QCD Corrections to B→ π Form Factors From Light-Cone Sum Rules

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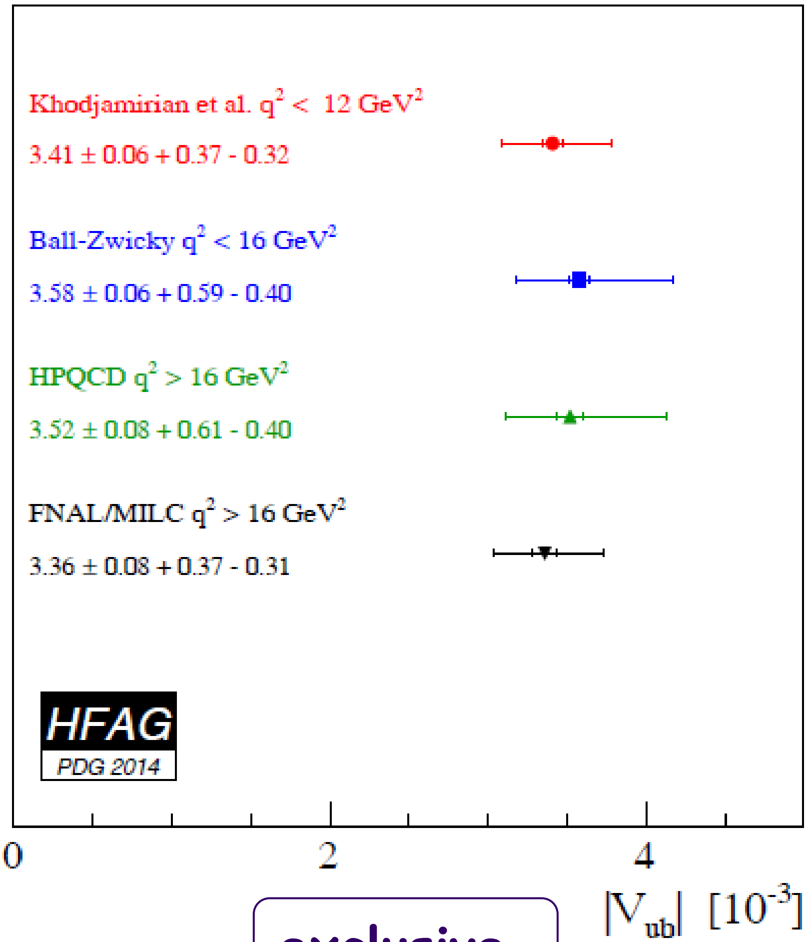
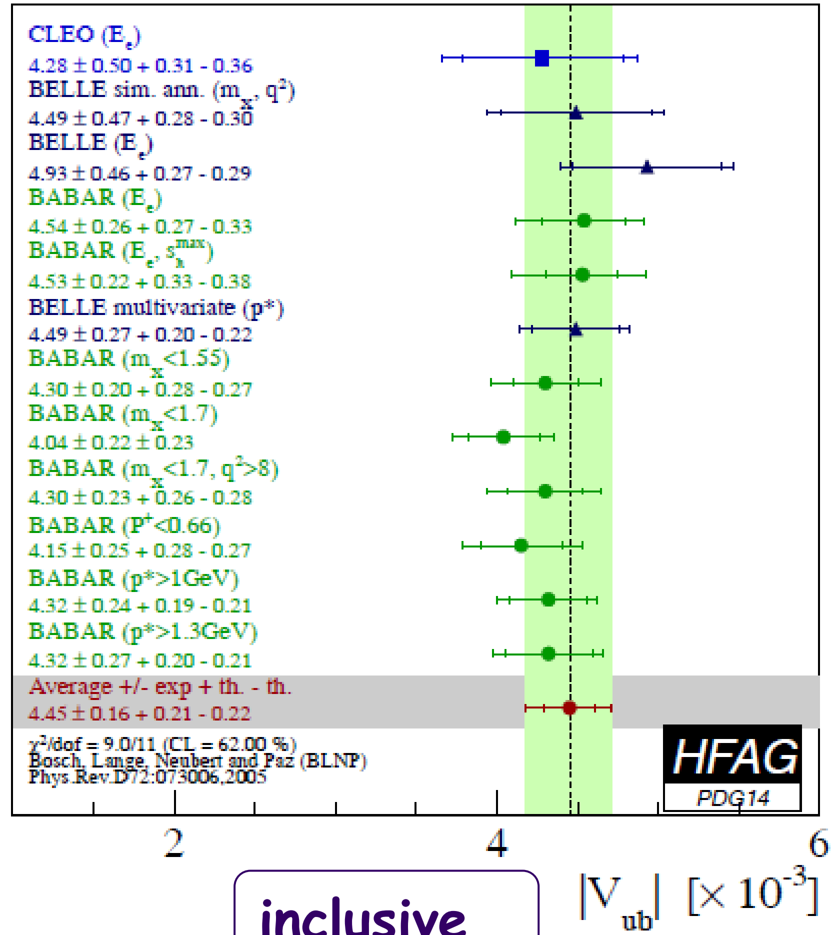
Vub Puzzle



exclusive



inclusive

For a recent review , see Int.J.Mod.Phys. A29 (2014) 1430040 by Wei Wang

Determination of $|V_{ub}|$ in exclusive processes

- Semi-leptonic and Leptonic decays:

$$B \to (\pi, \rho) l\nu,\ \lambda_b \to p l\nu,\ B \to \pi\pi l\nu, B \to \tau\nu_\tau,\ \text{etc.}$$

