XYZ

Are they cusp effects?

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Based on:

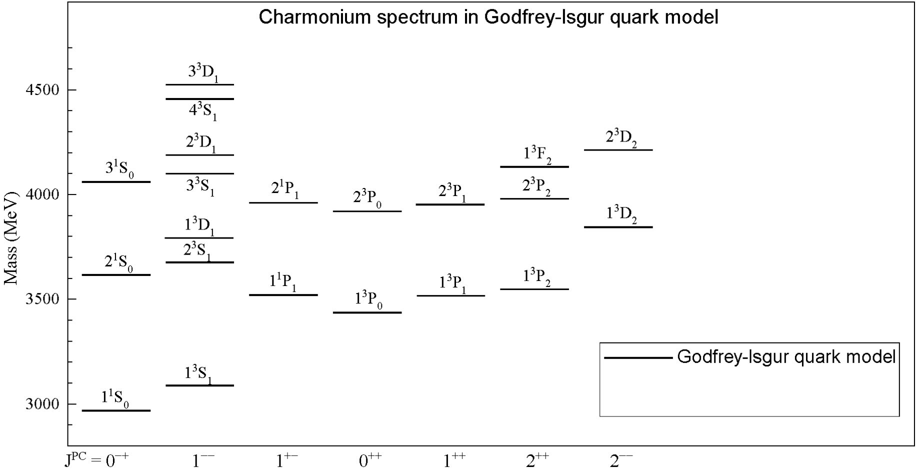
FKG, C. Hanhart, Q. Wang, Q. Zhao, Phys. Rev. D **91**, 051504(R) (2015) [arXiv:1411.5584[hep-ph]]

Outline

1. New charmonium(-like) states and two kinds of interpretations
2. Models of cusp effects
3. Distinguishing a physical state from a cusp: $Z_c(3900)$ as an example
4. Hadronic molecular candidates: $X(3872)$, $Y(4260)$ and $Z_c(3900)$
5. Summary

New charmonium(-like) states and two kinds of interpretations

Charmonia spectrum



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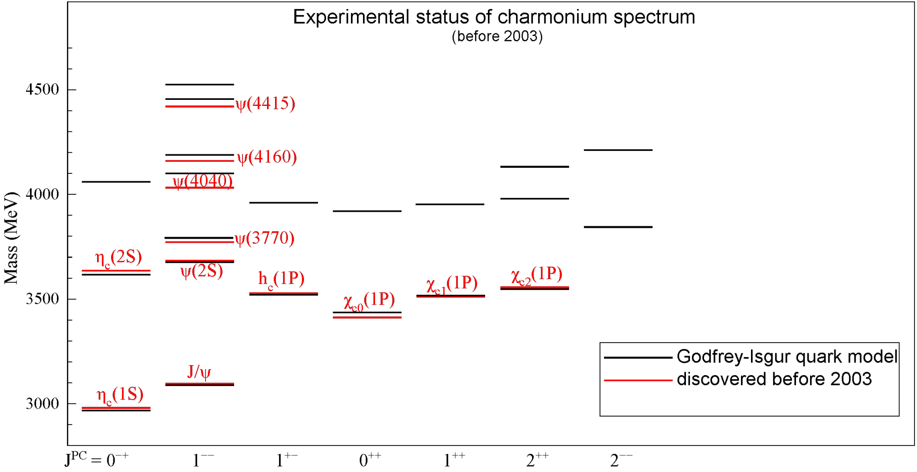

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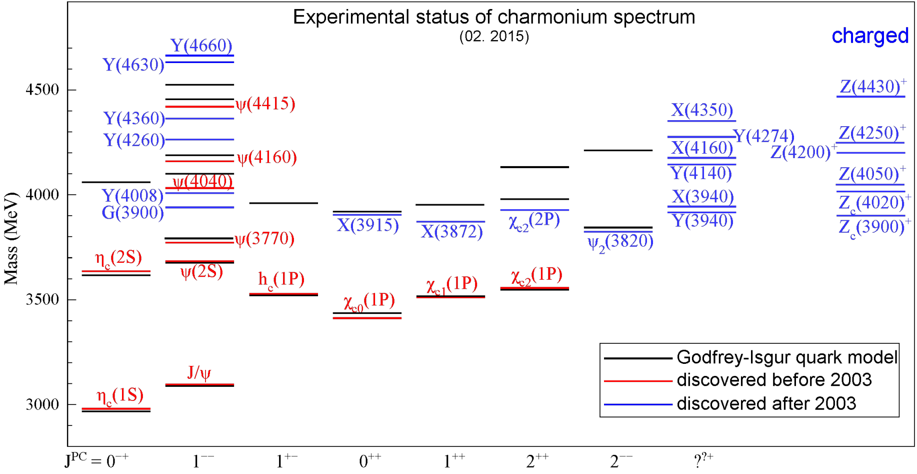

