On the symmetry improved CJT formalism in the O(4) linear sigma model

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I. Introduction

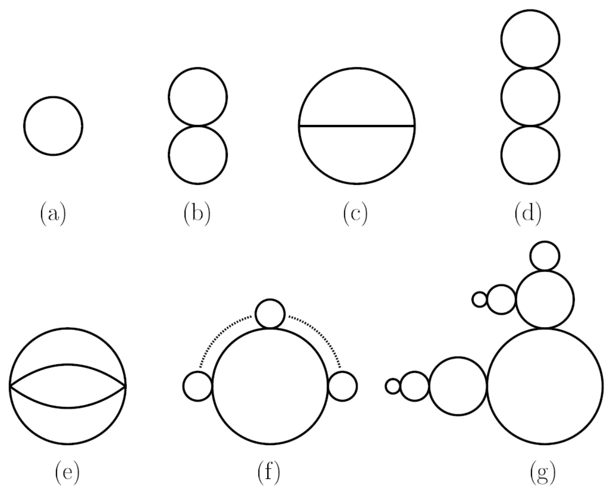
1.1 The conventional effective potential

The finite–temperature effective potential V, is defined through an effective action Γ, which is the generating functional of the **one particle irreducible** graphs.

a graph is called one–particle irreducible (1PI) if it cannot become disconnected by opening only one line, otherwise it is one–particle reducible).

1.2 Why we need the Cornwall-Jackiw-Tomboulis (CJT) formalism?

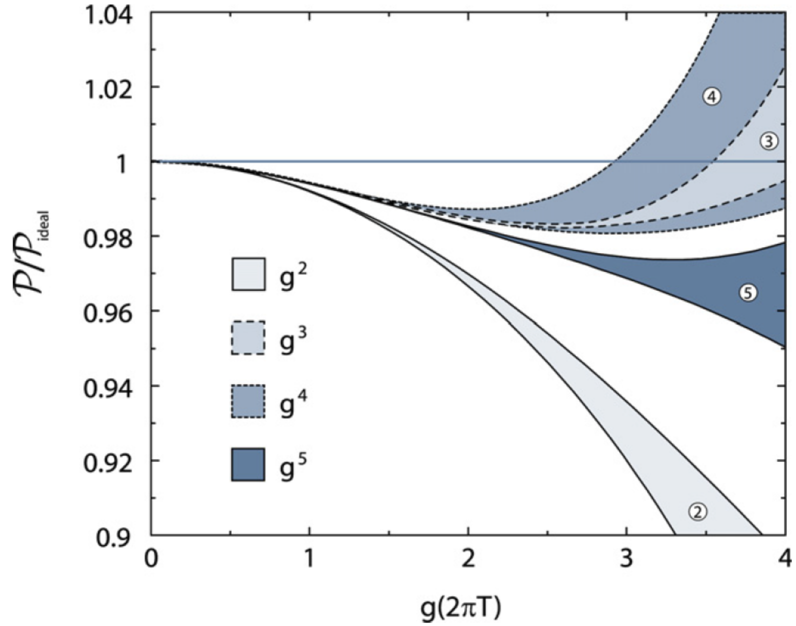
(1) Calculations using the loop expansion are very difficult beyond two loops.

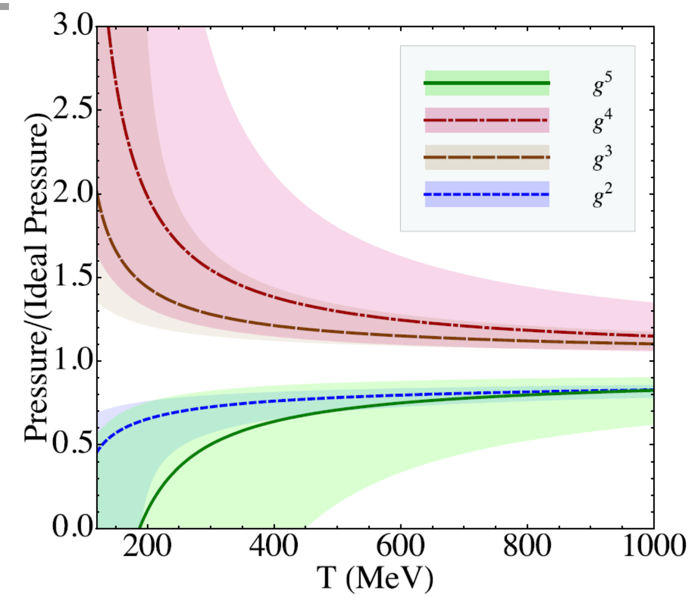


Examples of the various types of 1PI diagrams which contribute to the effective potential for a Φ^4 theory.

