Rare B decay in $\mathcal{F}$-$SU(5)$ Model

Wang Wenyu
With *Li Tianjun, Wang Xiao-Chuan, Xiong Zhao-hua*

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Beijing University of Technology

OUTLINE:

1. Introduction to $\mathcal{F}$-$SU(5)$ model;
2. Physics around the TeV scale;
3. Implication on B physics:
   - Effective Hamiltonian;
   - Analysis on Rare B decays.
4. Numerical results;
5. Summary.

Why $\mathcal{F}$-$SU(5)$ model ?

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  - Solve hierarchy problem;
  - Dark matter candidate;
  - Gauge coupling unification;
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  - Doublet-triplet splitting problem;
  - Dimension-five proton decay problem;
  - Hierarchy between GUT and string.

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- However, in the supersymmetric $SU(5)$ models which give the gauge coupling unification,
  - Doublet-triplet splitting problem;
  - Dimension-five proton decay problem;
  - Hierarchy between GUT and string.
- In the Flipped $SU(5) \times U(1)_X$ models, these problem can be solved through the missing partner mechanism.
- The testable can be constructed from the free fermionic string constructions at the Kac-Moody level one and locally from the F-theory model building.
- There is no Landau pole problem in Flipped $SU(5) \times U(1)_X$ models.

String-scale gauge coupling unification

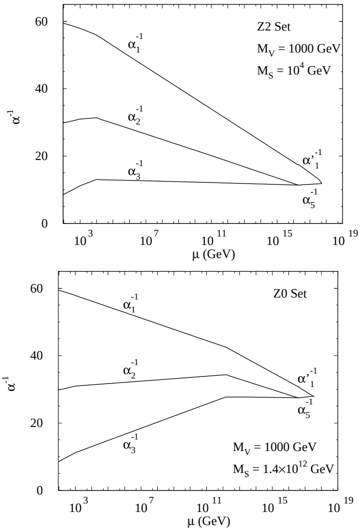

Figure: Two-loop string-scale gauge coupling unification for the simple flipped $SU(5) \times U(1)_X$ model and Split-SUSY. (from hep-ph/0610054)

Golden Strip in No-Scale, No-Parameter $\mathcal{F}$-$SU(5)$ Model.

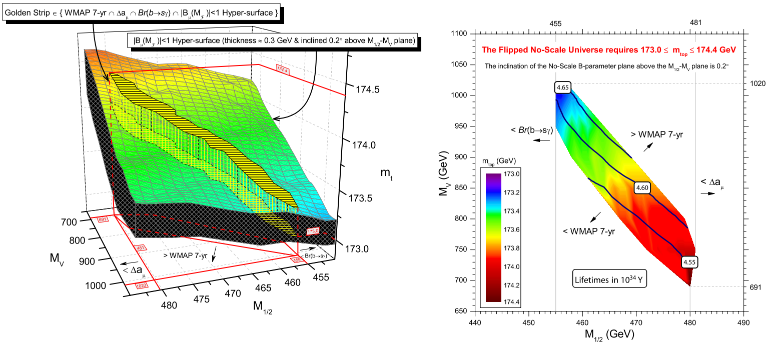

Figure: The Golden Strip of Correlated Gaugino, and Vector-like Mass In No-Scale, No-Parameter $\mathcal{F}$-$SU(5)$ model, (from arxiv: 1009.2981)

- The vector-like particles can be observed at the Large Hadron Collider (LHC), proton decay is within the reach of the future Hyper-Kamiokande and Deep Underground Science and Engineering Laboratory (DUSEL) experiments.

