Dashen Phase in Confining Sigma Model

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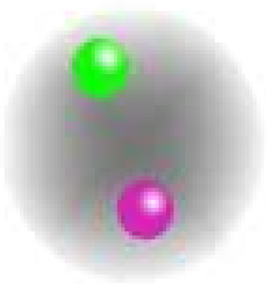
Outline

- U(1) problem and the pseudo-scalar spectrum
- The Theta parameter and CP
- Large Nc chiral dynamics
- Dashen phase
- Complex mass in confining sigma model
- Summary

The pseudo-scalar masses



- The standard picture of the pseudo-scalar (PS) meson spectrum in 2Flavor QCD:

  The first two controlled by ~ $m_{ud}=(m_u+m_d)/2$

$$\left\{\begin{array}{l}\bar{u}\gamma_5 d \sim \pi^+ \\ \bar{d}\gamma_5 u \sim \pi^-\end{array}\right.$$

- Naively, the last two do not mix---**Helicity Conserved** in gauge theories coupled to light fermions----Two neutral PS, mass controlled by

  (m_u , m_d) ?

$$\left\{\begin{array}{l}\bar{u}\gamma_5 u \sim ? \\ \bar{d}\gamma_5 d \sim ?\end{array}\right.$$

But this is not the case: η' (958) is not likely NG (<548)
Why η' is so heavy is blamed for **Axial Anomaly which** induces Strong **mixing** between two through the **'t Hooft vortex** due to nontrivial gauge field topology

$$\bar{u}\gamma_5 u + \bar{d}\gamma_5 d \sim \eta'$$

$$mass \sim \Lambda_{qcd}$$

G 't Hooft , Phys.Rev., D14:3432–3450, 1976

Heavy η' vs. the U(1) problem

- But where the Helicity Conservation has gone?

  Normaly, **violated**, by, **e.g.**

- **Instanton: flip the helicity of** $\bar{q}\gamma_5 q$

$$\partial_\mu j_5^\mu = 2iN_f q(x)$$

$$\bar{u}\gamma_5 u - \bar{d}\gamma_5 d \sim \pi^0$$

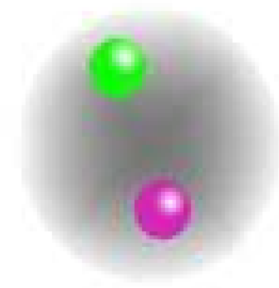
$$mass \sim m_{ud}$$

$$\psi \to e^{i\alpha\gamma_5}\psi$$

$$[d\psi][d\bar{\psi}] \to \exp\left\{-i\alpha g^2 N_f \int d^4x\, q(x)\right\}[d\psi][d\bar{\psi}]$$

$$q(x) = \frac{1}{16\pi^2} trG\tilde{G}$$

G 't Hooft , Phys.Rev., D14:3432–3450, 1976



Theta parameter

- The standard way to solve axial U(1) problem suggests nontrivial vaccum

$$|\theta\rangle = \sum_n e^{in\theta} |n\rangle$$

- In path integral formulation, this vacua is equivalent to a new top. Term in QCD Lagrangian

$$L^{QCD} = L^{QCD} - L^{\theta}$$

$$L^{\theta} = \theta q(x) = \frac{\theta g^2}{32\pi^2} tr G\tilde{G}$$

P and CP odd

- A **chiral rotation** is equi. to a theta shift

$$\theta \to \theta - 2\alpha N_f$$

- Conversely, theta can be rotated away, into Quark Mass Matrix m_f

$$m_f \to m_f e^{2i\alpha}$$

P and CP odd

CP violation, tiny

- This implies P and CP violation

$$\mathcal{L}_m = \frac{1}{2}\sum_f m_f \bar{\psi}_f (1+\gamma_5)\psi_f + \frac{1}{2}\sum_f m_f^* \bar{\psi}_f (1-\gamma_5)\psi_f = \sum_{f=1}^{N_f} \bar{\psi}_f (\mathrm{Re}\, m_f + \mathrm{i}\,\mathrm{Im}\, m_f \gamma_5)\psi_f .$$

- Main origin of the Imaginary quark mass comes from top. θ-term, which can not be rotated away by choice of α →

$$\theta \to \theta - 2\alpha N_f$$

$$m_f \to m_f e^{2i\alpha}$$

We see that

(1) For massless fermions (u,d,s) θ can be rotated away;

(2) For massive q, θ may be rotated to quark mass term , allowing mass to be complex