(2) In quantum thermal field theory, finite-order perturbative expansions break down at high temperatures and one needs to devise resummation methods to deal with this problem.

Jens O. Andersen and Michael Strickland, Annals Phys.317, 281 (2005)

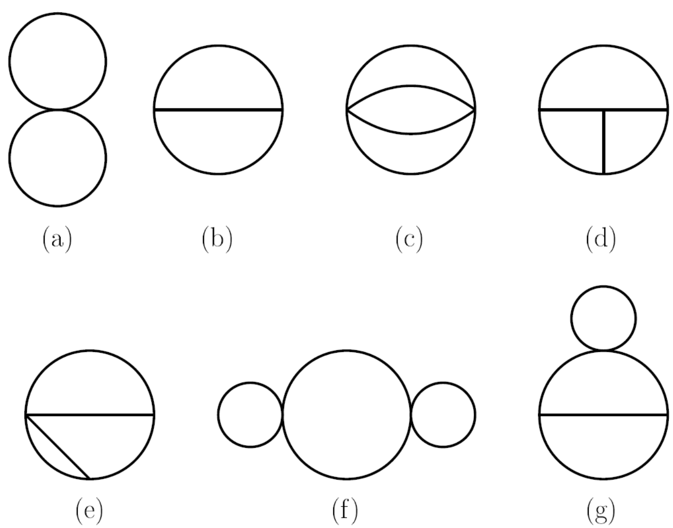




Running coupling expected is g ~ 2 or α_s ~

$$(\pi T \leq \mu \leq 4\pi T \;\&\; \alpha_s = g^2/4\pi)$$

(3) CJT formulism is a way to perform systematic selective **summations** by using the method of the effective action for composite operators. In this case, the effective action is the generating functional of the two –particle irreducible (2PI) vacuum graphs



Examples of two - particle irreducible graphs which contribute to the effective potential in the CJT method, up to three loops.

J. M. Cornwall, R. Jackiw and E. Tomboulis, *Effective Action For Composite Operators*, Phys. Rev. D10, 2428 (1974).

II. The symmetry improved CJT formalism

2.1 The QCD effective model with O(4) symmetry

The guiding principle for constructing models of the QCD:

Chiral $SU(N_f)_R \times SU(N_f)_L$ symmetry is an exact global symmetry of QCD with N_f **massless quark flavors**. In the low-T, this symmetry is spontaneously broken down to the flavor group $SU(N_f)_V$, As a consequence there exist $N_f^2 - 1$ pseudo scalar Nambu–Goldstone bosons and the QCD vacuum hosts a strong quark condensate:

$$\langle \bar{\psi} \psi \rangle = \langle \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R \rangle$$

Chiral Symmetry

The most general form of the **Ginzburg-Landau** free energy of the chiral field up to $\mathcal{O}(\Phi^4)$ ith $SU_L(2)\times SU_R(2)$ ~O(4) symmetry reads

$$\Omega_\chi = \frac{a_0}{2}\mathrm{Tr}\Phi^\dagger\Phi + \frac{b_1}{4!}(\mathrm{Tr}\Phi^\dagger\Phi)^2 + \frac{b_2}{4!}\mathrm{Tr}(\Phi^\dagger\Phi)^2 - \frac{c_0}{2}(\det\Phi + \det\Phi^\dagger),$$

With $\Phi = \sigma + i\pi_a\tau_a$

The model

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \sigma \, \partial^\mu \sigma + \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}\right) - U(\sigma, \vec{\pi}),$$