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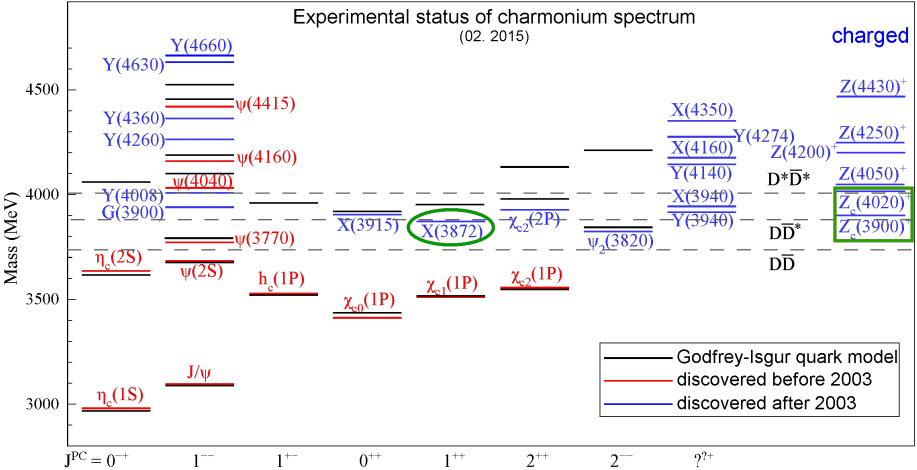
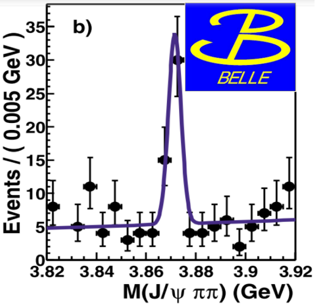


- $X(3872)$ Belle, PRL91(2003)262001



- Discovered in $B^{\pm} \to K^{\pm} J/\psi\pi\pi$, mass extremely close to the $D^0\bar{D}^{*0}$ threshold

$$M = (3871.69 \pm 0.17)\ \text{MeV}$$

$$M_{D^0} + M_{D^{*0}} = (3871.80 \pm 0.12)\ \text{MeV}$$

- $J^{PC} = 1^{++}$ LHCb (2013)

$\Rightarrow$ S-wave coupling to $D\bar{D}^*$

- Observed in the $D^0\bar{D}^{*0}$ mode as well

BaBar, PRD77(2008)011102

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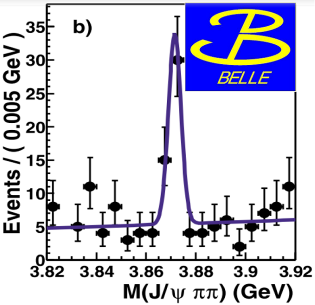

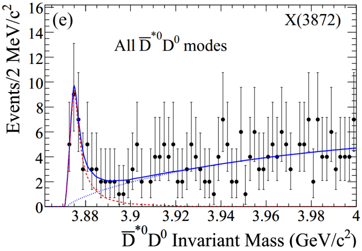

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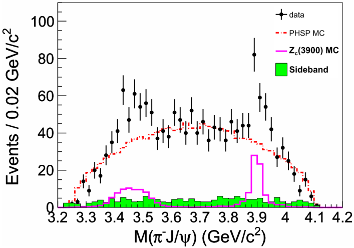

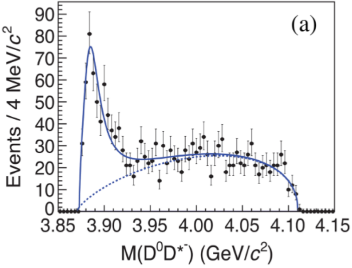

- Discovered by BESIII and Belle in $e^+e^- \to \pi\pi J/\psi$ at $\sqrt{s} = 4.26$ GeV

  Recall: the vector state $Y(4260)$

- A structure in $e^+e^- \to \pi^\pm[D\bar{D}^*]^\mp$ as well

- Mass from Breit-Wigner fits is close to the $D\bar{D}^*$ threshold

$$M = (3888.7 \pm 3.4)\ \mathrm{MeV}$$

assuming the two structures have the same origin

- $Z_c(4020)$
  - ☞ Discovered in $e^+e^- \to \pi^\pm\pi^\mp h_c$ BESIII, PRL111(2013)242001
  - ☞ A structure with similar mass observed in $e^+e^- \to \pi^\pm[D^*\bar{D}^*]^\mp$ BESIII, PRL112(2014)132001
  - ☞ Close to the $D^*\bar{D}^*$ threshold: $M = (4023.9 \pm 2.4)$ MeV
- $Z_b(10610)$ and $Z_b(10650)$
  - ☞ Discovered in 5 different channels:
    $\Upsilon(10860) \to \pi\pi\Upsilon(1S,2S,3S)/\pi h_b(1P,2P)$ Belle, PRL108(2012)122001
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- Poles in the S-matrix: genuine physical states
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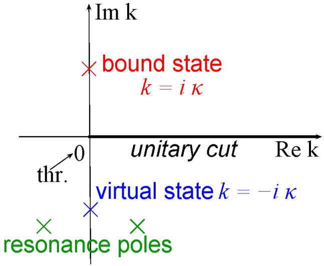
  - ☞ virtual states (real axis, 2nd RS)
  - ☞ resonances (2nd RS)



- The origins of the poles can be different:
  - ☞ normal $Q\bar{Q}$
  - ☞ hybrid states
  - ☞ tetraquarks
  - ☞ hadronic molecules
  - ☞ hadro-charmonia / hadro-bottomonia

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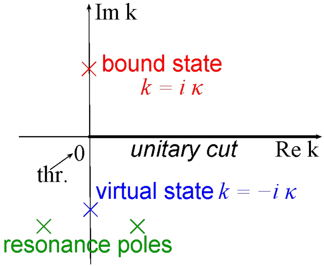

