中国科学院大学
University of Chinese Academy of Sciences

Oddballs in QCDSR

Liang Tang

2015.04.03

3rd workshop on the XYZ particles

Outline

1. An Introduction to Glueballs
2. Current Status of Glueballs
3. 0^{--} Oddballs via QCDSR
4. Experimentalists' Attentions for 0^{--} Oddballs.
5. Summary & Outlook

Based on Cong-Feng Qiao & Liang Tang, PRL113,221601(2014).

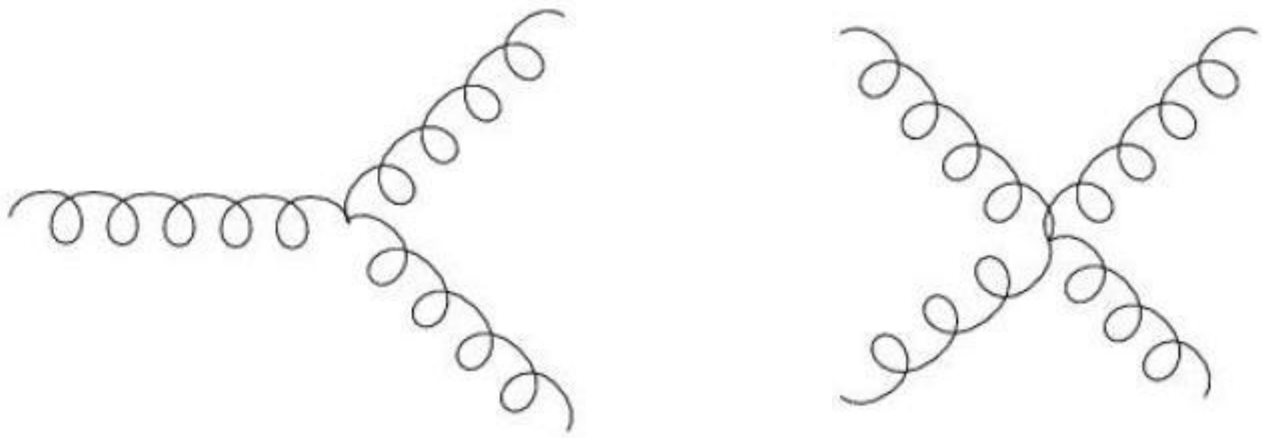
An Introduction to Glueballs

➢ The Lagrangian of QCD:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \sum_q \bar{\psi}_q (i\gamma^\mu D_\mu - m_q)\psi_q$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

➢ There exist interactions among gluons in QCD:



An Introduction to Glueballs

- Color structure
  - Quark= fundamental representation **3**
  - Gluon= Adjoint representation **8**
  - Observable particles=color singlet **1**
    - Mesons $3 \bigotimes 3 = 1 \bigoplus 8$
    - Baryons $3 \bigotimes 3 \bigotimes 3 = 1 \bigoplus 8 \bigotimes 8 \bigoplus 10$
    - Glueballs $\begin{cases} 8 \bigotimes 8 = 1 \bigoplus 8 \bigoplus 8 \bigoplus 10 \bigoplus \overline{10} \bigoplus 27 \\ 8 \bigotimes \cdots \bigotimes 8 = 1 \bigoplus 8 \bigoplus \cdots \end{cases}$
- Glueballs are predicted by QCD.
- No definite observations in the experiment until now.
  - lack knowledge of their production & decay properties
  - mixing with quark states adds difficulty to isolate them.

Current Status of Glueballs

- Theoretical Approaches

  - Lattice QCD
  - Flux tube model
  - MIT bag model
  - Coulomb gauge model
  - QCD Sum Rules (QCDSR)

