X(3872) Production

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Outline

- Experimental information on X(3872)
- Production of X(3872)
  - As $D^0\overline{D}^{*0}$ molecule state
  - As χ_{c1}'
  - As a mixing state of χ_{c1}' and $D^0\overline{D}^{*0} + c.c.$
- X(3872) production in B decays --- revisited
- Summary & Perspectives

Experimental information

- 1st observed by Belle Collaboration in

  $B \to J/\psi\pi^+\pi^- K$ Belle'03

- Mass, width and quantum numbers:

- $m_X = 3871.69 \pm 0.17$ MeV PDG'14

  $|m_X - m_{D^0 D^{*0}}| < 0.4\text{MeV} \quad \text{CL} = 90\%$ Tomaradze *et al.*'12

- $\Gamma < 1.2$ MeV $\quad \text{CL} = 90\%$ PDG'14

- $J^{PC} = 1^{++}$ LHCb'13

Experimental information

- Decay pattern:

- Observed decay modes:

  $J/\psi\rho,\ J/\psi\omega,\ D^0\bar{D}^{*0}/\bar{D}^0 D^{*0}/D\bar{D}\pi,\ J/\psi\gamma, \psi'\gamma$

  Relative ratios of these 4 modes: $1:1:10:0.3:1$ PDG'14

  Did not seen $\chi'_{c1}(3920-3960)$ in $B\to D\bar{D}^*K$ Belle'10

  ✓ $\mathrm{Br}(X\to J/\psi\rho) = \mathrm{Br}(X\to J/\psi\pi^+\pi^-) \equiv \mathrm{Br}_0 < 8\%$

Experimental information

- B-production:

$$1 \times 10^{-4} < \mathrm{Br}(B \to X(3872)K) < 3.2 \times 10^{-4}$$ BaBar'05

$$\mathrm{Br}(B \to X(3872)K)\mathrm{Br}_0 = (8.6 \pm 0.8) \times 10^{-6}$$ PDG'14

$$2.6\% < \mathrm{Br}_0 \equiv \mathrm{Br}(X \to J/\psi\pi^+\pi^-) < 8\%$$

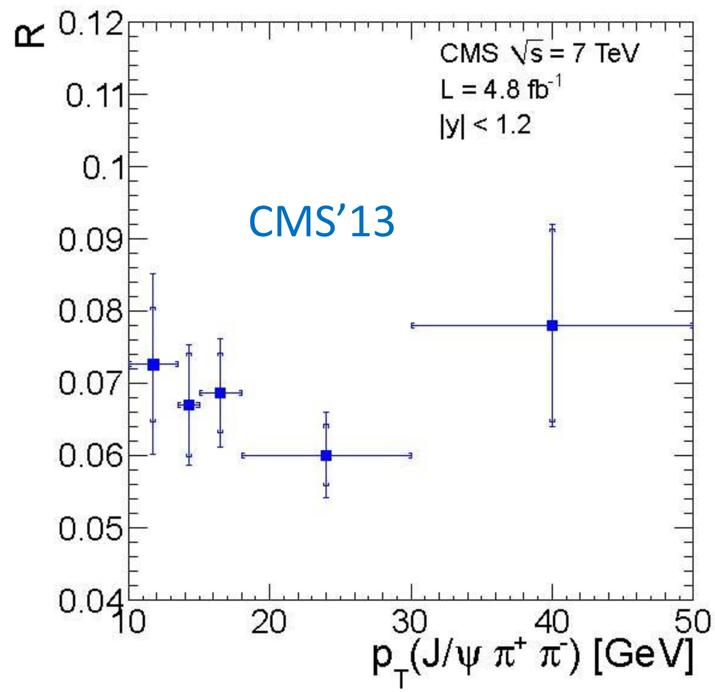
- Hadro-production

- Large production rate:

$$\frac{\sigma(p\bar{p}\to X)\mathrm{Br}_0}{\sigma(p\bar{p}\to\psi\prime)}\frac{\epsilon_{\psi\prime}}{\epsilon_X} = (4.8 \pm 0.8)\%$$ CDF'04

- Similar behaviors to ψ' production

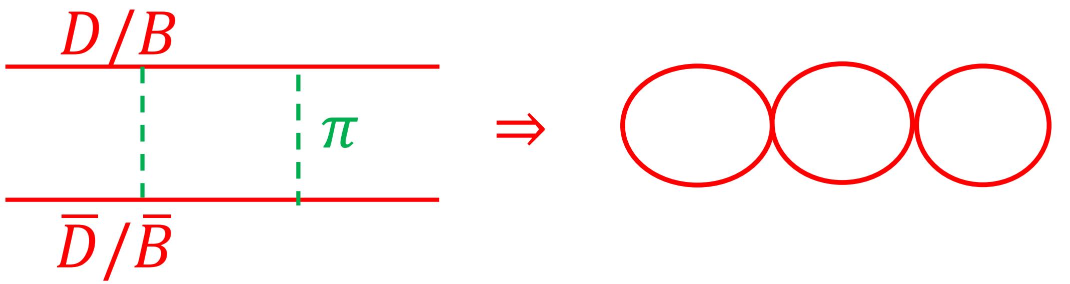
$$R = d\sigma(\psi')/d\sigma(X) \sim P_T$$



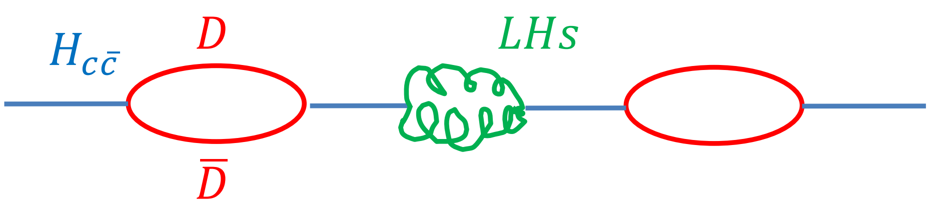
Production of X(3872)