(3) Strong CP violation, if exists, is very tiny,

$$d_n \sim \theta \frac{e m_f}{m_n^2} \sim \theta \frac{e m_\pi^2}{m_n^3} \approx 10^{-16} \theta e\,\mathrm{cm}.$$

Exp. Of neutron EDM:

$$|d_n| < 2.6 \times 10^{-26} \theta ecm$$

$$Exp: \quad |\theta| < 10^{-26}$$

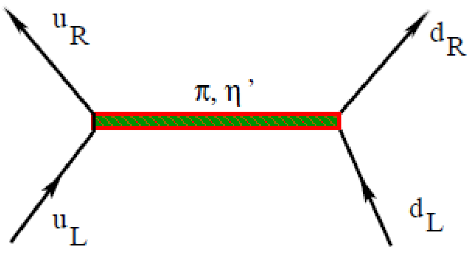
What can we learn from Axial U(1) ?

- Proposals for origin of tiny theta :
  - Spontanteou Breaking of CP,
  - vanishing of lightest u mass,
  - The axion,

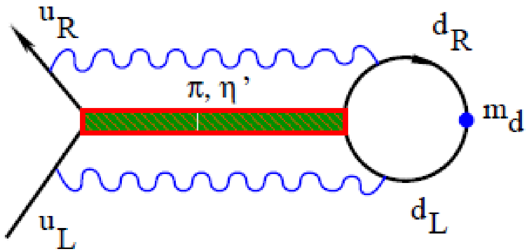
- To solve axial anomaly **$U(1)_A$**, the theta parameter is necessary, but should be tiny !!

Axial U(1) vs quark mass

- This possibility(zero up quark mass) is disfavored by a standard current algebra analysis: The exp. data are consistent with a nonzero mass.
- Also strongly supported by LQCD calculations.
- Arguments exist for nonvanishing of m_u



Due to the anomaly, spin-flip scattering of massless up and down quarks does not vanish



A small down quark mass induces an additive shift in the up quark mass through pseudoscalar meson exchange.

Topological susceptibility

- Large-Nc analysis by Witten-Veneziano: The problem should be solved at the lowest nonplanar (NLO of 1/N) level, using topological susceptibility

  Agrees with LQCD very well

$$\frac{4N_f}{f_s}\chi = m_{\eta'}^2 + m_\eta^2 - 2m_K^2$$

$(180)^2$

$$\chi = -i\int d^4x\langle 0|Tq(x)q(0)|0\rangle$$

- Instanton disfavored :
- (1) wrong large-Nc behavior
- (2) question open for instance liquid

$$m_{\eta'}^2 \sim 1/N; (LNQCD)$$

$$m_{\eta'}^2 \sim \exp[-cN]; (Ins\tan ce)$$

P. Di Vecchia, Phys. Lett. 885 (1979), 357
E. Witten, Ann. Phys 128, 363-375 (1980)

Large N chiral dynamics

- Based on Baluni 's **current algebra + current algebra theorem** for the Electric Dipole Moment of the neutron,
- Witten suggests the following chiral dynamics

$$L = \frac{F_\pi^{\ 2}}{4}\left[tr(\partial_\mu U^+ \partial^\mu U) + tr(\mathcal{M}U + \mathcal{M}^+ U^+) - \frac{a}{N}(i \ln \det U)^2 \right].$$

Soft meson amplitude

$$\mathcal{M} = e^{i\theta/3} M, \quad U \to e^{-i\theta/3} U$$

$$L = \frac{F_\pi^{\ 2}}{4}\left[tr(\partial_\mu U^+ \partial^\mu U) + tr(MU + MU^+) - \frac{a}{N}(i \ln \det U - \theta)^2 \right].$$

Minimizing

$$U \to \begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & e^{i\phi_3} \end{pmatrix}$$

$$V(\phi_i) = F_\pi^{\ 2}\left[-\sum \mu_i^{\ 2} \cos(\phi_i) + \frac{a}{2N}\left(\sum \phi_i - \theta\right)^2 \right].$$

P. Di Vecchia, Phys. Lett. 885 (1979), 357
E. Witten, Ann. Phys 128, 363-375 (1980)

Large N chiral dynamics

Minimization of chiral dynamics

$$\mu_i^2 \sin(\phi_i) = \tfrac{a}{N}(\theta - \sum \phi_i)$$

$$M = diag\{\mu_1^2, \mu_2^2, \mu_3^2\}$$

IF $\mu_1^2 = 0: \quad \phi_1 = \theta, \phi_{2,3} = 0,$

And theta can be removed by rotation $U \to \begin{pmatrix} e^{i\theta} & & \\ & 1 & \\ & & 1 \end{pmatrix} U$