Flux tube model, MIT bag model, Coulomb gauge model } Constituent Models

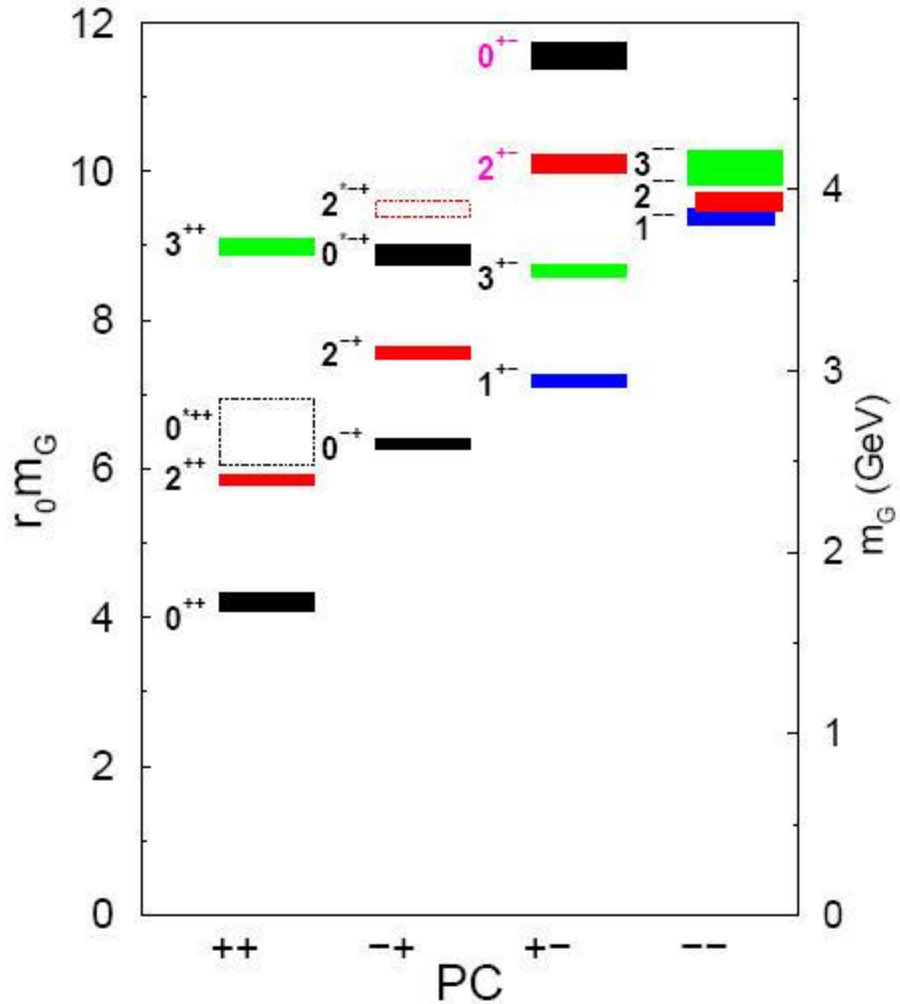
V.Mathieu, N.Kochelev&V.Vento, Int.J.Mod.Phys. E18,1(2009)

Current Status of Glueballs

- Results of Lattice QCD

J^{PC}	Other J	$r_0 m_G$	m_G (MeV)
0^{++}		4.21 (11)(4)	1730 (50)(80)
2^{++}		5.85 (2)(6)	2400 (25)(120)
0^{-+}		6.33 (7)(6)	2590 (40)(130)
0^{*++}		6.50 (44)(7)†	2670 (180)(130)
1^{+-}		7.18 (4)(7)	2940 (30)(140)
2^{-+}		7.55 (3)(8)	3100 (30)(150)
3^{+-}		8.66 (4)(9)	3550 (40)(170)
0^{*-+}		8.88 (11)(9)	3640 (60)(180)
3^{++}	6, 7, 9, ...	8.99 (4)(9)	3690 (40)(180)
1^{--}	3, 5, 7, ...	9.40 (6)(9)	3850 (50)(190)
2^{*-+}	4, 5, 8, ...	9.50 (4)(9)†	3890 (40)(190)
2^{--}	3, 5, 7, ...	9.59 (4)(10)	3930 (40)(190)
3^{--}	6, 7, 9, ...	10.06 (21)(10)	4130 (90)(200)
2^{+-}	5, 7, 11, ...	10.10 (7)(10)	4140 (50)(200)
0^{+-}	4, 6, 8, ...	11.57 (12)(12)	4740 (70)(230)

$$r_0^{-1} = 410 \pm 20 \text{ MeV}$$



Morningstar & Peardon, PRD60(1999)034509.

Current Status of Glueballs

- Results of Lattice QCD

R^{PC}	Possible J^{PC}	$r_0 M_G$	$r_0 M_G$
A_1^{++}	0^{++}	4.16(11)	4.21(11)
E^{++}	2^{++}	5.82(5)	5.85(2)
T_2^{++}	2^{++}	5.83(4)	5.85(2)
A_2^{++}	3^{++}	9.00(8)	8.99(4)
T_1^{++}	3^{++}	8.87(8)	8.99(4)
A_1^{-+}	0^{-+}	6.25(6)	6.33(7)
T_1^{+-}	1^{+-}	7.27(4)	7.18(3)
E^{-+}	2^{-+}	7.49(7)	7.55(3)
T_2^{-+}	2^{-+}	7.34(11)	7.55(3)
T_2^{+-}	3^{+-}	8.80(3)	8.66(4)
A_2^{+-}	3^{+-}	8.78(5)	8.66(3)
T_1^{--}	1^{--}	9.34(4)	9.50(4)
E^{--}	2^{--}	9.71(3)	9.59(4)
T_2^{--}	2^{--}	9.83(8)	9.59(4)
A_2^{--}	3^{--}	10.25(4)	10.06(21)
E^{+-}	2^{+-}	10.32(7)	10.10(7)
A_1^{+-}	0^{+-}	11.66(7)	11.57(12)

Chen *et al.*,PRD73(2006)014516.

Morningstar & Peardon, PRD60(1999)034509.

Mass(0^{--})=(5166±1000) MeV (Unquenched)

Gregory, *et al.*, JHEP1210(2012)170.