- $B \to \pi l \nu$ is the most reliable channel

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3 q^4 m_B^2} (q^2 - m_l^2)^2 |\vec{p}_\pi| \times \left[\left(1 + \frac{m_l^2}{q^2} \right) m_B^2 |\vec{p}_\pi|^2 |f_{B\pi}^+(q^2)|^2 + \frac{3 m_l^2}{8q^2} (m_B^2 - m_\pi^2)^2 |f_{B\pi}^0(q^2)|^2 \right]$$

- $B \to \pi$ form factors: Lattice, LCSR, PQCD

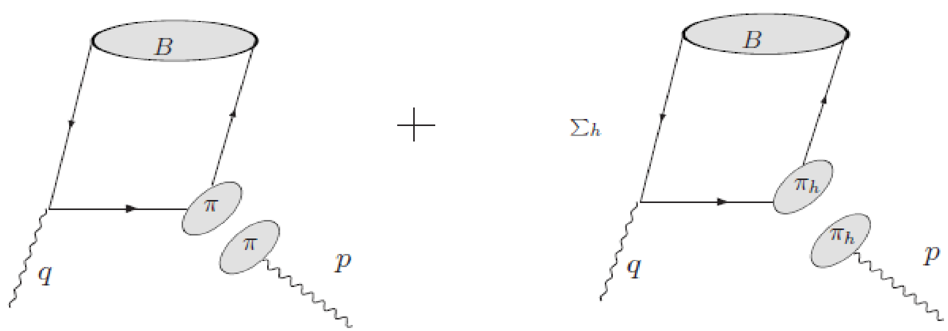
B meson LCSR at leading order

- The correlation function(Khodjamirian, Mannel ,Offen,2005; De Fazio, Feldman, Hurth 2005)

$$\begin{aligned}
\Pi_\mu(p,q) &= \int d^4x\, e^{ip\cdot x} \langle 0|T\left\{\bar{d}(x)\not{n}\gamma_5 u(x), \bar{u}(0)\gamma_\mu b(0)\right\}|\bar{B}(p+q)\rangle \\
&= \Pi(n\cdot p, \bar{n}\cdot p)n_\mu + \widetilde{\Pi}(n\cdot p, \bar{n}\cdot p)\bar{n}_\mu\,, \\
n\cdot p &= \frac{m_B^2+m_\pi^2-q^2}{m_B}, \qquad \bar{n}\cdot p \sim O(\Lambda_{\rm QCD})\,, \qquad p+q \equiv m_B v = \frac{m_B}{2}(n+\bar{n})\,.
\end{aligned}$$

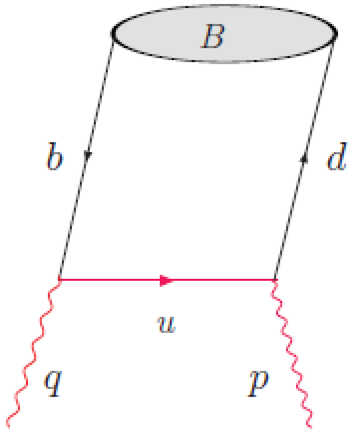
- Inserting complete set of Pion states

$\widetilde{\Pi}(n\cdot p, \bar{n}\cdot p) =$



$$\underbrace{\frac{f_\pi\,(n\cdot p)\,m_B}{2(m_\pi^2-p^2)}\left[\frac{n\cdot p}{m_B} f_{B\pi}^+(n\cdot p) + f_{B\pi}^0(n\cdot p)\right]}_{\text{relative sign changes for } \Pi(n\cdot p,\bar{n}\cdot p)} \qquad \int_{\omega_s}^{+\infty} d\omega' \,\frac{\widetilde{\rho}^h(n\cdot p,\omega')}{\omega'-\bar{n}\cdot p}$$

- OPE calculation of the correlation function



$$\widetilde{\Pi}(n\cdot p,\bar{n}\cdot p) = \tilde{f}_B m_B \int_0^{+\infty} d\omega' \frac{\phi_B^-(\omega')}{\omega' - \bar{n}\cdot p} + O(\alpha_s)$$

$$\Pi(n\cdot p,\bar{n}\cdot p) = O(\alpha_s)\,,$$

$$\Rightarrow f^0_{B\pi}(n\cdot p) = \frac{n\cdot p}{m_B} f^+_{B\pi}(n\cdot p) + O(\alpha_s)\,.$$

- Borel improved LCSRs

$$f^+_{B\pi}(q^2) = \frac{\tilde{f}_B(\mu)\, m_B}{f_\pi\, n\cdot p} \exp\left[\frac{m_\pi^2}{n\cdot p\, \omega_M}\right] \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \phi_B^-(\omega') + \mathcal{O}(\alpha_s)$$

