Factorization of Scalar Glueball Production from Heavy Quarkonium Decay

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- Factorization for $V(\psi(nS), \Upsilon(nS)) \to G(0^{++}) + \gamma$

  i.e. NRQCD+LCDA

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Motivation

various types of color singlet mesons and baryons:

$|q\bar{q}\rangle$ + … quarkonia √

$|qqq\rangle$ +... baryons √

$|q\bar{q}q\bar{q}\rangle$ +... tetraquarks

Zc(3900),Zc(4200),...

$|qqqq\bar{q}\rangle$ +... pentaquarks

Pc(4380),Pc(4450)

$|q\bar{q}g\rangle$ + … hybrids ?

$|gg\rangle$ +… glueballs ?

PDG, CPC38,090001(2014); BESIII,PRL110,252001(2013); Belle,PRL110,252002(2013);LHCb, arXiv:1507.03414.

Quark Model

$f_0(1500)$

? ? ?

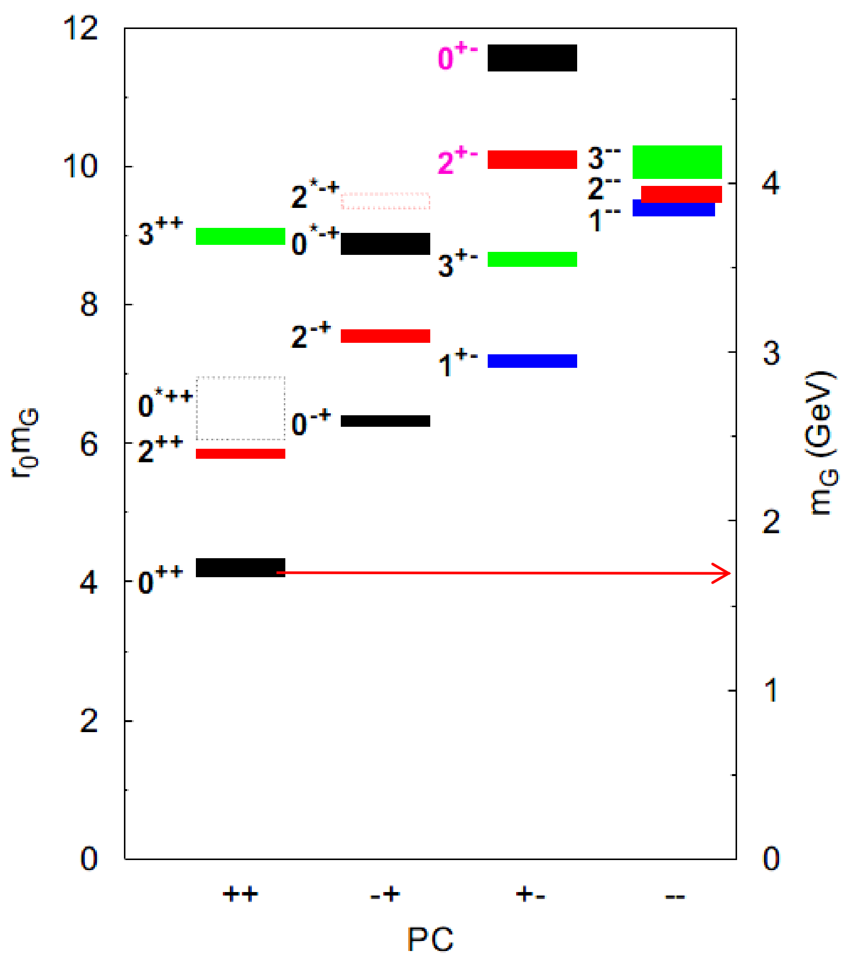
$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	−11.4	−24.5
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.1	36.4
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	32.1	30.5
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		

PDG, Chin.Phys.C38,090001(2014).

Properties for Glueball

- **bound states with 2 or more than 2 gluons**
- **consequence of non-abelian interaction**
- **populate the low energy region**
- **to be a narrow decay width**
- **chiral suppression for Scalar Glueball decays to quark-anti quark pair**