Rich phenomenology on low energy processes,

Since $\mathcal{F}$-$SU(5)$ model is very different from the other simple SM extensions in quark sector, and the 3×3 SM-like quark mixing matrix is now replaced by a 5×5 one which is no longer unitary, and there exists the tree-level $\bar{s}bZ$ interaction, the model has rich phenomenology on low energy processes:

- Precsion electro-weak observables such as U, S, T and $R_b,\ R_c$;
- Rare B decays induced by the flavor changing neutral current (FCNC) only occur at loop level in the SM and then are sensitive to new physics.
  - $B \to X_s \gamma$
  - $B \to X_s \ell^+ \ell^- (\ell = e, \mu)$
  - $B_s \to \mu^+ \mu^-$
  - $B_s \to \ell^+ \ell^- \gamma$
  - ...

The model:

The quantum numbers for the additional vector-like particles under the $SU(5) \times U(1)_X$ gauge symmetry are

$$XF = (\mathbf{10}, \mathbf{1}) \ , \ \overline{YF} = (\overline{\mathbf{10}}, -\mathbf{1}) \ ,$$
$$Xf = (\mathbf{5}, \mathbf{3}) \ , \ \overline{Yf} = (\overline{\mathbf{5}}, -\mathbf{3}) \ ,$$
$$Xl = (\mathbf{1}, -\mathbf{5}) \ , \ \overline{Yl} = (\mathbf{1}, \mathbf{5})$$

It is obvious that XF, $\overline{YF}$, Xf, $\overline{Yf}$, Xl, and $\overline{Yl}$ are standard vector-like particles with contents as follows

$$XF = (XQ, XD^c, XN^c) \ , \ \overline{YF} = (YQ^c, YD, YN) \ ,$$
$$Xf = (XU, XL^c) \ , \ \overline{Yf} = (YU^c, YL) \ ,$$
$$Xl = XE \ , \ \overline{Yl} = YE^c \ .$$

Under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, the quantum numbers for the extra vector-like particles are

$$XQ = (\mathbf{3}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{6}}) \ , \ YQ^c = (\bar{\mathbf{3}}, \mathbf{2}, -\frac{\mathbf{1}}{\mathbf{6}}) \ ,$$
$$XU = (\mathbf{3}, \mathbf{1}, \frac{\mathbf{2}}{\mathbf{3}}) \ , \ YU^c = (\bar{\mathbf{3}}, \mathbf{1}, -\frac{\mathbf{2}}{\mathbf{3}}) \ ,$$
$$XD = (\mathbf{3}, \mathbf{1}, -\frac{\mathbf{1}}{\mathbf{3}}) \ , \ YD^c = (\bar{\mathbf{3}}, \mathbf{1}, \frac{\mathbf{1}}{\mathbf{3}}) \ ,$$
$$XL = (\mathbf{1}, \mathbf{2}, -\frac{\mathbf{1}}{\mathbf{2}}) \ , \ YL^c = (\mathbf{1}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}}) \ ,$$
$$XE = (\mathbf{1}, \mathbf{1}, -\mathbf{1}) \ , \ YE^c = (\mathbf{1}, \mathbf{1}, \mathbf{1}) \ ,$$
$$XN = (\mathbf{1}, \mathbf{1}, \mathbf{0}) \ , \ YN^c = (\mathbf{1}, \mathbf{1}, \mathbf{0}).$$

At the GUT scale the superpotential is given by

$$\begin{aligned} W_{GUT} &= Y_{ij}^{D} F_i F_j h + Y_{ij}^{U\nu} F_i \bar{f}_j \bar{h} + Y_{ij}^{E} \bar{l}_i \bar{f}_j h + \mu h \bar{h} + Y_{kj}^{N} \phi_k \bar{H} F_j \\ &+ Y'^{D}_{j} X F F_j h + Y'^{U\nu}_{j} X F \bar{f}_j \bar{h} + Y"^{U\nu}_{i} F_i \overline{X f} \bar{h} + Y'^{E}_{j} \overline{X l} \bar{f}_j h \\ &+ Y"^{E}_{j} \bar{l}_j \overline{X f} h + Y'^{N}_{k} \phi_k \bar{H} X F + Y^{2D} X F X F h + Y'^{2D} \overline{Y F Y F} \bar{h} \\ &+ Y^{2U\nu} X F \overline{X f} \bar{h} + Y'^{2U\nu} \overline{Y F} Y f h + Y^{2E} \overline{X l X f} h + Y'^{2E} Y l Y f \bar{h} \\ &+ M_j^1 F_j \overline{Y F} + M_j^2 \bar{f}_j Y f + M_j^3 \bar{l}_j Y l \\ &+ M^4 X F \overline{Y F} + M^5 \overline{X f} Y f + M^6 \overline{X l} Y l \ . \end{aligned}$$