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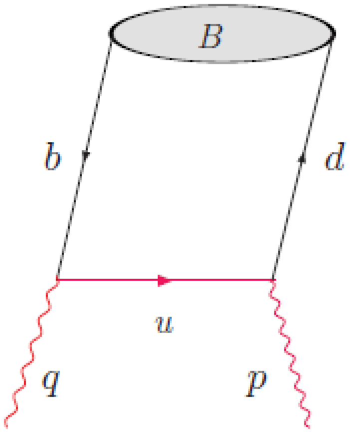

$$\begin{aligned}\widetilde{\Pi}(n\cdot p,\bar{n}\cdot p) &= \tilde{f}_B\, m_B \int_0^{+\infty} d\omega' \frac{\phi_B^-(\omega')}{\omega' - \bar{n}\cdot p} + O(\alpha_s)\,, \\ \Pi(n\cdot p,\bar{n}\cdot p) &= O(\alpha_s)\,, \\ \Rightarrow f^0_{B\pi}(n\cdot p) &= \frac{n\cdot p}{m_B} f^+_{B\pi}(n\cdot p) + O(\alpha_s)\,.\end{aligned}$$

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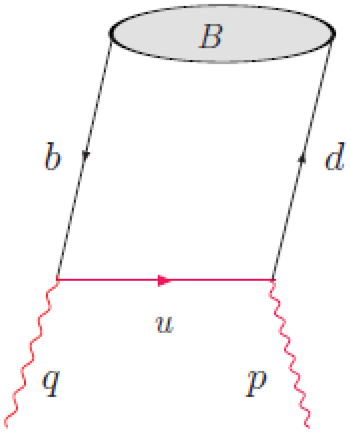
symmetry breaking

- OPE calculation of the correlation function



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$$\Pi(n\cdot p,\bar{n}\cdot p) = O(\alpha_s),$$

$$\Rightarrow f_{B\pi}^0(n\cdot p) = \frac{n\cdot p}{m_B} f_{B\pi}^+(n\cdot p) + O(\alpha_s).$$

- Borel improved LCSRs

$$f_{B\pi}^+(q^2) = \frac{\tilde{f}_B(\mu)\, m_B}{f_\pi\, n\cdot p} \exp\left[\frac{m_\pi^2}{n\cdot p\ \omega_M}\right] \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \phi_B^-(\omega') + \mathcal{O}(\alpha_s)$$

$$f_{B\pi}^0(q^2) = \frac{n\cdot p}{m_B} f_{B\pi}^+(q^2) + \mathcal{O}(\alpha_s).$$

symmetry breaking

- The convolution integrals involves $\phi_{b\bar{d}}^+(\omega)$ must be infrared finite
- The one loop contributions to $\Pi(n\cdot p,\bar{n}\cdot p)$ must be infrared finite

QCD corrections to the correlation functions

$$\Pi_\mu = \Pi_\mu^{(0)} + \Pi_\mu^{(1)} + \ldots = \Phi_B \otimes T$$
$$= \Phi_B^{(0)} \otimes T^{(0)} + \left[\Phi_B^{(0)} \otimes T^{(1)} + \Phi_B^{(1)} \otimes T^{(0)}\right] + \ldots .$$
$$\Downarrow$$
$$\Phi_B^{(0)} \otimes T^{(1)} = \Pi_\mu^{(1)} - \Phi_B^{(1)} \otimes T^{(0)} .$$

QCD corrections to the correlation functions

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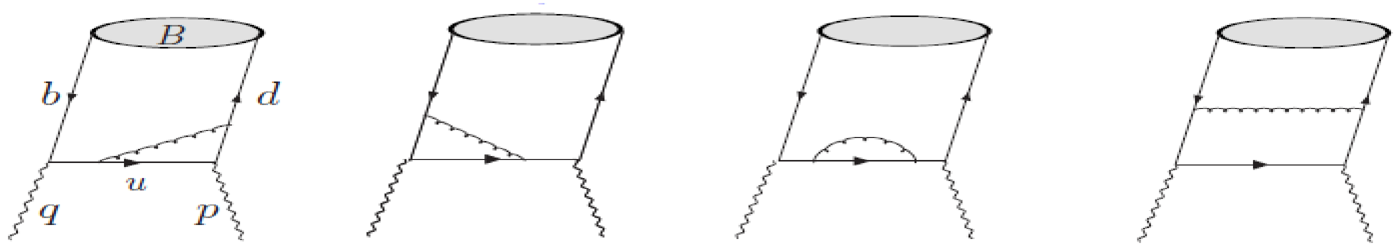
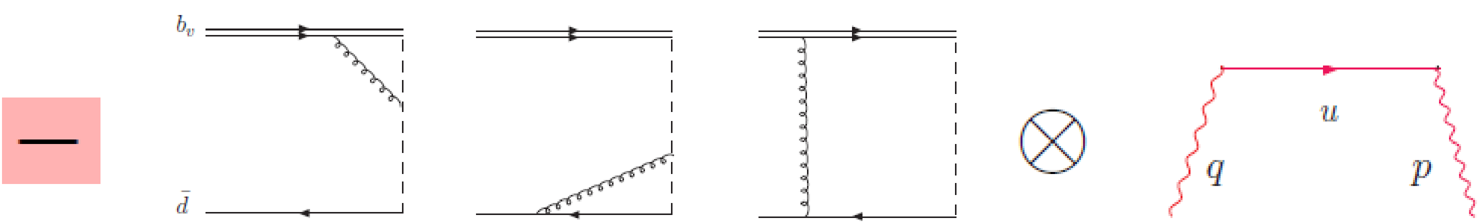
diagrammatical factorization





