BB*/DD* interaction in a Bethe-Salpeter equation approach

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OUTLINE

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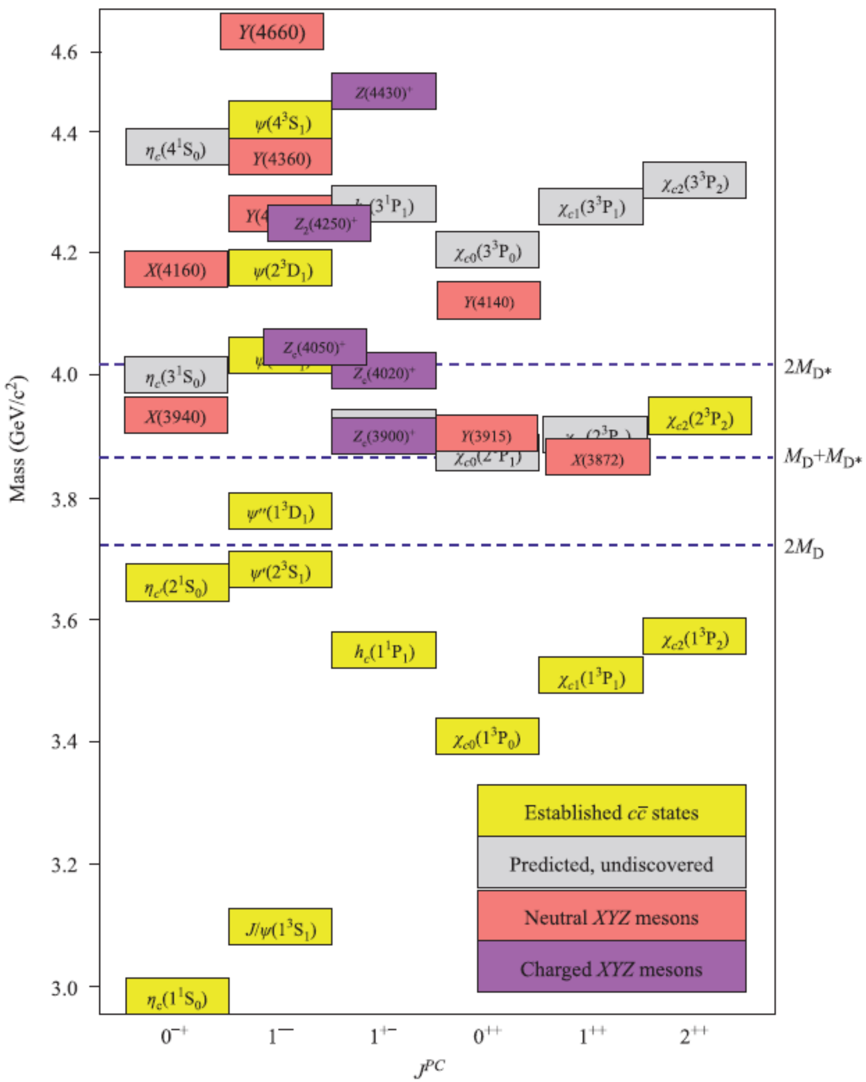
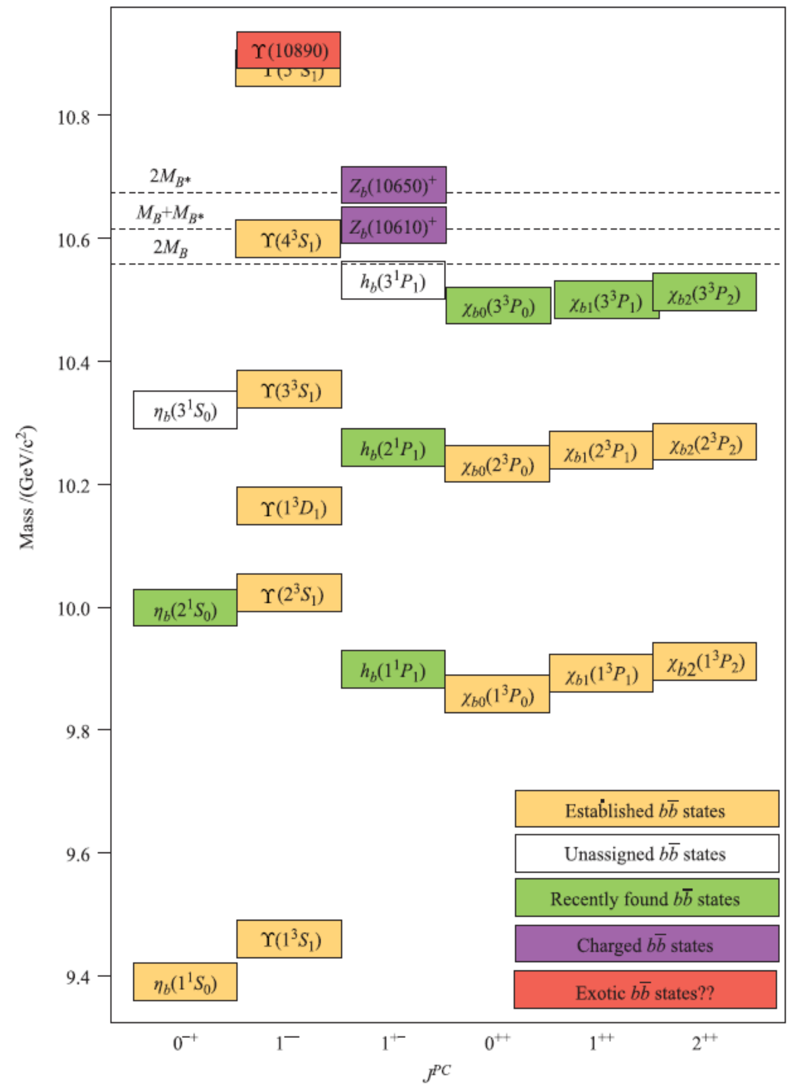
INTRODUCTION