Molecule

- QED molecule: eg. $H_2 \sim pp$
- QCD molecule:
  - Deuteron: $D \sim pn$
  - $Z_b \sim BB^*, \; Z_b' \sim B^*B^*; \; Z_c \sim DD^*, \; Z_c' \sim D^*D^*$



  - $X(3872) \sim DD^*$: with annihilation channel



$\Gamma_{LHs} \sim 1MeV \sim E_b$

X(3872) produced as DD^* molecule

- Naively, $\sigma(X) \sim |R(0)|^2 \sim k_0^3,\ k_0 = \sqrt{2\mu_{DD^*}|E_b|} < 40$ MeV
- Explicit calculations [Bignamini *et al*, PRL'09]:

$$\sigma_{\rm CDF}^{\rm th}(X) < 0.085\,{\rm nb} \quad v.s. \quad \sigma_{\rm CDF}^{\rm ex}(X){\rm Br}_0 = 3.1 \pm 0.7\,{\rm nb}$$

- DD^* scattering with cutoff Λ [Artoisenet & Braaten'10]:

$$\sigma_{\rm CDF}^{\rm th}(X) \sim \left(\frac{\Lambda}{m_\pi}\right)^2 \frac{k_0}{30\ {\rm MeV}} \times 6\ {\rm nb}$$

Similarly, for ${\rm Br}(B \to XK)$ [Braaten, Lu, Kusunoki'05-06]

$$\left(\frac{\Lambda}{m_\pi}\right)^2 \frac{k_0}{30\ {\rm MeV}} \times 0.2 \cdot 10^{-4} \quad v.s. \quad [(1-3)\times 10^{-4}]^{\rm ex}$$

Need $\Lambda = 3m_\pi$!

X(3872) produced as DD^* molecule

- $Z_c(3900)$ production in B decay: [Belle'14]

$$\mathrm{Br}(\bar{B}^0 \to Z_c^+ K^-) \times \mathrm{Br}(Z_c^+ \to J/\psi\pi^+) < 9 \times 10^{-7} \quad @\ 90\%\ \mathrm{C.L.}$$

If $\mathrm{Br}(Z_c^+ \to J/\psi\pi^+) = 0.1$, then

$$\mathrm{Br}(\bar{B}^0 \to Z_c^+ K^-) < 10^{-5}$$

- $\Lambda \le m_\pi$ if $Z_c(3900)$ and $X(3872)$ are similar in production
- The production rates of $X(3872)$ would be smaller than the data by one order or more in magnitude.

$X(3872)$ production as χ'_{c1}

- χ'_{c1} production

✓ Hard production is very similar to that of $\chi_{c1}(1P)$

- $\sigma(\chi'_{c1}) \sim |R'_{2P}(0)|^2$ $R'_{2P}(0) \approx R'_{1P}(0)$ Eichten & Quigg'95
- For the $b\bar{b}$ sector: $pp \to \chi_b$ @ LHC

$\sigma_{\chi_b}(1P) \sim \sigma_{\chi_b}(2P) \sim \sigma_{\chi_b}(3P)$ LHCb'14 & Han et al'14

- $\mathrm{Br}(B \to \chi_{c1} K) = (4-5) \times 10^{-4}$ PDG'14
- $\sigma(pp \to \chi_{c1}) \approx \sigma(pp \to \psi')$ Wang, Ma, Chao'10

$\chi'_{c1} - D^0\bar{D}^{*0}$ mixing model

Meng, Gao and Chao, PRD_87_074035 (2013) [hep-ph/0506222]

- Both the two components are substantial, and they may play different roles in the dynamics of $X(3872)$.

1. The short distance (the b- and $hadro$-) production and the quark annihilation decays of $X(3872)$ proceed dominantly through the χ'_{c1} component.
2. The $D^0\bar{D}^{*0}$ component is mainly in charge of the hadronic decays of $X(3872)$ into $DD\pi/DD\gamma$ as well as $J/\psi\rho$ and $J/\psi\omega$.
3. The long distance coupled-channel effects between the two components could renormalize the short distance dynamics by a product factor $Z_{c\bar{c}}$, the equivalent probability of χ'_{c1} in $X(3872)$.

$X(3872)$ production as a mixing state

- General factorization formula:

$$d\sigma(X(J/\psi\pi^+\pi^-)) = \sum_n d\hat{\sigma}((c\bar{c})_n) \cdot \langle O_n^{\chi'_{c1}} \rangle \cdot k, \qquad k = Z_{c\bar{c}}\mathrm{Br}_0$$

$$p_T, m_b, m_c \gg m_c v, m_c v^2, \Lambda_{QCD} \gg \epsilon, \Gamma_X \sim 1 \text{ MeV}$$

$c\bar{c}$ production $\quad \chi'_{c1}$production $\quad$ Binding & Decay(LD)

- B-procution: [Meng, Gao and Chao, PRD_87_074035 (2013) [hep-ph/0506222]]

$$\mathrm{Br}(B \to \chi'_{c1}K)/\mathrm{Br}(B \to \chi_{c1}K) = 0.75 \sim 1$$

- Consistent with the fitting result: [Kalashnikova & Nefediev PRD'09]

$$\mathrm{Br}^{\mathrm{fit}}(B \to \chi'_{c1}K) = (3.7\text{–}5.7) \times 10^{-4}$$

$$\mathrm{Br}(B \to X(J/\psi\pi^+\pi^-)K) = (8.6 \pm 0.8) \times 10^{-6} \qquad \text{PDG'14}$$

$$\therefore k = Z_{c\bar{c}}\mathrm{Br}_0 = 0.018 \pm 0.004$$

($Z_{c\bar{c}} = 28\%\text{–}44\%$ for $\mathrm{Br}_0 = 5\%$)