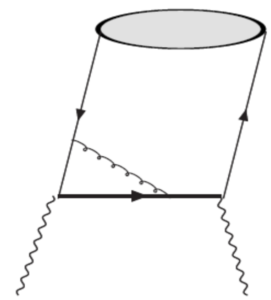
The correlation function at NLO

Strategy of the calculation: expansion by region

the leading region includes: Hard,Hard-collinear and soft region

- We confirmed the soft cancellation:

$$\Pi^{(1),\,s}_{\mu,\,weak} = \Phi^{(1)}_{b\bar{d},\,a} \otimes T^{(0)}$$



- The hard and hard-collinear contributions are absorbed to hard function and jet function respectively

$$\Pi \;=\; \tilde{f}_B(\mu)\, m_B \sum_{k=\pm} C^{(k)}(n\cdot p,\mu) \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, J^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega,\mu)$$

$$\widetilde{\Pi} \;=\; \tilde{f}_B(\mu)\, m_B \sum_{k=\pm} \widetilde{C}^{(k)}(n\cdot p,\mu) \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, \widetilde{J}^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega,\mu)$$

Hard function:
In agreement with the matching coefficient of QCD weak current to SCET I operator(Bauer et al, 2001)

Jet function:
In agreement with that computed in SCET sum rules, De Fazio et al 2005)

Cancellation of scale dependence

$$\frac{d}{d\ln\mu}\tilde{C}^{(-)}(n\cdot p,\mu) = -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{n\cdot p}+5\right]\tilde{C}^{(-)}(n\cdot p,\mu),$$

$$\frac{d}{d\ln\mu}\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) = \frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu^2}{n\cdot p\,\omega}\right]\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) + \frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma(\omega,\omega',\mu)\,\tilde{J}^{(-)}(\bar{n}\cdot p,\omega',\mu),$$

$$\frac{d}{d\ln\mu}\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right] = -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{\omega}-5\right]\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right] - \frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma(\omega,\omega',\mu)\,\left[\tilde{f}_B\,\phi_B^-(\omega',\mu)\right],$$

Up to NNLL, we need 3-loop cusp anomalous dimension, and two-loop anomalous dimension for $\tilde{f}_B(\mu)$

$$\Pi = m_B\left[U_2(\mu_{h2},\mu)\,\tilde{f}_B(\mu_{h2})\right]\int_0^\infty\frac{d\omega}{\omega-\bar{n}\cdot p}\,J^{(+)}\left(\frac{\mu^2}{n\cdot p\,\omega},\frac{\omega}{\bar{n}\cdot p}\right)\phi_B^{(+)}(\omega,\mu) + m_B\left[U_2(\mu_{h2},\mu)\,\tilde{f}_B(\mu_{h2})\right]C^{(-)}(n\cdot p,\mu)\int_0^\infty\frac{d\omega}{\omega-\bar{n}\cdot p}\,\phi_B^{(-)}(\omega,\mu),$$

$$\tilde{\Pi} = m_B\left[U_2(\mu_{h2},\mu)\,\tilde{f}_B(\mu_{h2})\right]\int_0^\infty\frac{d\omega}{\omega-\bar{n}\cdot p}\,\tilde{J}^{(+)}\left(\frac{\mu^2}{n\cdot p\,\omega},\frac{\omega}{\bar{n}\cdot p}\right)\phi_B^{(+)}(\omega,\mu) + m_B\left[U_1(n\cdot p,\mu_{h1},\mu)\,U_2(\mu_{h2},\mu)\right]\left[\tilde{f}_B(\mu_{h2})\,\tilde{C}^{(-)}(n\cdot p,\mu_{h1})\right] \times\int_0^\infty\frac{d\omega}{\omega-\bar{n}\cdot p}\,\tilde{J}^{(-)}\left(\frac{\mu^2}{n\cdot p\,\omega},\frac{\omega}{\bar{n}\cdot p}\right)\phi_B^{(-)}(\omega,\mu),$$

B→Pion form factors at NLO

$$f_\pi \, e^{-m_\pi^2/(n\cdot p\,\omega_M)} \left\{ \frac{n\cdot p}{m_B} f_{B\pi}^{+}(n\cdot p) \, , \, f_{B\pi}^{0}(n\cdot p) \right\}$$

$$= \tilde{f}_B(\mu) \int_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \Big[r\, \widetilde{C}^{(+)}(n\cdot p,\mu)\, \phi_{B,\mathrm{eff}}^{+}(\omega',\mu) + \widetilde{C}^{(-)}(n\cdot p,\mu)\, \phi_{B,\mathrm{eff}}^{-}(\omega',\mu)$$

$$\pm \frac{n\cdot p - m_B}{m_B} \left(C^{(+)}(n\cdot p,\mu)\, \phi_{B,\mathrm{eff}}^{+}(\omega',\mu) + C^{(-)}(n\cdot p,\mu)\, \phi_{B}^{-}(\omega',\mu) \right) \Big] \, .$$

