Spectra of mesons and baryons in Regge phenomenology

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Contents

- Motivation
- Mass relations and mass spectra
  - Charm-bottom mesons
  - Doubly and triply charmed baryons
  - Doubly and triply bottom baryons
- Summary

Motivation

- **Spectroscopy → interaction dynamics & structures**

  atomic spectrum →   atomic quantum theory

  nuclear spectrum →   shell model, collective motion

  hadron spectrum →   ? Important discovery

- **Many mesons and baryons discovered recently:**

  $B_c(2S)$ ATLAS   PRL 113, 212004

  $\Xi_b^*(5955)^-$ LHCb   PRL 14, 062004

  ……

Gell-Mann-Okubo formula

M. Gell-Mann, Phys. Rev. 125, 1067 (1964) S. Okubo, Prog. Theor. Phys. 27, 949 (1962)

$$M_\rho + M_\phi = 2M_{K^*}$$

$$4M_K = 3M_\pi + M_\eta \quad \rightarrow \quad 4M_K^2 = 3M_\pi^2 + M_\eta^2$$

$$(N + \Xi)/2 = (3\Lambda + \Sigma)/4,$$

$$\Omega - \Xi^* = \Xi^* - \Sigma^* = \Sigma^* - \Delta,$$

- Predicted the existence and mass of Ω^-

After *c* quark and *b* quark were found in 1970s, GMO formula was generalized to

$$2M_D = M_{cc} + M_{nn}, \quad 2M_{D_s} = M_{cc} + M_{ss},$$

$$2M_B = M_{bb} + M_{nn}, \quad 2M_{B_s} = M_{bb} + M_{ss}, \quad 2M_{B_c} = M_{bb} + M_{cc}.$$

These Eqs. do not agree with experiments, *e.g.*, for vector mesons,

$$2M_{D^*} > M_{J/\Psi} + M_\rho, \quad 2M_{B^*} > M_\Upsilon + M_\rho.$$

$$(4016) \quad (3872) \qquad (10650) \quad (10236)$$

In fact:

$$\frac{M_{i\bar{i}}+M_{j\bar{j}}}{2} < M_{i\bar{j}} < \sqrt{\frac{M_{i\bar{i}}^2+M_{j\bar{j}}^2}{2}}$$

$$\frac{M_{iiq}+M_{jjq}}{2} < M_{ijq} < \sqrt{\frac{M_{iiq}^2+M_{jjq}^2}{2}}$$

We can get a mass relation for mesons in a spin-parity multiplet:

$$M_{c\bar{b}}^2 = \frac{1}{4}\left(M_{c\bar{c}}^2+M_{b\bar{b}}^2\right)+\frac{1}{8M_{n\bar{n}}^2}\left(4M_{n\bar{c}}^2-M_{n\bar{n}}^2-M_{c\bar{c}}^2\right)\left(4M_{n\bar{b}}^2-M_{n\bar{n}}^2-M_{b\bar{b}}^2\right)$$
$$-\frac{1}{8M_{n\bar{n}}^2}\sqrt{\left(4M_{n\bar{c}}^2-M_{n\bar{n}}^2-M_{c\bar{c}}^2\right)^2-4M_{n\bar{n}}^2M_{c\bar{c}}^2}\sqrt{\left(4M_{n\bar{b}}^2-M_{n\bar{n}}^2-M_{b\bar{b}}^2\right)^2-4M_{n\bar{n}}^2M_{b\bar{b}}^2}$$

For the pseudoscalar 1^1S_0 meson multiplet: $\pi,\ D,\ B,\ \eta_c,\ \eta_b$

$B_c(1^1S_0)$	Exp.	Pre.
Mass (MeV)	6275.6±1.1	6274.6±3.4

Charm-bottom mesons

The masses of the bottom-charm mesons (in units of MeV).

States	Pre.	Exp.[2]	[4]	[7]	[23]	[8]	[9]	[10]	[11]	[12]	[13]
$B_c\ (1\,^1S_0)$	6274.6±3.4	6275.6±1.1	6.264	6.270	6283±79	6.263	6.270	6.253	6.264	6.247	6266
$B_c^*\ (1\,^3S_1)$	6355 ± 4	–	6.356	6.355	6356±80	6.354	6.332	6.317	6.337	6.308	6314
$B_{c2}^*\ (1\,^3P_2)$	$6798\ ^{+9}_{-10}$	–	6.814	6.782	6780±52	6.781	6.762	6.743	6.747	6.773	6797
$B_{c1}\ (1\,^1P_1)$	$6777\ ^{+6}_{-9}$	–	–	6761	–	–	6749	6729	6736	6757	6725
$B_{c0}^*\ (1\,^3P_0)$	6702±22	–	–	6697	–	–	6699	6683	6700	6689	6525
$B_c\ (2\,^1S_0)$	6836±34	$6842\pm4\pm5^a$	–	6863	–	–	6835	6867	6856	6853	6866
$B_c^*\ (2\,^3S_1)$	6912±23	–	–	6895	–	6894	6881	6902	6899	6886	6891

[a]The ATLAS Collaboration report recently [1].