Else IF none of the $(\mu\_i)^2$ vanishies, the physics depends on θ.
At large N with $(\mu\_i)^2$ fixed, the θ dependence disappears
In nature, however, since the η' is much heavier than PS, we are much close to the opposite: $\mu_i^2 \ll a/N$

$$\theta \simeq \sum \phi_i$$

When $\mu_i^2 \sim a/N$ (or both are much smaller than the other hadronic masses), neither θ nor the η' can be eliminated from the problem.

$$M^2 = \begin{pmatrix} \mu_1^2 & & \\ & \mu_2^2 & \\ & & \mu_3^2 \end{pmatrix} + \frac{a}{N}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

P. Di Vecchia, Phys. Lett. 885 (1979), 357
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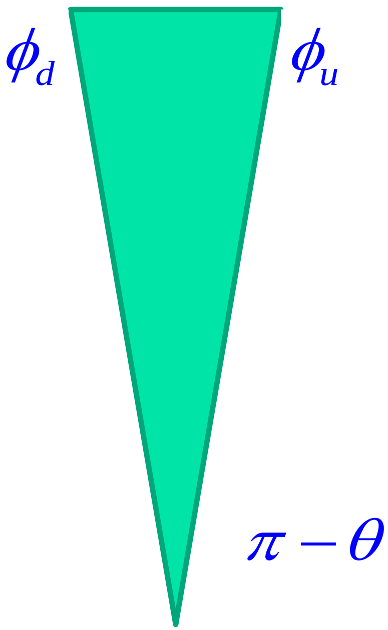
Large N chiral dynamics

- For general values , hard to solve Eq. (14) analytically.
- However, in the realistic situation

As $\mu_u^2, \mu_d^2 \ll \mu_s^2 \ll a/N: \quad \phi_1 = \theta, \phi_{2,3} = 0,$

$$\phi_u + \phi_d = \theta, \phi_s = 0,$$
$$\mu_u^2 \sin(\phi_u) = \mu_u^2 \sin(\phi_d)$$

And



$$\sin \phi_u = \frac{m_d \sin \theta}{(m_u^2 + m_d^2 + 2m_u m_d \cos \theta)^{1/2}},$$

$$\sin \phi_d = \frac{m_u \sin \theta}{(m_u^2 + m_d^2 + 2m_u m_d \cos \theta)^{1/2}}.$$

P. Di Vecchia, Phys. Lett. 885 (1979), 357
E. Witten, Ann. Phys 128, 363-375 (1980)

Periodicity and Analyticity in θ; Dashen's Phenomenon

Chiral dynamics

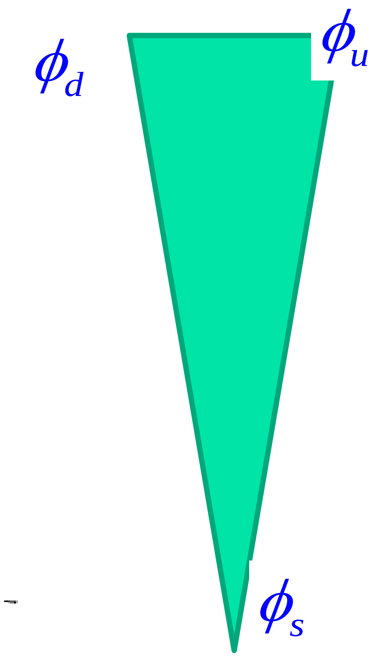
↓

$$\mu_i^2 \sin(\phi_i) = \tfrac{a}{N}(\theta - \sum \phi_i)$$

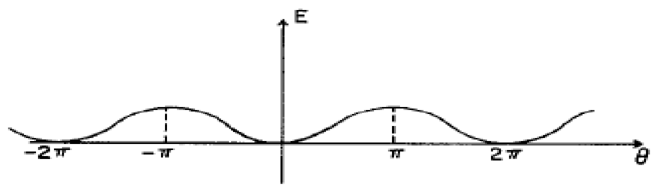
$$U = \begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & e^{i\phi_3} \end{pmatrix}$$

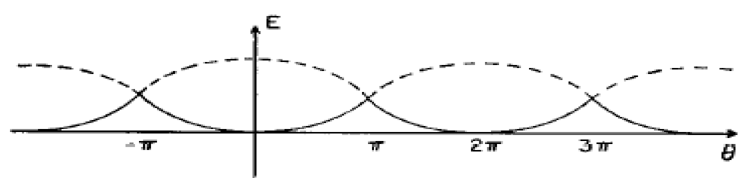
Periodical : $\theta \to \theta + 2\pi,$

analytical on theta if U has only one solution



If there is only one solution, it describes the physics for all θ, and therefore in this case the physics is analytic as a function of θ.





