Glauber divergence in the annihilation diagrams of *B* decays in the PQCD approach

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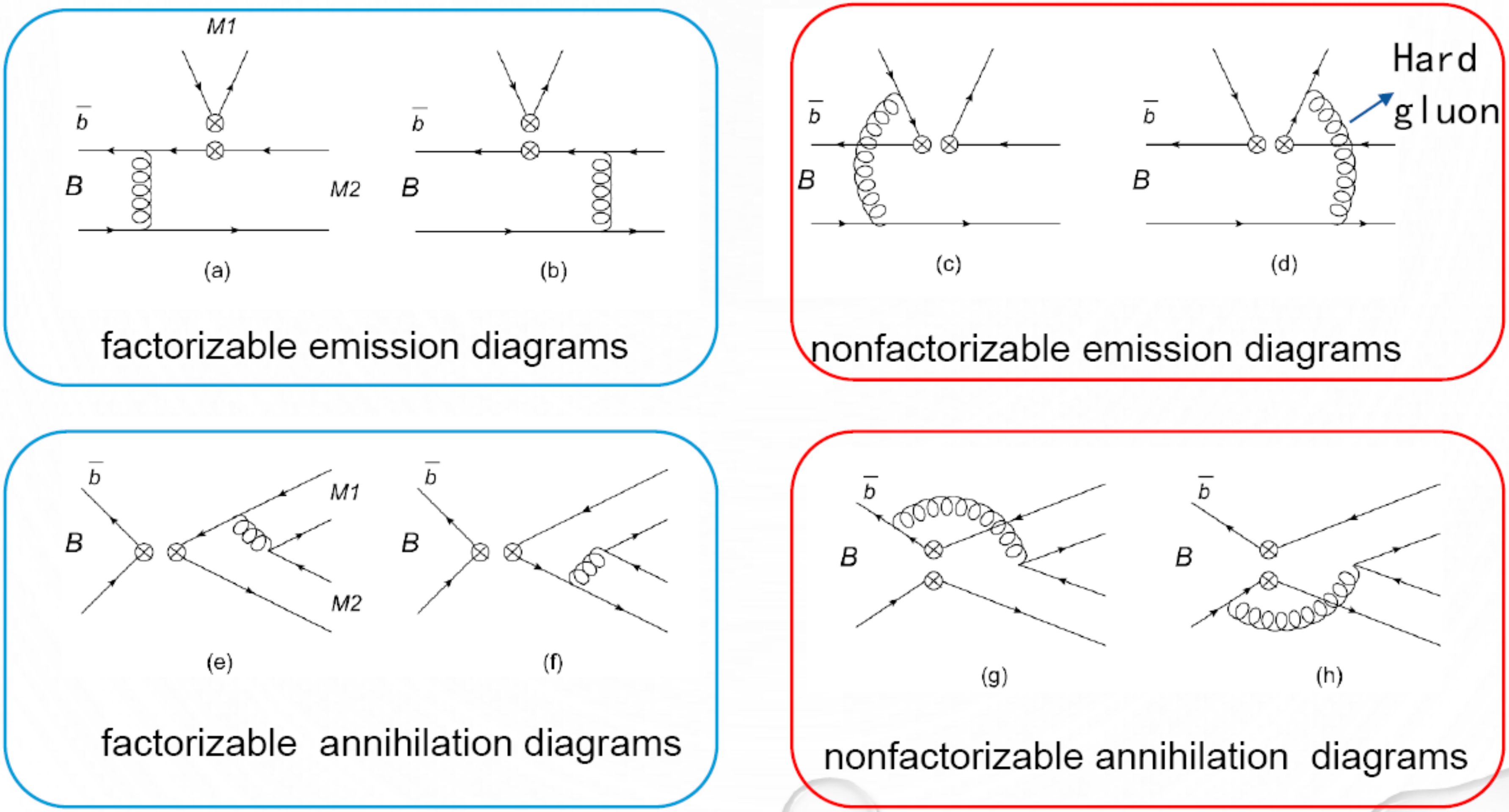
Outline

- Introduction
- Glauber divergence in the nonfactorizable annihilation diagrams.
- Numerical predictions to $B^0 \to \pi^0\pi^0, \pi^0\eta', \pi^0\rho^0, \pi^0\omega$
- Summary

Introduction

- The observed B->pipi, pi rho and rhorho branching ratios are several times larger, and consistent with the theoretical predictions.
- The difficulty for resolving puzzles do not only lies in the mismatch of the data and predictions, but also because similar process like B to rho rho sharing similar topological structures is consistent with the data.
- A recent proposal [PRD 83, 034023] making use of the Glauber gluon effects shows novel possibility to the puzzles.
- In this work we explore the Glauber gluons in the nonfactorizable annihilation amplitudes.

Feynman diagrams contributing to the $B \rightarrow M_1 M_2$ decays in the PQCD approach



Compared with QCDF:

(1)The k_T factorization formalism resolve the end point singularity.

(2)Annihilation diagrams can be calculated.