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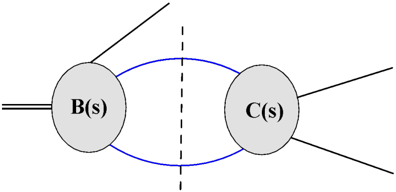
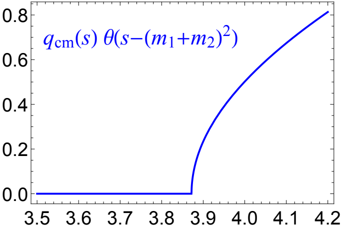
  there is always a cusp at an S-wave threshold if they couple. Unitarity $\Rightarrow$





$$= \quad \text{disc}\,\mathcal{A}(s) \propto C^*(s)\frac{q_{\rm cm}(s)}{\sqrt{s}}B(s)\theta(s-(m_1+m_2)^2)$$

Analyticity $\Rightarrow$ dispersion relation: $\mathcal{A}(s) = \frac{1}{2\pi i}\int_{s_0}^{\infty} ds' \frac{\text{disc}\,\mathcal{A}(s')}{s'-s-i\epsilon}$

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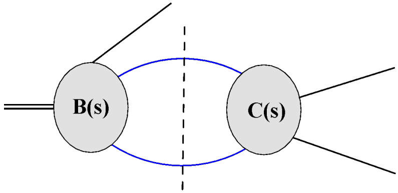

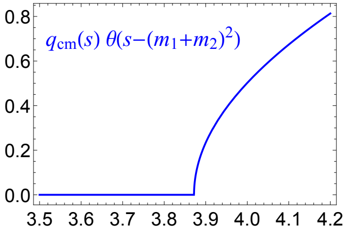

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Models of cusp effects

Cusp models — Bugg

- Bugg speculated that the $X(3872)$ could be a cusp effect PLB598(2004)8
  but no calculation
- Then he realized in 2008 that such a cusp model could not reproduce the data for $X(3872)$ in the $J/\psi\rho$ and $D\bar{D}^*$ channels, a pole is needed JPG35(2008)075005
- For the $Z_b(10610, 10650)$ EPL96(2011)11002
- could produce a narrow peak by using

$$\mathrm{Re}\,T(s) = \frac{1}{\pi}\,⨍_{s_{\rm th}} ds'\,\frac{g^2\rho(s')}{s'-s}, \quad \rho(s) = \frac{2q_{\rm cm}(s)}{\sqrt{s}} FF(s)$$

$$FF(s) = \exp\left(-q_{\rm cm}^2(s)R^2/3\right)$$

with $R = 1.41$ fm $\quad (1/R = 0.14$ GeV$)$

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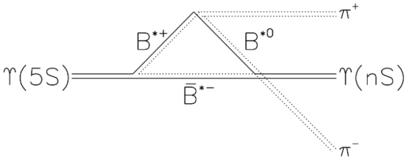

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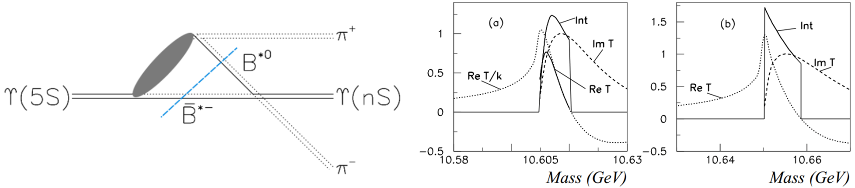

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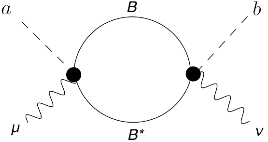
Cusp models — Swanson

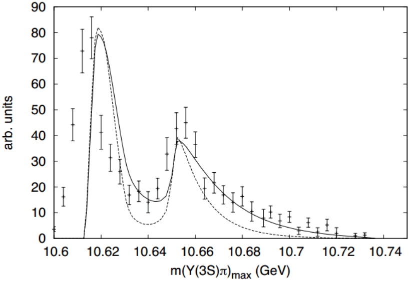
- Swanson's model for both Z_c and Z_b states PRD91(2015)034009

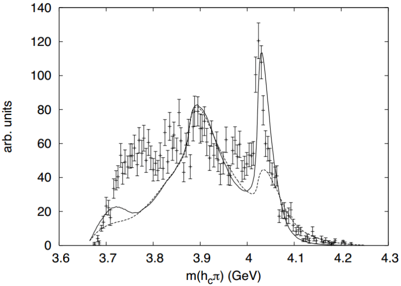


$$\Pi_{\alpha\beta}(s) = \frac{1}{\pi}\int_{s_{\rm th}} ds' \frac{\mathrm{Im}\,\Pi_{\alpha\beta}(s')}{s'-s-i\epsilon},$$

$$\mathrm{Im}\,\Pi_{\alpha\beta}(s) = \sum_i q_{\mathrm{cm},i}(s)F_{\alpha i}(s)F_{\beta i}(s), \qquad F_{\alpha i}(s) = g_{\alpha i}\exp\left(-\frac{s}{2\beta_{\alpha i}^2}\right)$$

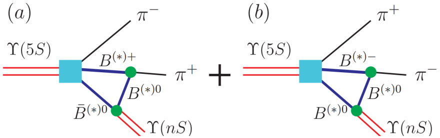
- Impressive agreement with data by adjusting 2 parameters for each channel ($g_{\alpha i}, \beta_{\alpha i}$)