$X(3872)$ as mixing state Production at $pp/p\bar{p}$ Colliders

Meng & Han & Chao, arXiv:1304.6710

- Similar to that of $\chi_{c1}(1P)$

  $d\sigma(\chi'_{c1}) \approx d\sigma(\chi_{c1})$ [MWC'11]

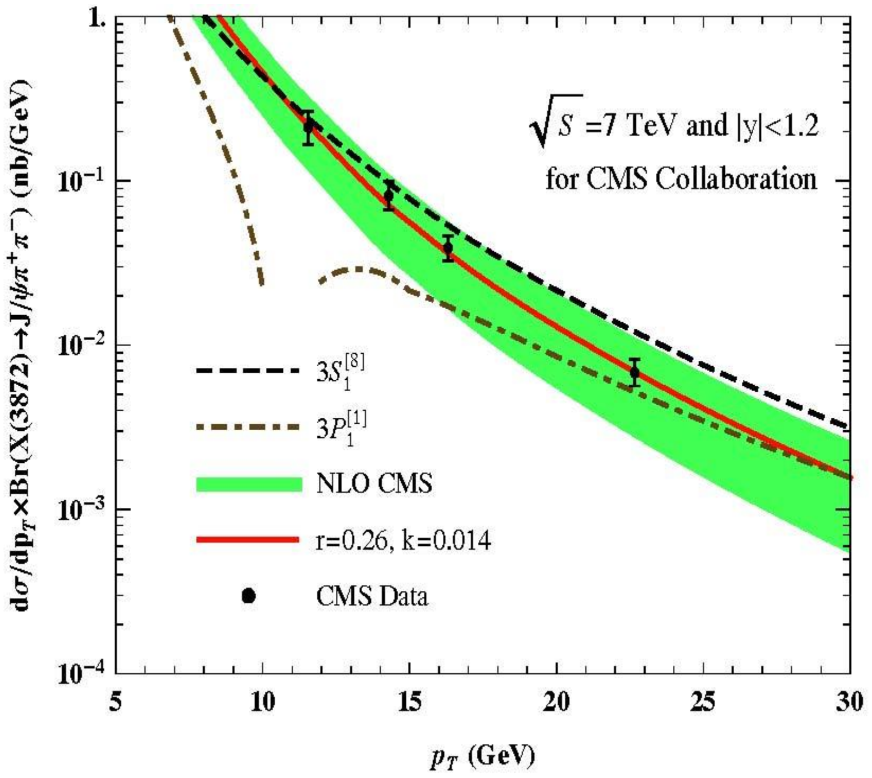
- Consistent with B-production

$$k = 0.014 \pm 0.007$$

$$(0.018 \pm 0.004)_{B-pro}$$

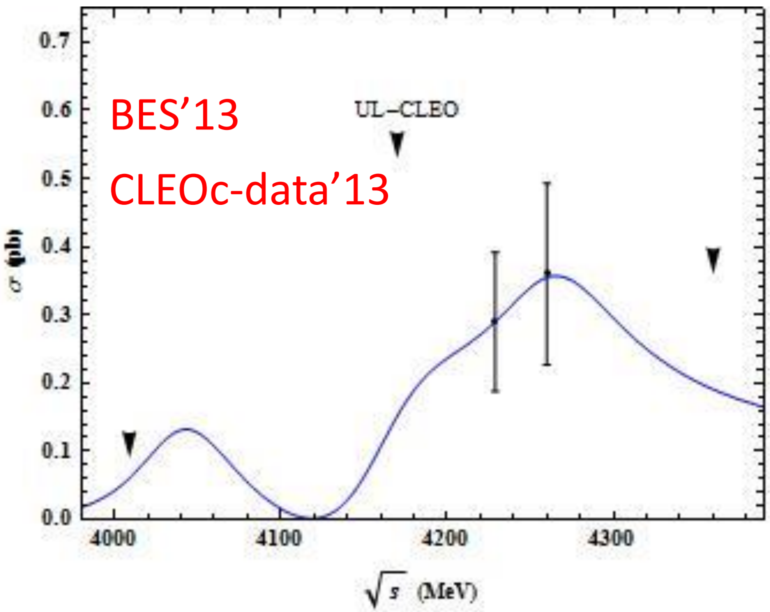
- Consistent with the P_T spectrum [CMS'13]

$$\chi^2/3 = 0.17$$



$e^+e^- \to \psi^n \to \gamma X(3872)$

Meng & Li & Chao, in preparation



m_i/MeV	Γ^i_{tot}/MeV	Γ^i_{ee}/keV
4260 [1]	100	0.5
4160	100	0.83
4040	80	0.86

[1] $\psi(4260) = \psi(4S)$ Li & Chao'09

- Molecule models: $DD_1(4260) \to \gamma[DD^*(3872)]$. Guo et al'13

$$\mathrm{Br}(Y \to \gamma X[J/\psi\pi\pi]) \sim \frac{50\ \mathrm{keV}}{100\ \mathrm{MeV}} \mathrm{Br}_0 \sim 2.5 \times 10^{-5}$$

$$\frac{\mathrm{Br}(Y\to\gamma X[J/\psi\pi\pi])}{\mathrm{Br}(Y\to J/\psi\pi\pi)} \sim 5 \times 10^{-3}$$ BES'13

$$\Gamma_{\mathrm{ee}} \cdot \mathrm{Br}(Y \to J/\psi\pi\pi) \sim 6\ \mathrm{eV} \quad \Rightarrow \quad \text{Need } \Gamma_{\mathrm{ee}} \sim 1\ \mathrm{keV!}$$

$$e^+e^- \to \psi^n \to \chi_{cJ}(2P)$$

Meng & Li & Chao, in preparation

- $\sigma > 0.15\ \mathrm{pb}$ @ 4170 MeV
- $\sigma\left(e^+e^- \to \gamma\chi'_{c2}(3930)\right) \sim 50\ \mathrm{pb}$ @ 4060 MeV