Olsen,Front. Phys. 10, 101401 (2015)

Structure near DD*(M_{DD*}=3871.81 MeV)

	Decay modes								
	$J/\psi\pi^+\pi^-$	$J/\psi\pi^+\pi^-\pi^0$ ($J/\psi\omega$)	$J/\psi\eta$	$D^0\bar{D}^0\pi^0$	$D^{*0}\bar{D}^0$	$\gamma J/\psi$	$\gamma\psi'$	Mass (MeV)	J^{PC}
Belle-1	■							$3872.0 \pm 0.6 \pm 0.5$	
Belle-2		■				■		–	
Belle-3				■				$3875.2 \pm 0.7^{+0.3}_{-1.6} \pm 0.8$	
Belle-4	■							$3871.46 \pm 0.37 \pm 0.07$	
Belle-5					■			$3872.9^{+0.3+0.5}_{-0.6-0.5}$	
Belle-6						■	■	–	
BaBar-1	■							3873.4 ± 1.4	
BaBar-2			□					–	
BaBar-3	■							–	
BaBar-4	■							$3871.3 \pm 0.6 \pm 0.1$ (B^-) $3868.6 \pm 1.2 \pm 0.2$ (B^0)	
BaBar-5				■				–	
BaBar-6						■		–	
BaBar-7					■			$3875.1^{+0.5}_{-0.7} \pm 0.5$	
BaBar-8	■							$3871.4 \pm 0.6 \pm 0.1$ (B^+) $3868.7 \pm 1.5 \pm 0.4$ (B^0)	
BaBar-9						■	■	–	
BaBar-10		■						$3873.0^{+1.8}_{-1.6} \pm 1.3$	2^{-+}
CDF-1	■							$3871.3 \pm 0.7 \pm 0.4$	
CDF-2	■							–	
CDF-3	■							–	$1^{++}/2^{-+}$
CDF-4	■							$3871.61 \pm 0.16 \pm 0.19$	
DØ	■							$3871.8 \pm 3.1 \pm 3.0$	
LHCb-1	■							–	1^{++}
LHCb-2	■							$3871.95 \pm 0.48 \pm 0.12$	
CMS	■							–	
BESIII						■		$3891.9 \pm 0.7 \pm 0.2$	
$m(D^0D^{*0}) = 3871.81 \pm 0.36$ MeV					PDG average mass of $X(3872)$: 3871.68 ± 0.17 MeV				

X(3872) M=3871.68±0.17MeV

Experiments	Mass (MeV)	Width (MeV)
BESIII [17]	3899.0 ± 3.6 ± 4.9	46 ± 10 ± 20
Belle [113]	3894.5 ± 6.6 ± 4.5	63 ± 24 ± 26
Xiao *et al.* [183]	3886 ± 4 ± 2	37 ± 4 ± 8