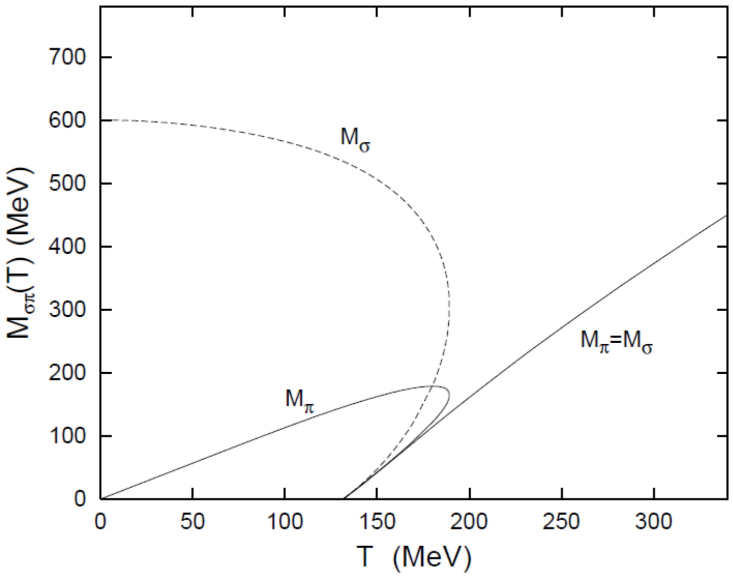
with $U(\sigma, \vec{\pi}) = \frac{m^2}{2}\left(\sigma^2 + \vec{\pi}^2\right) + \frac{\lambda}{24}\left(\sigma^2 + \vec{\pi}^2\right)^2.$

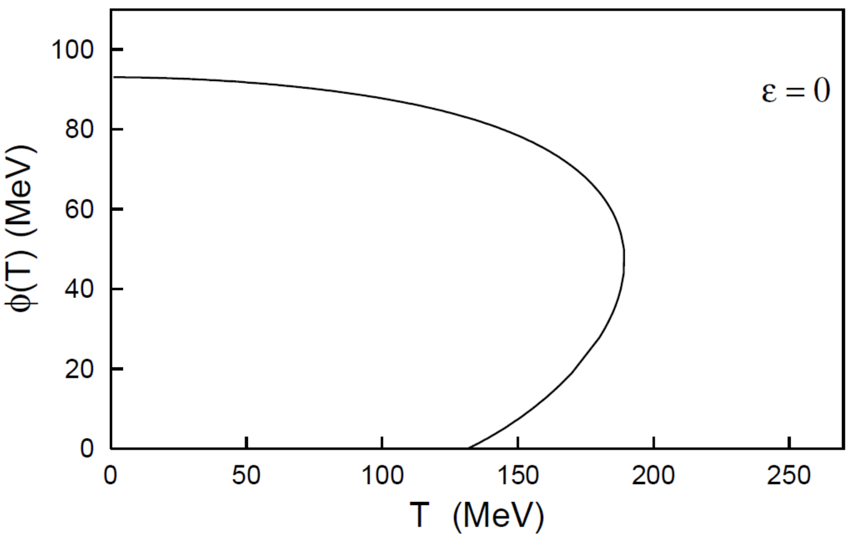
The parameters in chiral limit (**massless quark flavors or massless pions**)

$$\lambda = \frac{3m_\sigma^2}{f_\pi^2}, \qquad -2m^2 = m_\sigma^2 > 0.$$

The thermodynamic properties of the model at T in Hatree approximation

N. Petropoulos, *Linear sigma model and chiral symmetry at finite temperature*, J. Phys. G G25, 2225 (1999).





Two major problems: (1) in chiral limit, the CJT effective action violates the Goldstone theorem and gives massive pions in the spontaneous symmetry breaking phase; (2) it predicts a first order phase transition.

2.2 The SCJT effective potential at finite temperature

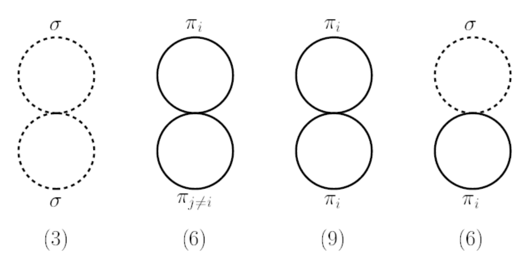
In Hartree approximation, the usual CJT formulism:

$$V(\phi, G_\sigma, G_\pi) = U(\phi) + \frac{1}{2}\int_\beta \ln G_\sigma^{-1}(\phi;k) + \frac{3}{2}\int_\beta \ln G_\pi^{-1}(\phi;k)$$
$$+ \frac{1}{2}\int_\beta [D_\sigma^{-1}(\phi;k)G_\sigma(\phi;k) - 1] + \frac{3}{2}\int_\beta [D_\pi^{-1}(\phi;k)G_\pi(\phi;k) - 1]$$
$$+ V_2(\phi, G_\sigma, G_\pi).$$