  $\mathrm{Br}(\chi'_{c2} \to D\bar{D}) \sim 70\%$
- $\sigma(e^+e^- \to \gamma\chi'_{c0}) \sim 50\ \mathrm{pb}$ @ 4170 MeV

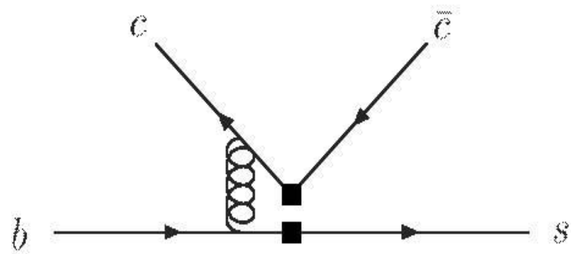
  Assuming χ'_{c0} =X(3915) & $\Gamma_{tot}(\chi'_{c0}) = 10\ \mathrm{MeV}$

  $\mathrm{Br}(\chi'_{c1} \to \gamma\psi') \sim 1\%$

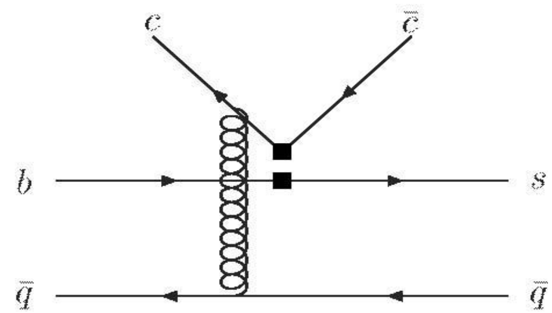
X(3872) production in B decays
--- revisited

$B \to \chi_{c1}^{(\prime)} K$ in QCD factorization

Meng, Gao and Chao, PRD'13; Meng and Chao, in preparation



$\times 4$



$\times 2$

- Vertex corrections $\sim R_P'(0) \cdot F^{B\to K}(m_{\chi_c}^2)$
- Spectator corrections $\sim \int_0^1 dx \frac{\Phi_B(x)}{x} \int_0^1 dy \frac{\Phi_K(y)}{y^2}$

End point singularity $\sim \frac{m_B}{\lambda_B} \int_0^1 dy \frac{6y(1-y)}{y^2}$

$$\Rightarrow \frac{m_B}{\lambda_B} \ln \frac{m_B}{\Lambda_h} \qquad \Lambda_h \sim 500\ \mathrm{MeV}$$

Violet factorization but numerical important

$B \to \chi_{c1}^{(\prime)} K$ in QCD factorization

Meng and Chao, in preparation

$\mathrm{Br} \times 10^4$	λ_B		PDG data [3]
	300 MeV	200 MeV	
$B^0 \to \chi_{c1} K^0$	1.79	3.56	3.93 ± 0.27
$B^0 \to \chi'_{c1} K^0$	1.78	3.65	
$B^+ \to \chi_{c1} K^+$	1.93	3.84	4.79 ± 0.23
$B^+ \to \chi'_{c1} K^+$	1.91	3.93	

- $\lambda_B = 200$ MeV: favored by fits in $B \to \pi\pi, \pi\rho, \ldots\ldots$
- $R_{2P/1P} = \frac{\mathrm{Br}(B \to \chi'_{c1} K)}{\mathrm{Br}(B \to \chi_{c1} K)} \approx 1$

  Almost independent to any parameters, such as λ_B, Λ_h

  Based on the factorization assumption

  Can be generalized to other B decay modes

X(3872) production in B decays

Meng and Chao, in preparation

- $\mathrm{Br}(B \to X(J/\psi\pi^+\pi^-) \ldots) = \mathrm{Br}(B \to \chi'_{c1} \ldots) \cdot k$
- Assuming $R_{2P/1P} = \frac{\mathrm{Br}(B\to\chi'_{c1}\ldots)}{\mathrm{Br}(B\to\chi_{c1}\ldots)} = 1$

$\mathrm{Br}(B^+ \to \chi_{c1}K^+) = (4.79 \pm 0.23) \times 10^{-4}$ PDG'14

$\mathrm{Br}(B \to X(J/\psi\pi^+\pi^-)K) = (8.6 \pm 0.8) \times 10^{-6}$ PDG'14

$$\therefore k = Z_{c\bar{c}}\mathrm{Br}_0 = 0.018 \pm 0.002$$

X(3872) production in B decays

Meng and Chao, in preparation

$X(J/\psi\pi^+\pi^-)$	$\mathbf{Br \times 10^6}$/predictions	$\mathbf{Br \times 10^6}$/data
$B^0 \to XK^0$	7.1 ± 0.9	4.3 ± 1.3 PDG'14
$B^+ \to XK^+$	8.6 ± 0.8	8.6 ± 0.8 PDG'14
$B^0 \to XK^+\pi^-$	6.8 ± 1.0	7.9 ± 1.4 Belle'15
$B^+ \to XK^0\pi^+$	9.9 ± 1.2	10.6 ± 3.1 Belle'15
$B^0 \to XK^{*0}$	4.4 ± 0.6	4.0 ± 1.5 Belle'15
$B^+ \to XK^{*+}$	5.4 ± 1.2	
$B^0 \to X\pi^0$	0.20 ± 0.05	
$B^+ \to X\pi^+$	0.40 ± 0.10	
$B_s \to X\phi$	3.6 ± 0.6	