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Theta=pi: Dashen's Phenomenon

Chiral dynamics →

$$U = U_0 V, \bar{\theta} = \theta + i \ln \det U_0$$

$$E = F_\pi^2 \left(-\frac{1}{2} \mathrm{Tr}\, AV - \frac{1}{2} \mathrm{Tr}\, AV^\dagger + \frac{a}{2N} (\bar{\theta}^2 + (i \ln \det V)^2) \right) + \tilde{E},$$

It is interesting that the CP violating part of the Hamiltonian is SU(3) invariant even though, since the quark masses may not be equal, W(3) is not necessarily a symmetry of the Hamiltonian.

$$\bar{M} = A + iB$$

$$B = \frac{3a}{N} \bar{\theta}$$

The Hermitian, CP conserving mass matrix A can always be diagonalized, and so, once a suitable definition is made, it conserves the three quark flavors u, d, and s. Since B, being a multiple of the identity, conserves all quantum numbers, we see that regardless of the values of the parameters the three flavor numbers are conserved.

Why QM works?

Constituent Mass(soft mass), corresponding to CS breaking .

- Mahohar-Georgi model in ChQT:
- 2 scales occurs(2 phases)

(250MeV) $\Lambda_{QCD} < Q < \Lambda_{\chi}$ (1GeV),

Confining | Mixing | Free

Constituent quark

Gluons

$G_\mu = G_\mu^a \lambda^a / 2$

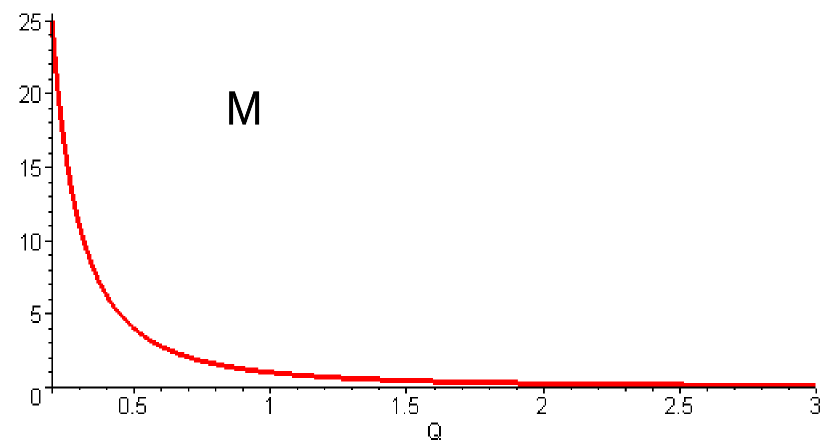
$$\mathcal{L}^{MG} = \bar{q}[i\partial\!\!\!/ - g\gamma^\mu G_\mu + iV\!\!\!/ + ig_A A\!\!\!/\gamma^5 - M]q + \frac{f^2}{4} Tr_F(\partial_\mu U^\dagger \partial^\mu U) - \frac{1}{2} Tr G_{\mu\nu} G^{\mu\nu} + \cdots$$

$$U = V^2 \qquad D\!\!\!/(U) = i\gamma^\mu(\partial_\mu + V_\mu + \gamma^5 A_\mu)$$

$$\{V_\mu, A_\mu\} = \frac{1}{2}\left(V^\dagger \partial_\mu V \pm V \partial_\mu V\right) \quad U = V^2 = \left[\exp(i\boldsymbol{\pi\tau}/f)\right]^2$$

Strong coupling (α_s) weak due to the presence of constituent mass

$$M = g\frac{\langle \bar{q}q \rangle}{Q^2} \approx 350MeV$$



$\mathcal{L}^{MG}$ invarinat under chiral $SU(3)_L \times SU(3)_R$
Non-renormalizable terms suppressed by

$$(\partial\pi / f)^{number}$$

Quark mass role in Baryons

The simplest fit for baryon masses:

$$M = m_1 + m_2 + m_3 + \sum_{i>j} A' \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j}$$