Zc(3900) M=3890±3 MeV

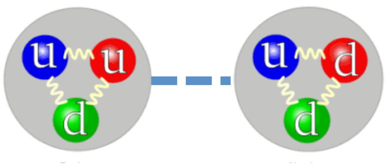
Structure near BB*(M_{DD*}=10604.4 MeV)

	$Z_b(10610)$	
Channels	Mass	Width
$\Upsilon(1S)\pi^\pm$	$10609 \pm 3 \pm 2$	$22.9 \pm 7.3 \pm 2$
$\Upsilon(2S)\pi^\pm$	$10616 \pm 2^{+3}_{-4}$	$21.1 \pm 4^{+2}_{-3}$
$\Upsilon(3S)\pi^\pm$	$10608 \pm 2^{+5}_{-2}$	$12.2 \pm 1.7 \pm 4$
$h_b(1P)\pi^\pm$	$10605.1 \pm 2.2^{+3.0}_{-1.0}$	$11.4^{+4.5}_{-3.9}{}^{+2.1}_{-1.2}$
$h_b(2P)\pi^\pm$	$10596 \pm 7^{+5}_{-2}$	$16^{+16}_{-10}{}^{+13}_{-4}$
Thresholds	$m_{B\bar{B}^*} = 10604.4$	

Zb(10610)
M=10607.2±2.0MeV

Hadronic molecules

Familiar examples:



Deuteron

weakly bound states of hadrons

Mass slightly below threshold: $m_H < m_{H1} + m_{H2}$

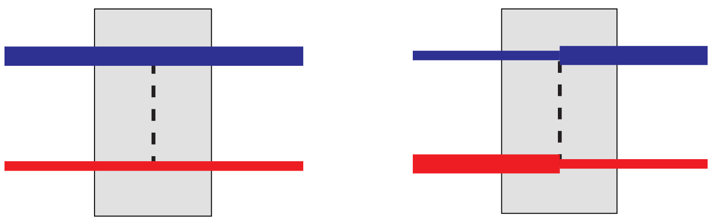
	X(3872)	Zc(3900)	Zb(10610)
OBE	Tornqvist, PLB 590,(2004) 209 Liu, et al., EPJC **56**(2008) 63 Lee, et al., PRD **80**(2009) 094005 Li and Zhu, PRD **86**(2012) 074022 ……	Sun et al. PRD84(2011)054002	Liu et al., EPJC 56(2008)63 Liu et al., EPJC61(2009)411 Sun et al. PRD84(2011)054002
QCD sum rule	Wang et al., EPJC74(2014)2891 Chen PRD88(2013)045027 Narison PRD83(2011)016004	Wang et al., EPJC74(2014)2891 Zhang et al., PRD87(2013)116004 Cui et al., JPG41(2014) 075003	Wang et al., EPJC74(2014)2891 Cui et al. PRD85(2012)074014 Zhang et al. PLB704(2011)312
Lattice	Prelovsek et al.,PRL111 (2013) 192001 Lee, et al. arXiv, 1411.1389(2014)	Prelovsek, et al., PLB727 (2013) 172 Prelovsek , et al. Phys.Rev. D91 (2015) 01450 Chen et al. Phys. Rev. D 89, 094506 (2014) Lee, et al. arXiv: 1411.1389(2014)	
	……		

OGE model

Sun et al. PRD84(2011)054002

$$-\left(\frac{\nabla^2}{\mu}+\epsilon\right)\psi = -V\psi$$

$$V(\mathrm{q}) = -\frac{1}{\sqrt{\prod_i 2M_i \prod_f 2M_f}}\mathcal{M}(J, J_Z)$$



$$\begin{cases} |Z^{(T)}_{B\bar{B}^*}{}^{+}\rangle = \frac{1}{\sqrt{2}}\left(|B^{*+}\bar{B}^0\rangle + cB^+\bar{B}^{*0}\right), \\ |Z^{(T)}_{B\bar{B}^*}{}^{-}\rangle = \frac{1}{\sqrt{2}}\left(|B^{*-}\bar{B}^0\rangle + cB^-\bar{B}^{*0}\right), \\ |Z^{(T)}_{B\bar{B}^*}{}^{0}\rangle = \frac{1}{2}\Big[\left(|B^{*+}B^-\rangle - B^{*0}\bar{B}^0\right) + c\left(B^+B^{*-} - B^0\bar{B}^{*0}\right)\Big], \end{cases}$$