Further remarks on the factorization assumption

Meng and Chao, in preparation

- $R_{K^*}(H) = \frac{\mathrm{Br}(B\to HK^*)\times\mathrm{Br}\left(K^*\to K^+\pi^-\right)}{\mathrm{Br}(B\to HK^+\pi^-)}$

- $R_{K^*}(\psi') = 0.68 \pm 0.01 \qquad v.s. \quad R_{K^*}(X) = 0.34 \pm 0.09$

  Belle'15: "......X(3872)......in marked contrast to ψ' case."

- While

$$R_{K^*}(J/\psi) = 0.69 \pm 0.05 \qquad \text{Belle'14}$$

$$R_{K^*}(\chi_{c1}) = 0.42 \pm 0.06 \qquad \text{PDG'14}$$

Nontrivial confirmation of the factorization assumption!

Further remarks on the factorization assumption

Meng and Chao, in preparation

- $R^{V}_{B_s/B}(H) = \frac{\mathrm{Br}(B_s \to H\phi)}{\mathrm{Br}(B \to HK^*)} \Rightarrow 1$

(In the heavy quark & chiral limit)

$$R^{V}_{B_s/B}(J/\psi) = 0.81 \pm 0.08$$

$$R^{V}_{B_s/B}(\psi') = 0.90 \pm 0.12$$

$$R^{V}_{B_s/B}(\chi_{c1}) = 0.83 \pm 0.13$$

Then we predict

$$\mathrm{Br}(B_s \to X(J/\psi\pi^+\pi^-)\phi) \sim 4 \times 10^{-6}$$

Further remarks on the factorization assumption

Meng and Chao, in preparation

- $R^{P}_{B_s/B}(H) = \frac{\mathrm{Br}\left(B_s \to H\eta^{(\prime)}\right)}{\mathrm{Br}(B \to HK)} \Rightarrow 0.5$

  (Assuming 50% of $\eta^{(\prime)}$ is $s\bar{s}$)

$$R^{P}_{B_s/B}(J/\psi) = 0.46 \pm 0.08 \qquad (0.39 \pm 0.06)$$

$$R^{P}_{B_s/B}(\psi') = 0.53 \pm 0.15$$

Then one can predict

$$\mathrm{Br}\left(B_s \to \chi_{c1}\eta^{(\prime)}\right) \sim 2 \times 10^{-4}$$

$$\mathrm{Br}\left(B_s \to X(J/\psi\pi^+\pi^-)\eta^{(\prime)}\right) \sim 4 \times 10^{-6}$$

Comparing with molecule model

- $R^{n/c} = \frac{\mathrm{Br}(B^0 \to XK^0, XK^+\pi^-, \dots)}{\mathrm{Br}(B^+ \to HK^+, XK^0\pi^+, \dots)} \xrightarrow{data} 0.5 - 0.75$
  - Molecule model [Braaten, Lu, Kusunoki'05-06]

$$R^{n/c} < 0.1 \sim 1/N_C^2$$

  - Mixing model

$$R^{n/c} = R^{n/c}(\chi_{c1}) = 0.7 - 0.8$$

- Pattern in different B decay modes
  - Similar to χ_{c1}, but not ψ'
  - Quite confusing in the molecule models

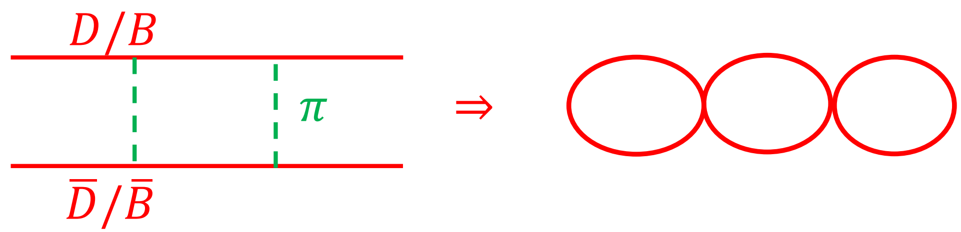
Summary & perspectives

- Data favor the mechanism that the X(3872) is produced at short distance mainly through the χ'_{c1}
  - Large production rates
  - Universality of $k = Z_{c\bar{c}}\mathrm{Br}_0 = 0.018 \pm 0.002$
  - Similar patterns to χ_{c1}
    - ✓ P_T spectrum in pp collision @ LHC
    - ✓ $R_{K^*}(X) \approx R_{K^*}(\chi_{c1})$
    - ✓ $R^{n/c} \sim 0.5 - 0.75$

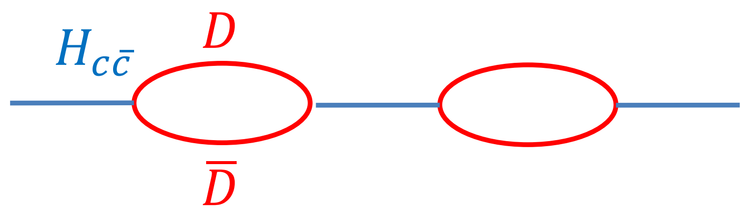
……

Summary & perspectives

- Partners of X(3872):



- Molecule model:
  - ✓ $X_2(4012) \sim D^*D^*$
    $J^{PC} = 2^{++}$ Nieves&Valderrama'12
  - ✓ $X_b(10580) \sim BB^*$ $J^{PC} = 1^{++}$ Guo et al'13



- Mixing model:
  - ✓ No X_2 unless $X_2 = \chi'_{c2} = Z(3930)$
  - ✓ No X_b unless $X_b = \chi''_{b1}(10510 - 10520)$ LHCb'14

Thank you!