Eikonal Approximation

- Approximation of straight-line propagation
- Omitting the transverse momentum
- For the coll-gluon attached to the outermost quark line(assuming gluon coll to Φ_1, so the gluon momentum is $(1, \lambda^2, \lambda)$):

$$\begin{aligned}
\frac{i(\not{l}+\not{p})}{(l+p)^2+i\varepsilon}\gamma^\mu &= \frac{2(l+p)^\mu}{(l+p)^2+i\varepsilon} - \gamma^\mu \frac{i(\not{l}+\not{p})}{(l+p)^2+i\varepsilon} \\
&= \frac{2i(l+p)^\mu}{l^2+2l\cdot p+i\varepsilon} \\
&= \frac{in_-^\mu}{l^+ + i\varepsilon/p^-}
\end{aligned}$$

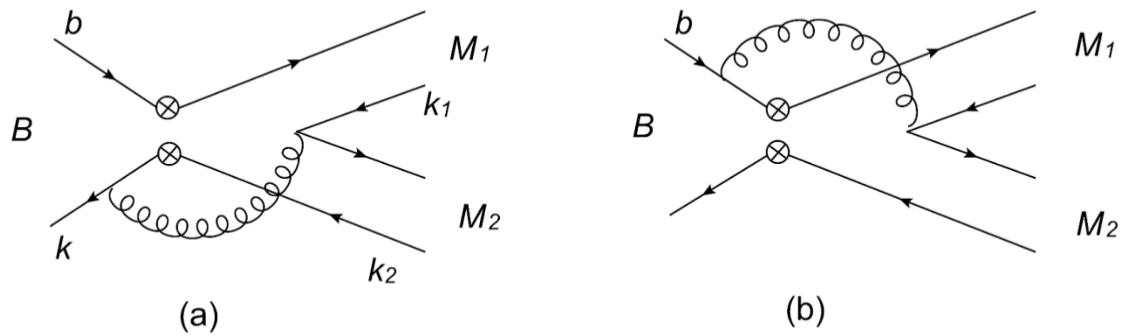
- For the coll-gluon attached to the inner quark line:

$$\begin{aligned}
\frac{i(\not{l}+\not{p})}{(l+p)^2+i\varepsilon}\gamma^\mu \frac{i\not{p}}{p^2+i\varepsilon} &= \left(\frac{i}{p^2+i\varepsilon} - \frac{i}{(l+p)^2+i\varepsilon}\right)\frac{(\not{l}+\not{p})\gamma^\mu \not{p}}{(l+p)^2-p^2+i\varepsilon'} \\
&= \left(\frac{i\not{p}}{p^2+i\varepsilon} - \frac{i(\not{l}+\not{p})}{(l+p)^2+i\varepsilon}\right)\frac{in_-^\mu}{l^+ + i\varepsilon'}
\end{aligned}$$

Nonfactorizable annihilation diagrams

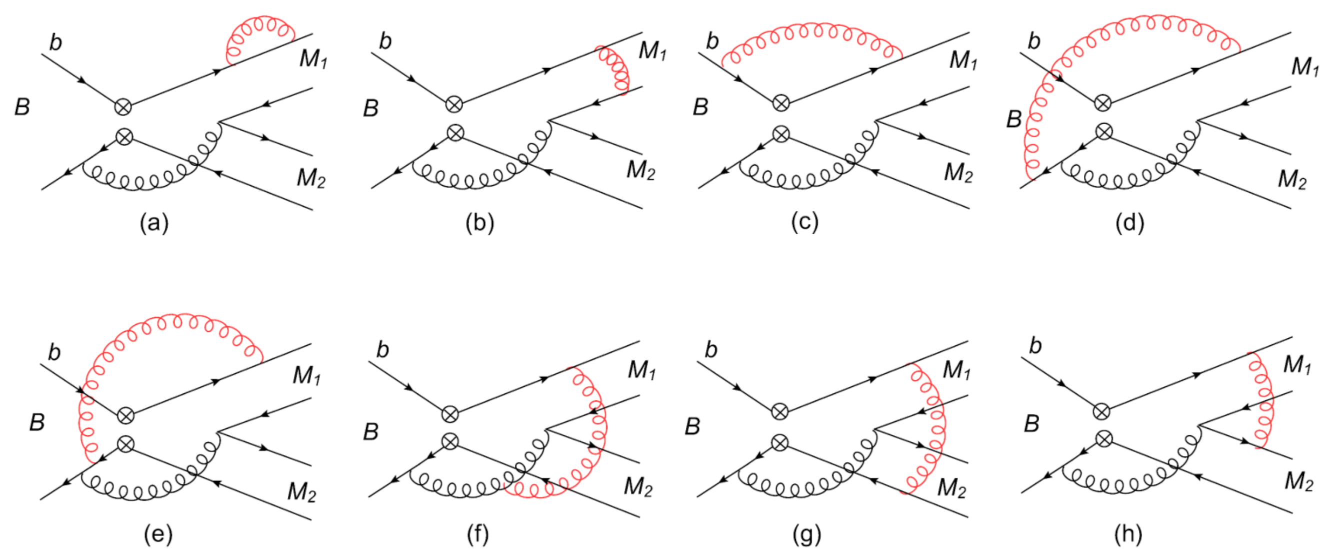
LO:

Diagrams with the hard gluon attached to the light quark of B meson.