Current Status of Glueballs

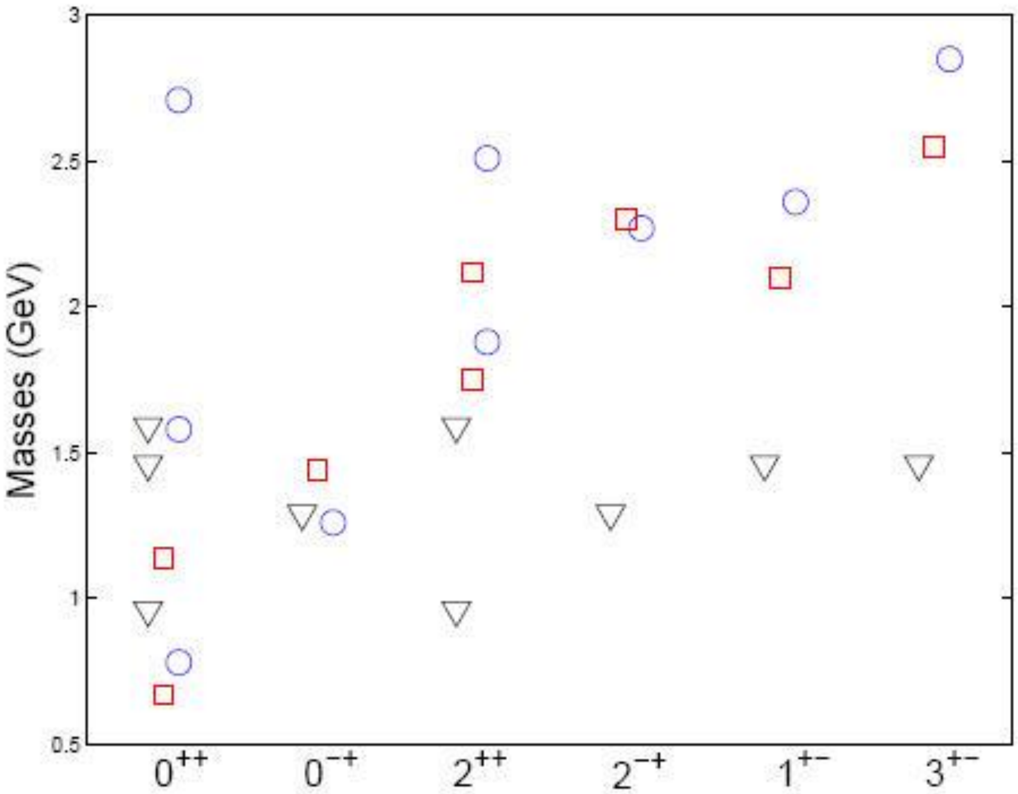
- Flux tube model

J^{PC}	Mass (GeV)
0^{++}	1.52
1^{+-}	2.25
0^{++}	2.75
$0^{++}, 0^{+-}, 0^{-+}, 0^{--}$	2.79
2^{++}	2.84
$2^{++}, 2^{++}, 2^{++}, 2^{++}$	2.84
1^{+-}	3.25
3^{+-}	3.35

Isgur & Parton, PRD31(1985)2910.

Current Status of Glueballs

- MIT bag model



▽ =Jaffe &Johnson, PLB60,201(1976).

◯=Carlson *et al.*, PRD27 (1983)1556.

□ =Chanowitz &Sharpe, NPB222(1983)211.

Current Status of Glueballs

- Coulomb Gauge model

Model	J^{PC}	0^{-+}	1^{--}	2^{--}	3^{--}	5^{--}	7^{--}
	color	f	d	d	d	d	d
	S	0	1	2	3	3	3
	L	0	0	0	0	2	4
H^g_{eff} (this work)		3900	3950	4150	4150	5050	5900
H_M (this work)		3400	3490	3660	3920	5150	6140

Llances-Estrada, Bicudo &Cotanch, PRL96(2006)081601

Current Status of Glueballs

- QCD Sum Rules

Two-gluon glueballs in QCDSR

	Novikov *et.al.*	Forkel	Bagan *et.al.*	Huang *et.al*
0^{++}	0.7-0.9 GeV	1.25 GeV	1.7 GeV	1.66 GeV
0^{-+}	-	2.2 GeV	-	-

Novikov et al., NPB165(1980)67.
Bagan&Steele, PLB243(1990)43.
Forkel, PRD64(2001)034015.
Huang, Jin&Zhang, PRD59(1999)034026.

Tri-gluon glueballs in QCDSR

	0^{++}	0^{-+}	1^{-+}	1^{--}	2^{++}
Latorre *et. al.*	3.1 GeV	-	-	-	-
Liu *et. al.*	1.45 GeV	-	1.87 GeV	2.4 GeV	2.0 GeV
Hao *et. al.*	-	1.9-2.7 GeV	-	-	-

Latorre *et al*., PLB191(1987)437. Liu, CPL15(1998)784. Hao et al., PLB642(2006)53.