Back Up

χ'_{c1} or molecule: Decay pattern

- χ'_{c1} decay modes
  - Radiative decay modes

	Barnes & Godfry'04	Barnes et al'05	Li & Chao'09
$\Gamma_{\psi\gamma}$/keV	11	59	45
$\Gamma_{\psi'\gamma}$/keV	64	88	60
$\Gamma_{\psi'\gamma}/\Gamma_{\psi\gamma}$	5.8	1.5	1.3

$\chi'_{c1} \to \gamma\psi'$ node-allowed; $\chi'_{c1} \to \gamma J/\psi$ node-surppressed

  - Light hadron decay mode

$$\Gamma(\chi'_{c1} \to LHs) \sim \Gamma(\chi_{c1} \to LHs) \sim 0.6\ \mathrm{MeV}$$

  - Coupled-channel induced decay modes Meng & Chao'07

$$\Gamma(DD^*/D^0\bar{D}^0\pi) \sim 0.5\text{-}1\ \mathrm{MeV}, \quad \Gamma(J/\psi\rho(\omega)) \sim 50\text{-}100\ \mathrm{keV}$$

$$\mathrm{Br}_0 \equiv \mathrm{Br}(X \to J/\psi\pi^+\pi^-) \sim 5\%$$

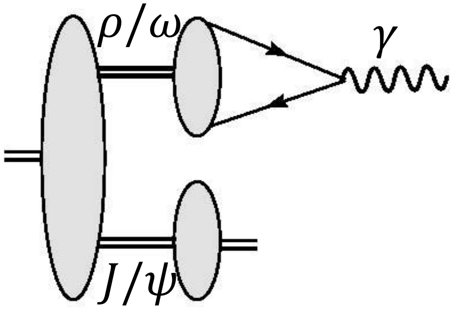
χ'_{c1} or molecule: Decay pattern

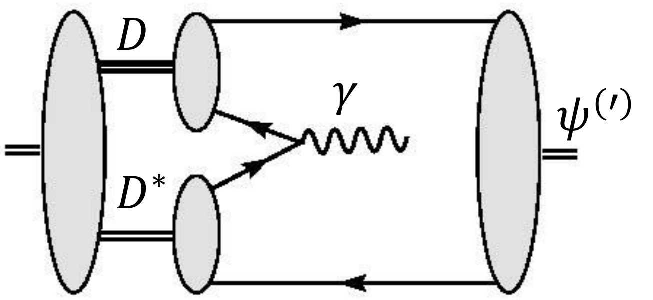
➢ Molecule Decay pattern

- $DD\pi$ decay mode [Swanson; Voloshin; Fleming, mehen, ……]

$$\Gamma(X \to D^0\bar{D}^0\pi) \sim 2\Gamma(D^{*0} \to D^0\pi) \sim 100\ \mathrm{keV}$$

- Radiative decays: [Swanson'04]





$$\Gamma(X \to J/\psi\gamma) \approx 8\ \mathrm{keV} \qquad \Gamma(X \to \psi'\gamma) \approx 0.03\ \mathrm{keV}$$

$$\frac{\Gamma(X\to\psi'\gamma)}{\Gamma(X\to D^0\bar{D}^0\pi)} \sim 10^{-4} \quad v.s. \quad (10^{-1})_{ex}$$ [PDG'14]

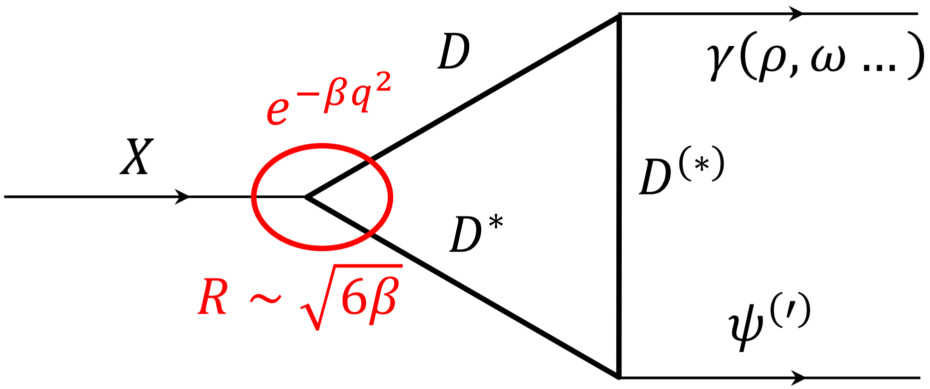
- $J/\psi\rho(\omega)$ decay mode [Swanson'04]

$$\Gamma(X \to J/\psi\rho(\omega)) \sim 1\text{-}2\ \mathrm{MeV}$$

χ'_{c1} or molecule: Decay pattern

- Molecule Decay pattern

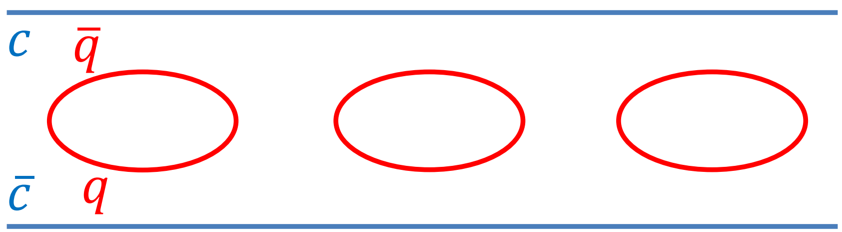
- Radiative decays: additional remarks



β/GeV^{-2}	R/fm	$\Gamma_{J/\psi\gamma}$/keV	$\Gamma_{\psi'\gamma}$/keV	$\Gamma_{\psi\rho}$/keV
0.4	0.31	13.4	3	
3	0.84	0.7	0.17	50
10	1.55	0.18	0.05	