Where

$$V_2(\phi, G_\sigma, G_\pi) = 3\frac{\lambda}{24}\Big[\int_\beta G_\sigma(\phi;k)\Big]^2 + 6\frac{\lambda}{24}\Big[\int_\beta G_\sigma(\phi;k)\Big]\Big[\int_\beta G_\pi(\phi;k)\Big]$$
$$+ 15\frac{\lambda}{24}\Big[\int_\beta G_\pi(\phi;k)\Big]^2.$$

$$\int \frac{d^4k}{(2\pi)^4} f(k) \to \frac{1}{\beta}\sum_n \int \frac{d^3\vec{k}}{(2\pi)^3} f(i\omega_n, \vec{k}) \equiv \int_\beta f(i\omega_n, \vec{k}),$$



Minimizing the effective potential with respect to full propagators we obtain the following system of non linear gap equations:

$$G_\sigma^{-1} = D_\sigma^{-1} + \frac{\lambda}{2}\int_\beta G_\sigma(\phi;k) + \frac{\lambda}{2}\int_\beta G_\pi(\phi;k),$$

$$G_\pi^{-1} = D_\pi^{-1} + \frac{\lambda}{6}\int_\beta G_\sigma(\phi;k) + \frac{5\lambda}{6}\int_\beta G_\pi(\phi;k).$$

G

The full propagators assume the simple form

$$G_\sigma^{-1} = \omega_n^2 + \vec{k}^2 + M_\sigma^2,$$

$$G_\pi^{-1} = \omega_n^2 + \vec{k}^2 + M_\pi^2,$$

The gap equations:

(1) $$M_\sigma^2 = m^2 + \frac{\lambda}{2}\phi^2 + \frac{\lambda}{2}F(M_\sigma) + \frac{\lambda}{2}F(M_\pi),$$

(2) $$M_\pi^2 = m^2 + \frac{\lambda}{6}\phi^2 + \frac{\lambda}{6}F(M_\sigma) + \frac{5\lambda}{6}F(M_\pi).$$

$$F(M) = \int_\beta \frac{1}{\omega_n^2 + \vec{k}^2 + M^2}.$$

By minimizing the potential with respect to the expectation value for the sigma fieldφ,we obtain one more the stationarityequation:

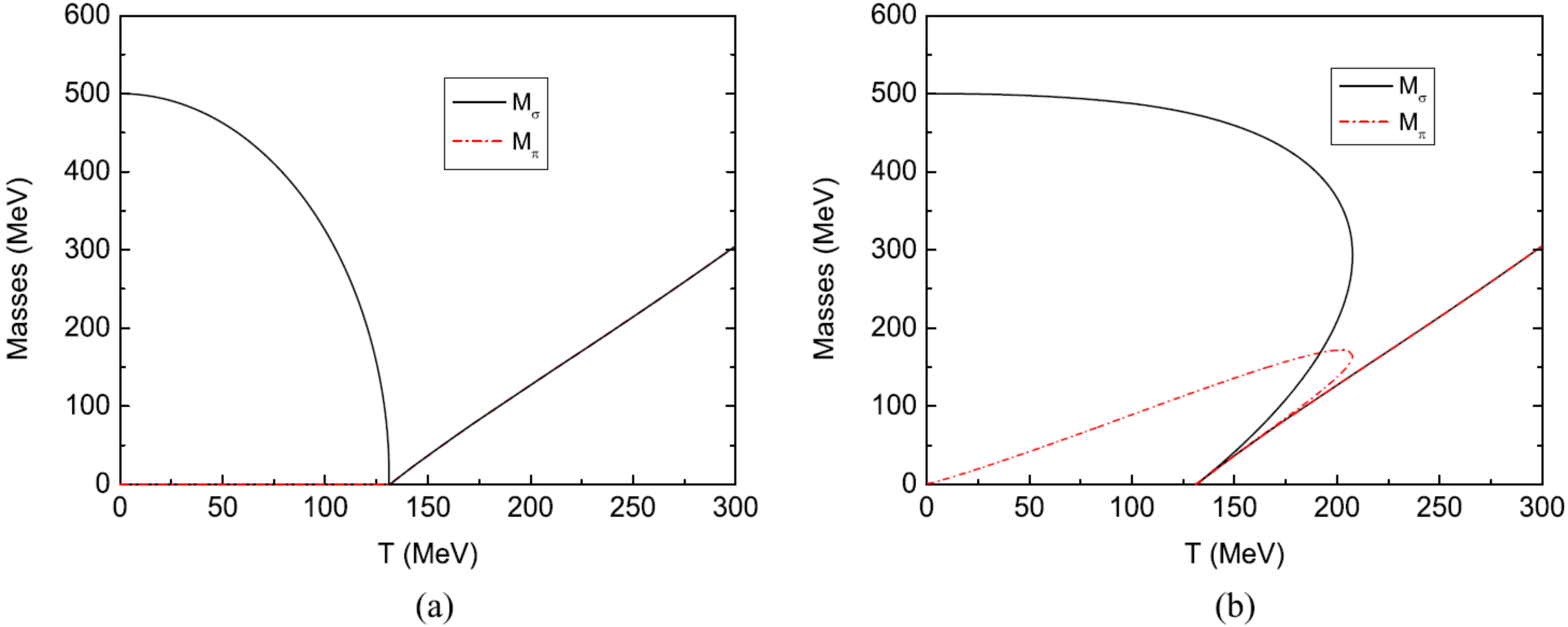
(3) $$m^2 + \frac{\lambda}{6}\phi^2 + \frac{\lambda}{2}F(M_\sigma) + \frac{\lambda}{2}F(M_\pi) = 0.$$

For **the SCJT formulism**, the above equation (3) has been replaced by the constraint:

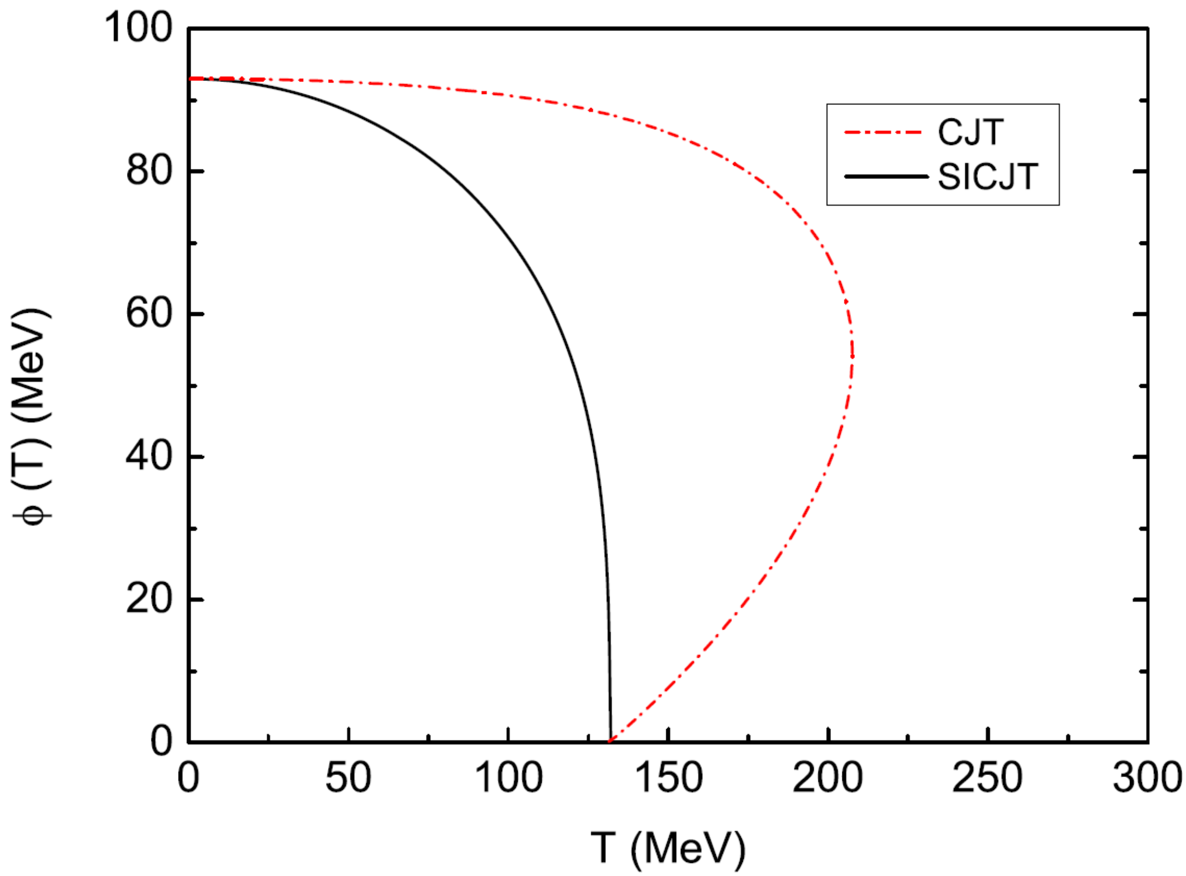
(3’) $$\phi M_\pi^2 = 0$$

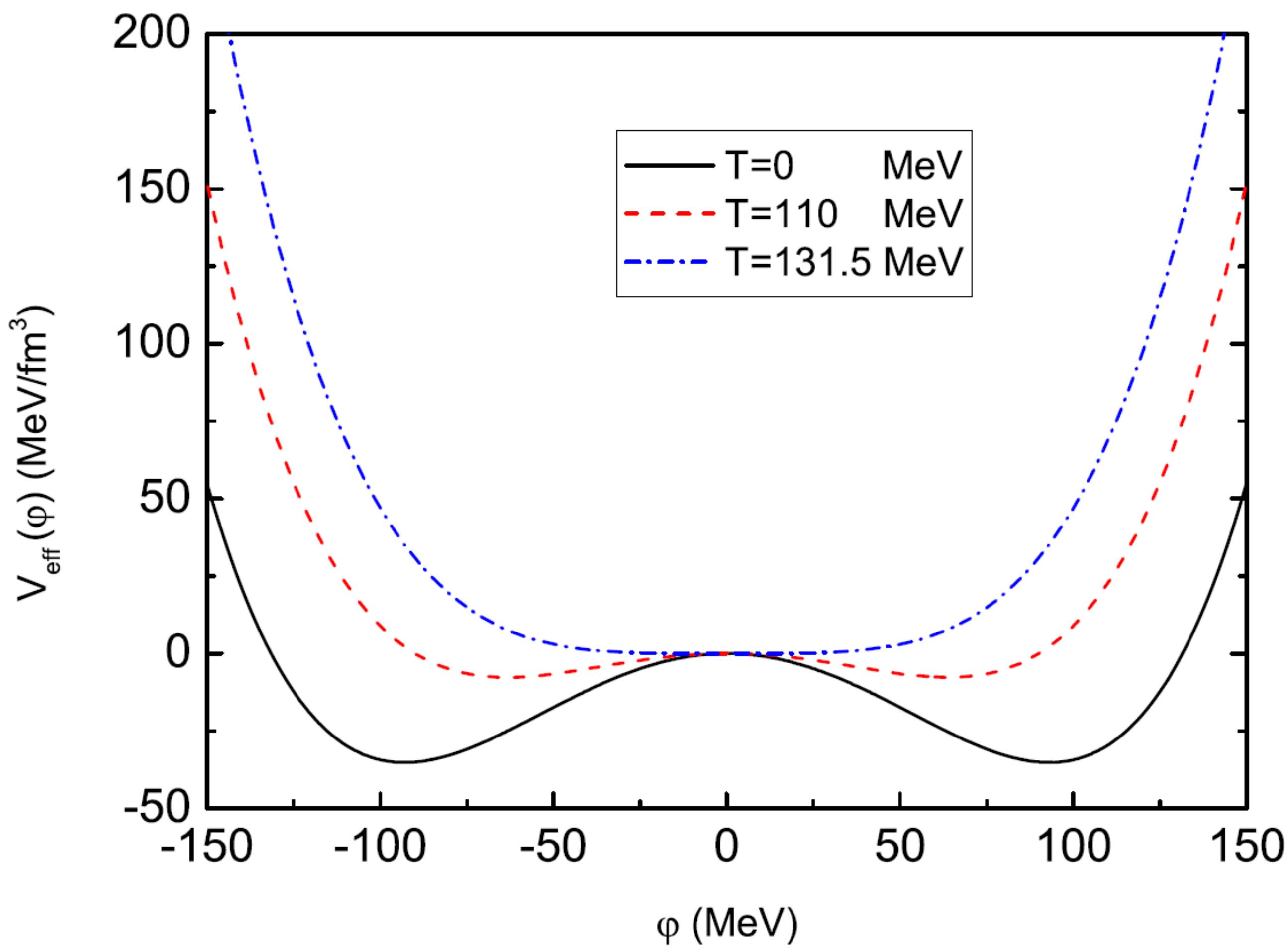
Apostolos Pilaftsis and Daniele Teresi, *Symmetry Improved CJT Effective Action*, Nucl.Phys. B874 (2013) 594-619