Doubly and triply charmed baryons

Ξ_{cc}^{+} **MASS**

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
3518.9±0.9 OUR AVERAGE					
3518 ±3	6	[1] OCHERASHVI...	05	SELX	Σ^- nucleus ≈ 600 GeV
3519 ±1	16	[2] MATTSON	02	SELX	Σ^- nucleus ≈ 600 GeV

[1] OCHERASHVILI 05 claims "an excess of 5.62 events over ... 1.38 ± 0.13 events" for a significance of 4.8 σ in pD^+K^- events.

[2] MATTSON 02 claims "an excess of 15.9 events over an expected background of 6.1 ± 0.5 events, a statistical significance of 6.3 σ" in the $\Lambda_c^+ K^- \pi^+$ invariant-mass spectrum. The probability that the peak is a fluctuation increases from 1.0×10^{-6} to 1.1×10^{-4} when the number of bins searched is considered.

However, the J^P number has not been determined experimentally.

Moreover, it has not been confirmed by other experiments (notably by LHCb, BABAR, BELLE, and FOCUS).

Doubly and triply charmed baryons

Many light baryons and singly charmed baryons have been well established. (3 or 4 stars in Baryon Summary Table)

Therefore, we focus on searching mass relations which can express the mass of a doubly charmed baryon as a function of masses of the well established light baryons and singly charmed baryons.

$$M^2_{\Omega^{*+}_{cc}} = \frac{2M^2_{\Omega^{*0}_c}\left(M^2_{\Omega^-} - 4M^2_{\Xi^{*0}} + 3M^2_{\Sigma^{*+}}\right) - M^2_{\Omega^-}\left(M^2_{\Omega^-} - 6M^2_{\Xi^{*+}_c} - 6M^2_{\Xi^{*0}} + 2M^2_{\Sigma^{*+}}\right) - \left(4M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}\right)\left(2M^2_{\Xi^{*+}_c} + 2M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}\right)}{2\left(M^2_{\Omega^-} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}}\right)} + \frac{\sqrt{M^4_{\Omega^-} - 2M^2_{\Omega^-}\left(4M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}}\right) + \left(M^2_{\Sigma^{*+}} - 4M^2_{\Xi^{*0}}\right)^2}\sqrt{\left(M^2_{\Omega^-} - 2M^2_{\Xi^{*+}_c} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}}\right)^2 - 4M^2_{\Omega^{*0}_c}\left(M^2_{\Omega^-} + 2M^2_{\Xi^{*+}_c} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}} - M^2_{\Omega^{*0}_c}\right)}}{2\left(M^2_{\Omega^-} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}}\right)}$$

$$M^2_{\Omega^{++}_{ccc}} = \frac{2M^2_{\Omega^-}\left(9M^2_{\Xi^{*+}_c} + 7M^2_{\Xi^{*0}} - 2M^2_{\Sigma^{*+}}\right) - M^4_{\Omega^-} - 3\left(4M^2_{\Omega^{*0}_c}\left(M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}\right) + \left(4M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}\right)\left(2M^2_{\Xi^{*+}_c} + 2M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}\right)\right)}{2\left(M^2_{\Omega^-} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}}\right)} + \frac{3\sqrt{M^4_{\Omega^-} + \left(M^2_{\Sigma^{*+}} - 4M^2_{\Xi^{*0}}\right)^2 - 2M^2_{\Omega^-}\left(4M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}}\right)}\sqrt{4M^4_{\Omega^{*0}_c} + \left(M^2_{\Omega^-} - 2M^2_{\Xi^{*+}_c} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}}\right)^2 - 4M^2_{\Omega^{*0}_c}\left(M^2_{\Omega^-} + 2M^2_{\Xi^{*+}_c} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}}\right)}}{2\left(M^2_{\Omega^-} - 2M^2_{\Xi^{*0}} + M^2_{\Sigma^{*+}}\right)}.$$

Masses of the doubly and triply baryons

$J^P = \frac{1}{2}^+$	Ξ_{cc}^{+}	Ξ_{cc}^{++}	Ω_{cc}^{+}	
	3519.1	3519.5	3652. 4	
$J^P = \frac{3}{2}^+$	Ξ_{cc}^{*+}	Ξ_{cc}^{*++}	Ω_{cc}^{*+}	Ω_{ccc}^{++}
	3694. 6	3695. 9	3808. 9	4834. 3

$$M_{\Omega_{cc}^{*+}} - M_{\Xi_{cc}^{*+}} \approx 3809 - 3695 = 114 \ \text{MeV}$$

$$M_{\Omega_{cc}^{+}} - M_{\Xi_{cc}^{+}} \approx 3650 - 3520 = 130 \ \text{MeV}$$

$M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^{+}}$ = 0.4±0.3 MeV   1.5±2.7 MeV, PLB **698**, 251

2.3±1.7 MeV, PRD **78**, 073013

Table 1. The masses of the doubly charmed baryons lying on the Ω_{cc}^{*+} trajectory (in units of MeV).