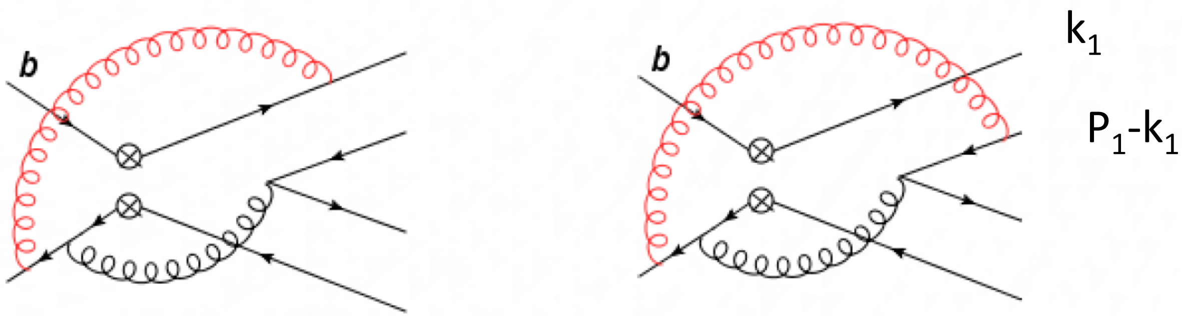
NLO:

We add a radiative gluon emitted by the valence quark in M_1
The radiative gluon can also be emitted from other 3 (anti)quarks in M_1 and M_2



Soft divergence in NLO diagrams

The soft divergences can always be canceled due to the *mechanism* of color transparency.



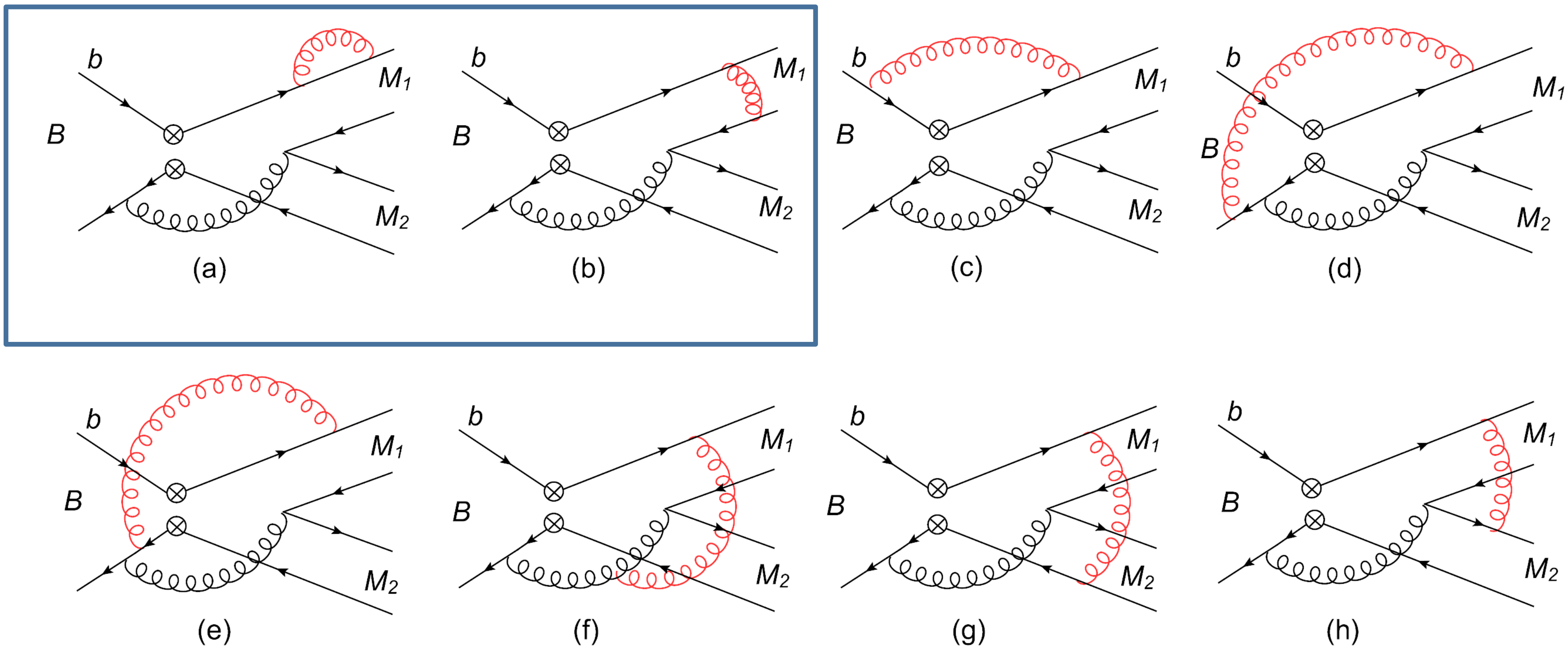
$$\gamma^\mu \frac{i \not{l} + P_1 - k_1}{(l + P_1 - k_1)^2} \rightarrow \frac{i(P_1^\mu - k_1^\mu)}{l^-(P_1 - k_1)^+} = \frac{i n^\mu}{n \cdot l},$$

$$-\frac{i \not{l} + k_1}{(l + k_1)^2} \gamma^\mu \rightarrow -\frac{i k_1^\mu}{l^- k_1^+} = -\frac{i n^\mu}{n \cdot l},$$

Collinear divergence in NLO diagrams

As two-particle reducible diagrams, they can be factorized into the M_1 meson wave function.