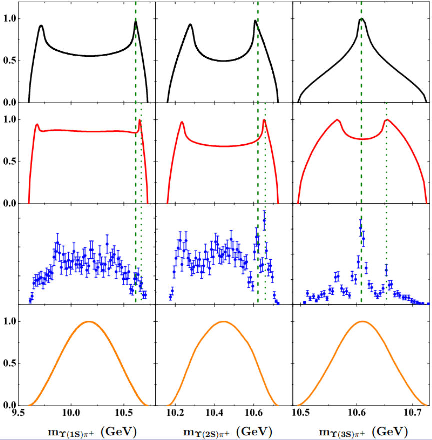


Cusp models — Chen, Liu, Matsuki

D.-Y.Chen, X.Liu, PRD84(2011)094003; PRD84(2011)034032;
Chen, Liu, Matsuki, PRD84(2011)074032; PRD88(2013)036008; PRL110(2013)232001; …



Form factor $\left(\frac{\Lambda^2 - m^2_{B^{(*)}}}{\Lambda^2 - q^2}\right)^2$



$\Leftarrow B\bar{B}^* + c.c.$ loops

$\Leftarrow B^*\bar{B}^*$ loops

$\Leftarrow$ Belle data

$\Leftarrow B\bar{B}$ loops

Cusp models — Blitz, Lebed

S. Blitz, R. Lebed, arXiv:1503.0480 [hep-ph]

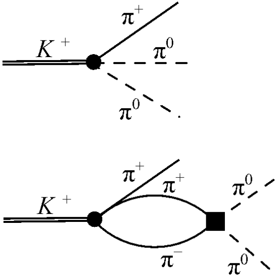
- They proposed cusp effects due to diquark–antidiquark “thresholds”
- But diquarks and antidiquarks are NOT asymptotic states and cannot go on-shell, thus cannot produce any cusp!

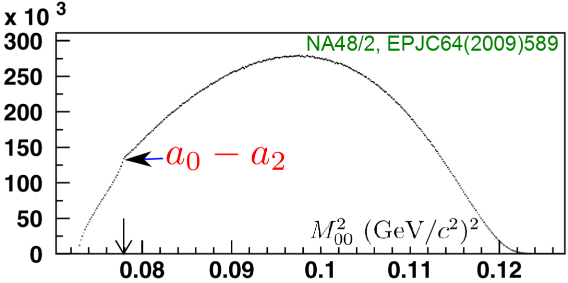
Distinguishing a physical state from a cusp: $Z_c(3900)$

Cusp effects are well-known

- Opening of an S-wave threshold can produce a structure. What can be learned?
- Cusp effect has been well-known for a long time:
  - ☞ example of the cusp in $K^\pm \to \pi^\pm\pi^0\pi^0$
  - ☞ the strength of the cusp is determined by the interaction strength!

Meißner, Müller, Steininger (1997); Cabibbo (2004); Colangelo, Gasser, Kubis, Rusetsky (2006); ...





- Logic:
  first, fit to data with the one-loop expression which produces a cusp;
  then, try to understand the implications of the resulting parameter values

- Example: $Y(4260) \to D\bar{D}^*\pi$:

$$\mathcal{A}_{\text{1-loop}} = g_Y \left[1 - C\, G_\Lambda(E)\right]$$

regularize the loop with a Gaussian form factor with a cutoff Λ

$$G_\Lambda(E) = \int \frac{d^3q}{(2\pi)^3} \frac{f_\Lambda(q)}{E - m_1 - m_2 - q^2/(2\mu)}, \quad f_\Lambda(q) = \exp\left(-\frac{2q^2}{\Lambda^2}\right)$$

three parameters: g_Y, C, Λ

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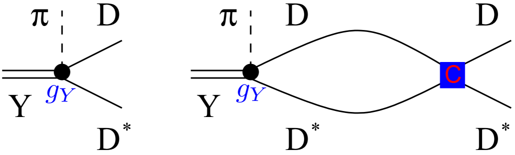
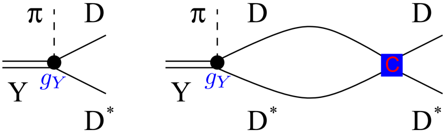
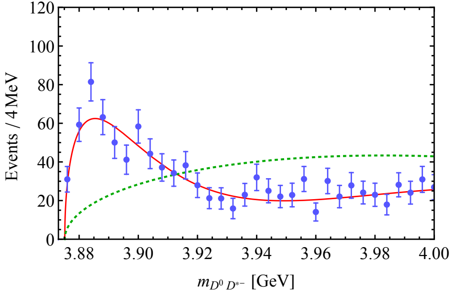


$Z_c(3900)$ as an example (II)





- Implicit assumption of using $g_Y \left[1 - C\, G_\Lambda(E)\right]$ as the decay amplitude: the $D\bar{D}^*$ interaction is perturbative
- The two-loop contribution is large ⇒ nonperturbative $\quad C\, G_\Lambda(E_{\rm th}) = -1.3!$
- Resumming all the bubbles by $$\frac{g_Y}{1 + C\, G_\Lambda(E)}$$ with the parameters determined from the 1-loop fit gives a bound state pole very close to the threshold (binding energy: 0.6 MeV)

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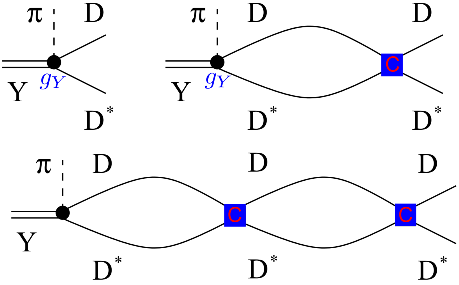