J^P	$\frac{3}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{9}{2}^-$
Pre.	$3808.9^{+36.3}_{-35.7}$	$4058.4^{+59.7}_{-58.6}$	$4293.5^{+80.7}_{-79.2}$	$4516.3^{+99.8}_{-97.9}$
Exp. [1]				
[2]	3876	4152	4230	
[3]	3730	4134	4204	
[4]	3808.4±4.3		4313±23	
[5]	3872	4303		
[6]	3760±170			
[7]	3762±17			
[8]	3765			
[9]	3764			
[10]	3850 ± 25			
[11]	3746			
[12]	3840±60			
[13]	3734±14±8±97			
[14]	3820±80			
[15]	3721			
[16]	3797			
[21]	3822±20±22			
[22]	3773±38			
[24]	3758			
[25]	3780±160			
[26]	3765±43±17±5			
[27]	3700			
[30]	3735±33±18±43			
[31]	3810±60			
[34]	3651-3782			
[35]	3847			
[36]	3800			
[37]	3795			
[38]	3690			
[39]	3710			
[40]	3770			
[41]	3769			

The masses of the doubly charmed baryons Ξ_{cc}^{+}, Ξ_{cc}^{++} and the orbital excited baryons lying on the Ξ_{cc}^{++} trajectory.

J^P	$\frac{1}{2}^+$ (Ξ_{cc}^{+}, Ξ_{cc}^{++})	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{7}{2}^-$
Pre.	$3520.2^{+40.6}_{-38.8}$, $3520.6^{+43.6}_{-39.8}$	$3786.0^{+65.0}_{-64.8}$	$4034.0^{+88.6}_{-86.9}$	$4267.6^{+108.8}_{-106.9}$
Exp. [1]	3518.9±0.9			
[2]	3676	3921	4047	
[3]	3478	3834	4050	
[4]	3518.9±0.9		4047±19	
[5]	3620	3969		
[6]	3710±140			
[7]	3522±16			
[8]	3642	3920		
[9]	3511			
[10]	3610 ± 3			
[11]	3510			
[12]	3635			
[13]	3549±13±19±92			
[14]	3660±70			
[15]	3520			
[17]	3550±80			
[20]	3524			
[21]	3610±23±22			
[22]	3558±39			
[23]	3627±12			
[24]	3532			
[26]	3596±39±20±7			
[27]	3527			
[28]	3520-2560			
[29]	3513±23±24			
[30]	3568±14±19±1			
[31]	4260±190			
[32]	3470±50			
[33]	3480±50			
[34]	3478-3604, 3468-3583			
[35]	3584, 3579			
[36]	3557			
[37]	3538			
[38]	3480			
[39]	3519			
[40]	3547			
[41]	3579			

Doubly and triply bottom baryons

The masses of doubly and triply charmed baryons (in units of MeV).

Ξ_{bb}^0	Ξ_{bb}^-	Ω_{bb}	Ξ_{bb}^{*0}	Ξ_{bb}^{*-}	Ω_{bb}^*	Ω_{bbb}
10164.4	10166.8	10286.8	10319.2	10322.4	10437.0	14797.4