No Glauber



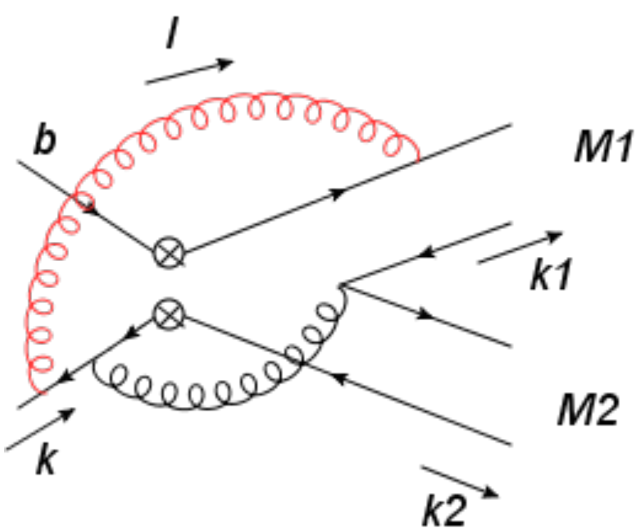
No Glauber

Cancel with (h)

No Glauber

Cancel with (f)

$$I = -i g^4 N_c C_F^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2} \frac{1}{(\not{P}_2 - k_2 + k_1)^2} \mathrm{tr}\Big[\Phi_B \cdot \gamma_\mu \cdot \frac{\not{l} - \not{k}}{(l-k)^2} \cdot \gamma_\nu \cdot \frac{\not{l} + \not{P}_2 - \not{k}_2 + \not{k}_1 - \not{k}}{(l + P_2 - k_2 + k_1 - k)^2} \cdot \Gamma \cdot \Phi_{M2} \cdot \gamma^\nu \cdot \Phi_{M1} \cdot \gamma^\mu \cdot \frac{\not{l} - \not{P}_1 + \not{k}_1}{(l - P_1 + k_1)^2} \cdot \Gamma \Big]$$

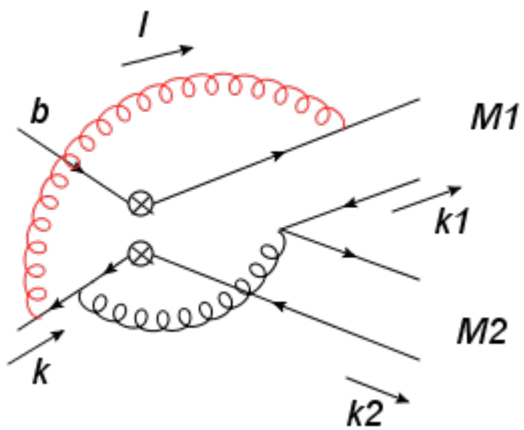


Momenta of mesons

$$P_1 = (P_1^+, 0, \mathbf{0})$$
$$P_2 = (0, P_2^-, \mathbf{0})$$
$$k_1 = (k_1^+, 0, \boldsymbol{k}_{1T})$$
$$k_2 = (0, k_2^-, \boldsymbol{k}_{2T})$$
$$k = (k^+, k^-, \boldsymbol{k}_T)$$

$$P_1^+, k_1^+, P_2^-, k_2^- \sim O(m_B)$$

$$|\boldsymbol{k}_{iT}|, k^\mu \sim O(\Lambda_{QCD})$$



Consider the l^+ poles

$$l^+ = k^+ + |l_T - k_T|^2 / 2(l^- - k^-) + i\epsilon(+i\epsilon)$$

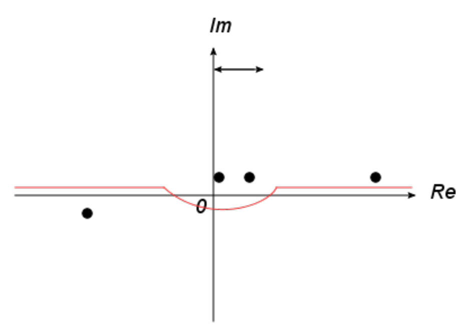
$$l^+ = -k_1^+ + k^+ + |l_T - k_{2T} + k_{1T} - k_T|^{2} \ldots$$

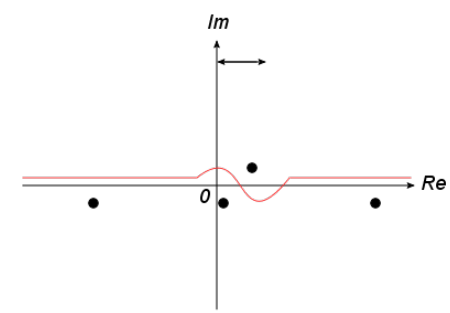
$$l^+ = P_1^+ - k_1^+ + |l_T + k_{1T}|^2 / 2l^- + i\epsilon(\ldots$$

$$l^+ = |l_T|^2 / 2l^- + i\epsilon(-i\epsilon)$$

for the range of

$-p_2^- + k_2^- + k^- < l^- < 0$ $\qquad$ $0 < l^- < k^-$





It contains four denominators

$$[(l-k)^2 + i\varepsilon][(l+P_2 - k_2 + k_1 - k)^2 + i\varepsilon][(l-P_1 + k_1)^2 + i\varepsilon][l^2 + i\varepsilon]$$

There is no pinch singularity for other range because all the poles are in the same half plane