Current Status of Glueballs

- Production studies of glueballs via Lattice QCD. For example:
  - *Scalar glueball in radiative J/ψ decay on lattice*,
    Long-Cheng Gui, *et al*., (CLQCD Collaboration),
    Phys. Rev. Lett. 110, 021601 (2013).
  - *Lattice study of radiative J/ψ decay to a tensor glueball*,
    Yi-Bong Yang, *et al*., (CLQCD Collaboration),
    Phys. Rev. Lett. 111 (2013) 9, 091601.
- Decay analysis of glueballs. For example:
  - *Comment on "Chiral Suppression of Scalar-Glueball Decay"*,
    Kuang-Ta Chao, Xiao-Gang He, and Jian-Ping Ma,
    Phys. Rev. Lett. 98, 149103 (2007).
  - *On Two-Body Decays of A Scalar Glueball*,
    Kuang-Ta Chao, Xiao-Gang He, and Jian-Ping Ma,
    Eur. Phys. J. C55: 417-421(2008).

0^{--} Oddballs : Why important?

- Oddballs

Oddballs: glueballs with exotic quantum numbers

$$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+} \text{ and so on}$$

Physics at BESIII, Editors Kuang-Ta Chao & Yifang Wang, Int. JMPA24,1,(2009).

V.Mathieu, N.Kochelev&V.Vento, Int.J.Mod.Phys. E18,1(2009).

- $C = -1$ →Trigluon glueballs.

- Exotic quantum numbers ➡ Do not mix with $q\bar{q}$

- 0^{--} oddball may be the lowest lying one. Besides, it has the simplest Lorentz structure.

0^{--} Oddballs : Why important?

- It can be produced in the decay of heavy vector quarkonium or quarkoniumlike states.

[yellow] = exotic state (not easy to be detected, only $\pi_1(1400)$ with 1^{-+} in PDG)

[red] = unfavorable production channel

Like in football [yellow] + [yellow] = [red]

$$1^{--} \to G_{0^{--}} + 1^{++}$$

$$1^{--} \to G_{2^{+-}} + 1^{-+}/2^{-+}/3^{-+}$$

$$1^{--} \to G_{0^{+-}} + 1^{-+}$$

$$1^{--} \to G_{3^{-+}} + 2^{+-}$$

$$1^{--} \to G_{1^{-+}} + 0^{--}$$

$$1^{++} = f_1(1285)/\chi_{c1}(3511)/\chi_{b1}(10255)$$

0^{--} Oddballs :QCDSR

➤QCDSR

- The two-point correlation function

$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\Big\{j_{0^{--}}(x), j_{0^{--}}(0)\Big\}|0\rangle\ ,$$

- The QCD side of the correlation function

$$\Pi^{\rm QCD}(Q^2) = a_0 Q^{12}\ln\frac{Q^2}{\mu^2} + b_0 Q^8\langle\alpha_s G^2\rangle + \left(c_0 + c_1\ln\frac{Q^2}{\mu^2}\right)Q^6\langle g_s G^3\rangle + d_0 Q^4\langle\alpha_s G^2\rangle^2\ .$$

- The phenomenological side of the correlation function

$$\frac{1}{\pi}{\rm Im}\Pi^{\rm phe}(s) = f_G^2 M_{0^{--}}^{12}\delta(s - M_{0^{--}}^2) + \rho(s)\theta(s - s_0)\ .$$

0^{--} Oddballs :QCDSR

- The dispersion relation

$$\Pi(Q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s+Q^2} + \Big(\Pi(0) - Q^2\Pi'(0) + \frac{1}{2}Q^4\Pi''(0) - \frac{1}{6}Q^6\Pi'''(0)\Big) ,$$

- The Borel transformation

$$\hat{B}_\tau \equiv \lim_{\substack{Q^2\to\infty, n\to\infty \\ \frac{Q^2}{n}=\frac{1}{\tau}}} \frac{(-Q^2)^n}{(n-1)!}\left(\frac{d}{dQ^2}\right)^n ,$$

- The quark-hadron duality approximation

$$\frac{1}{\pi}\int_{s_0}^\infty e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds \simeq \int_{s_0}^\infty \rho(s)e^{-s\tau}ds ,$$

0^{--} Oddballs :QCDSR

- The moments

$$L_0(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds \ ,$$
$$L_1(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} se^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds \ ,$$

- The mass function

$$M^i_{0^{--}}(\tau, s_0) = \sqrt{\frac{L_1(\tau, s_0)}{L_0(\tau, s_0)}}$$

- Ratios to constrain the windows of τ

$$R_i^{\mathrm{OPE}} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\langle \mathrm{g_s G^3}\rangle}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds}$$