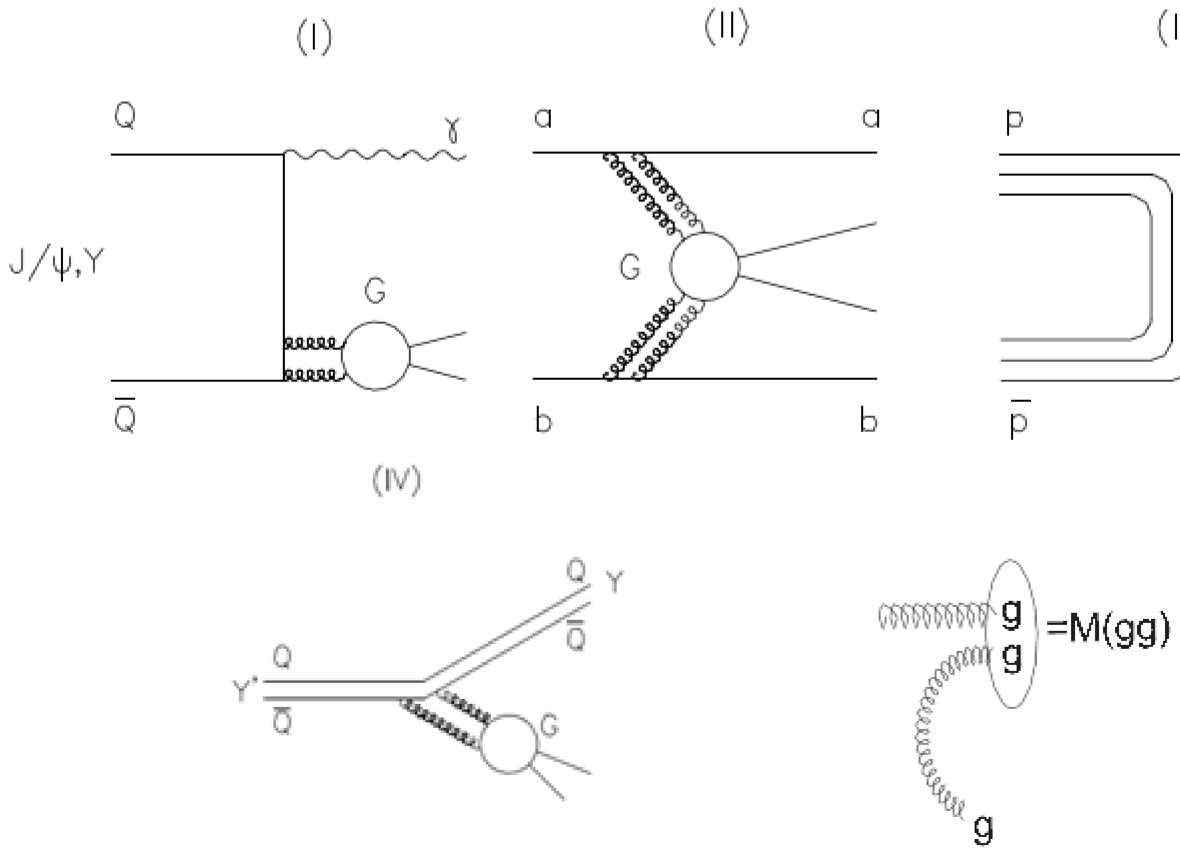
Glueball mass in Lattice QCD



J^{PC}	Other J	$r_0 m_G$	m_G (MeV/c^2)
0^{++}		4.21 (11)(4)	1730 (50) (80)
2^{++}		5.85 (2) (6)	2400 (25) (120)
0^{-+}		6.33 (7) (6)	2590 (40) (130)
0^{*++}		6.50 (44)(7)†	2670 (180)(130)
1^{+-}		7.18 (4) (7)	2940 (30) (140)
2^{-+}		7.55 (3) (8)	3100 (30) (150)
3^{+-}		8.66 (4) (9)	3550 (40) (170)
0^{*-+}		8.88 (11)(9)	3640 (60) (180)
3^{++}	6, 7, 9, ...	8.99 (4) (9)	3690 (40) (180)
1^{--}	3, 5, 7, ...	9.40 (6) (9)	3850 (50) (190)
2^{*-+}	4, 5, 8, ...	9.50 (4) (9)†	3890 (40) (190)
2^{--}	3, 5, 7, ...	9.59 (4) (10)	3930 (40) (190)
3^{--}	6, 7, 9, ...	10.06 (21)(10)	4130 (90) (200)
2^{+-}	5, 7, 11, ...	10.10 (7) (10)	4140 (50) (200)
0^{+-}	4, 6, 8, ...	11.57 (12)(12)	4740 (70) (230)

Y. Chen et al., Phys.Rev.D73,014516(2006).

Glue-rich processes



W. Ochs, J. Phys. G 40 (2013) 043001.

Data & explanation

→ production

$$\frac{\Gamma(J/\psi\to f_0(1710)\gamma)}{\Gamma(J/\psi\to f_0(1500)\gamma)} \sim 16.5$$ but $f_0(1500)$ not seen in $\gamma\gamma$

except in $\gamma\gamma \to \pi^0\pi^0$

→ decay

$$\frac{\Gamma(f_0(1710)\to\pi\pi)}{\Gamma(f_0(1710)\to K\bar{K})} = 0.31 \pm 0.05 \quad \frac{\Gamma(f_0(1500)\to\pi\pi)}{\Gamma(f_0(1500)\to K\bar{K})} = 4.1 \pm 0.5$$