1-4%

m_u=363
m_s=583

$$A' = (m_u / \hbar)^2 50 MeV$$

The NRp estimate for baryon masses:[PRD12(1975)147]

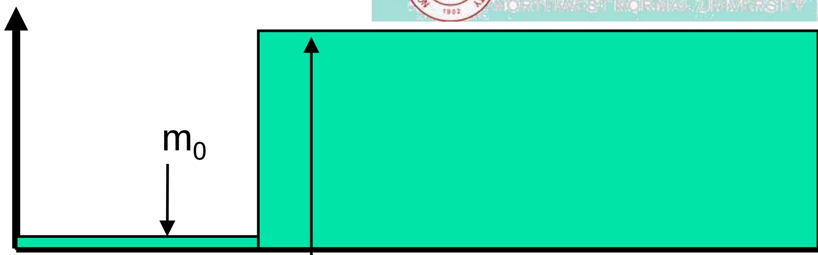
$$M = M_0 + \sum_i \left[\Delta m_i + a\left(\frac{1}{m_i} - \frac{1}{m_P}\right)\right]$$
$$+ \sum_{i>j} (\alpha Q_i Q_j - \frac{2}{3}\alpha_s) \left[b - \frac{c}{m_i m_j} - d\left(\frac{1}{m_i^{\,2}} + \frac{1}{m_j^{\,2}} + \frac{4}{3}\frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j}\right)\right]$$

m_u=300
m_P=336

Smaller

Fine fit, so why RQM?
(1) More constraints on models(including ChSymmetry)
(2) Less parameters for spin-interaction

Quark mass in bag pictures



- The MIT bag-RQM, degrees (quark and/or gluons), Confinement put in by Bag boundary condition/effective mass

$$\mathcal{L}^{MIT} = [\bar{q}(i\gamma^\mu \partial_\mu - m(r))q] - B$$

$$m(r) = \begin{cases} m_0, & r < R \\ M \to \infty, & r \ge R \end{cases}$$

Mass $M = E(m_0, R) = [m_0^2 + (x/R)^2]^{1/2}$

scale 2-5MeV 300MeV

$$q = N(x)\begin{pmatrix} \sqrt{\frac{E+m_0}{E}}\, j_0(kr) \\ i\sqrt{\frac{E-m_0}{E}}\, j_1(kr)\sigma\cdot\hat{r} \end{pmatrix} y_{ljm} e^{-iEt} \chi_F \chi_C$$

The consititent mass(~1/R) mainly from BC while the current mass contributes a few MeV: A mechanism for mass splitting

$$M = \langle n | \int_{Bag} d^3x T^{00} | n \rangle = nE_\kappa + \frac{4\pi}{3}R^3 B \overset{\kappa=-1}{=} n\frac{2.04}{R} + B\frac{4\pi}{3}R^3$$

$$M = \left[n\left(m^2 + \frac{x^2}{R^2}\right)^{1/2} + n_s\left(m_s^2 + \frac{x_s^2}{R^2}\right)^{1/2} \right] + \left[BV - \frac{Z_0}{R} \right] + \Delta E_M + \Delta E_E,$$

valence quark kinetic energy; zero point energy; color interaction energy

(a) (b)

$$\Delta E_M = -g^2 \sum_{a,j>i} \int_{Bag} d^3x \mathbf{B}_i^a \mathbf{B}_j^a = \frac{4\alpha_c}{3} \sum_{i\neq j} \vec{\sigma}_i \cdot \vec{\sigma}_j \frac{\mu_{ij}}{R}$$

