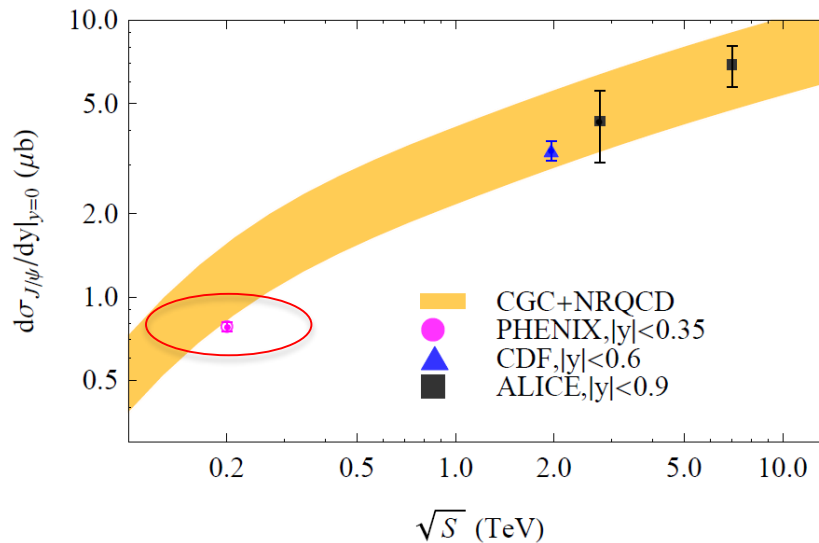
- Dipole distribution at initial scale ($x = x_0 = 0.01$): using MV model
- All parameters are fixed from fits to the HERA DIS data Albacete, Dumitru, Fujii, Nara, 1209.2001
- $R_p = 0.48\text{fm}$ to match with collinear PDF at large x

➤ NRQCD CO matrix elements

- Taken from fitting high p_T data Chao, YQM, Shao, Wang, Zhang, 1201.2675

J/ψ @ p+p: \sqrt{S} dependence

➤ Good agreement with data



◇ Worst agreement with RHIC data at central rapidity

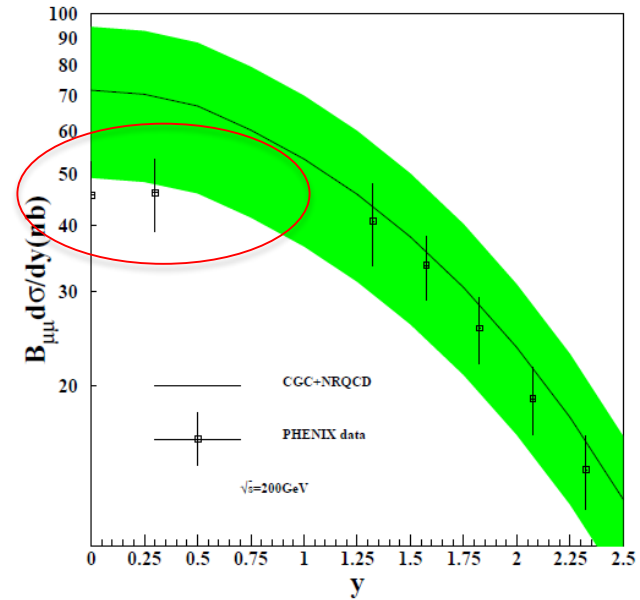
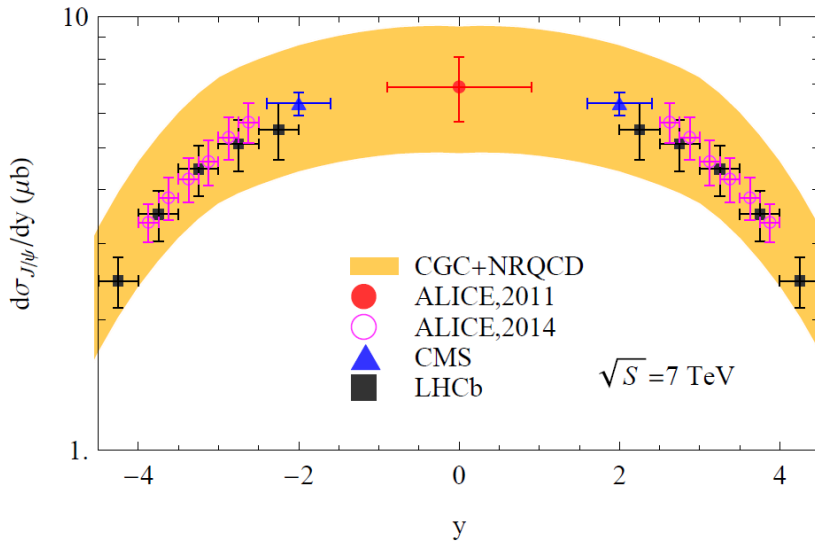
➤ CS contribution is found to be only 10%