Symmetry breaking effect

The effective wave functions

$$\phi_{B,\mathrm{eff}}^{+}(\omega',\mu) = 0 + \frac{\alpha_s C_F}{4\pi} \int_{\omega'}^{\infty} \frac{d\omega}{\omega} \, \phi_B^{+}(\omega,\mu) \, ,$$

$$\phi_{B,\mathrm{eff}}^{-}(\omega',\mu) = \phi_B^{-}(\omega',\mu) + \frac{\alpha_s C_F}{4\pi} \left\{ \int_0^{\omega'} d\omega \left[\frac{1}{\omega-\omega'} \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} - 4\ln\frac{\omega'-\omega}{\omega'} \right) \right]_{+} \phi_B^{-}(\omega,\mu) \right.$$

$$\left. - \int_{\omega'}^{\infty} d\omega \left[\ln^2\frac{\mu^2}{n\cdot p\,\omega} - \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} + 3 \right) \ln\frac{\omega-\omega'}{\omega'} + 2\ln\frac{\omega}{\omega'} + \frac{\pi^2}{6} - 1 \right] \frac{d\phi_B^{-}(\omega,\mu)}{d\omega} \right\} .$$

End point singularity

A short discussion on 3-particle B meson DAs

- The 3-particle B meson DAs can contribution at leading power, but not in leading order. This contribution is very small in phenomenological study.

- The 3-particle B meson DAs can compensate the factorization scale dependence

- The NLO corrections to the diagrams with 3-particle B meson DAs should be infrared finite.

Comparison with previous work

- For the first time we proved the factorization of the correlation function, and calculated both the hard and jet functions
- We firstly give the right evolution behavior on the hard and jet function

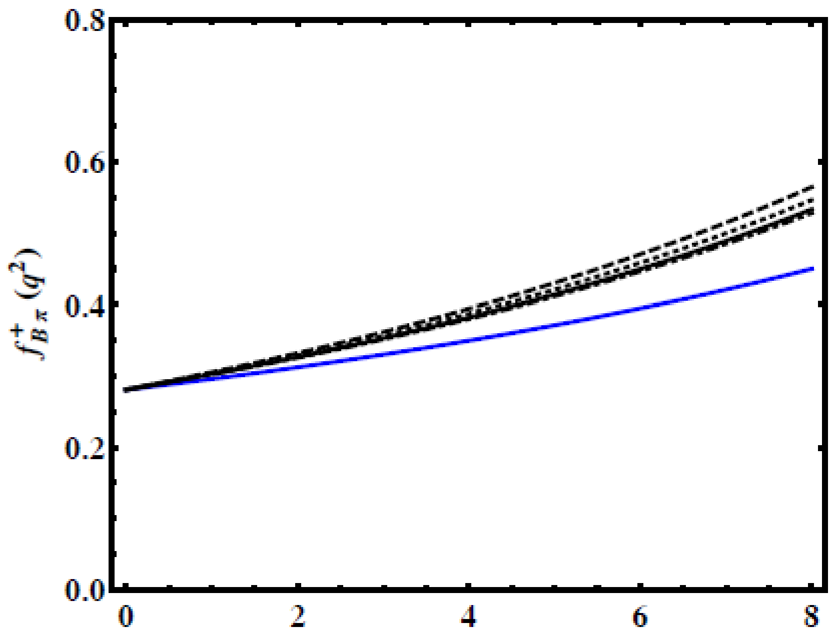
The B meson LCDA

$$\phi^+_{B,\mathrm{I}}(\omega,\mu_0) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0}\,, \qquad \text{[Grozin and Neubert, 1997]}$$

$$\phi^+_{B,\mathrm{II}}(\omega,\mu_0) = \frac{1}{4\pi\,\omega_0}\,\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2(\sigma_B-1)}{\pi^2}\ln k\right], \; k = \frac{\omega}{1\,\mathrm{GeV}}\,, \qquad \text{[Braun et al, 2004]}$$

$$\phi^+_{B,\mathrm{III}}(\omega,\mu_0) = \frac{2\omega^2}{\omega_0\omega_1^2}\, e^{-(\omega/\omega_1)^2}\,, \; \omega_1 = \frac{2\omega_0}{2\sqrt{\pi}}\,, \qquad \text{[De Fazio, Feldmann, Hurth, 2008]}$$

$$\phi^+_{B,\mathrm{IV}}(\omega,\mu_0) = \frac{\omega}{\omega_0\omega_2}\,\frac{\omega_2-\omega}{\sqrt{\omega(2\omega_2-\omega)}}\,, \; \omega_2 = \frac{4\omega_0}{4-\pi}\,, \qquad \text{[De Fazio, Feldmann, Hurth, 2008]}$$