$$R_i^{\mathrm{PC}} = \frac{L_0(\tau, s_0)}{L_0(\tau, \infty)} \ .$$

0^{--} Oddballs

➢ Interpolating currents of 0^{--} oddballs

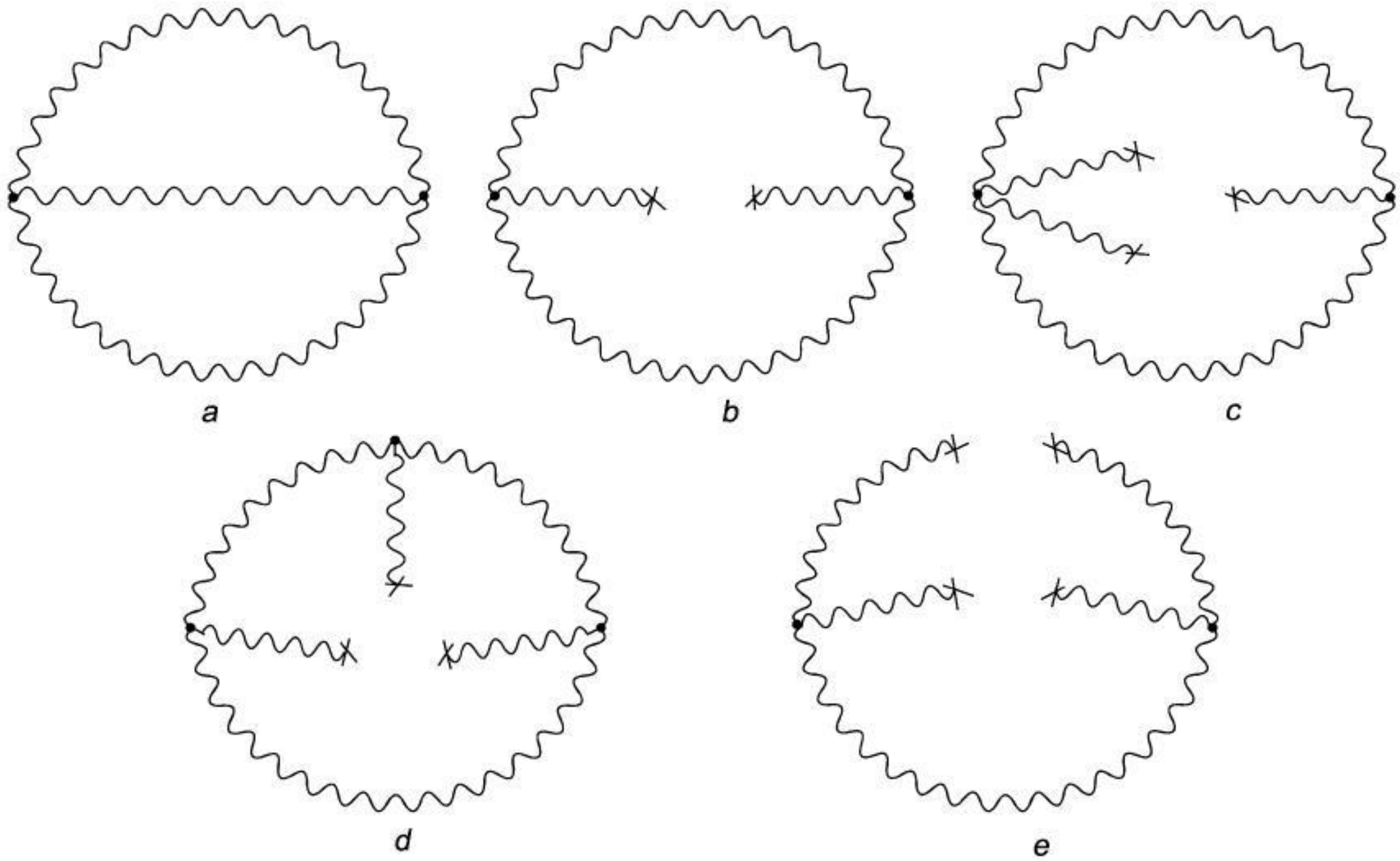
- Constraints: quantum number, gauge invariance, Lorentz invariance and $SU_c(3)$ symmetry

$$j^A_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,,$$
$$j^B_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,,$$
$$j^C_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,,$$
$$j^D_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,,$$

where $g^t_{\alpha\beta} = g_{\alpha\beta} - \partial_\alpha\partial_\beta/\partial^2 \quad \tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\kappa\tau}G^a_{\kappa\tau}$

0^{--} Oddballs

- Typical Feynman diagrams of trigluon glueballs



0^{--} Oddballs

- Wilson coefficients in the QCD-side

$$
\begin{aligned}
a_0^i &= \frac{487\alpha_s^3}{143 \times 2^6 \times 3^3 \pi}\ ,\ b_0^i = -\frac{5\pi}{36}\alpha_s^2\ , c_0^A = -\frac{205}{12}\pi\alpha_s^3\ , \\
c_1^A &= -\frac{775}{144}\pi\alpha_s^3\ ,\ c_0^B = -\frac{2065}{48}\pi\alpha_s^3\ , c_1^B = -\frac{1075}{96}\pi\alpha_s^3\ , \\
c_0^C &= \frac{2275}{72}\pi\alpha_s^3\ ,\ c_1^C = \frac{2125}{144}\pi\alpha_s^3\ ,\ c_0^D = -\frac{1045}{144}\pi\alpha_s^3\ , \\
c_1^D &= -\frac{25}{32}\pi\alpha_s^3\ ,\ d_0^j = 0\ ,\ d_0^D = -\frac{5}{9}\pi^3\alpha_s\ ,
\end{aligned}
$$

where, i=A, B, C, D; j=A, B, C; with A, B, C and D corresponding to the above four currents.