The numerical results:



Solution of the gap equations in the case of chiral limit. (a) is for SCJT formalism. (b) is for the traditional CJT formalism.


100
80
60
40
20
0
0
50
100
150
200
250
300
T (MeV)
φ (T) (MeV)
CJT
SICJT



The Symmetry improved CJT effective potential

2.3 Comparison with large-N approximation

The O(N) model

$$\mathcal{L}_N = \frac{1}{2}(\partial_\mu \mathbf{\Phi})^2 - \frac{1}{2}m^2\mathbf{\Phi}^2 - \frac{1}{6N}\lambda\mathbf{\Phi}^4$$

The CJT effective potential

$$V(\phi, G_\sigma, G_\pi) = \frac{1}{2}m^2\phi^2 + \frac{1}{6N}\lambda\phi^4 \qquad + \frac{1}{2}\int_\beta \ln G_\sigma^{-1}(\phi;k) + \frac{N-1}{2}\int_\beta \ln G_\pi^{-1}(\phi;k)$$
$$+ \frac{1}{2}\int_\beta [D_\sigma^{-1}(\phi;k)G_\sigma(\phi;k) - 1] + \frac{N-1}{2}\int_\beta [D_\pi^{-1}(\phi;k)G_\pi(\phi;k) - 1]$$
$$+ V_2(\phi, G_\sigma, G_\pi),$$

$$V_2(\phi, G_\sigma, G_\pi) = 3\frac{\lambda}{6N}\left[\int_\beta G_\sigma(\phi;k)\right]^2 + 2(N-1)\frac{\lambda}{6N}\left[\int_\beta G_\sigma(\phi;k)\right]\left[\int_\beta G_\pi(\phi;k)\right]$$
$$+ (N^2-1)\frac{\lambda}{6N}\left[\int_\beta G_\pi(\phi;k)\right]^2.$$

The gap equations in large-N approximation

$$M_\sigma^2 = m^2 + \frac{2\lambda}{N}\phi^2 + \frac{2\lambda}{3}F(M_\pi),$$
$$M_\pi^2 = m^2 + \frac{2\lambda}{3N}\phi^2 + \frac{2\lambda}{3}F(M_\pi),$$
$$0 = \left(m^2 + \frac{2\lambda}{3N}\phi^2 + \frac{2\lambda}{3}F(M_\pi)\right)\phi.$$

in chiral limit →

(3') $\phi M_\pi^2 = 0$

The differences and advantages of the SCJT formalism

(1) the constraint (3′) can be automatically realized in the case of large-N approximation in the O(N) linear sigma model, whereas, it should be artificially introduced in the framework of the SCJT formalism.

(2) The sigma contribution is reserved in the case of the SCJT formalism, however, it is ignored in the case of large-N approximation.