$$= \frac{4}{3}\alpha_c \frac{\mu_{ij}}{R}[n(n-6) + S(S+1) + 3I(I+1)]$$

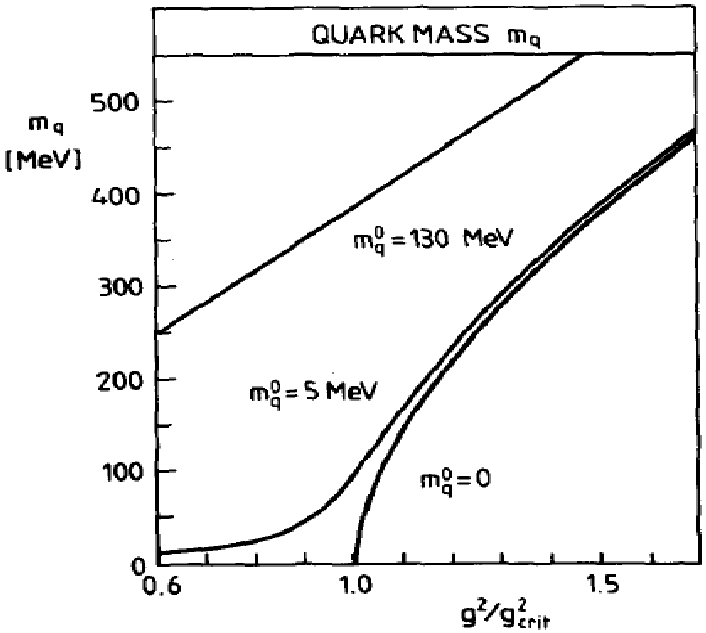
Mass role in ChQM

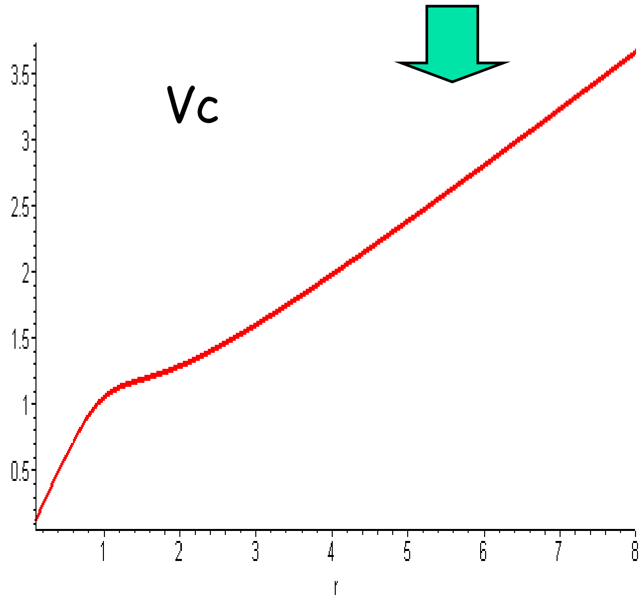
- The consti. mass varying Δm_i and the EM effects breaks the flavor SU(3) mainly;
- The mass varying dominates for p-n spliting, in ground states

$$\mathcal{L}^{ChQM} = \bar{q}_i[i\partial\!\!\!/ - (m_i + S(x))U_5 + \gamma^0 V_c]q_i + \mathcal{L}^U$$

$$S(x) = S_0 + r/a^2$$
$$V_c = \alpha(r)/r, e.g.,$$
$$\alpha(r) = \alpha_0 \tanh((r/d)^n),$$

a=0.3/0.2;L=0.75/0.2;
M=0.3;S0=0;kappa=-1;
d=1.0;alpha0=0.8;





Chromo-magnetic interaction in ChQM

$$q=\frac{N}{r}\begin{bmatrix} G(r)\Omega_{l,M-1/2} \\ F(r)\Omega_{l,M+1/2} \end{bmatrix}$$

- The radial Eq. of Motion of a quark:
  With Y determined by Y equations

$$\frac{dG}{dx}+\frac{\kappa}{x}G=\left[LE_q+LM\cos Y-LVc\right]F$$

$$-\frac{dF}{dx}+\frac{\kappa}{x}F=\left[LE_q-LM\cos Y-LVc\right]G$$

$$H=N^2\int dx\left\{FG_x-GF_x+\frac{2\kappa}{x}GF+L(M+S)\cos Y(G^2-F^2)\right.$$

$$\left.+LV_c(G^2+F^2)+\frac{4\pi L^4B}{LN^2}x^2(1-\cos(Y))\right\}+E^\pi,$$

$$Y_x=dY/dx, etc.,$$

The Y profile determined by a dynamics, eg., the Coupled Skyme lagrangian here, it can be set by comparing with ChPT

Complex mass in quark model

- The radial Eq. of Motion of a quark in Heavy-light meson:
  With Y determined by Y equations

$$q=\frac{N}{r}\begin{bmatrix} G(r)\Omega_{l,M-1/2} \\ F(r)\Omega_{l,M+1/2} \end{bmatrix}$$