There are symmetries within Wilson coefficients a_0^i , b_0^i and d_0^j . The position and number of $\tilde{G}$ do not influence the perturbative and $\langle \alpha_s G^2 \rangle$ contributions, whereas they influence $\langle g_s G^3 \rangle$ term. Since $\langle \alpha_s G^2 \rangle^2$ involves no loop contribution, d_0^j are governed by the number of $\tilde{G}$.

0^{--} Oddballs

- Figures for case-A

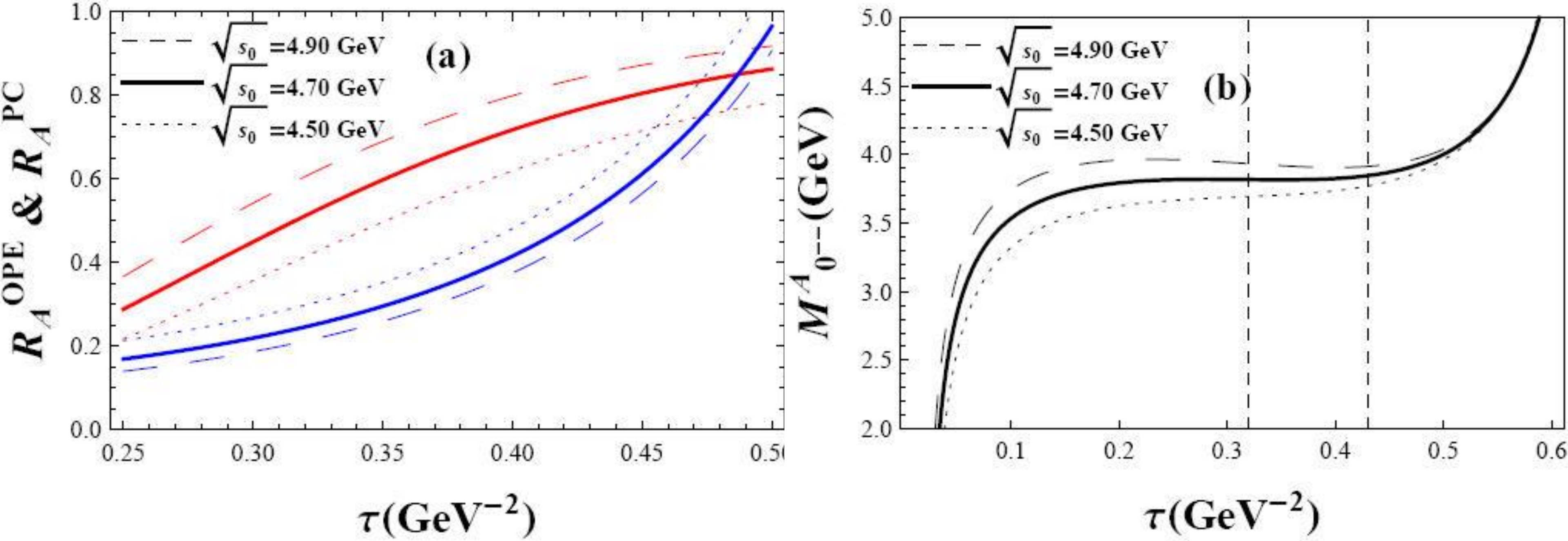


FIG. 1: (a) The ratios R_A^{OPE} and R_A^{PC} in case-A as functions of Borel parameter τ for different values of $\sqrt{s_0}$, where blue lines represent R_A^{OPE} and red lines denote R_A^{PC}. (b) The mass $M^A_{0^{--}}$ as function of the Borel parameter τ for different values of $\sqrt{s_0}$, where the two vertical lines indicate the upper and lower limits of the valid Borel window.