SCJT

$$M_\sigma^2 = m^2 + \frac{\lambda}{2}\phi^2 + \frac{\lambda}{2}F(M_\sigma) + \frac{\lambda}{2}F(0),$$

$$0 = m^2 + \frac{\lambda}{6}\phi^2 + \frac{\lambda}{6}F(M_\sigma) + \frac{5\lambda}{6}F(0).$$

Large N

$$M_\sigma^2 = m^2 + \frac{2\lambda}{N}\phi^2 + \frac{2\lambda}{3}F(M_\pi),$$

$$M_\pi^2 = m^2 + \frac{2\lambda}{3N}\phi^2 + \frac{2\lambda}{3}F(M_\pi),$$

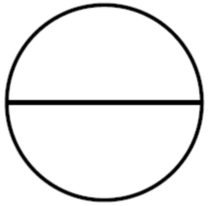
(3) In the large-N expansion, the Goldstone theorem is satisfied only at the leading order, unless the external propagator is used, the Goldstone bosons become massive within quantum loops, while the Goldstone theorem holds to an arbitrary level of truncation in the SCJT formalism.

III. Summary

- We have discussed the effective masses of the mesons and the effective potential at finite temperature in the framework of the O(4) linear sigma model in chiral limit by adopting the symmetry improved CJT formalism.
- The symmetry improved CJT formalism naturally cures the two major problems of the naive CJT method, namely, the breakdown of the Goldstone theorem in symmetry-broken phase and the existence of first-order phase transition.

Future studies

(1) Any attempts to go beyond the usual Hartree approximation are worthy to do in order to identify their effects in the novel CJT formalism, e.g. the sunset type diagrams.



(2) The O(4) linear sigma model with two quarks could be combined with the Polyakov loop which allows to investigate both the chiral and the deconfinement phase transition.

Thanks!

RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 57, NUMBER 1 1 JANUARY 1998

Soft modes associated with chiral transition at finite temperature

S. Chiku and T. Hatsuda
Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan
(Received 23 June 1997; published 26 November 1997)

Using a novel resummation procedure of thermal loops, real-time correlations in the scalar and pseudoscalar channels are studied in the O(4) linear σ model at finite temperature. A threshold enhancement of the spectral function in the scalar channel is shown to be a noticeable precritical phenomenon of the chiral phase transition. [S0556-2821(98)50101-4]

PACS number(s): 12.38.Mh, 11.10.Wx, 12.38.Lg

RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 29, NUMBER 2 15 JANUARY 1984

Remarks on the chiral phase transition in chromodynamics

Robert D. Pisarski and Frank Wilczek
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 27 October 1983)

The phase transition restoring chiral symmetry at finite temperatures is considered in a linear σ model. For three or more massless flavors, the perturbative ϵ expansion predicts the phase transition is of first order. At high temperatures, the $U_A(1)$ symmetry will also be effectively restored.

$$S[\phi] = \int d^4x \mathcal{L}\{\phi(x)\}$$

$$Z[j] = \langle 0_{\rm out} \mid 0_{\rm in}\rangle_j \equiv \int d\phi \exp\{i(S[\phi] + \phi j)\}$$

the connected generating functional W[j] defined as,

$$Z[j] \equiv \exp\{iW[j]\}$$

the effective action Γ

$$\Gamma[\overline{\phi}] = W[j] - \int d^4x \frac{\delta W[j]}{\delta j(x)} j(x) \qquad \overline{\phi}(x) = \frac{\delta W[j]}{\delta j(x)}$$

$$\left.\frac{\delta \Gamma[\overline{\phi}]}{\delta \overline{\phi}}\right|_{j=0} = 0$$

$$Z[j] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \ldots d^4x_n j(x_1) \ldots j(x_n) G_{(n)}(x_1, \ldots, x_n)$$

$$iW[j] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \ldots d^4x_n j(x_1) \ldots j(x_n) G^c_{(n)}(x_1, \ldots, x_n)$$

the effective action can be expanded

$$\Gamma[\overline{\phi}] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \ldots d^4x_n \overline{\phi}(x_1) \ldots \overline{\phi}(x_n) \Gamma^{(n)}(x_1, \ldots, x_n)$$

Γ are the one-particle irreducible (1PI) Green functions.

In a translationally invariant theory,

$$\overline{\phi}(x) = \phi_c$$

define the effective potential

$$\Gamma[\phi_c] = -\int d^4x V_{\text{eff}}(\phi_c)$$