$M_{\Xi_{bb}^0} - M_{\Xi_{bb}^-}$ = -2.4±2.0 MeV

-6.3±1.7 MeV, PLB **698**, 251

-5.3±1.1 MeV, PRD **78**, 073013

$M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^+}$ = 0.4±0.3 MeV

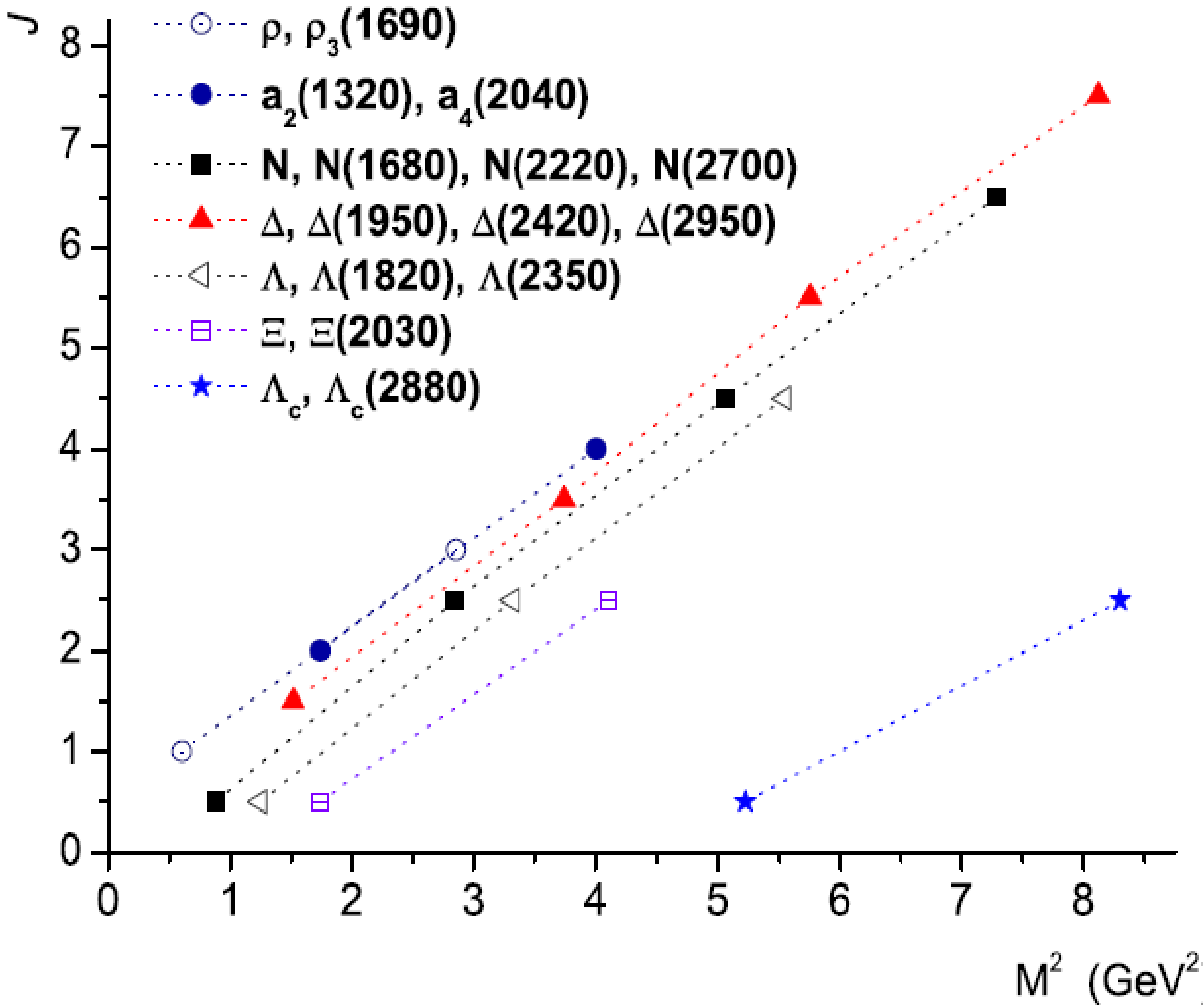
1.5±2.7 MeV, PLB **698**, 251

2.3±1.7 MeV, PRD **78**, 073013

Regge phenomenology

- Regge phenomenology was derived from the analysis of the properties of the scattering amplitude in the complex angular momentum plane.(T. Regge, Nuovo Cim. 1959)
- In 1962, it was introduced to study hadron spectra by Chew and Frautschi. (PRL 8, 41)
- Regge theory is concerned with almost all aspects of strong interactions, including the particle spectrum, the forces between particles, and the high energy behavior of scattering amplitudes.
- One of the most distinctive features of Regge theory is the Regge trajectory by which the mass and the spin of a hadron are related.

We plot Regge trajectories for some mesons and baryons in the (J,M^2) plane.



The quasi-linear Regge trajectory is usually parameterized as Eq. (1)

$$J = a(0) + \alpha' M^2 \tag{1}$$

For meson, Regge slope and intercepts have the following relations:

the additivity of intercepts $$a_{i\bar{i}}(0) + a_{j\bar{j}}(0) = 2a_{i\bar{j}}(0) \tag{2}$$

the additivity of inverse slopes $$\frac{1}{\alpha'_{i\bar{i}}} + \frac{1}{\alpha'_{j\bar{j}}} = \frac{2}{\alpha'_{i\bar{j}}} \tag{3}$$

Equations (2) and (3) were derived in a model based on the topological expansion and the *qq*-string picture of hadrons (QGSM) [*Kaidalov 1982*]. This model provides a microscopic approach to describe Regge phenomenology in terms of quark degrees of freedom.

For baryons, the additivity of intercepts and the additivity of inverse
slopes are written as follows:

$$a_{iiq}(0) + a_{jjq}(0) = 2a_{ijq}(0) \quad (4)$$

$$\frac{1}{\alpha'_{iiq}} + \frac{1}{\alpha'_{jjq}} = \frac{2}{\alpha'_{ijq}} \quad (5)$$

There are also relations about the factorization of slopes for mesons and baryons:

$$\alpha'_{i\bar{i}} \cdot \alpha'_{j\bar{j}} = {\alpha'_{i\bar{j}}}^2 , \quad (6)$$

$$\alpha'_{iiq} \cdot \alpha'_{jjq} = {\alpha'_{ijq}}^2 , \quad (7)$$

which follow from the factorization of residues of the *t*-channel poles.