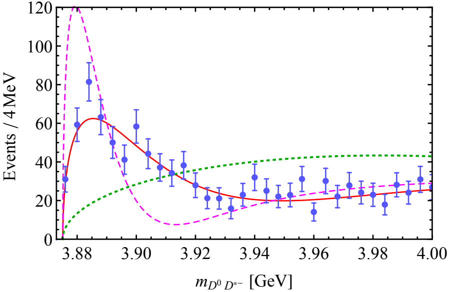

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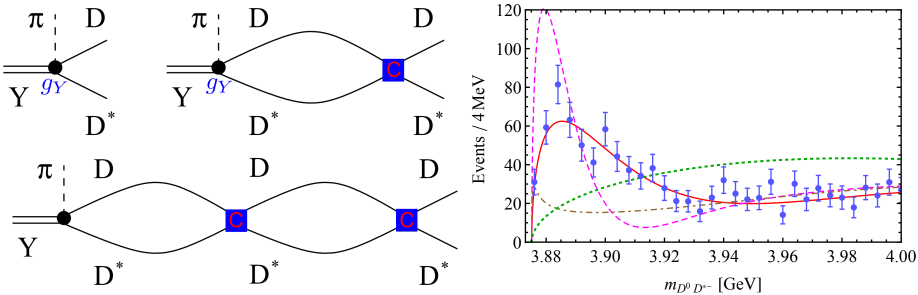

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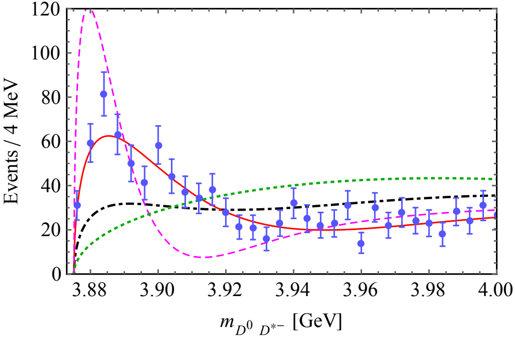

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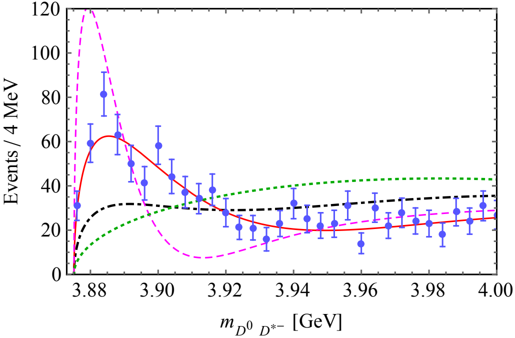

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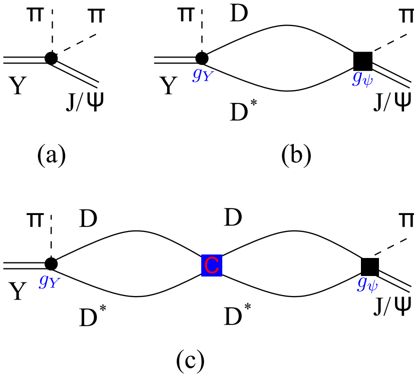
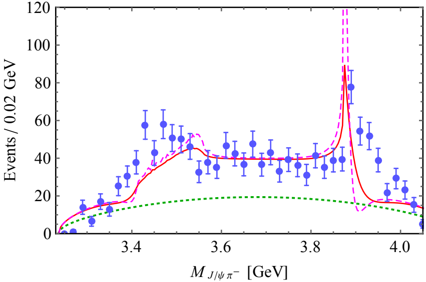
$Z_c(3900)$ as an example (IV)

For $Y(4260) \to J/\psi\pi^+\pi^-$

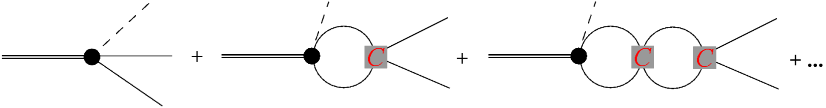


- Inelastic channel: $g_{Y\psi} - g_Y G_\Lambda(E) g_\psi$
  g_ψ cannot be determined separately
- Compare with the elastic case:
  $$g_Y[1 - CG_\Lambda(E)]$$
- Fit to the data with Λ fixed above

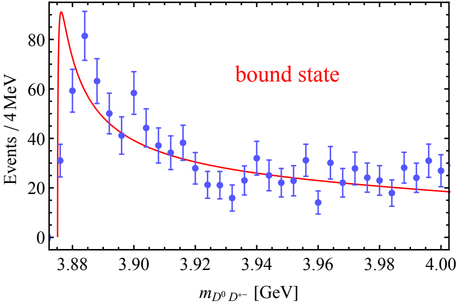


- A near-threshold peak in inelastic channel cannot distinguish a cusp effect from a pole

Naive fit with resummed amplitude



- fix $\Lambda = 0.8$ GeV and fit C to the data

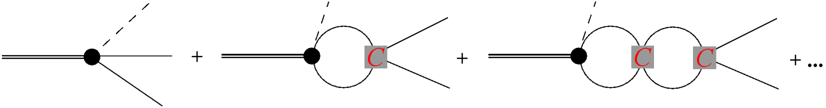


has a bound state pole with a binding energy of 1.6 MeV $[C = 0.92 \text{ fm}^2]$

- Caution:

more complications in a realistic fit, e.g.,

possible effect of triangle singularity, data for angular distribution . . .