$$|Z^{(S)}_{B\bar{B}^*}{}^{0}\rangle = \frac{1}{2}\Big[\left(|B^{*+}B^-\rangle + B^{*0}\bar{B}^0\right) + c\left(B^+B^{*-} + B^0\bar{B}^{*0}\right)\Big],$$

$$\begin{aligned} \mathcal{L}_{HH\mathbb{P}} &= ig\langle H_b^{(Q)}\gamma_\mu A^\mu_{ba}\gamma_5\bar{H}_a^{(Q)}\rangle + ig\langle \bar{H}_a^{(\bar{Q})}\gamma_\mu A^\mu_{ab}\gamma_5 H_b^{(\bar{Q})}\rangle, \\ \mathcal{L}_{HH\mathbb{V}} &= i\beta\langle H_b^{(Q)}v_\mu(\mathcal{V}^\mu_{ba}-\rho^\mu_{ba})\bar{H}_a^{(Q)}\rangle + i\lambda\langle H_b^{(Q)}\sigma_{\mu\nu}F^{\mu\nu}(\rho)\bar{H}_a^{(Q)}\rangle \\ &\quad -i\beta\langle \bar{H}_a^{(\bar{Q})}v_\mu(\mathcal{V}^\mu_{ab}-\rho^\mu_{ab})H_b^{(\bar{Q})}\rangle + i\lambda\langle H_b^{(\bar{Q})}\sigma_{\mu\nu}F'^{\mu\nu}(\rho)\bar{H}_a^{(\bar{Q})}\rangle, \\ \mathcal{L}_{HH\sigma} &= g_s\langle H_a^{(Q)}\sigma\bar{H}_a^{(Q)}\rangle + g_s\langle \bar{H}_a^{(\bar{Q})}\sigma H_a^{(\bar{Q})}\rangle, \end{aligned}$$

Table 6. A summary of the $\mathrm{B\bar{B}^*}$, $\mathrm{B^*\bar{B}^*}$, $\mathrm{D\bar{D}^*}$, $\mathrm{D^*\bar{D}^*}$ systems. Here, we use ✓ and × to mark the corresponding systems with and without the bound states solution when taking a reasonable Λ value, respectively. The criteria of the choice of the reasonable Λ may be strongly biased.

$I^G(J^P)$	system	remark	experiment [36]	system	remark	experiment
$1^+(1^+)$	$Z^{(\mathrm{T})}_{\mathrm{B\bar{B}^*}}$	✓	$\mathrm{Z_b(10610)}$	$Z^{(\mathrm{T})}_{\mathrm{D\bar{D}^*}}$	×	
$0^-(1^{+-})$	$Z^{(\mathrm{S})}_{\mathrm{B\bar{B}^*}}$	✓		$Z^{(\mathrm{S})}_{\mathrm{D\bar{D}^*}}$	✓	
$1^-(1^+)$	$Z^{(\mathrm{T})\prime}_{\mathrm{B\bar{B}^*}}$	×		$Z^{(\mathrm{T})\prime}_{\mathrm{D\bar{D}^*}}$	×	
$0^+(1^{++})$	$Z^{(\mathrm{S})\prime}_{\mathrm{B\bar{B}^*}}$	✓		$Z^{(\mathrm{S})\prime}_{\mathrm{D\bar{D}^*}}$	✓	$X(3872)$ [58]
$1^-(0^+)$	$Z^{(\mathrm{T})}_{\mathrm{B^*\bar{B}^*}}$ [0]	✓		$Z^{(\mathrm{T})}_{\mathrm{D^*\bar{D}^*}}$ [0]	×	
$0^+(0^{++})$	$Z^{(\mathrm{S})}_{\mathrm{B^*\bar{B}^*}}$ [0]	✓		$Z^{(\mathrm{S})}_{\mathrm{D^*\bar{D}^*}}$ [0]	✓	$Y(3930)$ [63–65]
$1^+(1^+)$	$Z^{(\mathrm{T})}_{\mathrm{B^*\bar{B}^*}}$ [1]	✓	$\mathrm{Z_b(10650)}$	$Z^{(\mathrm{T})}_{\mathrm{D^*\bar{D}^*}}$ [1]	×	
$0^-(1^{+-})$	$Z^{(\mathrm{S})}_{\mathrm{B^*\bar{B}^*}}$ [1]	✓		$Z^{(\mathrm{S})}_{\mathrm{D^*\bar{D}^*}}$ [1]	✓	
$1^-(2^+)$	$Z^{(\mathrm{T})}_{\mathrm{B^*\bar{B}^*}}$ [2]	×		$Z^{(\mathrm{T})}_{\mathrm{D^*\bar{D}^*}}$ [2]	×	
$0^+(2^{++})$	$Z^{(\mathrm{S})}_{\mathrm{B^*\bar{B}^*}}$ [2]	✓		$Z^{(\mathrm{S})}_{\mathrm{D^*\bar{D}^*}}$ [2]	✓	$Y(3940)$ [63–65]