Eikonalization

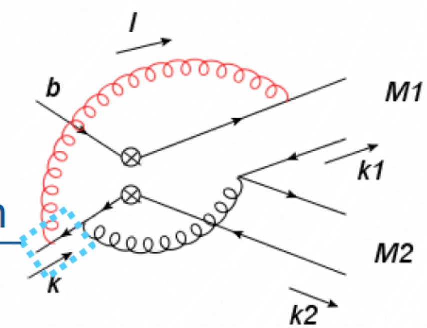
We can deform the contour of l^+ , and make it remains $O(m_B)$.
Then we have the hierarchy

$$l^+ k^- \sim O(m_B \Lambda_{QCD}) \gg |l_T - k_T|^2 \sim O(\Lambda_{QCD}^2)$$

Using the eikonal approximation (Omitting the transverse momentum)

$$\gamma_\mu \cdot \frac{\not l - \not k}{(l-k)^2 + i\epsilon} \approx \frac{-n_{-\mu}}{-l^+ + i\varepsilon}$$

eikonalization



$$I = -i g^4 N_c C_F^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2} \frac{1}{(P_2 - k_2 + k_1)^2}$$
$$tr[\Phi_B \cdot \gamma_\mu \cdot \frac{\not l - \not k}{(l-k)^2} \cdot \gamma_\nu \cdot \frac{\not l + \not P_2 - \not k_2 + \not k_1 - \not k}{(l+P_2-k_2+k_1-k)^2} \cdot \Gamma \cdot \Phi_{M2} \cdot \gamma^\nu \cdot \Phi_{M1} \cdot \gamma^\mu \cdot \frac{\not l - \not P_1 + \not k_1}{(l-P_1+k_1)^2} \cdot \Gamma]$$

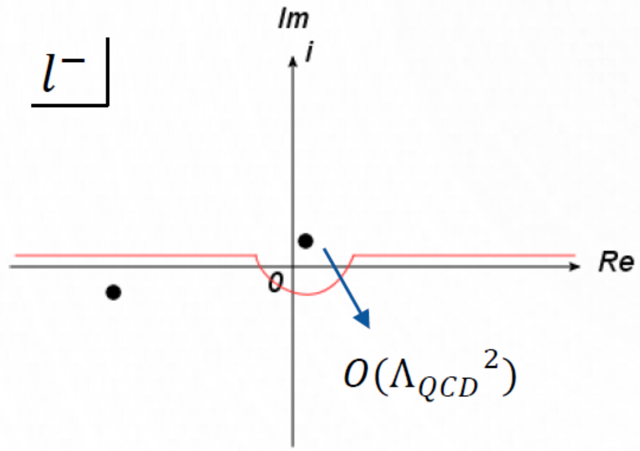
Employing the principal-value prescription

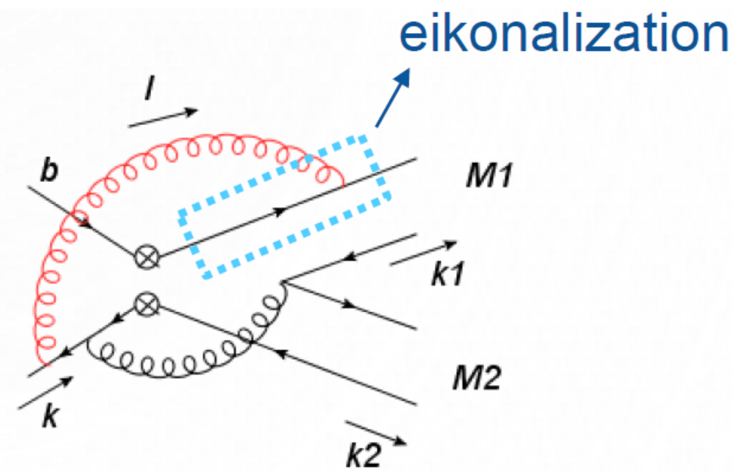
$$\frac{1}{-n_{-}\cdot l+\mathrm{i}\varepsilon}=\frac{PV}{-l^{+}}-i\pi\delta(l^{+})$$

Consider the l^{-} poles from the denominators with $l^{+}=0$

$$l^{-}=-P_{2}{}^{-}+k_{2}{}^{-}+k^{-}+\frac{\left|l_{T}-k_{2T}+k_{1T}-k_{T}\right|^{2}}{2(k_{1}^{+}-k^{+})}-i\epsilon$$

$$l^{-}=\frac{\left|l_{T}+k_{1T}\right|^{2}}{2(-P_{1}^{+}+k^{+})}+i\epsilon$$





We can deform the contour of l^-, such that l^- remains $O(\Lambda_{QCD})$.

The hierarchy is

$$l^-(P_1^+ - k^+) \sim O(\Lambda_{QCD} m_B) \gg \left|l_T + k_{1T}\right|^2 \sim O(\Lambda_{QCD}{}^2)$$

Then we can eikonalize the propagator $(l - P_1 + k_1)^2$

After another eikonalization

$$I = -ig^4 N_c C_F^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2} \frac{1}{(P_2 - k_2 + k_1)^2} tr[\, \Phi_B . \gamma_\mu . \frac{l-k}{(l-k)^2} . \gamma_\nu .$$
$$\frac{l + P_2 - k_2 + k_1 - k}{(\mathrm{l+P_2-k_2+k_1-k})^2} . \Gamma . \Phi_{M2} . \gamma^\nu . \Phi_{M1} . \gamma^\mu . \frac{l - P_1 + k_1}{(l - P_1 + k_1)^2} . \Gamma]$$

⇨ $\frac{n_{+\mu}}{-l^- + \mathrm{i}\varepsilon}$

We have

$$I^{(1)} = -ig^4 N_c C_F^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(P_2 - k_2 + k_1)^2} tr[\, \Phi_B . \gamma_\nu .$$
$$\frac{l + P_2 - k_2 + k_1 - k}{(\mathrm{l+P_2-k_2+k_1-k})^2} . \Gamma . \Phi_{M2} . \gamma^\nu . \Phi_{M1} . \Gamma] \times \frac{1}{-l^- + i\epsilon} \frac{1}{-l_T^2 + i\epsilon} (-i\pi\delta(l^+))$$