Using Eqs. (1) and (2), we have Eq. (8)

$$\alpha'_{i\bar{i}} M^2_{i\bar{i}} + \alpha'_{j\bar{j}} M^2_{j\bar{j}} = 2\alpha'_{i\bar{j}} M^2_{i\bar{j}}. \quad (8)$$

$$\frac{1}{\alpha'_{i\bar{i}}} + \frac{1}{\alpha'_{j\bar{j}}} = \frac{2}{\alpha'_{i\bar{j}}}. \quad (3)$$

Combining the relations (3) and (8), we can get relations between the slope ratio and meson masses.

$$\frac{\alpha'_{j\bar{j}}}{\alpha'_{i\bar{i}}} = \frac{1}{2M^2_{j\bar{j}}} \times [(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}}) + \sqrt{(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}})^2 - 4M^2_{i\bar{i}}M^2_{j\bar{j}}}] \quad (9)$$

For baryons, one can have

$$\frac{\alpha'_{jjq}}{\alpha'_{iiq}} = \frac{1}{2M^2_{jjq}} \times [(4M^2_{ijq} - M^2_{iiq} - M^2_{jjq}) + \sqrt{(4M^2_{ijq} - M^2_{iiq} - M^2_{jjq})^2 - 4M^2_{iiq}M^2_{jjq}}]$$

b-baryons

Quadratic mass equalities

For mesons, from Eqs. (2) and (3), we can redefined Regge slope and Regge intercept as the following:

$$a_{i\bar{j}}(0) = a_{n\bar{n}}(0) - \lambda_i - \lambda_j,$$

$$\frac{1}{\alpha'_{i\bar{j}}} = \frac{1}{\alpha'_{n\bar{n}}} + \gamma_i + \gamma_j,$$

where $\lambda_i = a_{n\bar{n}}(0) - a_{n\bar{i}}(0)$, $\gamma_i = \frac{1}{\alpha'_{n\bar{i}}} - \frac{1}{\alpha'_{n\bar{n}}}$.

For baryons, from Eqs. (4) and (5), we can redefined Regge slope and Regge intercept as the following:

$$a_{ijq}(0) = a_{nnn}(0) - \lambda_i - \lambda_j - \lambda_q,$$

$$\frac{1}{\alpha'_{ijq}} = \frac{1}{\alpha'_{nnn}} + \gamma_i + \gamma_j + \gamma_q,$$

where $\lambda_i = a_{nnn}(0) - a_{nni}(0)$, $\gamma_i = \frac{1}{\alpha'_{nni}} - \frac{1}{\alpha'_{nnn}}$.

To evaluate the deviation of Relations (13) and (16), from the equalities that would be obtained by changing the signs of inequalities to equal signs, we introduce a parameter δ, which is denoted by δ_{ij}^{m} for mesons,

$$\begin{aligned}\delta_{ij}^{m} &= M_{i\bar{i}}^{2} + M_{j\bar{j}}^{2} - 2M_{i\bar{j}}^{2} \\ &=(\alpha'_{n\bar{n}}M_{n\bar{n}}^{2} + 2\lambda_i)(\frac{1}{\alpha'_{n\bar{n}}} + 2\gamma_i) + (\alpha'_{n\bar{n}}M_{n\bar{n}}^{2} + 2\lambda_j)(\frac{1}{\alpha'_{n\bar{n}}} + 2\gamma_j) \\ &\quad - 2(\alpha'_{n\bar{n}}M_{n\bar{n}}^{2} + \lambda_i + \lambda_j)(\frac{1}{\alpha'_{n\bar{n}}} + \gamma_i + \gamma_j) \\ &=2(\lambda_i - \lambda_j)(\gamma_i - \gamma_j).\end{aligned}$$

and δ_{ij}^{b} for baryons,

$$\begin{aligned}\delta_{ij}^{b} &=M_{iiq}^{2} + M_{jjq}^{2} - 2M_{ijq}^{2} \\ &=(\alpha'_{nnn}M_{nnn}^{2} + 2\lambda_i + \lambda_q)(\frac{1}{\alpha'_{nnn}} + 2\gamma_i + \gamma_q) + (\alpha'_{nnn}M_{nnn}^{2} + 2\lambda_j + \lambda_q)(\frac{1}{\alpha'_{nnn}} + 2\gamma_j + \gamma_q) \\ &\quad - 2(\alpha'_{nnn}M_{nnn}^{2} + \lambda_i + \lambda_j + \lambda_q)(\frac{1}{\alpha'_{nnn}} + \gamma_i + \gamma_j + \gamma_q) \\ &=2(\lambda_i - \lambda_j)(\gamma_i - \gamma_j).\end{aligned} \tag{31}$$

From Eq. (31), we can see that δ_{ij}^{b} is independent of *q.*

(A.) For the $\frac{3}{2}^+$ multiplet, considering the difference of u-quark and d-quark, Eqs. (54) can be re written as follow:

(A0) When $i = u$, $j = d$, $q = u, d, s, c, b$,

$$\delta_{ud}^{\frac{3}{2}^+} = M^2_{\Delta^{++}} + M^2_{\Delta^0} - 2M^2_{\Delta^+} = M^2_{\Delta^+} + M^2_{\Delta^-} - 2M^2_{\Delta^0} = M^2_{\Sigma^{*+}} + M^2_{\Sigma^{*-}} - 2M^2_{\Sigma^{*0}} = M^2_{\Sigma_c^{*++}} + M^2_{\Sigma_c^{*0}} - 2M^2_{\Sigma_c^{*+}} = M^2_{\Sigma_b^{*+}} + M^2_{\Sigma_b^{*-}} - 2M^2_{\Sigma_b^{*0}}. \quad (1a)$$

(A1) When $i = u, d$, $j = s$, $q = u, d, s, c, b$,

$$\delta_{us}^{\frac{3}{2}^+} = M^2_{\Delta^{++}} + M^2_{\Xi^{*0}} - 2M^2_{\Sigma^{*+}} = M^2_{\Delta^+} + M^2_{\Xi^{*-}} - 2M^2_{\Sigma^{*0}} = M^2_{\Sigma^{*+}} + M^2_{\Omega^-} - 2M^2_{\Xi^{*0}} = M^2_{\Sigma_c^{*++}} + M^2_{\Omega_c^{*0}} - 2M^2_{\Xi_c^{*+}} = M^2_{\Sigma_b^{*+}} + M^2_{\Omega_b^{*-}} - 2M^2_{\Xi_b^{*0}}; \quad (1b)$$

$$\delta_{ds}^{\frac{3}{2}^+} = M^2_{\Delta^0} + M^2_{\Xi^{*0}} - 2M^2_{\Sigma^{*0}} = M^2_{\Delta^-} + M^2_{\Xi^{*-}} - 2M^2_{\Sigma^{*-}} = M^2_{\Sigma^{*-}} + M^2_{\Omega^-} - 2M^2_{\Xi^{*-}} = M^2_{\Sigma_c^{*0}} + M^2_{\Omega_c^{*0}} - 2M^2_{\Xi_c^{*0}} = M^2_{\Sigma_b^{*-}} + M^2_{\Omega_b^{*-}} - 2M^2_{\Xi_b^{*-}}. \quad (1c)$$

(A2) When $i = u, d$, $j = c$, $q = u, d, s, c, b$,

$$\delta_{uc}^{\frac{3}{2}^+} = M^2_{\Delta^{++}} + M^2_{\Xi_{cc}^{*++}} - 2M^2_{\Sigma_c^{*++}} = M^2_{\Delta^+} + M^2_{\Xi_{cc}^{*+}} - 2M^2_{\Sigma_c^{*+}} = M^2_{\Sigma^{*+}} + M^2_{\Omega_{cc}^{*+}} - 2M^2_{\Xi_c^{*+}} = M^2_{\Sigma_c^{*++}} + M^2_{\Omega_{ccc}^{++}} - 2M^2_{\Xi_{cc}^{*++}} = M^2_{\Sigma_b^{*+}} + M^2_{\Omega_{bcc}^{*+}} - 2M^2_{\Xi_{bc}^{*+}}; \quad (1d)$$

$$\delta_{dc}^{\frac{3}{2}^+} = M^2_{\Delta^0} + M^2_{\Xi_{cc}^{*++}} - 2M^2_{\Sigma_c^{*+}} = M^2_{\Delta^-} + M^2_{\Xi_{cc}^{*+}} - 2M^2_{\Sigma_c^{*0}} = M^2_{\Sigma^{*-}} + M^2_{\Omega_{cc}^{*+}} - 2M^2_{\Xi_c^{*0}} = M^2_{\Sigma_c^{*0}} + M^2_{\Omega_{ccc}^{++}} - 2M^2_{\Xi_{cc}^{*+}} = M^2_{\Sigma_b^{*-}} + M^2_{\Omega_{bcc}^{*+}} - 2M^2_{\Xi_{bc}^{*0}}. \quad (1e)$$

(A3) When $i = s$, $j = c$, $q = u, d, s, c, b$,

$$\delta_{sc}^{\frac{3}{2}^+} = M^2_{\Xi^{*0}} + M^2_{\Xi_{cc}^{*++}} - 2M^2_{\Xi_c^{*+}} = M^2_{\Xi^{*-}} + M^2_{\Xi_{cc}^{*+}} - 2M^2_{\Xi_c^{*0}} = M^2_{\Omega^-} + M^2_{\Omega_{cc}^{*+}} - 2M^2_{\Omega_c^{*0}} = M^2_{\Omega_c^{*0}} + M^2_{\Omega_{ccc}^{++}} - 2M^2_{\Omega_{cc}^{*+}} = M^2_{\Omega_b^{*-}} + M^2_{\Omega_{bcc}^{*+}} - 2M^2_{\Omega_{bc}^{*0}}. \quad (1f)$$