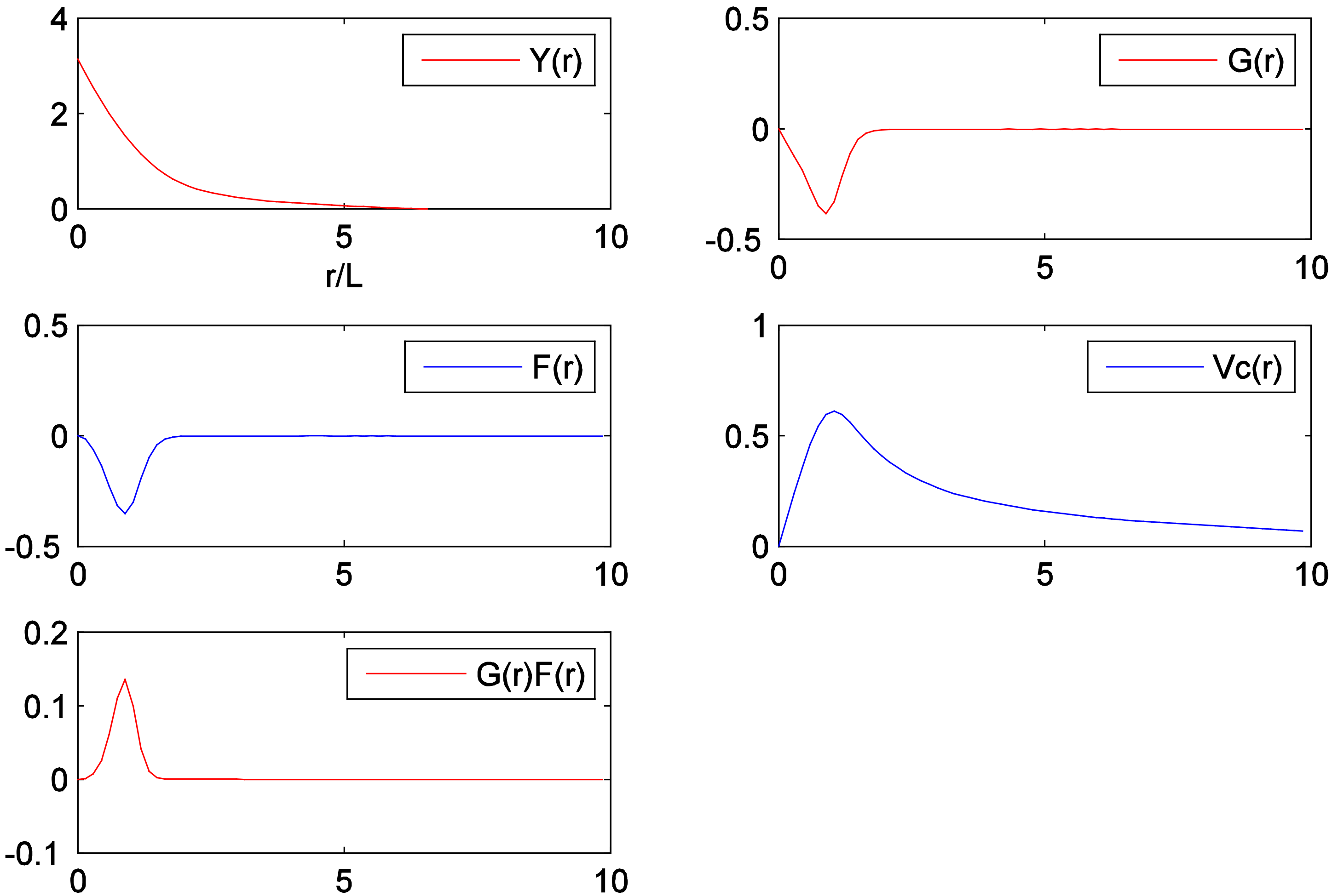
$$H=N^2\int dx\left\{FG_x-GF_x+\frac{2\kappa}{x}GF+L(M+S)\cos Y(G^2-F^2)\right.$$
$$\left.+LV_c(G^2+F^2)+\frac{4\pi L^4 B}{LN^2}x^2(1-\cos(Y))\right\}+E^{\pi},$$
$$Y_x=dY/dx, etc.,$$

$$\frac{dG}{dx}+\frac{\kappa}{x}G=\left[LE_q+LM\cos Y-LVc\right]F$$
$$-\frac{dF}{dx}+\frac{\kappa}{x}F=\left[LE_q-LM\cos Y-LVc\right]G$$