The first line is the SSM superpotential, the second line is the Yukawa mixing terms between the SM fermions and vector-like particles, the third and fourth lines are the SM-like superpotential for vector-like multiplets, and the fifth and sixth lines are bilinear mass terms.

After the $SU(5) \times U(1)_X$ gauge symmetry breaking down to the SM gauge symmetry, we obtain the superpotential as follows

$$
\begin{aligned}
W_{EW} &= (Y_{ij}^D - Y_{ji}^D)(D^c)_i Q_j \cdot H_d + Y_{ij}^{U\nu} U_j^c Q_i \cdot H_u - Y_{ij}^{U\nu} N_i^c L_j \cdot H_u \\
&- Y_{ij}^E E_i^c L \cdot H_d - Y_j^{\prime D}(X D^c Q_j \cdot H_d + D_j^c X Q \cdot H_d) + Y\prime_j^{U\nu} U_j^c X Q \cdot H_u \\
&- Y_j^{\prime U\nu} X N^c L \cdot H_u + Y_i^{"U\nu} X U^c Q \cdot H_u - Y_i^{"U\nu} N_i^c X L \cdot H_u - Y_j^{\prime E} X E^c L \cdot H_d \\
&- Y_j^{"E} E_j^c X L \cdot H_d - 2Y^{2D} X D^c X Q \cdot H_d - 2Y^{\prime 2D} Y D Y Q^c \cdot H_u \\
&+ Y^{2U\nu} X U^c X Q \cdot H_u - Y^{2U\nu} X N^c X L \cdot H_u - Y^{\prime 2U\nu} Y U Y Q^c \cdot H_d \\
&+ Y^{\prime 2U\nu} Y N Y L^c \cdot H_d - Y^{2E} X E^c X L \cdot H_d - Y^{\prime 2E} Y E Y L^c \cdot H_u \\
&- 2M_j^1 \left[D_j^c Y D + Q \cdot Y Q^c + N_j^c Y N\right] + M_j^2 \left[U^c Y U + L \cdot Y L^c\right] + M_j^3 E_j^c Y E \\
&- 2M^4 \left[X D^c Y D + X Q \cdot Y Q^c + X N^c Y N\right] + M^5 \left[X U^c Y U + X L \cdot Y L^c\right] \\
&+ M^6 X E^c Y E \, .
\end{aligned}
$$

At low energy scale, the sparticles decouple rapidly when M_S increases, thus we will concentrate on the contributions from new vector-like quark multiplets $XU, \; YU^c, \; XD, \text{ and } YD^c$ in our study.

The down-type quark mass matrix is

$$M_D = \begin{pmatrix} (Y_{11}^D + Y_{11}^D)v_d & (Y_{12}^D + Y_{21}^D)v_d & (Y_{13}^D + Y_{31}^D)v_d & Y'^D_1 v_d & -2M_1^1 \\ (Y_{21}^D + Y_{12}^D)v_d & (Y_{22}^D + Y_{22}^D)v_d & (Y_{23}^D + Y_{32}^D)v_d & Y'^D_2 v_d & -2M_2^1 \\ (Y_{31}^D + Y_{13}^D)v_d & (Y_{32}^D + Y_{23}^D)v_d & (Y_{33}^D + Y_{33}^D)v_d & Y'^D_3 v_d & -2M_3^1 \\ Y'^D_1 v_d & Y'^D_2 v_d & Y'^D_3 v_d & 2Y^{2D} v_d & -2M^4 \\ 2M_1^1 & 2M_2^1 & 2M_3^1 & 2M^4 & -2Y'^{2D} v_u \end{pmatrix},$$