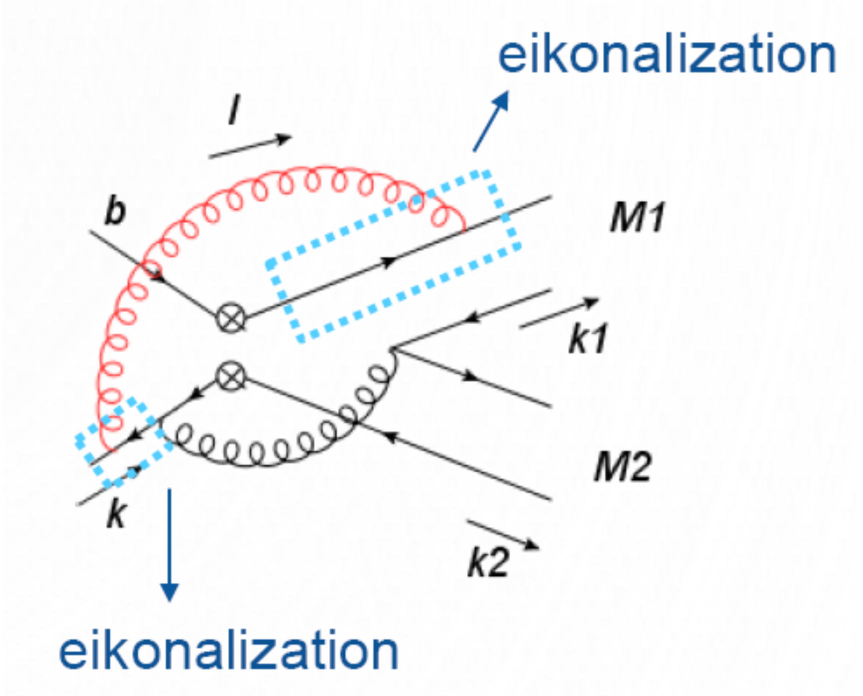
⇩

Glauber divergence

Glauber divergence

Finally We get an imaginary logarithmic divergence, called Glauber divergence.

$$I^{(1)} = \mathrm{i}\frac{\alpha_s}{\pi}\frac{C_F}{2}\int \frac{d^2 l_T}{l_T^2} M^{(0)}(l_T)$$



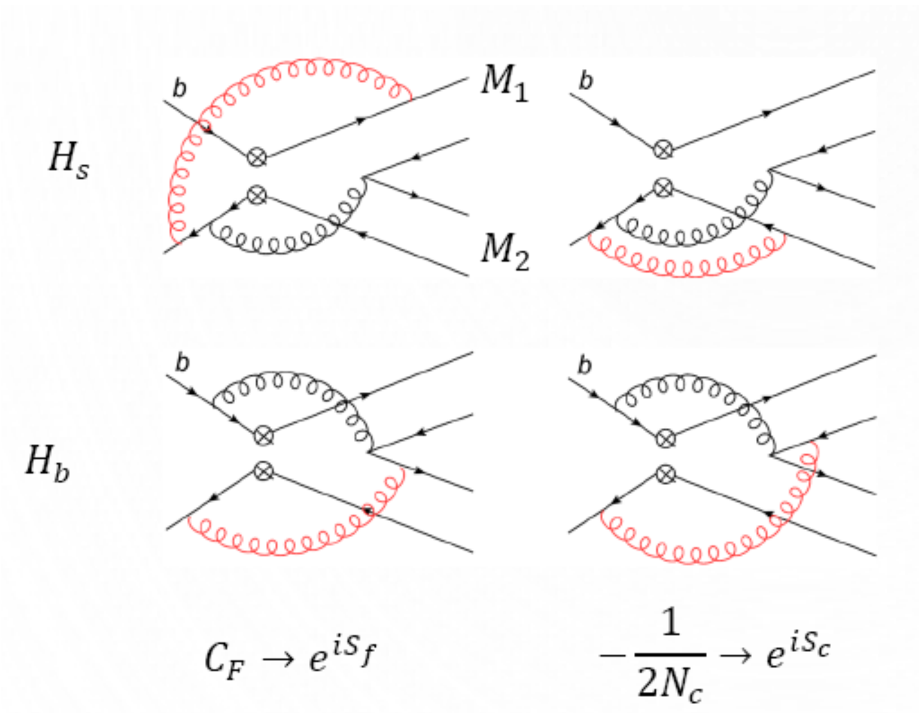
•It is proved that we can construct a soft factor

$$S(\mathbf{b}) = exp[i\frac{\alpha_s}{\pi}C_F\int\frac{d^2l_T}{l_T^2}e^{-i\mathbf{l_T}\cdot\mathbf{b}}]$$

to collect the Glauber gluon effects to all orders.

•However, the soft factor collecting all the Glauber gluons originates from the nonperturbative region. So the **b** dependence of the soft factor can be obtained only by nonperturbative calculations or experimental feed.