Specrum: Charmonium

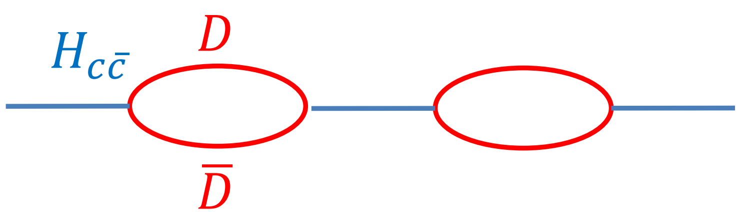
- Quark-level picture



Quark-pairs creation $\Rightarrow$ Screening the linear potential

$\Rightarrow$ Screened (unquenched) potential model [Chao & Ding & Qin'92]

- Hadron-level picture



Coupled-Channel models $\Rightarrow$ mixing between $H_{c\bar{c}}$ and $D\bar{D}$

Which have been considered even in the Cornell model [E. Eichten et al'78].

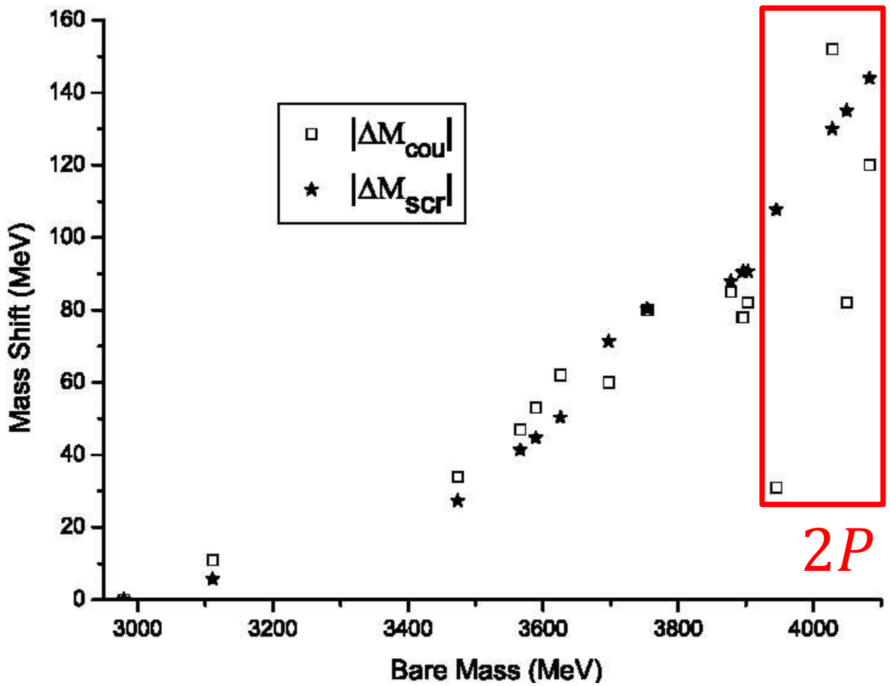
Specrum: Screened potential model

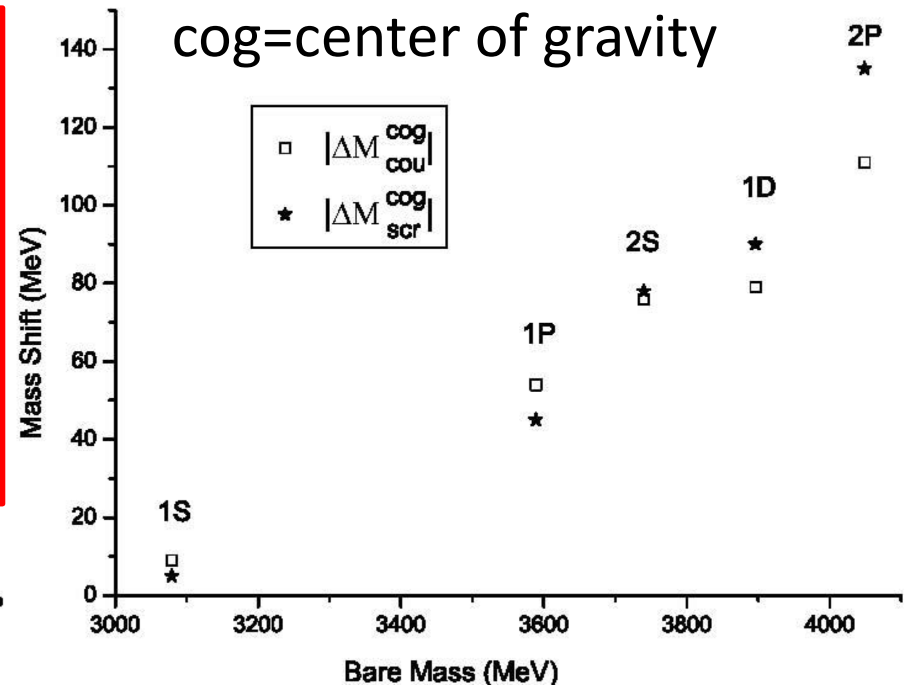
Li & Chao, PRD_79_094004 (2009)