→explanation: mixing matrix $N \equiv n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $S \equiv s\bar{s}$,

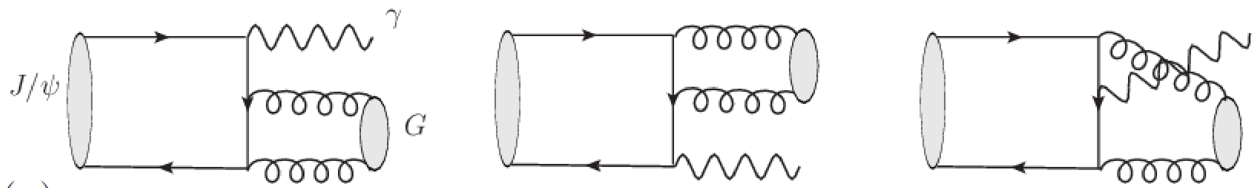
$$\begin{pmatrix} |f_0(1370)\rangle \\ |f_0(1500)\rangle \\ |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} 0.78 & 0.51 & -0.36 \\ -0.54 & 0.84 & 0.03 \\ 0.32 & 0.18 & 0.93 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}$$

Demanding a solid frame to systematically describe its productions and decays

H.Y. Cheng et al. arXiv:1503.06827;BESIII, PhysRevD.91.052006(2015).

Factorization for $V(\psi(nS), \Upsilon(nS)) \rightarrow G(0^{++}) + \gamma$

Radiative heavy quarkonium decay
e.g. $J/\psi(P) \rightarrow G(k) + \gamma(q)$



→ Momentum fraction at light-cone frame, LC factorization is valid

$$\frac{k^-}{k^+} = \frac{m_G^2}{m_{J/\psi}^2} \sim 0.20, \qquad \frac{\Lambda_{QCD}}{k^+} = \frac{\sqrt{2}\Lambda_{QCD}}{m_{J/\psi}} \sim 0.14,$$

~0.03 (in Upsilon decay) ~0.04 (in Upsilon decay)

assume that $m_G \approx 1.7GeV$, $\Lambda_{QCD} \approx 0.3GeV$.

→ There are constituent gluons , and also binding gluons which excite quark anti-quark sea in Glueball, as in the case of binding gluons, quark anti-quark sea in Proton. These effects should be considered in a valid factorization.

Naïve factorization

For the process $J/\psi(P) \to G(k) + \gamma(q)$,

→ S-matrix elements

$$\langle \gamma G(K)|S|J/\psi\rangle = -ieQ_c\varepsilon^{*\mu}(q)\int d^4x e^{iq\cdot x}\langle G(k)|\bar{c}(x)\gamma_\mu c(x)|J/\psi(P)\rangle ,$$

$$= i\frac{1}{24}eQ_c g_s^2\varepsilon^{*\alpha}(q)(2\pi)^4\delta(P-k-q)\langle 0|\chi^\dagger\sigma^\beta\psi|J/\psi\rangle \times \int\frac{d^4q_1}{(2\pi)^4}\Gamma^{\mu\nu}(k,q_1)M_{\alpha\beta\mu\nu}(P,k,q_1) ,$$

$$\Gamma^{\mu\nu}(k,q_1) = \int d^4x e^{-iq_1\cdot x - i(k-q_1)\cdot y}\langle G(k)|A^{a,\mu}(x)A^{a,\nu}(y)|0\rangle ,$$

X.G. He, H.Y Jin, J.P. Ma, Phys.Rev.D66,074015(2002).

→Twist-2 Glueball light-cone distribution amplitude (LCDA)

$$\Gamma^{\mu\nu}(k,q_1)|_{twist-2} = (2\pi)^4\delta(q_1^-)\delta^2(q_{1\perp})\frac{1}{u(u-1)}g_\perp^{\mu\nu}F_0(u) ,$$

$$F_0(u) = \frac{1}{2\pi(k^+)^2}\int dx^- e^{-i(1-2u)k^+x^-}\langle G(k)|G^a_{+\mu}(-x^-)G^a_{+\nu}(x^-)|0\rangle .$$

→ However, this kind of factorization will be revised at next-to-leading order

LCDAs of Glueball

Light-cone distribution amplitudes (LCDAs) is defined by the non-local operators matrix elements, which include the non-perturbative effects of Hadron. The parameter u is the paton momentum fraction in Hadron.