fitting $f^+_{B\pi}(q^2=0) = 0.28 \pm 0.03$ from pion LCSR $\Rightarrow$

Model-I: $\omega_0 = 360^{+40}_{-30}$ MeV ,

Model-II: $\omega_0 = 375^{+40}_{-35}$ MeV ,

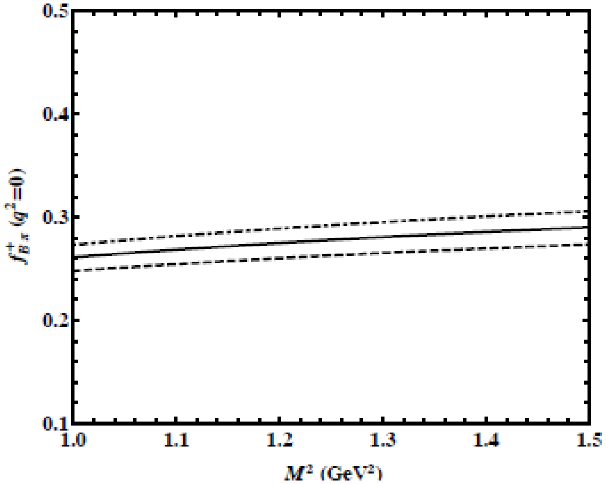
Model-III: $\omega_0 = 395^{+35}_{-30}$ MeV ,

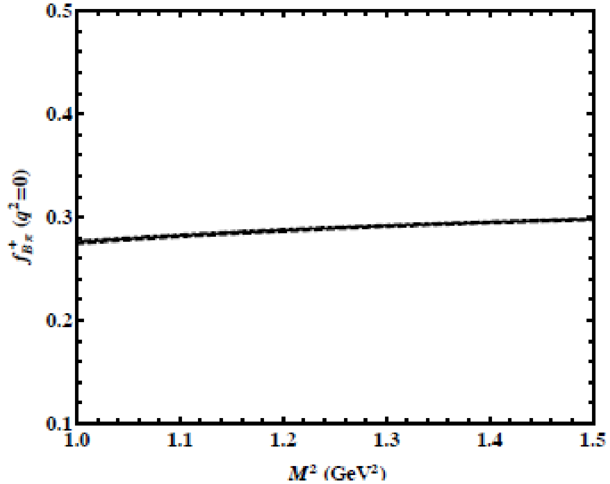
Model-IV: $\omega_0 = 310^{+40}_{-30}$ MeV .

blue curve from pion LCSR, solid, dotted, dashed and dot-dashed curves from Model-I, II, III and IV.