and the up-type quark matrix is

$$M_U = \begin{pmatrix} Y_{11}^{U\nu} v_u & Y_{21}^{U\nu} v_u & Y_{31}^{U\nu} v_u & Y'^{U\nu}_1 v_u & M_1^2 \\ Y_{12}^{U\nu} v_u & Y_{22}^{U\nu} v_u & Y_{32}^{U\nu} v_u & Y'^{U\nu}_2 v_u & M_2^2 \\ Y_{13}^{U\nu} v_u & Y_{23}^{U\nu} v_u & Y_{33}^{U\nu} v_u & Y'^{U\nu}_3 v_u & M_3^2 \\ Y"^{U\nu}_1 v_u & Y"^{U\nu}_2 v_u & Y"^{U\nu}_3 v_u & Y^{2U\nu} v_u & M^5 \\ -2M_1^1 & -2M_2^1 & -2M_3^1 & -2M^4 & Y'^{2U\nu} v_d \end{pmatrix}$$

where v_u and v_d are the vacuum expectation values (VEVs) for H_u and H_d. These two matrixes can be diagonalized by unitary matrices U and V,

$$V_d^\dagger M_D U_d = \text{diag.}[m_d, m_s, m_b, m_{d_x}, m_{d_y}],$$
$$V_u^\dagger M_U U_u = \text{diag.}[m_u, m_c, m_t, m_{u_x}, m_{u_y}].$$

$$V_{\text{CKM}}^{ij} = \sum_{m=1}^{4} U_u^{*mi} U_d^{mj}$$

The Feynman Rule:

Charged W boson, Goldstone boson, and charged Higgs boson with quarks $\overline{u_l} d_j \chi^+ (\chi = W,\ G,\ h)$ and for Z boson $\overline{d_j} d_l Z$

$$i\frac{g}{\sqrt{2}}\gamma^\mu \left[g_L^\chi(l,j)P_L + g_R^\chi(l,j)P_R\right], \quad (\chi = W,\ Z)\ , \tag{1}$$

$$i\frac{g}{\sqrt{2}} \left[g_L^\chi(l,j)P_L + g_R^\chi(l,j)P_R\right], \quad (\chi = G,\ h)\ ,$$

$$\begin{aligned}
g_L^W(i,j) &= \sum_{m=1}^{4} U_u^{*mi} U_d^{m,j}, \quad g_R^W(i,j) = V_u^{*5i} V_d^{5j}, \\
g_L^Z(i,j) &= -\frac{1}{\sqrt{2}\cos\theta_W}\left[\left(1 - \frac{2}{3}\sin^2\theta_W\right)\delta^{ij} - U_d^{*5i}U_d^{5j}\right], \\
g_R^Z(i,j) &= -\frac{1}{\sqrt{2}\cos\theta_W}\left[-\frac{2}{3}\sin^2\theta_W\,\delta^{ij} + V_d^{*5i}V_d^{5j}\right], \\
g_L^G(i,j) &= \left(\sum_{k,m=1}^{4} Y_{km}^{U\nu} V_u^{*ki} U_d^{mj} + 2Y'^{2D} V_u^{*5i} U_d^{5j}\right)\frac{v_u}{m_W}, \\
g_R^G(i,j) &= -\left(\sum_{k,m=1}^{4} (Y_{mk}^D + Y_{km}^D) V_d^{*kj} U_u^{mi} - 2Y'^{U\nu} V_d^{*5j} U_d^{5i}\right)\frac{v_d}{m_W}, \\
g_L^h(i,j) &= \left(\sum_{k,m=1}^{4} Y_{km}^{U\nu} V_u^{*ki} U_d^{mj} + 2Y'^{2D} V_u^{*5i} U_d^{5j}\right)\frac{v_d}{m_W}, \\
g_R^h(i,j) &= \left(\sum_{k,m=1}^{4} (Y_{mk}^D + Y_{km}^D) V_d^{*kj} U_u^{mi} - 2Y'^{U\nu} V_d^{*5j} U_d^{5i}\right)\frac{v_u}{m_W}.
\end{aligned}$$