State		Expt.	Theor. of ours		Theor. of Ref.[5]	
			Mass	$\langle r^2 \rangle^{\frac{1}{2}}$	NR	GI
1S	$J/\psi(1^3S_1)$	3096.916 ± 0.011	3097	0.41	3090	3098
	$\eta_c(1^1S_0)$	2980.3 ± 1.2	2979		2982	2975
2S	$\psi'(2^3S_1)$	3686.093 ± 0.034	3673	0.91	3672	3676
	$\eta_c'(2^1S_0)$	3637 ± 4	3623		3630	3623
3S	$\psi(3^3S_1)$	4039 ± 1	4022	1.38	4072	4100
	$\eta_c(3^1S_0)$		3991		4043	4064
4S	$\psi(4^3S_1)$	4263^{+8}_{-9}	4273	1.87	4406	4450
	$\eta_c(4^1S_0)$		4250		4384	4425
5S	$\psi(5^3S_1)$	4421 ± 4	4463	2.39		
	$\eta_c(5^1S_0)$		4446			
6S	$\psi(6^3S_1)$		4608	2.98		
	$\eta_c(6^1S_0)$		4595			
1P	$\chi_2(1^3P_2)$	3556.20 ± 0.09	3554	0.71	3556	3550
	$\chi_1(1^3P_1)$	3510.66 ± 0.07	3510		3505	3510
	$\chi_0(1^3P_0)$	3414.75 ± 0.31	3433		3424	3445
	$h_c(1^1P_1)$	3525.93 ± 0.27	3519		3516	3517
2P	$\chi_2(2^3P_2)$	$3929 \pm 5 \pm 2$	3937	1.19	3972	3979
	$\chi_1(2^3P_1)$		3901		3925	3953
	$\chi_0(2^3P_0)$		3842		3852	3916
	$h_c(2^1P_1)$		3908		3934	3956

Specrum: SPM v.s. CCM

Li & Meng & Chao, PRD_80_014012 (2009)





- SPM ≈ CCM in the global features.
- CCM is more adept in investigating the open-charmed threshold effects. (Especially for the 2P states)

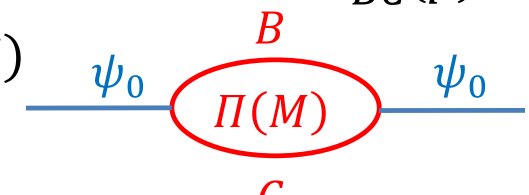
Specrum: S-wave threshold v.s. X(3872)

Li & Meng & Chao, PRD_80_014012 (2009)

$M - M_0 + \Pi(M) = 0 \qquad \Pi = \sum_{BC} \int d^3p \frac{\left|\langle BC, \vec{p} | H_{QPC} | \psi_0 \rangle\right|^2}{E_{BC}(\vec{p}) - M - i\epsilon}$

$\left|\langle BC, \vec{p} | H_{QPC} | \psi_0 \rangle\right|^2 \sim \Gamma(\psi \to BC)$

$\sim (M - M_B - M_C)^{\frac{2L+1}{2}}$

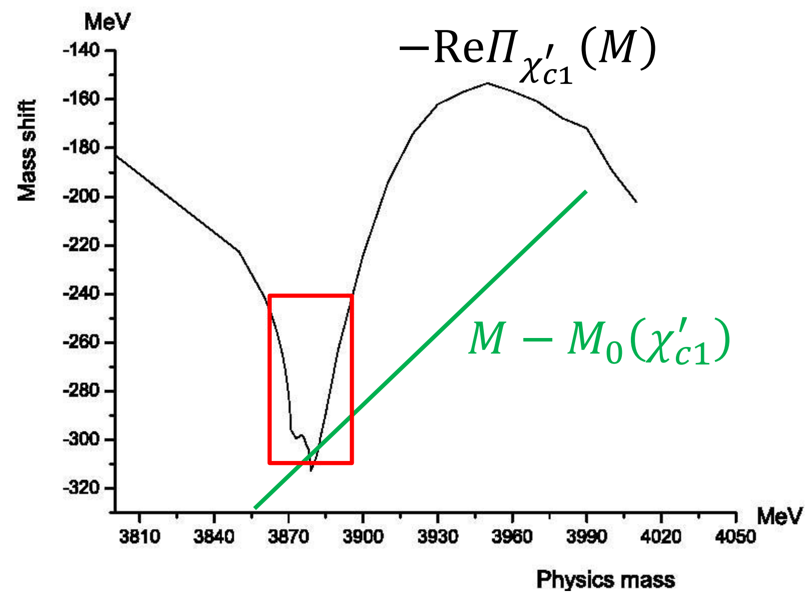


- S-wave threshold effect: $L = 0 \qquad E = M - M_B - M_C \Rightarrow 0$

$\Pi(E) \sim \sqrt{E},\ \mathrm{d}\Pi(E)/\mathrm{d}E \sim 1/\sqrt{E}$

$\Rightarrow$ S-wave cusp

$\Rightarrow$ “attracting” the mass of the bare state to the threshold



- $M_{\chi'_{c1}} \sim th_{DD^*}$:

$\Delta M \sim 15\ \mathrm{MeV} \Leftrightarrow \Delta\mathrm{Re}\Pi \sim 70\ \mathrm{MeV}$

$\Leftrightarrow \Delta M_0 \sim 85\ \mathrm{MeV}$