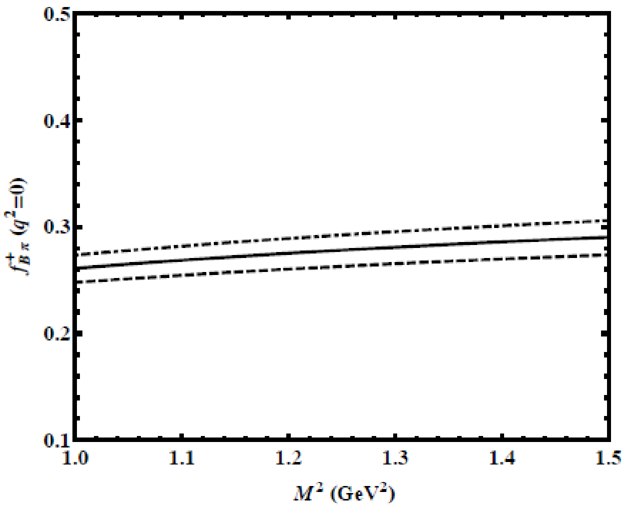
Dependence on Borel parameter

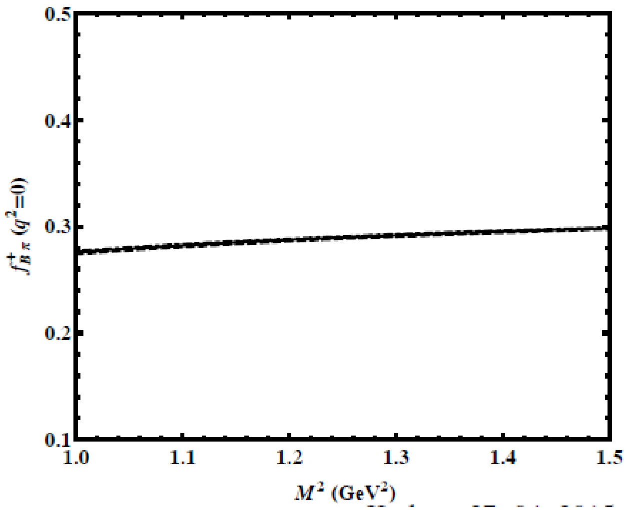
Dependence on threshold parameter





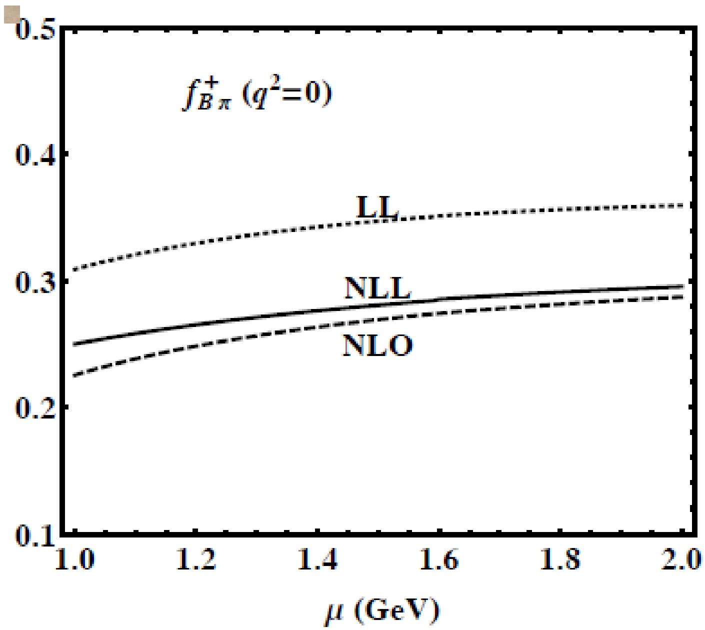
Pion decay constant from experiment

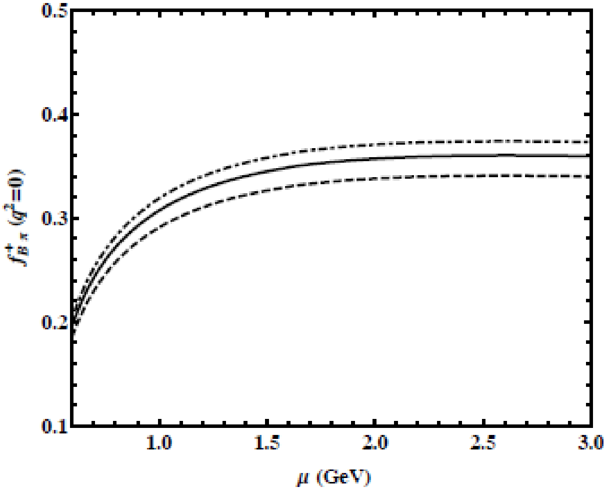


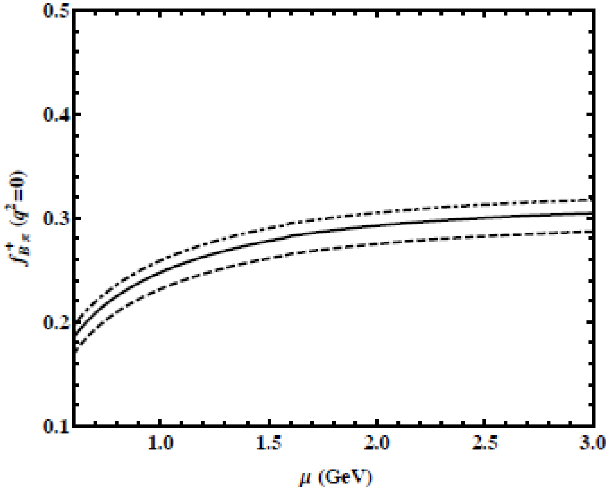


Pion decay constant from QCD sum rules

Dependence on factorization scale

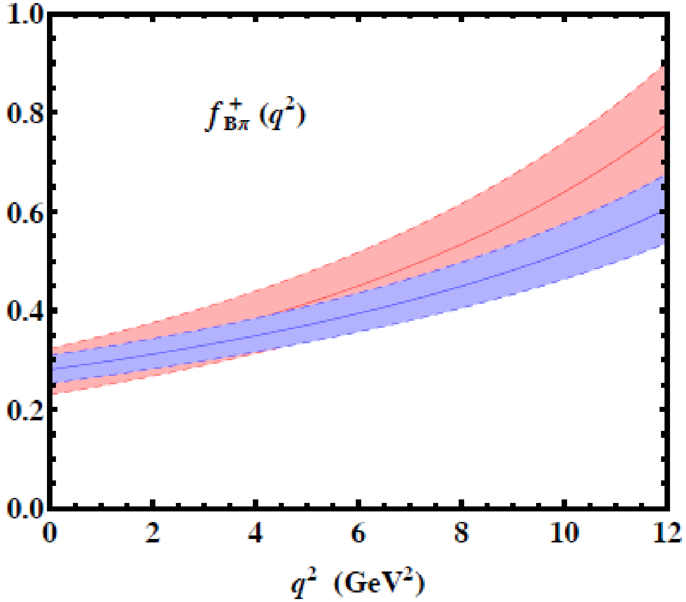


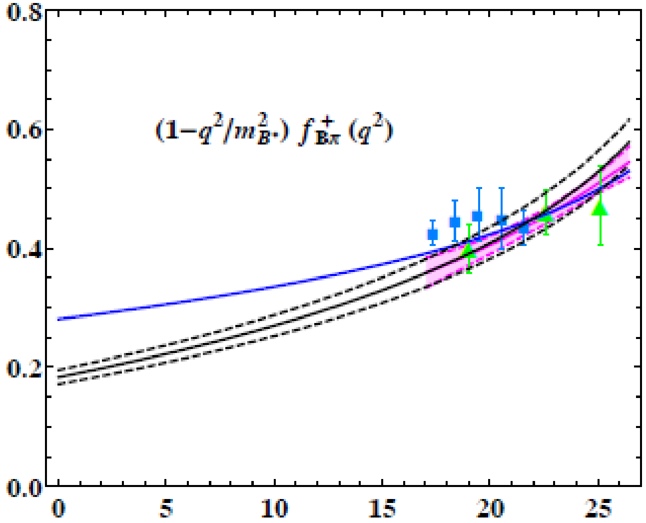




q^2 dependence of the form factors

fitting $f^{+}_{B\pi}(q^2 = 17.34\,\text{GeV}^2)$ from MILC 2015 data $\Rightarrow$ Model-I: $\omega_0 = 530^{+30}_{-30}$ MeV.



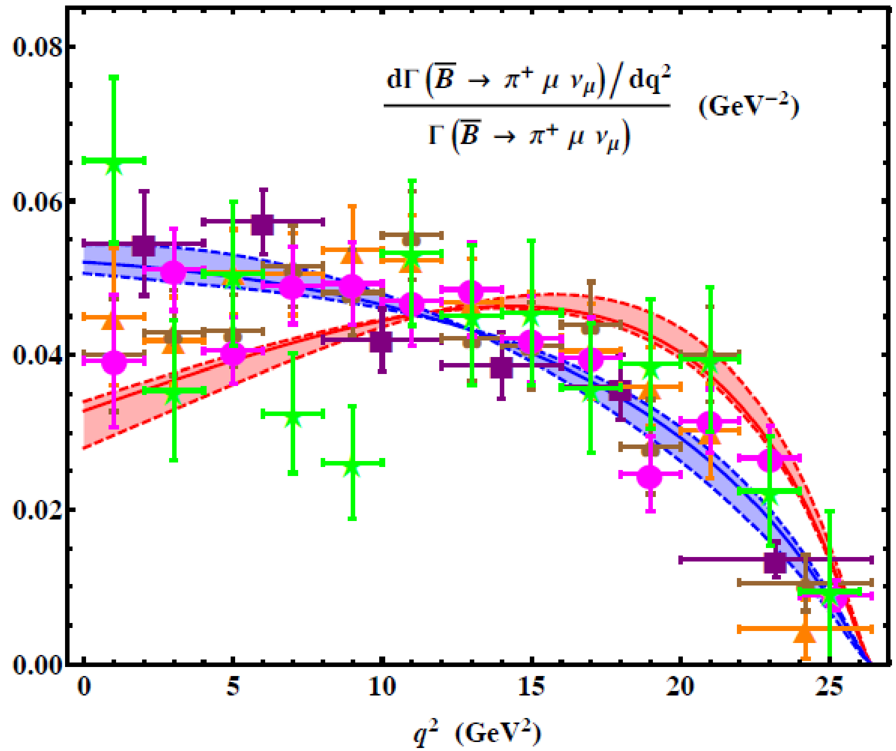


Pink band: B-meson LCSR @ NLO,
Blue band: pion LCSR @ NLO.

black: B-meson LCSR @ NLO,
blue: pion LCSR @ NLO,
box: HPQCD 2006,
triangle: RBC/UKQCD 2015,
magenta band: MILC 2015.

Normalized differetial q^2 distribution

$$\frac{d\Gamma}{dq^2}(B \to \pi \mu \nu_\mu) = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |\vec{p}_\pi|^3 \ |f_{B\pi}^+(q^2)|$$