→ Introduce a two-component LCDA for Glueball

$$\mathbf{\Phi}(u) \equiv \begin{pmatrix} \phi_q(u) \\ \phi_g(u) \end{pmatrix},$$

With $\phi_q(u) = \int \frac{dz^-}{2\pi} \frac{e^{i(2u-1)k^+z^-/2}}{N_q} \langle G(k)|\overline{\Psi}_i(-z^-/2)L_{ij}(-z^-/2, z^-/2)\Psi_j(z^-/2)|0\rangle,$

describes the quark anti-quark sea effects.

$$\phi_g(u) = \int \frac{dz^-}{2\pi} \frac{e^{i(2u-1)k^+z^-/2}}{N_g u(1-u)} g_\perp^{\mu\nu} \langle G(k)|G^{a,+\mu}(-z^-/2)L_{ab}(-z^-/2, z^-/2)G^{b,+\nu}(z^-/2)|0\rangle,$$

describes the constituent gluons and binding gluons effects.

where gauge link is defined as $L(x,y) = P\, e^{ig\int_0^1 ds(x-y)_\mu A^\mu((x-y)s+y)},$

Factorization formula

For $J/\psi(P) \to G(k) + \gamma(q)$,

The amplitude can be factored into

$$iM = ieQ_c g_s^2 \langle 0|\chi^\dagger \boldsymbol{\sigma}\psi|J/\psi\rangle \times \int_0^1 du \int_0^1 dt(\varepsilon_{J/\psi}\cdot\varepsilon_\gamma m_c^2 \boldsymbol{H}_0(u,v,\mu) + \varepsilon_{J/\psi}\cdot q\,\varepsilon_\gamma \cdot P\boldsymbol{H}_1(u,v,\mu))\boldsymbol{\Phi}(t,\mu)\,,$$

with

$$\langle 0|\chi^\dagger \boldsymbol{\sigma}\psi|J/\psi\rangle = \Gamma_{J/\psi}(v,\mu)\langle 0|\chi^\dagger \boldsymbol{\sigma}\psi|J/\psi\rangle^r\,,$$
$$\boldsymbol{\Phi}(t,\mu) = \boldsymbol{\Gamma}(u,t,\mu)\boldsymbol{\Phi}^r(t,\mu)\,,$$

Hard kernels

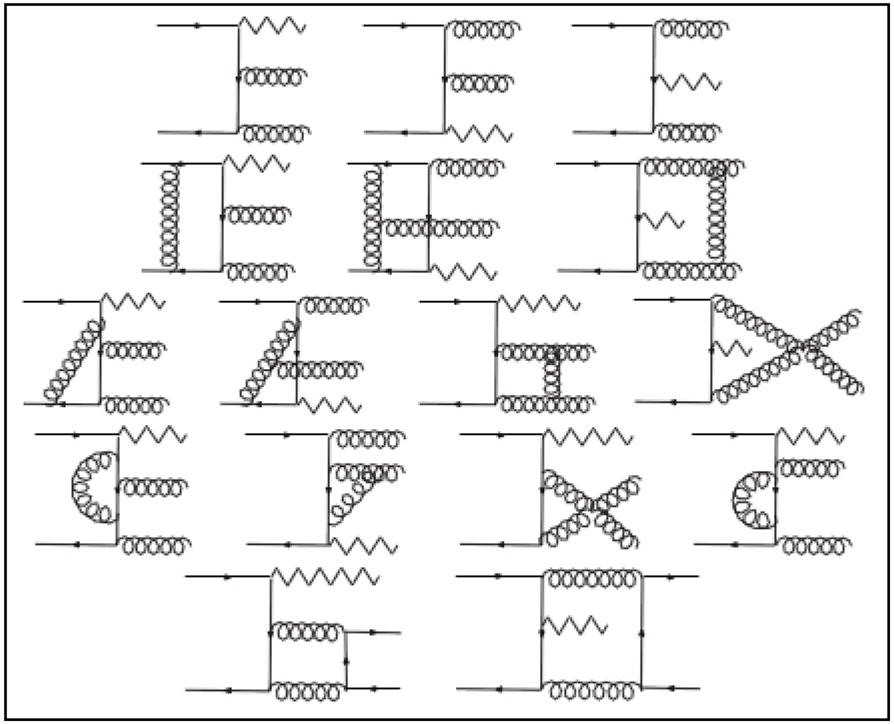
The hard kernels and soft factors can be calculated independently

$$\rightarrow \quad \boldsymbol{H}_0^{(0)} = \left(0, -\frac{\sqrt{2}}{N_c m_c^3 \sqrt{m_{J/\psi}} u(1-u)}\right), \quad \boldsymbol{H}_1^{(0)} = \left(0, \frac{1}{N_c m_c^3 \sqrt{2 m_{J/\psi}} u\bar{u}}\right).$$