We can see that the TeV-scale $\mathcal{F}$-$SU(5)$ model has two points for rich physics to be explored:

- Since the quark mass matrices are not the same as two Higgs doublet model (2HDM) or the Minimal Supersymmetric Standard Model (MSSM), the loop-level FCNC will be changed by the Yukawa interactions, and then may change the prediction of process $b \to s\gamma$ significantly.
- The last terms in above Equations, which we call the "tail terms", will cause the tree-level FCNC processes induced by $b \to s\ell^+\ell^-$ and then the stringent constraints on the model parameter space will be expected.

Implication on B physcs: the Hamiltionian

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} [C_i(\mu) O_i(\mu) + C_i^{'}(\mu) O_i^{'}(\mu)] \ ,$$

in which the operators in SM are:

$$\begin{aligned}
O_1 &= (\bar{s}_i c_j)_{V-A} (\bar{c}_j b_i)_{V-A} \\
O_2 &= (\bar{s} c)_{V-A} (\bar{c} b)_{V-A} \\
O_3 &= (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V-A} \\
O_4 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} \\
O_5 &= (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V+A} \\
O_6 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} \\
O_7 &= \frac{e}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) b_i F_{\mu\nu} \\
O_8 &= \frac{g}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) T_{ij}^a b_j G_{\mu\nu}^a \\
O_9 &= (\bar{s} b)_{V-A} (\bar{l} l)_V \\
O_{10} &= (\bar{s} b)_{V-A} (\bar{l} l)_A
\end{aligned}$$

New operators and the implication

The chirality-flipped operators O_i' are obtained from O_i by the replacement $\gamma_5 \to -\gamma_5$ in quark current.

- CKM matrix is replaced by a 5×5 matrix

$$V_{\mathrm{CKM}}^{ij} = \sum_{m=1}^{4} U_u^{*mi} U_d^{mj}$$

  In our analysis we take a reasonable assumption that the deviation from unitary is not large.
- The effective coefficient $C_9^{eff}(\mu_b)$ have the same as the SM.
- The coefficient of operator $O_2' = (\overline{s}c)_{V+A}(\overline{c}b)_{V-A}$, is proportional to the elements of quark mixing matrix V_u^{5j} or U_d^{5i}. $C_9^{',eff}(\mu_b)$ receives contributions mainly from the tree-level diagrams,

- For $b \to s\gamma$, the new contributions mainly come from the new type Yukawa interactions, and for $b \to s\ell^+\ell^-$, the new contributions mainly arise from the new operators $O'_{9,10}$.

- The Wilson coefficient C_7 at the matching scale is

$$
\begin{aligned}
C_7 &= \frac{1}{V_{tb}V_{ts}^*}\sum_{i=1}^{5}\{A(x_i)g_L^{W*}(i,2)g_L^{W}(i,3) - B(x_i)\frac{m_W}{m_b}g_L^{W*}(i,2)g_R^{G}(i,3) \\
&+ g_L^{G*}(i,2)[C(x_i)g_L^{G}(i,3) - \frac{m_{u_i}}{m_b}D(x_i)g_R^{G}(i,3)] \\
&+ \frac{x_i}{y_i}g_L^{h*}(i,2)[C(y_i)g_L^{h}(i,3) - \frac{m_{u_i}}{m_b}D(y_i)g_R^{h}(i,3)]\},
\end{aligned}
$$