•At present, we ignore the **b** dependence of S(**b**) and consider S(**b**) as a free parameter



We postulate the soft factor associated with pion is significant, for it is a Nambu-Goldstone boson.

$$B^0 \to \pi^0\pi^0, \pi^0\eta', \pi^0\rho^0, \pi^0\omega$$

We focus on the above channels in our numerical calculations:

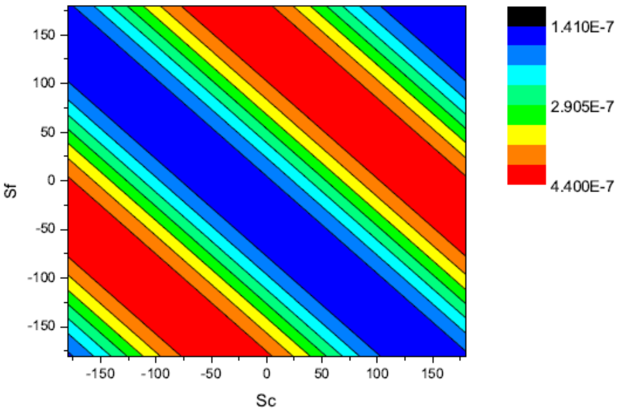
(1)The nonfactorizable annihilation diagrams can play some role because

- Tree dominated
- No color-allowed tree

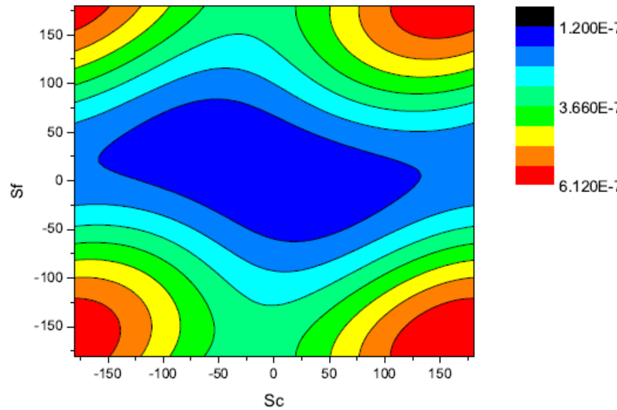
(2)The theoretical results from the PQCD approach are much smaller than Experimental data

Mode	Experimental data	LO PQCD
$B(\pi^0\pi^0)$	1.62 ± 0.31	0.171
$B(\pi^0\eta')$	1.2 ± 0.6	0.129
$B(\pi^0\rho^0)$	2.0 ± 0.5	0.121
$B(\pi^0\omega)$	< 0.5	0.122

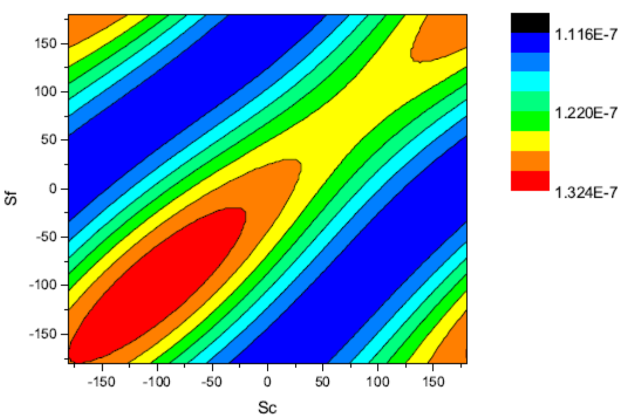
TABLE I: Experimental data and LO PQCD predictions for the branching ratio of the $\pi^0\pi^0, \pi^0\eta', \pi^0\rho^0, \pi^0\omega$ decay modes (in unit of 10^{-6}).



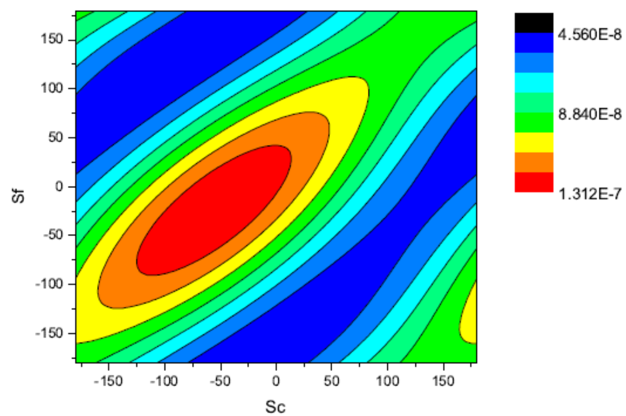
(a) $B^0 \to \pi^0\pi^0$



(b) $B^0 \to \pi^0\rho^0$



(c) $B^0 \to \pi^0\eta\prime$



(d) $B^0 \to \pi^0\omega$

Soft factor dependences of branching ratios in some *B* decays when considering the Glauber divergence in the nonfactorizable annihilation diagrams

From the above pictures, we can see the best region is

$$S_f = S_c(\text{around } -50^o)$$

Our results from B decays are consistent with that of D decays in the factorization-assisted topological-amplitude approach.

Their global fit results are: $(S_f = S_c = S_\pi)$

$D \to PP \quad S_\pi = -29^o$ PRD 86,036016

$D \to VV \quad S_\pi = -55^o \ (-49^o)$ PRD 89,054006

Summary

- We analyzed the nonfactorizable annihilation diagrams of B meson decay processes and found that there also exist the uncancelled Glauber divergences similar to that in the nonfactorizable emission amplitudes.