Integrated quantity

$$\Delta\zeta(0, q_0^2) = \frac{G_F^2}{24\pi^3} \int_0^{q_0^2} dq^2 \ |\vec{p}_\pi|^3 \ |f_{B\pi}^+(q^2)|^2$$

$$\Delta\zeta(0, 12\,\mathrm{GeV}^2) = 5.89 \ {}^{+1.12}_{-1.10}\Big|_{\omega 0} \ {}^{+0.30}_{-0.29}\Big|_{\sigma_B^{(1)}} \ {}^{+0.60}_{-1.22}\Big|_{\mu} \ {}^{+0.21}_{-0.21}\Big|_{\mu_{h1(2)}} \ {}^{+0.34}_{-0.53}\Big|_{M,s0} \ {}^{+0.52}_{-0.25}\Big|_{\overline{M},\bar{s}0} \ \mathrm{ps}^{-1}$$

Determined Vub

$$|V_{ub}| = \left(3.05^{+0.54}_{-0.38}|_{\mathrm{th.}} \pm 0.09|_{\mathrm{exp.}}\right) \times 10^{-3}$$

Summary and outlook

- We calculated the NLO corrections to the B→Pion form factors from B meson LCSRs, the key point is factorization of the correlation function;
- $\lambda_B(\omega_0)$ is an important input parameter but hard to determine;
- Understanding the subleading correction is important;
- This framework can be extended to various heavy-to-light transition processes .

Thanks