- The Wilson coefficient C_9, C_{10}, C_9', and C_{10}' at the matching scale is

$$\begin{aligned} C_9 &= \frac{P(x_t)-Q(x_t)}{\sin^2\theta_W} + 4Q(x_t) \\ &- \frac{2\pi}{\alpha_{em}} \frac{U_d^{*52}U_d^{53}}{V_{tb}V_{ts}^*}\left(\frac{1}{4}-\sin^2\theta_W\right) \\ &+ \frac{1}{V_{tb}V_{ts}^*}\left\{\sum_{i=3}^{5}\left[R(x_i)g_L^{W*}(i,2)g_L^W(i,3) + S(x_i)g_R^{G*}(i,2)g_L^G(i,3)\right]\right. \\ &+ \sum_{i=1}^{5}\frac{m_W}{m_{u_i}}T(x_i)\left[g_L^{W*}(i,2)g_L^G(i,3) + g_R^{G*}(i,2)g_L^W(i,3)\right] \\ &+ \left.\frac{x_i}{y_i}S(y_i)g_R^{h*}(i,2)g_L^h(i,3)]\right\} + \frac{4}{9}. \end{aligned}$$

$$C_{10} = -\frac{P(x_t)-Q(x_t)}{\sin^2\theta_W} + \frac{2\pi}{\alpha_{em}}\frac{1}{4}\frac{U_d^{*52}U_d^{53}}{V_{tb}V_{ts}^*},$$

$$C_9' = \left(\frac{1}{4}-\sin^2\theta_W\right)\frac{2\pi}{\alpha_{em}}\frac{V_d^{*52}V_d^{53}}{V_{tb}V_{ts}^*},$$

$$C_{10}' = -\frac{2\pi}{\alpha_{em}}\frac{1}{4}\frac{V_d^{*52}V_d^{53}}{V_{tb}V_{ts}^*}.$$

- The contributions from loop diagrams to $C_{9,10}'$ can be neglected safely.

Calculation of Branching ratios:

1. $B \to X_s \gamma$

$$\mathrm{Br}(B \to X_s \gamma) = \mathrm{Br}^{\mathrm{ex}}(B \to X_c e \overline{\nu_e}) \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(z)} |C_7^{eff}(\mu_b)|^2.$$

2. $B \to X_s \ell^+ \ell^-$

$$\begin{aligned} \frac{d\Gamma(B \to X_s \ell^+ \ell^-)}{ds} &= \frac{G_F^2 m_b^5}{768\pi^5} \alpha_{em}^2 |V_{tb} V_{ts}^*|^2 (1-s)^2 (1 - \frac{4r}{s})^{1/2} \\ &\times \left\{ 4|C_7^{eff}|^2 (1 + \frac{2}{s}) + (|C_9^{eff}|^2 + |C_9'|^2)(1+2s) \right. \\ &+ \left. (|C_{10}|^2 + |C_{10}'|^2)(1+2s) + 12 Re(C_7^{eff} C_9^{eff*}) \right\} , \end{aligned}$$

where $s = (p_{\ell^+} + p_{\ell^-})^2 / m_b^2$.

3. $B_s \to \mu^+ \mu^-$

$$\Gamma(B_s \to \mu^+ \mu^-) = \kappa \frac{\alpha_{em}^2 G_F^2}{16\pi^3} |V_{tb} V_{ts}^*|^2 f_{B_s}^2 m_{B_s} m_\mu^2 |C_{10} - C_{10}'|^2 , \quad (2)$$

where f_{B_s} is the decay constant for B_s determined by $\langle 0 | \overline{q} \gamma_\mu \gamma_5 b | B_q \rangle = -i f_{B_q} p_\mu$.