(A4) When $i = u, d$, $j = b$, $q = u, d, s, c, b$,

$$\delta_{ub}^{\frac{3}{2}^+} = M^2_{\Delta^{++}} + M^2_{\Xi_{bb}^{*0}} - 2M^2_{\Sigma_b^{*+}} = M^2_{\Delta^+} + M^2_{\Xi_{bb}^{*-}} - 2M^2_{\Sigma_b^{*0}} = M^2_{\Sigma^{*+}} + M^2_{\Omega_{bb}^{*-}} - 2M^2_{\Xi_b^{*0}} = M^2_{\Sigma_c^{*++}} + M^2_{\Omega_{bbc}^{*0}} - 2M^2_{\Xi_{bc}^{*+}} = M^2_{\Sigma_b^{*+}} + M^2_{\Omega_{bbb}^{-}} - 2M^2_{\Xi_{bb}^{*0}}. \quad (1g)$$

$$\delta_{db}^{\frac{3}{2}^+} = M^2_{\Delta^0} + M^2_{\Xi_{bb}^{*0}} - 2M^2_{\Sigma_b^{*0}} = M^2_{\Delta^-} + M^2_{\Xi_{bb}^{*-}} - 2M^2_{\Sigma_b^{*-}} = M^2_{\Sigma^{*-}} + M^2_{\Omega_{bb}^{*-}} - 2M^2_{\Xi_b^{*-}} = M^2_{\Sigma_c^{*0}} + M^2_{\Omega_{bbc}^{*0}} - 2M^2_{\Xi_{bc}^{*0}} = M^2_{\Sigma_b^{*-}} + M^2_{\Omega_{bbb}^{-}} - 2M^2_{\Xi_{bb}^{*-}}. \quad (1h)$$

(1h)

(A5) When $i = s$, $j = b$, $q = u, d, s, c, b$,

$$\delta_{sb}^{\frac{3}{2}^+} = M^2_{\Xi^{*0}} + M^2_{\Xi^{*0}_{bb}} - 2M^2_{\Xi^{*0}_{b}} = M^2_{\Xi^{*-}} + M^2_{\Xi^{*-}_{bb}} - 2M^2_{\Xi^{*-}_{b}} = M^2_{\Omega^{-}} + M^2_{\Omega^{*-}_{bb}} - 2M^2_{\Omega^{*-}_{b}} = M^2_{\Omega^{*0}_{c}} + M^2_{\Omega^{*0}_{bbc}} - 2M^2_{\Omega^{*0}_{bc}} = M^2_{\Omega^{*-}_{b}} + M^2_{\Omega^{-}_{bbb}} - 2M^2_{\Omega^{*-}_{bb}}. \tag{1i}$$

(A6) When $i = c$, $j = b$, $q = u, d, s, c, b$,

$$\delta_{cb}^{\frac{3}{2}^+} = M^2_{\Xi^{*++}_{cc}} + M^2_{\Xi^{*0}_{bb}} - 2M^2_{\Xi^{*+}_{bc}} = M^2_{\Xi^{*+}_{cc}} + M^2_{\Xi^{*-}_{bb}} - 2M^2_{\Xi^{*0}_{bc}} = M^2_{\Omega^{*+}_{cc}} + M^2_{\Omega^{*-}_{bb}} - 2M^2_{\Omega^{*0}_{bc}} = M^2_{\Omega^{++}_{ccc}} + M^2_{\Omega^{*0}_{bbc}} - 2M^2_{\Omega^{*+}_{bcc}} = M^2_{\Omega^{*+}_{bcc}} + M^2_{\Omega^{-}_{bbb}} - 2M^2_{\Omega^{*0}_{bbc}}. \tag{1j}$$

We can have

$$(M^2_{\Omega^{*+}_{cc}} - M^2_{\Xi^{*++}_{cc}}) + (M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}) = (M^2_{\Omega^{*0}_{c}} - M^2_{\Sigma^{*++}_{c}}); \tag{1k}$$

$$(M^2_{\Omega^{*+}_{cc}} - M^2_{\Xi^{*+}_{cc}}) + (M^2_{\Xi^{*-}} - M^2_{\Sigma^{*-}}) = (M^2_{\Omega^{*0}_{c}} - M^2_{\Sigma^{*0}_{c}}). \tag{1l}$$

$$(M^2_{\Omega^{*-}_{bb}} - M^2_{\Xi^{*0}_{bb}}) + (M^2_{\Xi^{*0}} - M^2_{\Sigma^{*+}}) = (M^2_{\Omega^{*-}_{b}} - M^2_{\Sigma^{*+}_{b}}); \tag{1m}$$

$$(M^2_{\Omega^{*-}_{bb}} - M^2_{\Xi^{*0}_{bb}}) + (M^2_{\Xi^{*-}} - M^2_{\Sigma^{*-}}) = (M^2_{\Omega^{*-}_{b}} - M^2_{\Sigma^{*-}_{b}}). \tag{1n}$$

$$(M^2_{\Omega^{*0}_{cbb}} - M^2_{\Xi^{*0}_{bb}}) + (M^2_{\Xi^{*++}_{cc}} - M^2_{\Sigma^{*++}_{c}}) = (M^2_{\Omega^{*+}_{bcc}} - M^2_{\Sigma^{*+}_{b}}); \tag{1o}$$

$$(M^2_{\Omega^{*0}_{cbb}} - M^2_{\Xi^{*-}_{bb}}) + (M^2_{\Xi^{*+}_{cc}} - M^2_{\Sigma^{*0}_{c}}) = (M^2_{\Omega^{*+}_{bcc}} - M^2_{\Sigma^{*-}_{b}}); \tag{1p}$$

$$(M^2_{\Omega^{*0}_{cbb}} - M^2_{\Omega^{*-}_{bb}}) + (M^2_{\Omega^{*+}_{cc}} - M^2_{\Omega^{*0}_{c}}) = (M^2_{\Omega^{*+}_{ccb}} - M^2_{\Omega^{*-}_{b}}). \tag{1q}$$