0^{--} Oddballs

- Figures for case-B

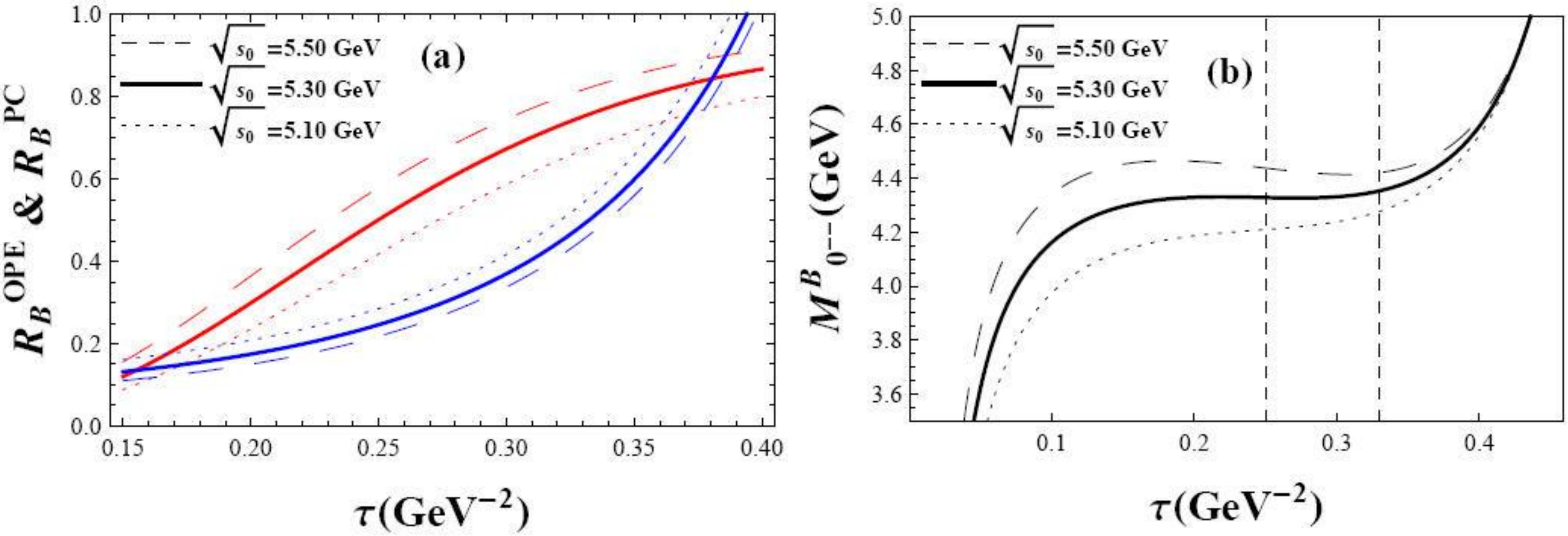


FIG. 2: The same caption as in Figure 1, but for case-B.

0^{--} Oddballs

- Figures for case-C

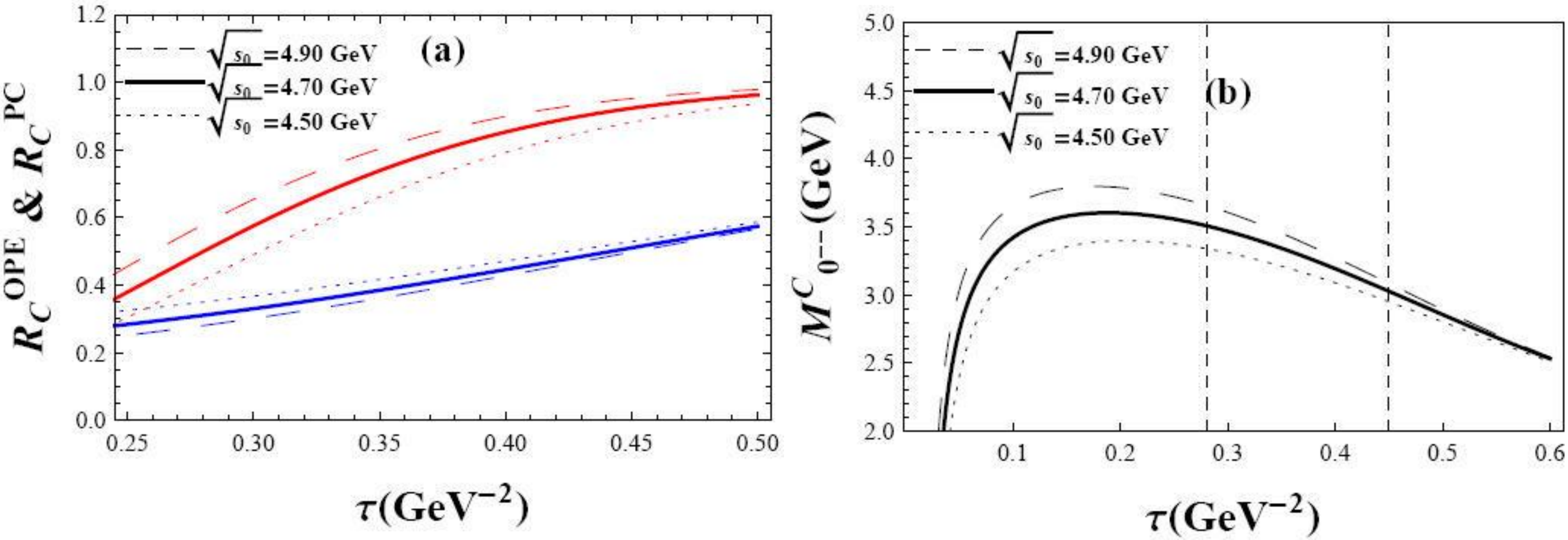


FIG. 3: The same caption as in Figure 1, but for case-C.