Other interpretations

	X(3872)	Zc(3900)	Zb(10610)
Tetraquark	Hogaasen et al., PRD73(2006)054013 Ebert et al., PLB634,(2006)214 Matheus et al. PRD75(2007)014005 ……	Qiao et al. EPJC74(2014)3122 Deng et al. PRD90(2014)054009	Wang, NPA930(2014)63-85 Qiao et al. EPJC74(2014)3122 Ali et al. PRD85(2012)054011 Cui et al. PRD85(2012)074014
Cusp effect	Dugg, PLB598(2004)8	Swanson PRD91(2015)034009	Swanson PRD91(2015)034009
ISPE		Chen, PRD88(2011)036008	Chen, et al., PRD84(2011)094003 Chne, et al.,PRD84(2011)074016
Threshold effect	Rosner, PRD74(2006)076006		
……			

BETHE-SALPETER EQUATION

The **Bethe–Salpeter equation** is a general quantum field theoretical tool, thus applications for it can be found in any quantum field theory.

- positronium (bound state of an electron–positron pair),
- excitons (bound state of an electron–hole pair),
- meson as quark-antiquark bound-state,
- ……

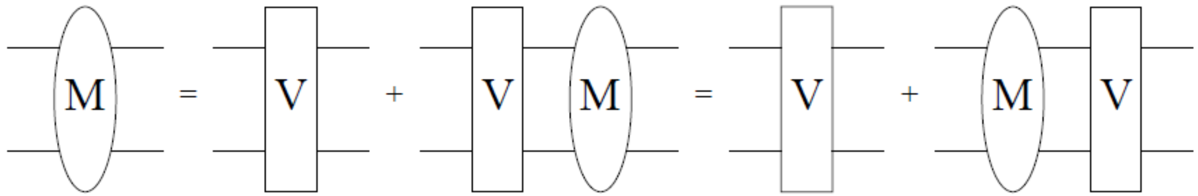
Hadronic molecular state:

- KK*: X.H.Guo et al.
- Zc & Zb: K. W. Wei et al. JHEP 2012(2012)56
  K. W. Wei et al. EPJC73(2013)2561
- Oset et al. Dias and Oset, arXiv: 1410.1785
  Aceti et al. PRD (2014)016003
  ……

No cross diagram

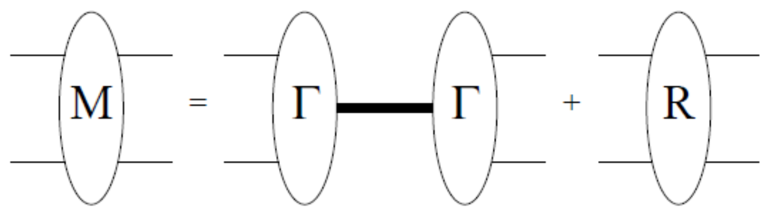
BS equation For vertex

Bethe-Salpeter equation:

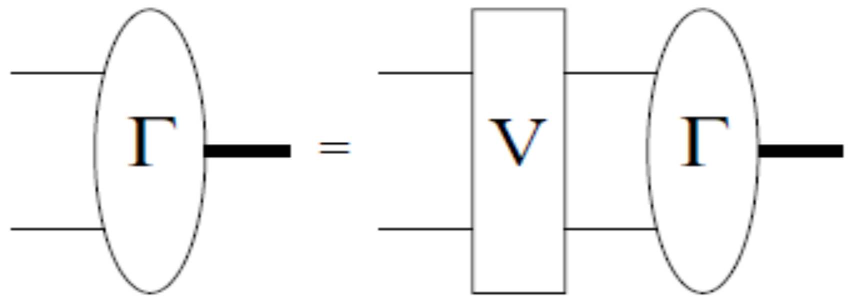


$$\mathcal{M} = V - V G_{BS} \mathcal{M} = V - \mathcal{M} G_{BS} V$$

Bound state:



$$\mathcal{M} = \frac{|\Gamma >< \Gamma|}{P^2 - M^2} + \mathcal{R},$$



The BSE of the vertex function Γ for a system composed of a vector meson and a pseudoscalar meson (marked as constituent 1 and 2) is written explicitly as

$$|\Gamma^{\mu}\rangle = \mathcal{V}^{\mu\nu}\ G_{\nu\mu'}\ |\Gamma^{\mu'}\rangle,$$

where the propagator is

$$G^{\mu'\mu} = G_1^{\mu'\mu}\ G_2 = \frac{-P_1^{\mu'\mu}}{(k_1^2 - m_1^2)(k_2^2 - m_2^2)} \equiv P_1^{\mu'\mu} G_0,$$

where $P_1^{\mu\nu} = -g^{\mu\nu} + \frac{k_1^{\mu} k_1^{\nu}}{m_1^2}$ and $k_{1,2}$ and $m_{1,2}$ are the momentum and mass for constituent 1 or 2.

Flavor structure

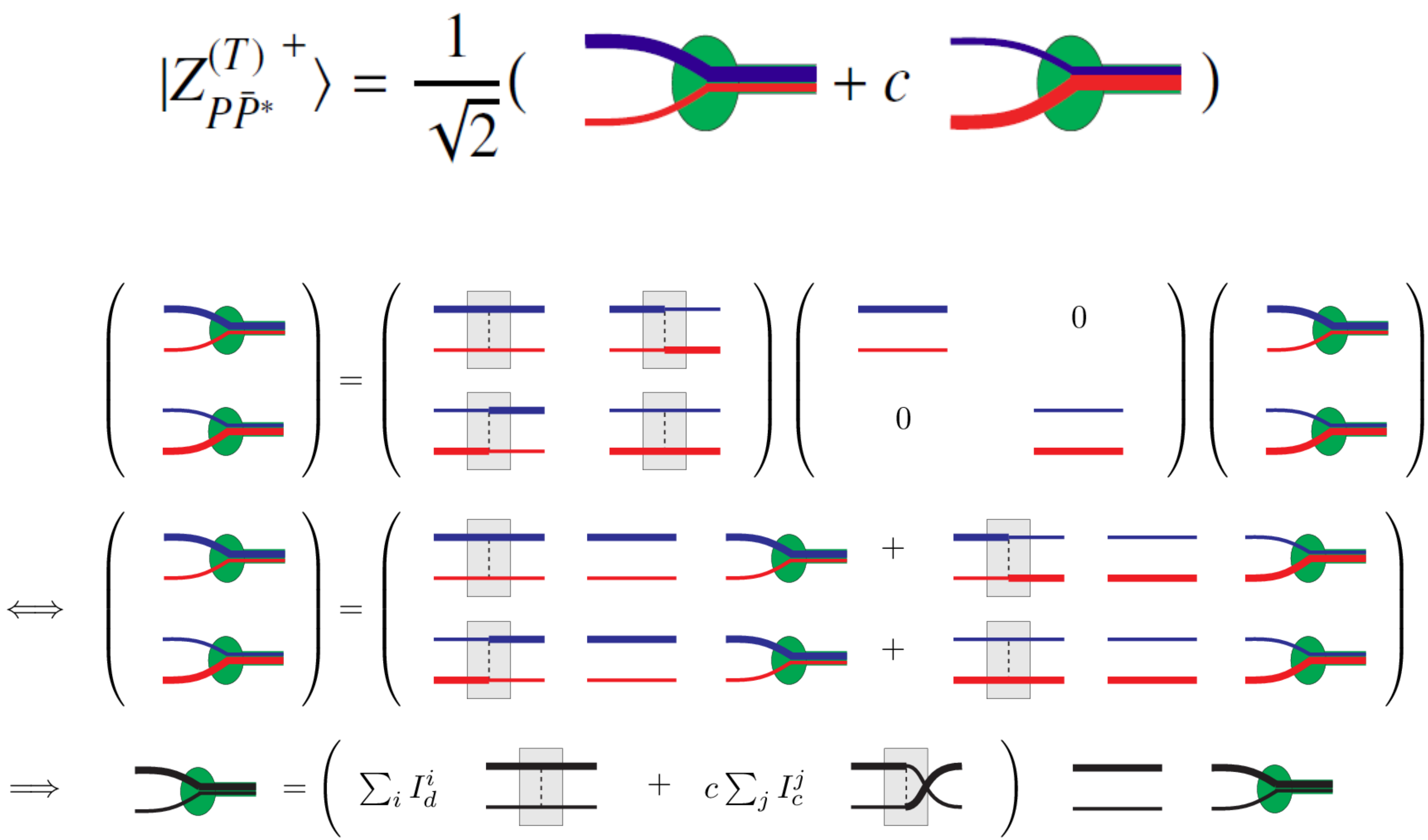
The $P\bar{P}^*$ systems (we mark B and D as P) can be categorized as the isovector (T) and isoscalar (S) states under SU(3) symmetry with the corresponding flavor wave functions