Q

Quark configuration in ChQM

Length scale L=3.75GeV^{-1}



Static N in ChQM

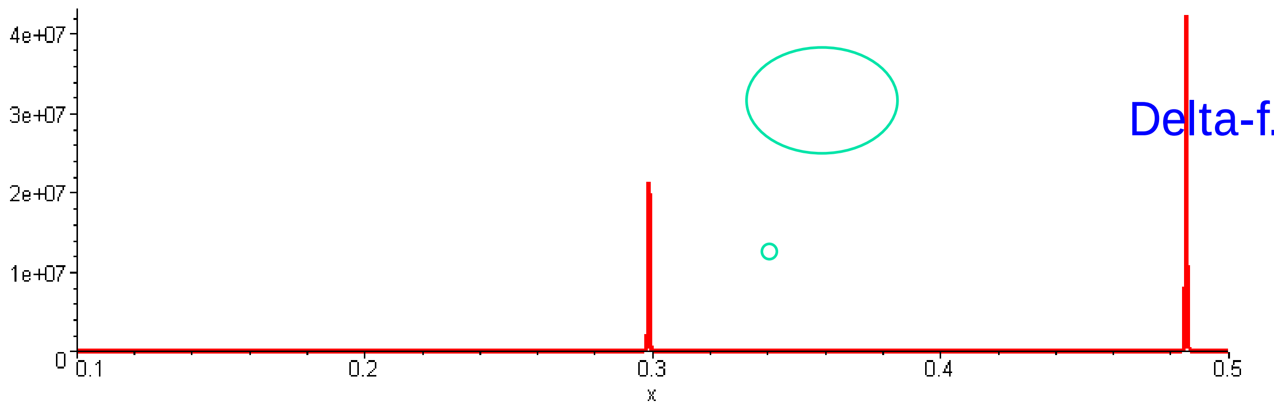
D. Jia, L.Yu,R. Wan, arXiv:1308.0700v1

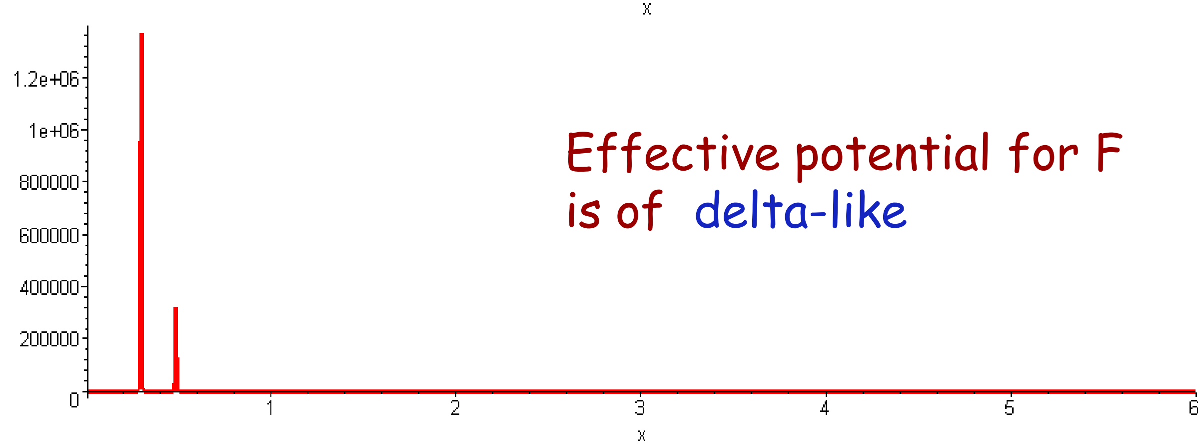
Table I

Quantities	MIT[5]	Skyrme[3]	HCB[12]	PCQM[8]	This Work	Exp.
$R[fm]$	1.0	1.0	0.6	$0.55 \sim 0.65$	0.909	
e	$[\alpha c = 2.2]$	5.45	4.5		2.80	
$f_\pi[fm]$	$[Z = 1.84]$	93	93	88	93	93
$a[GeV^{-1}]$					0.296	
$B^{1/4}[MeV]$	146		150	$[B_0 = 1400]$	134	
$m_p[MeV]$	938	939	1425	938.3	938.27	938.27
$\langle r^2\rangle_{ch}^{1/2}[fm]$	0.73	0.59	0.48	0.85	0.8723	0.877
$\|\mu_p\|[\mu_N]$	1.93	1.87	2.19	2.6	2.704	2.793

Pair creation from sea

- The the effective potential for upper component is confining but
- That for lower component is not, delta-like





Effective potential for F is of delta-like

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Summary

- Complex mass is required by anomaly
- Imaginary(small) part of mass is possible, for decay of meson

- Thanks !!!