- We then include the contributions from the annihilation diagrams with the extracted Glauber factors into the predictions of the branching ratios in the B meson decay processes: $B^0 \to \pi^0\pi^0, \pi^0\eta', \pi^0\rho^0, \pi^0\omega$

- The best fit region for explaining the data is $S_f = S_c(\text{around } -50^o)$

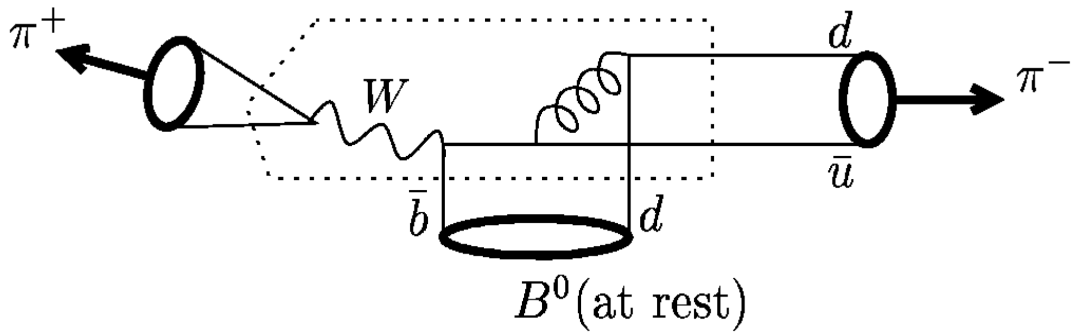
- we argue that the discrepancies between experimental data and theoretical predictions can still be considered as in the framework of the standard model and no new physics explanations are needed.

- The future calculations on the Glauber factor may provide more reliable predictions on the non-leptonic decays

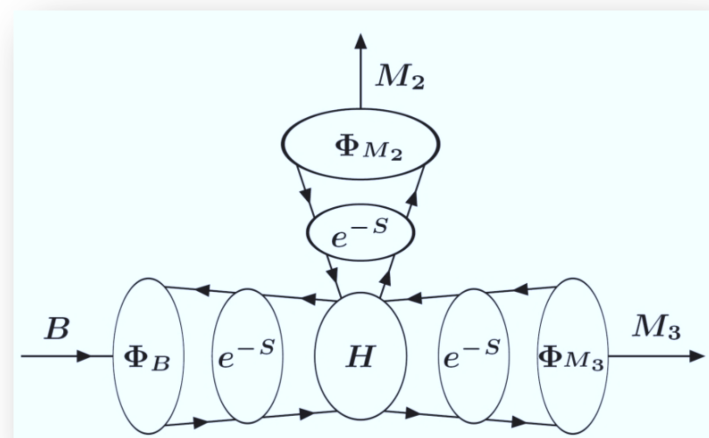
Thank You!

The perturbative QCD (PQCD) approach based on the k_T factorization theorem has been applied to two-body nonleptonic B meson decays successfully.

The physical picture of PQCD [PRD 63, 074009]



The decay amplitude can be factorized into the convolution of the Wilson coefficients , the hard scattering kernel and the light-cone wave functions of mesons characterized by different scales.



$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3$$
$$\times Tr\left[C(t)\Phi_B(x_1,b_1)\Phi_{M_2}(x_2,b_2)\Phi_{M_3}(x_3,b_3)H(x_i,b_i,t)S_t(x_i)e^{-S(t)}\right]$$

- Wilson coefficient: short distance contributions
- Hard kernel: process dependent
- Sudakov factor: collect the double logs from the combination of the soft and collinear divergences
- Wave function: extracted from experimental data

The factorization of the Glauber divergences into a soft factor up to all orders has been proved. (H-n. Li and S. Mishima, 2011)

At LO

$$I_a^{(0)} = \int d^2b S^{(0)}(\mathbf{b}) \mathcal{M}_a^{(0)}(\mathbf{b}),$$

At NLO

$$I_a^{(1)} \approx \int d^2b S^{(1)}(\mathbf{b}) \mathcal{M}_a^{(0)}(\mathbf{b}),$$

$$S^{(1)}(\mathbf{b}) = i\frac{\alpha_s}{\pi} C_F \int \frac{d^2 l_T}{l_T^2} e^{-i\mathbf{l}_T\cdot\mathbf{b}}.$$

$\Rightarrow$

$$G^{(j)} = \sum_{i=0}^{j} S^{(i)} \otimes \mathcal{M}_a^{(j-i)},$$

By analyzing the different attachments of gluons to lines in diagrams and by employing Ward identity. It is proved that

$$G^{(N+1)} = \sum_{i=1}^{N+1} S^{(i)} \otimes \mathcal{M}_a^{(N+1-i)} + \mathcal{M}_a^{(N+1)}$$

$$= \sum_{i=0}^{N+1} S^{(i)} \otimes \mathcal{M}_a^{(N+1-i)},$$

with $S^{(0)} = 1$.

•The Glauber divergence may not cause trouble, if the LO amplitude is real.

$$|\mathcal{M}|^2 = |\mathcal{M}^{(0)}|^2 + 2\,\mathrm{Re}[\mathcal{M}^{(0)}\mathcal{M}^{(1)*}].$$

•The Glauber divergence does not exist in the collinear factorization.

•The LO amplitude is complex in the k_T factorization, since partons carry transverse momenta, and internal lines go on mass shell at finite momentum fractions

Development in PQCD

- NLO corrections:

  pion EM form factor

  B leptonic decays

  pi->pi

  B->pi form factor

- Three body B decays

$$\mathcal{A} = \phi_B \otimes H \otimes \phi_{h_1 h_2} \otimes \phi_{h_3},$$

- Glauber effects in both B decays and pion-induced Drell-Yan processes