$$\begin{cases} |Z_{P\bar{P}^*}^{(T)}{}^{+}\rangle = \frac{1}{\sqrt{2}}\left(|P^{*+}\bar{P}^{0}\rangle + cP^{+}\bar{P}^{*0}\right), \\ |Z_{P\bar{P}^*}^{(T)}{}^{-}\rangle = \frac{1}{\sqrt{2}}\left(|P^{*-}\bar{P}^{0}\rangle + cP^{-}\bar{P}^{*0}\right), \\ |Z_{P\bar{P}^*}^{(T)}{}^{0}\rangle = \frac{1}{2}\Big[\big(|P^{*+}P^{-}\rangle - P^{*0}\bar{P}^{0}\big) + c\big(P^{+}P^{*-} - P^{0}\bar{P}^{*0}\big)\Big], \end{cases}$$

$$|Z_{P\bar{P}^*}^{(S)}{}^{0}\rangle = \frac{1}{2}\Big[\big(|P^{*+}P^{-}\rangle + P^{*0}\bar{P}^{0}\big) + c\big(P^{+}P^{*-} + P^{0}\bar{P}^{*0}\big)\Big],$$

where $c = \pm$ corresponds to C-parity $C = \mp$ respectively.

Isolate the flavor factor



direct cross

Quasipotential approximation

$$\mathcal{M} = \mathcal{V} + \mathcal{V} G \mathcal{M}$$

$$\Updownarrow$$

$$\begin{aligned} \mathcal{M} &= U - U g \mathcal{M} \\ U &= V - V \Delta G U, \\ \Delta G &= G_{BS} - g \end{aligned}$$

g:quasipotential propagator
U: quaipotential kernel.

$$g(p; W, \nu) = \frac{2\pi}{E_p} \delta(p_0 - \nu(E_p - W/2)) \frac{f(4E_p^2, W^2, \nu)}{4E_p^2 - W^2 - i\epsilon}$$

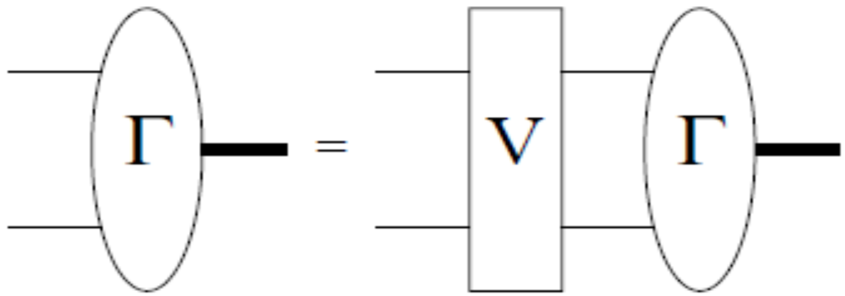
$$f(x, y, \nu) = \frac{x}{y} \cdot \frac{x + y}{(1 + \nu^2)x + (1 - \nu^2)y}$$

$f(x^2, y^2)$	$\delta(p^0)$	$\delta(p^0 - (E_p - W/2))$
1	Blankenbecler-Sugar	Erkelenz-Holinde
$\frac{x}{y}$	Todorov	--
$\frac{x+y}{2x}$	--	Kadychevsky
$\frac{x+y}{2y}$	Thompson	Gross

The propagator written down in the center of mass frame where $p = (W, \mathbf{0})$

$$G_0 = 2\pi i \frac{\delta^+(k_1^2 - m_1^2)}{k_2^2 - m_2^2} = 2\pi i \frac{\delta^+(k_1^0 - E_1))}{2E_1[(W - E_1)^2 - E_2^2]},$$

The heavier constituent is treated as on shell



For D*

$$P_1^{\mu\nu} = \sum_\lambda \epsilon_{1\lambda}^{\mu} \epsilon_{1\lambda}^{\nu}$$

$$|\Gamma_\lambda\rangle = \mathcal{V}_{\lambda\lambda'}\ G_0\ |\Gamma_{\lambda'}\rangle,$$

with $|\Gamma_\lambda\rangle = \epsilon_{1\lambda}^{\mu}|\Gamma_\mu\rangle$ and $\mathcal{V}_{\lambda\lambda'} = \epsilon_{1\lambda}^{\mu} \cdot \mathcal{V}_{\mu\nu} \cdot \epsilon_{1\lambda'}^{\nu}$.