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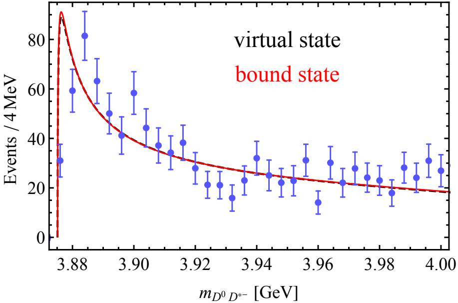

or a virtual state pole with the same mass $[C = 0.68 \text{ fm}^2]$

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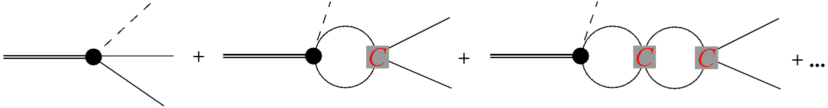

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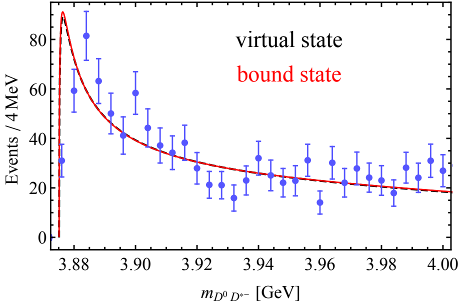

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Hadronic molecular candidates:

$X(3872)$, $Y(4260)$ and $Z_c(3900)$

$X(3872)$, $Y(4260)$ and $Z_c(3900)$ (I)

- Suppose that the $X(3872)$, $Y(4260)$ and $Z_c(3900)$ are hadronic molecules:
  - ☞ $X(3872)$: $J^{PC} = 1^{++}$, $D\bar{D}^*$
  - ☞ $Y(4260)$: $J^{PC} = 1^{--}$, $D_1(2420)\bar{D}$ [two D_1's, should be the narrow one]
  - ☞ $Z(3900)$: $J^{PC} = 1^{+-}$, $D\bar{D}^*$
- Features of hadronic molecules:
  - ☞ spin partners with similar fine splitting of their components

$$M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)} \simeq M_{D^*} - M_D \simeq M_{Z_c(4020)} - M_{Z_c(3900)}$$

  - ☞ couple strongly to their components. If the binding energy is small,

$$g^2 \approx 16\pi(1-Z)(m_1+m_2)^2\sqrt{\frac{2E_B}{\mu}} \leq 16\pi(m_1+m_2)^2\sqrt{\frac{2E_B}{\mu}} = g^2_{\text{h.m.}}$$

$1-Z$: compositeness

  - ☞ can decay through the decays of their components
  - ☞ seemingly unrelated processes may be related; …

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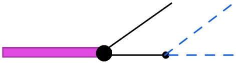

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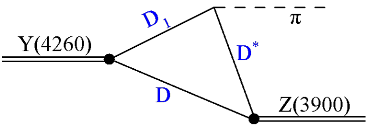
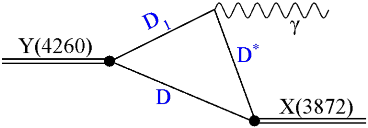
$X(3872)$, $Y(4260)$ and $Z_c(3900)$ (II)

Wang et al, PRL111(2013)132003; Cleven et al, PRD90(2014)074039; FKG et al, PLB725(2013)127

- Production of $X(3872)$ and $Z_c(3900)$ in $Y(4260)$ decays





- Loops are enhanced when the binding energies are small:

$$\mathcal{A} \sim \mathcal{O}\left(\frac{v^5}{(v^2)^3}\right) V_{D_1 D^*}(q_{\pi/\gamma}) = \mathcal{O}\left(\frac{1}{v}\right) V_{D_1 D^*}(q_{\pi/\gamma})$$

  - ☞ S-wave vertices for couplings to $Y(4260)$, $X(3872)$ and $Z_c(3900)$
  - ☞ intermediate mesons are nonrelativistic. v: velocity
  - ☞ power counting: three-momentum $\sim \mathcal{O}(v)$, energy $\sim \mathcal{O}(v^2)$

$$\text{loop measure}: \quad v^5, \qquad \text{propagator}: \quad \frac{1}{v^2}$$

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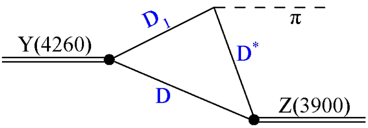

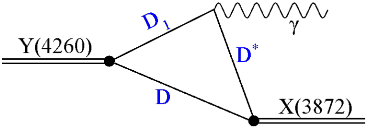

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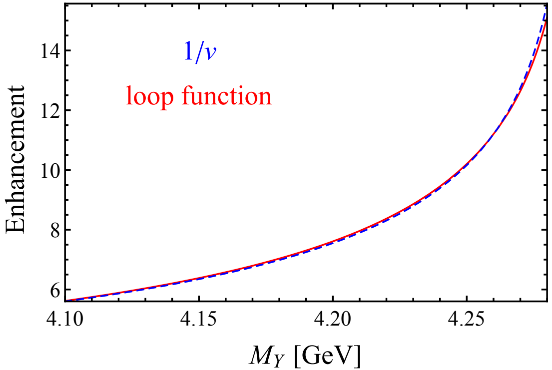
Explicit check of the power counting

Check the $1/v$ with explicit calculation of the three-point loop function (normalized to $1/v$ at an arbitrary point) for $Y(4260) \to Z_c(3900)\pi$:



- power counting is well satisfied
- a large enhancement to the reaction rate

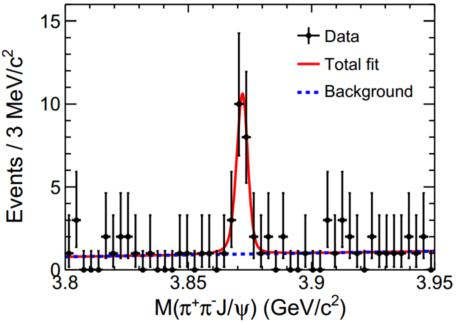
$X(3872)$, $Y(4260)$ and $Z_c(3900)$ (III)