1. $B_s \to \ell^+\ell^-\gamma$

$$\frac{d\Gamma}{ds} = \left|\frac{\alpha_{em}^{3/2} G_F}{4\sqrt{6\pi}} V_{tb} V_{ts}^*\right|^2 \frac{m_{B_s}^7}{(2\pi)^3} s(1-s)^3 \left[|K|^2 + |L|^2 + |M|^2 + |N|^2\right] , \quad (3)$$

where $s = p^2/m_{B_s}^2$ is normalized dileptonic mass squraed, and

$$K = \frac{1}{m_{B_s}^2} \left\{[C_9^{eff}(\mu_b) + C_9'] G_1(p^2) - 2C_7^{eff}(\mu_b) \frac{m_b}{p^2} G_2(p^2)\right\} ,$$

$$L = \frac{1}{m_{B_s}^2} \left\{[C_9^{eff}(\mu_b) - C_9'] F_1(p^2) - 2C_7^{eff}(\mu_b) \frac{m_b}{p^2} F_2(p^2)\right] ,$$

$$M = \frac{C_{10} + C_{10}'}{m_{B_s}^2} G_1(p^2), \quad N = \frac{C_{10} - C_{10}'}{m_{B_s}^2} F_1(p^2) , \quad (4)$$

with G_i and F_i being the form factors.

Numerical results:

As the first glance on B physics in $\mathcal{F}$-$SU(5)$ model, we focus on the implication of mass scale of the vector-like quark on B physics, this will give us the most important information of the model. Thus in the numerical study we scan the mass m_{u_x} in the range $180\ \mathrm{GeV} \sim 2000\ \mathrm{GeV}$, and m_{u_y} in the range $40 \sim 60$ GeV heavier than m_{u_x}.

1. The constraints on CKM matrix element measurements are not from rare B decays but from tree-level B decays.

Table: The CKM matrix elements constrained by the tree-level B decays.

	absolute value	relative error	direct measurement from
V_{ud}	0.97418 ± 0.00027	0.028%	nuclear beta decay
V_{us}	0.2255 ± 0.0019	0.84%	semi-leptonic K-decay
V_{ub}	0.00393 ± 0.00036	9.2%	semi-leptonic B-decay
V_{cd}	0.230 ± 0.011	4.8%	semi-leptonic D-decay
V_{cb}	0.0412 ± 0.0011	2.7%	semi-leptonic B-decay
V_{tb}	> 0.74		(single) top-production

1. we use the following bounds on the rare B decays

$$Br(b \to c e \overline{\nu}_e) = (10.74 \pm 0.16) \times 10^{-2} ,$$
$$Br(\overline{B} \to X_s \gamma) = (3.06 \pm 0.23) \times 10^{-4} ,$$
$$Br(B \to X_s \ell^+ \ell^-) = (4.5 \pm 1) \times 10^{-6} ,$$
$$Br(B_s \to \mu^+ \mu^-) < 4.5 \times 10^{-9} \quad (95\% C.L.) .$$

2. We scan the two parameters randomly and choose two typical points ($\tan\beta = 2, \quad m_{h^+} = 3000$ GeV) and ($\tan\beta = 40, \quad m_{h^+} = 500$ GeV) for the demonstration.

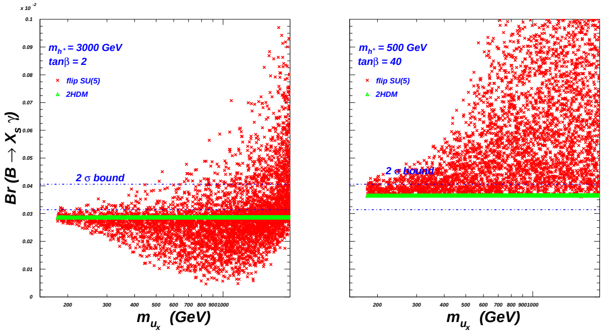

Figure: Comparison of $B \to X_s \gamma$ versus m_{u_x} in the $\mathcal{F}$-$SU(5)$ model (red cross) and 2HDM (green triangle).