(B.) Similarly, for the $\frac{1}{2}^+$ multiplet, we can express $\delta_{ij}^{\frac{1}{2}^+}$ as follow:

(B0) When $i=u$, $j=d$, $q=u,d,s,c,b$,

$$\delta_{ud}^{\frac{1}{2}^+}=M^2_{\Sigma^+}+M^2_{\Sigma^-}-2M^2_{\Sigma^0}=M^2_{\Sigma_c^{++}}+M^2_{\Sigma_c^0}-2M^2_{\Sigma_c^+}=M^2_{\Sigma_b^+}+M^2_{\Sigma_b^-}-2M^2_{\Sigma_b^0}.$$

(B1) When $i=u,d$, $j=s$, $q=u,d,s,c,b$,

$$\delta_{us}^{\frac{1}{2}^+}=M^2_{\Sigma_c^{++}}+M^2_{\Omega_c^0}-2M^2_{\Xi_c'^+}=M^2_{\Sigma_b^+}+M^2_{\Omega_b^-}-2M^2_{\Xi_b'^0}$$

$$\delta_{ds}^{\frac{1}{2}^+}=M^2_{\Sigma_c^0}+M^2_{\Omega_c^0}-2M^2_{\Xi_c'^0}=M^2_{\Sigma_b^-}+M^2_{\Omega_b^-}-2M^2_{\Xi_b'^-}$$

....

(B6) When $i=c$, $j=b$, $q=u,d,s,c,b$,

$$\delta_{cb}^{\frac{1}{2}^+}=M^2_{\Xi_{cc}^{++}}+M^2_{\Xi_{bb}^0}-2M^2_{\Xi_{bc}'^+}=M^2_{\Xi_{cc}^+}+M^2_{\Xi_{bb}^-}-2M^2_{\Xi_{bc}'^0}=M^2_{\Omega_{cc}^+}+M^2_{\Omega_{bb}^-}-2M^2_{\Omega_{bc}'^0}.$$

$$\delta_{ub,d}^{\frac{1}{2}^+}+\delta_{db,u}^{\frac{1}{2}^+}=M^2_{N^+}+M^2_{\Xi_{bb}^-}-2\left(\frac{3M^2_{\Lambda_b^0}+M^2_{\Sigma_b^0}}{4}\right)+M^2_{N^0}+M^2_{\Xi_{bb}^0}-2\left(\frac{3M^2_{\Lambda_b^0}+M^2_{\Sigma_b^0}}{4}\right).$$

$$(M^2_{\Omega_{bb}^-}-M^2_{\Xi_{bb}^0})+(M^2_{\Xi^0}-M^2_{\Sigma^+})=(M^2_{\Omega_b^-}-M^2_{\Sigma_b^+});$$

$$(M^2_{\Omega_{bb}^-}-M^2_{\Xi_{bb}^-})+(M^2_{\Xi^-}-M^2_{\Sigma^-})=(M^2_{\Omega_b^-}-M^2_{\Sigma_b^-}).$$

$$\delta_{ub,s}^{\frac{1}{2}^+}+\delta_{sb,u}^{\frac{1}{2}^+}=(M^2_{\Omega_{bb}^-}+M^2_{\Xi_{bb}^0})+(M^2_{\Xi^0}+M^2_{\Sigma^+})-(3M^2_{\Xi_b^0}+M^2_{\Xi_b'^0});$$

$$\delta_{db,s}^{\frac{1}{2}^+}+\delta_{sb,d}^{\frac{1}{2}^+}=(M^2_{\Omega_{bb}^-}+M^2_{\Xi_{bb}^-})+(M^2_{\Xi^-}+M^2_{\Sigma^-})-(3M^2_{\Xi_b^-}+M^2_{\Xi_b'^-}).$$

Summary

- The mass relations obtained from Regge phenomenology are suitable to describe the existing hadron spectra with high accurcy.
- We numerically prove that the factorization of slopes is wrong for heavy mesons.
- We support that $\Xi_{cc}(3520)^{+}$ is the ground baryon with $J^{P} = ½^{+}$.
- The mass of $a_0(1450)$ is too high to be the $1^{3}P_{0}$ state.
- The mass relations and the predictions may be useful for the discovery and the J^{P} assignment of the unobserved states.

Thank you for your attentions !