$$\boldsymbol{H}_0^{(1)} = \frac{1}{\epsilon} \frac{\sqrt{2}\alpha_s}{2\pi N_c m_c^3 \sqrt{m_{J/\psi}} u\bar{u}} \left(\frac{n_f(2u-1)(u\ln u + \bar{u}\ln\bar{u})}{2u\bar{u}} - \epsilon\, n_f H_0^a, \right.$$
$$\left. \frac{C_A(2u^2-2u+1)(u\ln u + \bar{u}\ln\bar{u})}{u\bar{u}} + \frac{\beta_0}{2} + \frac{C_F}{4v}\left(\pi^2\epsilon\left(\frac{\mu}{2m_c}\right)^{2\epsilon} - i\pi v^{-2\epsilon}\right) - \epsilon\, H_0^b \right)$$

$$\boldsymbol{H}_1^{(1)} = -\frac{1}{2}\boldsymbol{H}_0^{(1)}|_{H_0^a \to H_1^a,\, H_0^b \to H_1^b},$$

Typical diagrams



Another 9

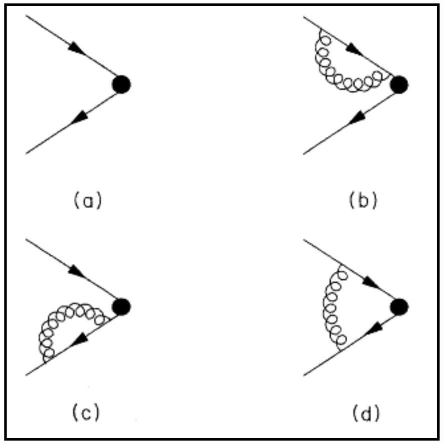
21

29

4

Renormalization

One for parent heavy quarkonium, the another for Glueball



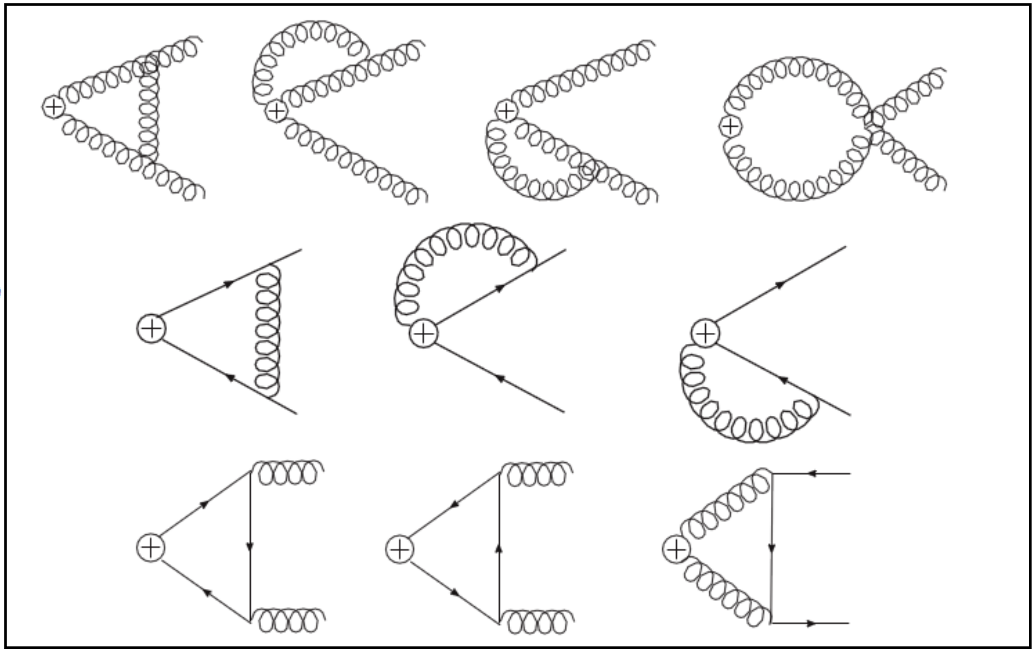
→ for heavy quarkonium

$$\Gamma^{(0)}_{J/\psi} = 1\,, \qquad \Gamma^{(1)}_{J/\psi} = \frac{\alpha_s C_F}{4\pi v}\left(\pi^2 - i\pi(\frac{1}{\epsilon} + \ln\frac{\mu^2}{4m_c^2 v^2})\right)\,.$$

→ for scalar Glueball

$$\mathbf{\Gamma}^{(0)}(u,t,\mu) = \begin{pmatrix} \delta(u-t) & 0 \\ 0 & \delta(u-t) \end{pmatrix}\,.$$

$$\mathbf{\Gamma}^{(1)}(u,t,\mu) = \frac{\alpha_s}{2\pi}(\frac{\mu}{\mu_0})^{2\epsilon}\frac{1}{\epsilon}\begin{pmatrix} S^{(1)}_{qq} & S^{(1)}_{qg} \\ S^{(1)}_{gq} & S^{(1)}_{gg} \end{pmatrix}\,,$$



(where the explicit expressions are listed in the next slice.)