- C_7 determined in both $\mathcal{F}$-$SU(5)$ model and 2HDM will approach to the SM value when the charged Higgs boson is much heavier than EW scale. Nevertheless, the contributions from the fourth and fifth generation up-type vector-like quarks in 2HDM can be suppressed by small V^{5i} and V^{4i} due to the unitarity condition of 5×5 matrix;
- Because the summed indices are only from 1 to 4 in the $\mathcal{F}$-$SU(5)$ model, the unitary condition of the CKM matrix can not be maintained. When the vector-like particle mass approaches to the charged Higgs boson mass, the suppression from 5×5 CKM mixing matrix will be released and then the non-decoupling effects will be sizable. In fact, the non-decoupling effects are a very special part of the $\mathcal{F}$-$SU(5)$ model at EW scale and can be tested at the LHC and other B physics detectors.

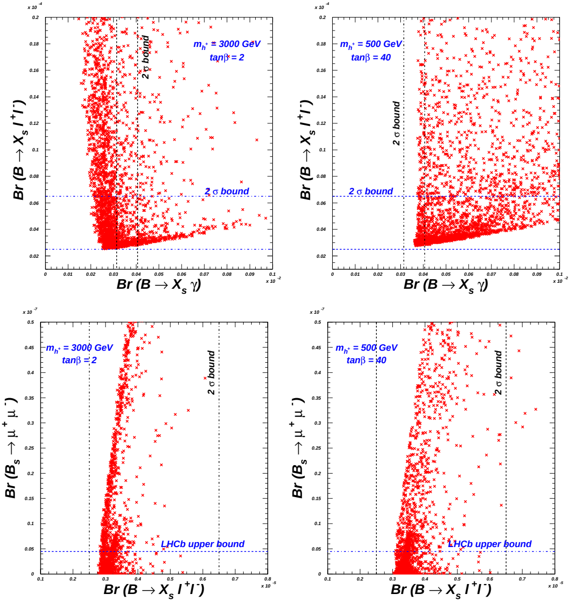

Figure: Branching ratios of $B \to X_s\gamma$, $B_s \to \mu^+\mu^-$ versus $B \to X_s\ell^+\ell^-$ in the $\mathcal{F}$-$SU(5)$ model.

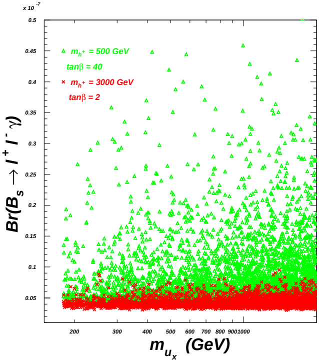

Figure: Branching ratio of $B_s \to \ell^+\ell^-\gamma$ with the combined constraints from $B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$ and $B_s \to \mu^+\mu^-$. Red cross stands for the type inputs $(\tan\beta = 2, \;\; m_{h^+} = 3000\text{GeV})$ and green triangle for $(\tan\beta = 40, \;\; m_{h^+} = 500\text{GeV})$ in the $\mathcal{F}$-$SU(5)$ model, respectively.

Summary

The quark mass spectra, Feynman rules, the new operators in low energy effective theory and the correspondence Wilson coefficients, etc in the $\mathcal{F}$-$SU(5)$ model are studied:

1. There exists the $\overline{s}bZ$ interaction at tree level, and the Yukawa interactions are changed. The new operators O'_9 and O'_{10} must be introduced in effective Hamiltonian.
2. The effects of vector-like quarks on rare B decays such as $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$ do not decouple in some allowed parameter space, especially when the vector-like quark mass is comparable to the charged Higgs boson mass.
3. Under the constraints from $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$, there exist scenarios in the model the latest measurement for $B_s \to \mu^+\mu^-$ can be explained naturally, and the branching ratio of $B_s \to \ell^+\ell^-\gamma$ can be up to $(4 \sim 5) \times 10^{-8}$.

Thank you !