If these three states are hadronic molecules, then

- the $Z_c(3900)$ can be easily produced in the $Y(4260)$ decays, in line with the BESIII and Belle observations
- Prediction: FKG et al, PLB725(2013)127

  the $X(3872)$ can be easily produced in $Y(4260) \to \gamma X(3872)$

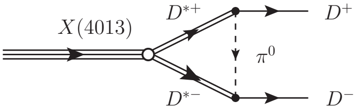
  BESIII observation of $e^+e^- \to \gamma X(3872) \to \gamma J/\psi\pi^+\pi^-$ at $\sqrt{s} = 4.26$ GeV:



BESIII, PRL112(2014)092001

Assuming the $X(3872)$ as a 1^{++} $D\bar{D}^*$ bound state, then

- heavy quark spin symmetry $\Rightarrow$ predicted in many papers
  very likely, a 2^{++} $D^*\bar{D}^*$ bound state $X_2(4013)$
- decay dominantly into $D\bar{D}$ and $D\bar{D}^* + c.c.$.
  decay width calculated
  M. Albaladejo, FKG, C. Hidalgo-Duque, J. Nieves, M. Pavón Valderrama, arXiv:1504.0xxx [hep-ph]



  large uncertainty, but of the order of a few MeV
- for BESIII, if search for the $X_2(2013)$ in $e^+e^- \to \gamma D\bar{D}$, then the best energy region is 4.4 – 4.5 GeV FKG, U.-G. Meißner, Z. Yang, PLB740(2015)42

Summary

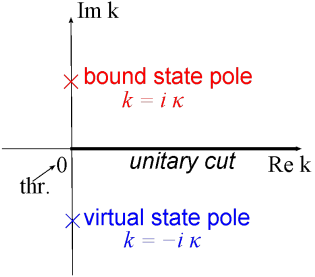
Summary

- There are always cusps at S-wave thresholds.
  Near-threshold peaks (cusps) may provide information on interaction strength
- To distinguish a cusp from a pole, one needs to study the elastic channel.
  A pronounced narrow peak in line shape of the elastic channel cannot be explained by just the kinematical cusp. It requires a pole
- Hadronic molecular interpretation of the $X(3872)$, $Y(4260)$ and $Z_c(3900)$

THANK YOU FOR YOUR ATTENTION!

Backup slides

Bound state and virtual state (I)



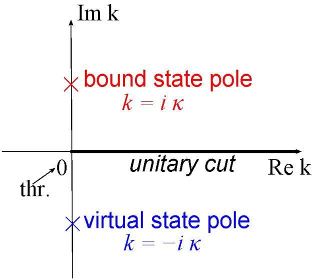
Suppose the scattering length is very large, the S-wave scattering amplitude

$$f_0(k) = \frac{1}{k\cot\delta_0(k) - ik} \simeq \frac{1}{-1/a - ik}$$

- bound state pole: $1/a = \kappa$
- virtual state pole: $1/a = -\kappa$

- If the same binding energy, cannot be distinguished above threshold (k is real):

$$|f_0(k)|^2 \sim \frac{1}{\kappa^2 + k^2}$$



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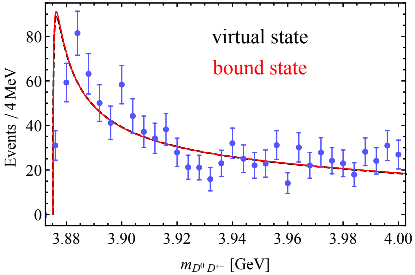
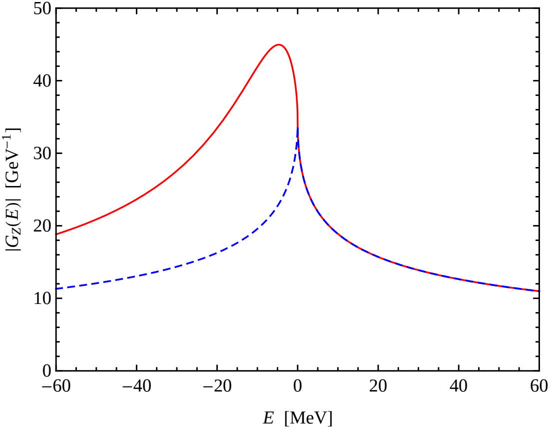


- Bound state and virtual state with a small binding energy should be distinguished in inelastic channel



A bound state and virtual state with a 5 MeV binding energy, a small residual width to the inelastic channel is allowed. Cleven et al, EPJA47(2011)120

Suppose the physical state $|\psi\rangle$ contains a two-hadron continuum state $|h_1 h_2\rangle = |\mathbf{q}\rangle$ and something else $|\psi_0\rangle$

The time-independent Schrödinger Equation

$$(\hat{H}_0 + \hat{V})|\psi\rangle = -E_B|\psi\rangle$$

here H_0 is the free Hamiltonian, $\hat{H}_0|\mathbf{q}\rangle = q^2/(2\mu)$, and $E_B > 0$ is the binding energy.