→ explicit expression for the matrix elements at NLO

$$
\begin{aligned}
S^{(1)}_{qq}(u,t) &= C_F \frac{u}{t}\left(1+\frac{1}{t-u}\right)_+ \theta(t-u) + (u\to\bar{u}, t\to\bar{t})\ , \\
S^{(1)}_{qg}(u,t) &= 2n_f T_F \frac{u}{t^2\bar{t}}\left(2u-t-1\right)\theta(t-u) - (u\to\bar{u}, t\to\bar{t})\ , \\
S^{(1)}_{gq}(u,t) &= C_F \frac{u}{t}\left(2t-u\right)\theta(t-u) - (u\to\bar{u}, t\to\bar{t})\ , \\
S^{(1)}_{gg}(u,t) &= C_A \frac{u^2}{t^2}\left(\frac{1}{(t-u)_+} + \frac{\beta_0}{4C_A}\delta(u-t) + 2\left(\bar{u}+t(1+2\bar{u})\right)\right)\theta(t-u) \\
&\quad + (u\to\bar{u}, t\to\bar{t})\ , \\
F(x,y)_+ &= F(x,y) - \delta(x-y)\int_0^1 dz F(z,y)\,.
\end{aligned}
$$

→Renormalization group evolution equation for LCDA

$$
\mu^2 \frac{\partial}{\partial\mu^2}\mathbf{\Phi}(u,\mu^2) = \boldsymbol{V}\left(u,t,\alpha_s(\mu^2)\right)\otimes\mathbf{\Phi}(t,\mu^2)\,,
$$

$$
\boldsymbol{V} = -\mathbf{\Gamma}^{-1}\otimes\left(\mu^2\frac{\partial}{\partial\mu^2}\mathbf{\Gamma}\right) = \frac{\alpha_s(\mu^2)}{2\pi}\begin{pmatrix} S^{(1)}_{qq} & S^{(1)}_{qg} \\ S^{(1)}_{gq} & S^{(1)}_{gg}\end{pmatrix} + \mathcal{O}(\alpha_s^2)\,.
$$

Resummation-LCDAs of Glueball

→ eigenfunction- Gegenbauer series

$$\Phi_q(u,\mu^2) \;=\; 6u(1-u)f_q \sum_{n=1,3,\ldots} a_n^q(\mu^2) C_n^{3/2}(2u-1)\,,$$

$$\Phi_g(u,\mu^2) \;=\; 30u^2(1-u)^2 f_g \left(1+\sum_{n=3,5,\ldots} a_n^g(\mu^2) C_{n-1}^{5/2}(2u-1)\right)\,,$$

→Resummed Gegenbauer momenta

$$a_n^q(\mu^2) \;=\; \frac{1}{\delta\sigma}\left(a_n^+(\mu_0^2)\left[\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right]^{2\lambda_n^+/\beta_0} - a_n^-(\mu_0^2)\left[\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right]^{2\lambda_n^-/\beta_0}\right)\,,$$

$$a_n^g(\mu^2) \;=\; \frac{1}{\delta\sigma\gamma_n^{gq}}\left(a_n^+(\mu_0^2)\left(\lambda_n^+-\gamma_n^{gg}\right)\left[\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right]^{2\lambda_n^+/\beta_0} - a_n^-(\mu_0^2)\left(\lambda_n^--\gamma_n^{gg}\right)\left[\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right]^{2\lambda_n^-/\beta_0}\right)$$

where $\lambda_n^\pm$ are the eigenvalues

$$\lambda_n^\pm = \frac{1}{2}\left(\gamma_n^{gg}+\gamma_n^{qq}\pm\delta\right)\,,$$

with $\delta=\sqrt{(\gamma_n^{gg}-\gamma_n^{qq})^2+4\gamma_n^{gq}\gamma_n^{qg}}$. And $a_n^\pm$ are the eigenvectors, with

$$a_n^\pm(\mu^2) = a_n^q(\mu^2)\gamma_n^{gq} - a_n^g(\mu^2)(\lambda_n^\pm-\gamma_n^{gg})\,.$$