◇ Large p_{\perp} : suppressed by $\frac{1}{p_T^2}$

◇ Small p_{\perp} : suppressed by phase space

J/ψ @ p+p: y dependence

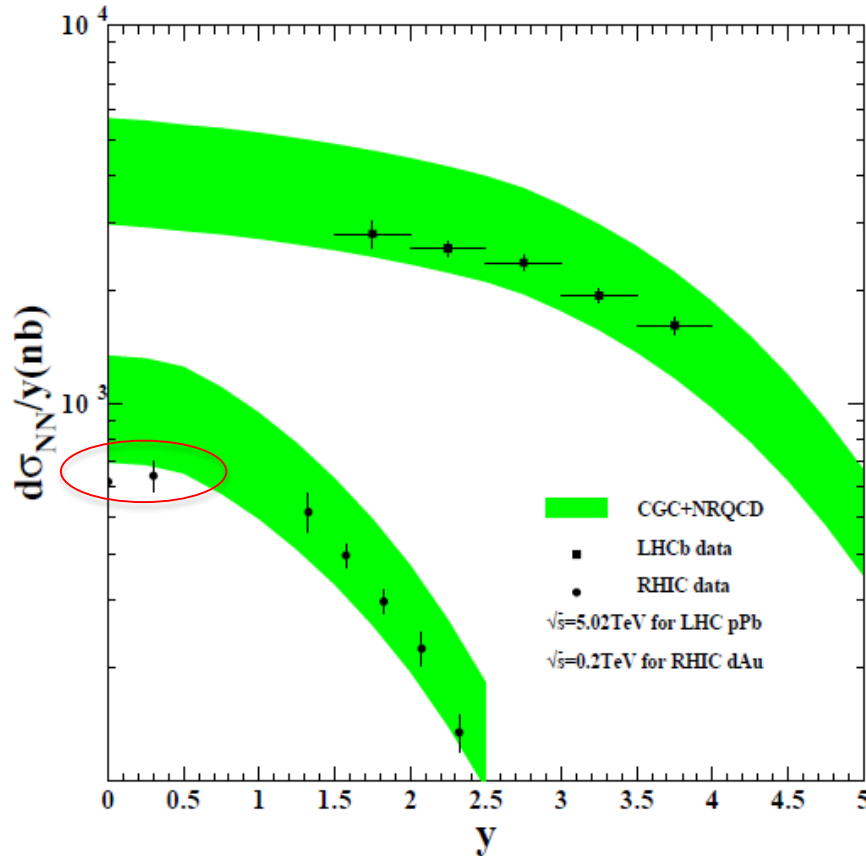
➤ Good agreement with data



◇ Worst agreement with RHIC data at central rapidity

J/ψ @ p+A: y dependence

➤ Good agreement with data



◇ Worst agreement with RHIC data at central rapidity

Parameters for p+A

YQM, Venugopalan, Zhang, 1503.07772

- **Two free parameters: $Q_{s0,A}$ and R_A**
- **Self-consistent condition: $R_{pA} \rightarrow 1$ at high p_T limit**

$$R_{pA} = \frac{d\sigma_{pA}}{A \times d\sigma_{pp}} \xrightarrow{\text{high } p_{\perp}} \frac{R_A^2 \tilde{N}_{Y_A}^A(\mathbf{p}_{\perp})}{AR_p^2 \tilde{N}_{Y_p}^A(\mathbf{p}_{\perp})} \approx \frac{R_A^2 Q_{s0,A}^{2\gamma}}{AR_p^2 Q_{s0,p}^{2\gamma}} = 1$$

◇ $\gamma = 1$ in MV model, $Q_{s0,p}$ and R_p are known from p+p case

- $Q_{s0,A}^2 = N \times Q_{s0,p}^2$

Dusling, Gelis, Lappi, Venugopalan, 0911.2720

- ◇ Fitting HERA DIS data, $N \approx 3$ for $\gamma = 1.113$, and $N \approx 1.5$ for $\gamma = 1$
- ◇ Set $N = 2$ as a tentative choice

- **Many uncertainties can be cancelled in the ratio**

$$R_{pA} = \frac{d\sigma_{pA}}{A \times d\sigma_{pp}}$$

- **Calculate R_{pA} for each NRQCD channel**

- ◇ Combining curves of all channels to provide the prediction for J/ψ
- ◇ Results are independent of NRQCD matrix elements

R_{pA} calculated in this way is almost parameter-independent