Multiplying by $\langle\mathbf{q}|$, we get the momentum-space wave function

$$\langle\mathbf{q}|\psi\rangle = -\frac{\langle\mathbf{q}|\hat{V}|\psi\rangle}{E_B + q^2/(2\mu)}$$

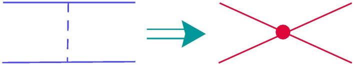
The probability of finding the physical state in the continuum state is

$$\lambda^2 = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\,|\langle\mathbf{q}|\psi\rangle|^2 = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\,\frac{|\langle\mathbf{q}|\hat{V}|\psi\rangle|^2}{[E_B + q^2/(2\mu)]^2}$$

Denoting $g_{\rm NR}^2(\mathbf{q}) = |\langle \mathbf{q}|\hat{V}|\psi\rangle|^2$, we have

$$\lambda^2 = 4\mu^2 \int \frac{d\Omega_{\mathbf{q}}}{(2\pi)^3} \int_0^\infty dq q^2 \frac{g_{\rm NR}^2(\mathbf{q})}{(q^2+2\mu E_B)^2}$$

If the binding energy is very small, so that the binding momentum $\sqrt{2\mu E_B} \ll 1/r$ with r the range of forces, we have an expansion



$$g_{\rm NR}^2(\mathbf{q}) = q^{2L} g_{\rm NR}^2(0) + \mathcal{O}\left(r\sqrt{2\mu E_B}\right)$$

here L is the orbital angular momentum.

The integral is only convergent for S-wave. Therefore,

the probability of finding the physical state in an S-wave two-hadron state with a small binding energy is related to the coupling constant $g_{\rm NR}(0)$

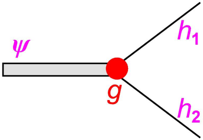
Landau (1960), Weinberg (1963,1965), Baru et al (2004),...

$$\boxed{\lambda^2 \approx \frac{\mu^2}{2\pi\sqrt{2\mu E_B}} g_{\rm NR}^2(0)}$$

From nonrelativistic quantum mechanics to relativistic QFT:

$$g = \sqrt{2m_1}\sqrt{2m_2}\sqrt{2(m_1+m_2)}\,g_{\rm NR}(0)$$

here g is the coupling constant in the relativistic Lagrangian



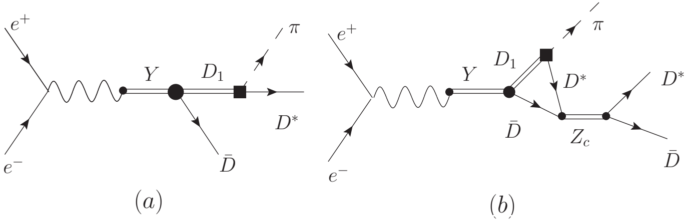
$$\mathcal{L} = g\psi^\dagger h_1 h_2 + h.c.$$

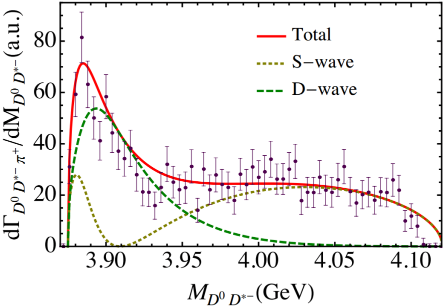
Therefore, the coupling constant contains the structure information

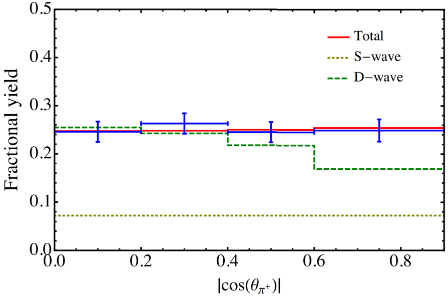
$$\boxed{g^2 \approx 16\pi\lambda^2(m_1+m_2)^2\sqrt{\frac{2E_B}{\mu}} \leq 16\pi(m_1+m_2)^2\sqrt{\frac{2E_B}{\mu}}}$$

It is bounded from above!

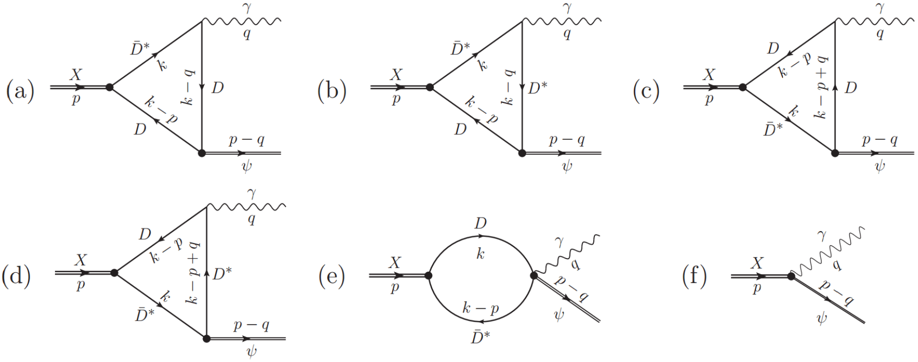
$Y(4260) \to D\bar{D}^*\pi$ with an explicit Z_c pole:







FKG, Hanhart, Kalashnikova, Meißner, Nefediev, PLB742(2015)394



The ratio $\frac{\mathcal{B}(X(3872) \to \psi'\gamma)}{\mathcal{B}(X(3872) \to J/\psi\gamma)} = 2.46 \pm 0.64 \pm 0.29$ LHCb, NPB886(2014)665

is insensitive to the molecular component of the $X(3872)$:

- ☞ loops are sensitive to unknown couplings $g_{\psi DD}/g_{\psi' DD}$
- ☞ loops are divergent, needs a counterterm (short-distance physics)