LCDA for scalar Glueball

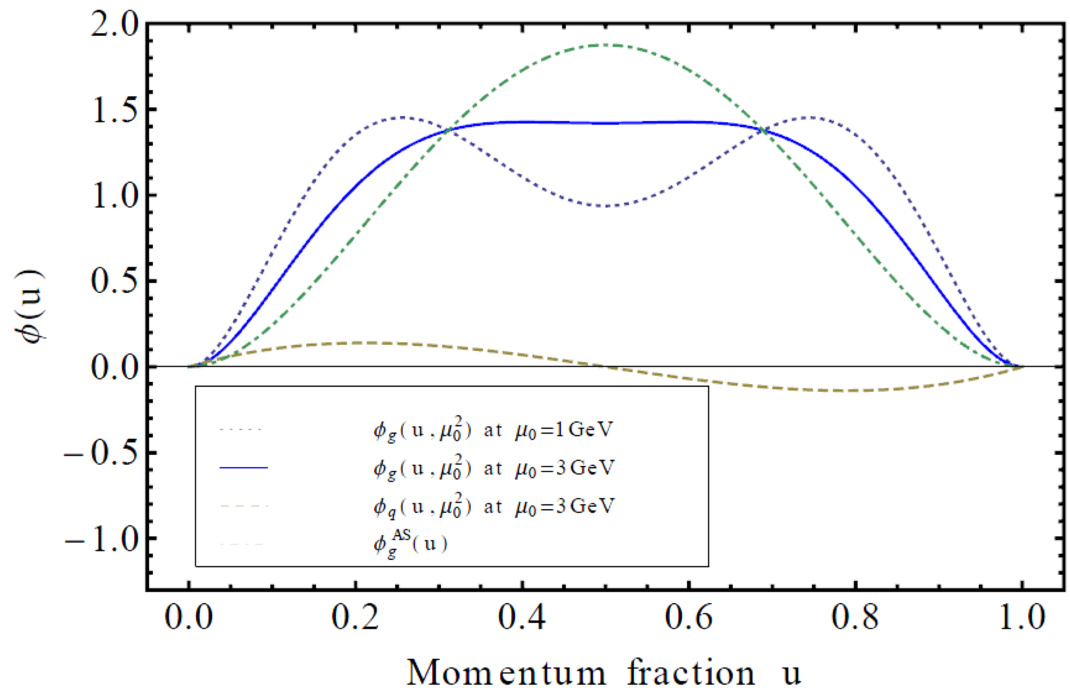


FIG. 3: The light-cone distribution amplitude for scalar Glueball, where we only consider the first Gegenbauer momentum and resum the corresponding large logatithms in $a_3^g(\mu)$ and $a_1^q(\mu)$, using $\mu_0 = 1\text{GeV}$ and $a_3^g(\mu_0) = 0.2\text{GeV}$, $a_1^q(\mu_0) = 0$ as input. The asymptotic form of $\phi_g(u)$ is $30 f_g u^2(1-u)^2$.

Phenomenological application

→ Extract the decay constant

For scalar Glueball with mass about 1.7GeV, CLQCD gives

$$\mathcal{B}(J/\psi \to G + \gamma) \ = \ (3.8 \pm 0.9) \times 10^{-3}.$$

CLQCD, L.C. Gui et al. PRL 110,021601(2013).

one can extract the decay constant

$$f_g = 0.0386^{+0.0097}_{-0.0049} GeV,$$

and predicts

$$\mathcal{B}(\psi(2S) \to G + \gamma) \ = \ (5.9^{+3.4}_{-1.4}) \times 10^{-4},$$
$$\mathcal{B}(\Upsilon \to G + \gamma) \ = \ (1.3^{+0.7}_{-0.3}) \times 10^{-4}.$$

→ Study the mixing effects

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.819(89) & 0.290(91) & -0.495(118) \\ -0.399(113) & 0.908(37) & -0.128(52) \\ 0.413(87) & 0.302(52) & 0.859(54) \end{pmatrix} \begin{pmatrix} |n\bar{n}\rangle \\ |s\bar{s}\rangle \\ |G\rangle \end{pmatrix}.$$

Lee&Weingarten, PRD61,014015(2000).