0^{--} Oddballs

- Figures for case-D

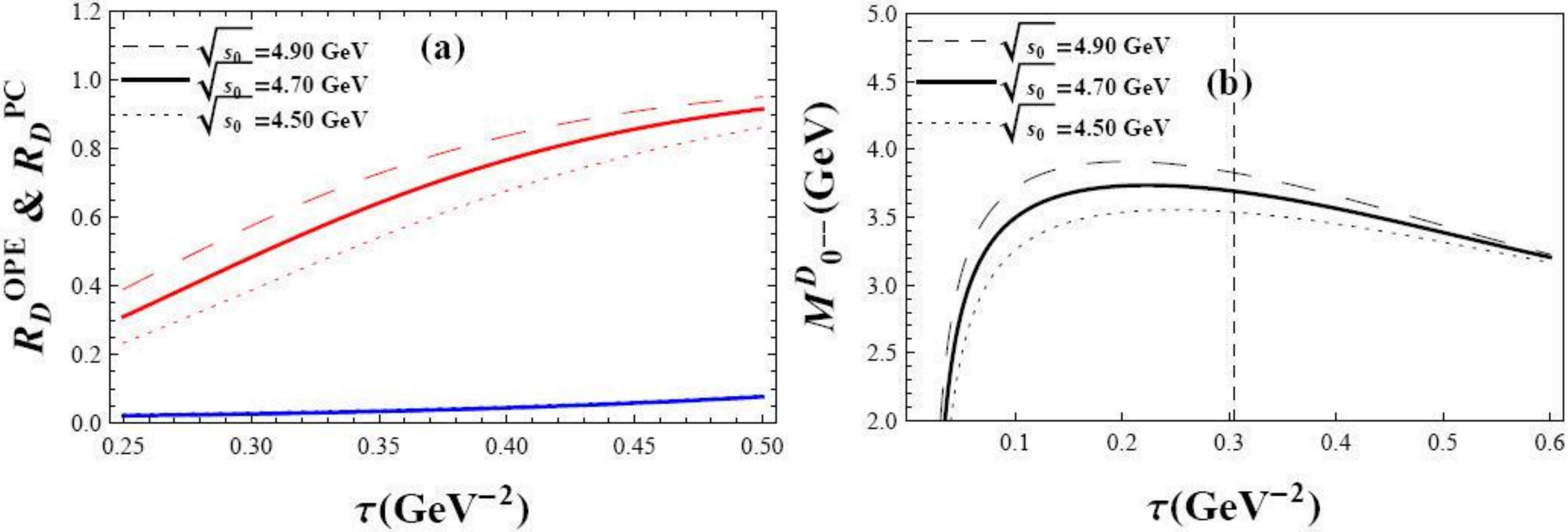


FIG. 4: The same caption as in Figure 1, but for case-D. Here the single vertical line indicates the lower limit of the valid Borel window while the upper limit is out of the region.

0^{--} **Oddballs**

- Masses of 0^{--} oddballs

$$M^A_{0^{--}} = 3.81 \pm 0.12\,\mathrm{GeV},$$

$$M^B_{0^{--}} = 4.33 \pm 0.13\,\mathrm{GeV},$$

- Nominate the above two 0^{--} oddballs as follows

$$M^A_{0^{--}} \Rightarrow G_{0^{--}}(3810)$$

$$M^B_{0^{--}} \Rightarrow G_{0^{--}}(4330)$$

0^{--} Oddballs

- Compare the lighter one with Flux tube model:

$$G_{0^{--}}(3810) > 2.79\,\mathrm{GeV}$$

Isgur & Parton, PRD31(1985)2910.

- Compare the heavier one with Lattice QCD:

$$G_{0^{--}}(4330) < (5166 \pm 1000)\,\mathrm{MeV}$$

Gregory, *et al.*, JHEP1210(2012)170.

0^{--} Oddballs

- Proposed production channels (Taking the lighter one as an example)

$$X(3872) \to \gamma + G_{0^{--}}(3810), \qquad \Upsilon(1S) \to f_1(1285) + G_{0^{--}}(3810),$$

$$\Upsilon(1S) \to \chi_{c_1} + G_{0^{--}}(3810), \qquad \chi_{b_1} \to J/\psi + G_{0^{--}}(3810),$$

$$\chi_{b_1} \to \omega + G_{0^{--}}(3810).$$

- Proposed decay channels

$$G_{0^{--}}(3810) \to \gamma + f_1(1285),$$

$$G_{0^{--}}(3810) \to \omega + f_1(1285).$$

$$G_{0^{--}}(3810) \to \gamma + \chi_{c_1},$$

Experimentalists' Attentions

- BESIII Collaboration, Changzheng Yuan, Ronggang Ping, Jinzhi Zhang, Xiaorui Lu, *et al*.
- Belle Collaboration, Chengping Shen.
- LHCb Collaboration, Paolo Gandini.
- phys.org,``*Long-searched-for glueball could soon be detected*", by Lisa Zyga.

Summary

- We obtained two stable 0^{--} oddballs with masses about 3.81 and 4.33 GeV.

- Oddballs can in principle mix with hybrids and tetraquark states, though naively the OZI suppression may hinder the mixing in certain degree.

- We briefly analyzed the 0^{--} oddball optimal production and decay mechanism. They are expected to be measured in BESIII, BELLEII, Super-B, PANDA, and LHCb experiments.

Outlook

- We are calculating the other oddballls ($0^{+-}, 1^{-+}, 2^{+-}$ & 3^{-+}), (unfinished)

- We hope to see that more works on their production and decay can be exploited in the theory.

- We hope that they will be observed by experiments in the future.

Thank you！