leads to

Branching ratio (10^{-4})	This work	PDG2014 [31]
$\mathcal{B}(J/\psi \to f_0(1370) + \gamma)$	9.3 ± 2.2	–
$\mathcal{B}(J/\psi \to f_0(1500) + \gamma)$	0.62 ± 0.15	1.01 ± 0.32
$\mathcal{B}(J/\psi \to f_0(1710) + \gamma)$	28.0 ± 6.6	> 15.6
$\mathcal{B}(\psi(2S) \to f_0(1370) + \gamma)$	$1.45^{+0.83}_{-0.34}$	–
$\mathcal{B}(\psi(2S) \to f_0(1500) + \gamma)$	$0.97^{+0.56}_{-0.23}$	–
$\mathcal{B}(\psi(2S) \to f_0(1710) + \gamma)$	$4.4^{+2.5}_{-1.0}$	> 0.9
$\mathcal{B}(\Upsilon \to f_0(1370) + \gamma)$	$0.32^{+0.18}_{-0.08}$	–
$\mathcal{B}(\Upsilon \to f_0(1500) + \gamma)$	$0.02^{+0.01}_{-0.01}$	< 0.15
$\mathcal{B}(\Upsilon \to f_0(1710) + \gamma)$	$0.96^{+0.55}_{-0.23}$	< 2.6

Summary

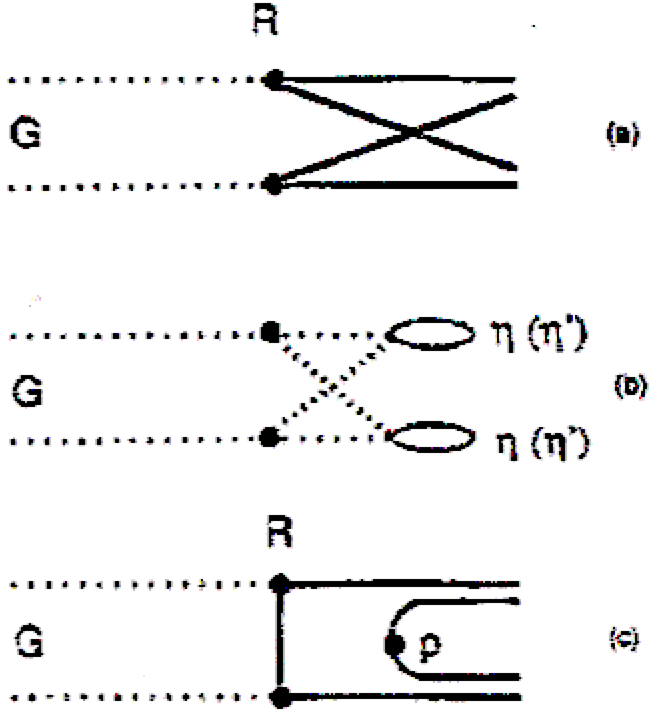
- **Glueball is a consequence of QCD, its hunting need more investigation on both theoretical approach and experimental techniques.**

- **A factorization of a heavy quarkonium to scalar Glueball is established, which is valid in all order of strong coupling constant.**

- **A symstematic analyse for all kinds of quantum number and production mechanism should be done in order to confirm Glueball**

Thank You !

Additional materials



Glueball exclusive two-body decay

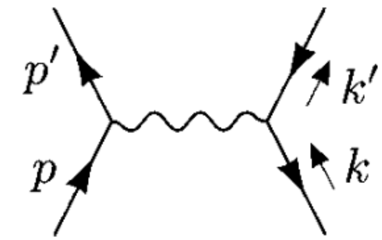
Static potential-an example

Definition: $$\langle p'|iT|p\rangle = -i\widetilde{V}(\mathbf{q})\,(2\pi)\delta(E_{\mathbf{p}'} - E_{\mathbf{p}}), \qquad (\mathbf{q} = \mathbf{p}' - \mathbf{p}).$$

Leading-order results: $$V_{q\bar{q}}(r) \sim -f\,\frac{\alpha_S}{r}$$

$$f_{q\bar{q}} = T^a_{ij}T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right) \quad \rightarrow C_A C_F = 4$$

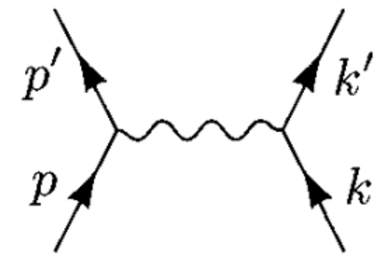
Color-Singlet
(i=l,j=k),
Attractive



$$V_{qq}(r) \sim +f\,\frac{\alpha_S}{r}$$

$$f_{qq} = T^a_{ij}T^a_{lk} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right) \quad \rightarrow 1$$

Diquark
(i=l,j=k),
Repulsive



$$V_{gg}(r) \sim -f\,\frac{\alpha_s}{r}$$

$$f_{gg} = f^{aec}f^{bed} = \frac{8}{21}\left(\delta_{ab}\delta_{cd} - \frac{1}{N_c^2 - 1}\delta_{ac}\delta_{bd}\right) \quad \rightarrow 2C_A^2 C_F = 24$$

Color-Singlet
(a=b,c=d),
Attractive