BSE for wave function

After moving factor *A* to potential kernel and vertex,

$$A = \sqrt{2E_2(\boldsymbol{k})/[W - E_1(\boldsymbol{k}) + E_2(\boldsymbol{k})]}$$

$$\bar{\mathcal{V}}_{\lambda\lambda'} = A\mathcal{V}_{\lambda\lambda'}A', \quad |\bar{\Gamma}_\lambda\rangle = A|\Gamma_\lambda\rangle, \quad \text{and} \quad \bar{G}_0 = G_0/A^2$$

The vertex function can be related to the Bethe-Salpeterbound state wave function as $|\psi_\lambda\rangle = \bar{G}_0|\bar{\Gamma}_\lambda\rangle$

$$\bar{G}_0^{-1}\,|\psi_\lambda\rangle = \bar{\mathcal{V}}_{\lambda\lambda'}|\psi_{\lambda'}\rangle$$

Normalization

The normalization of the wave function can be obtained by the normalization of the vertex,

$$\begin{aligned} 1 &= i\frac{\langle\Gamma|G - G\cdot\mathcal{V}\cdot G|\Gamma\rangle}{p^2 - M^2} = i\frac{\langle\bar{\Gamma}|\bar{G}_0 - \bar{G}_0\bar{\mathcal{V}}\bar{G}_0|\bar{\Gamma}\rangle}{p^2 - M^2} \\ &= i\langle\bar{\Gamma}|(\bar{G}_0 - \bar{G}_0\bar{\mathcal{V}}\bar{G}_0)'|\bar{\Gamma}\rangle = i\langle\bar{\Gamma}|(\bar{G}_0)' - \bar{G}_0(\bar{\mathcal{V}})'\bar{G}_0|\bar{\Gamma}\rangle \\ &= i\langle\psi|(-iN^2 - \bar{\mathcal{V}}')|\psi\rangle. \end{aligned}$$

Here $\psi(k) \to 0$ when $k \to \infty$. If k is small A is stable compared with $\bar{G}_0$ at $W \approx m_1 + m_1$. As usual we assume the dependence of $\mathcal{V}$ on W is small. Hence $(\bar{\mathcal{V}})'$ is negligible.

The normalization of wave functions of bound state can be introduced as with

$$|\phi\rangle = N|\psi\rangle, \quad \text{and} \quad N = \frac{\sqrt{2E_1 2E_2}}{\sqrt{(2\pi)^5 2W}} \quad \text{with} \quad \int d^3k|\phi|^2 = 1.$$

The integral equation can be written explicitly as

$$(W - E_1(\boldsymbol{k}) - E_2(\boldsymbol{k}))\phi_\lambda(\boldsymbol{k}) = \int \frac{d\boldsymbol{k}'}{(2\pi)^3} V_{\lambda\lambda'}(\boldsymbol{k}, \boldsymbol{k}')\phi_{\lambda'}(\boldsymbol{k}')$$

with

$$V = \frac{i\bar{\mathcal{V}}(\boldsymbol{k}, \boldsymbol{k}')}{\sqrt{2E_1(\boldsymbol{k})2E_2(\boldsymbol{k})2E_1(\boldsymbol{k}')2E_2(\boldsymbol{k}')}}.$$

Partial wave expansion

$$\phi_\lambda^J(|\boldsymbol{k}|) = \sqrt{\frac{2J+1}{4\pi}} \int d\Omega D^J_{\lambda_R,\lambda}(\phi,\theta,-\phi)\phi_{\lambda,\lambda_R}(\boldsymbol{k})$$

$$V^J_{\lambda\lambda'}(|\boldsymbol{k}|,|\boldsymbol{k}|') = 2\pi \int d\cos\theta_{k,k'} d^J_{\lambda,\lambda'}(\theta_{k,k'}) V_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{k}')$$

One dimension integral eqaution

$$(W - E_1 - E_2)\phi_\lambda^J(|\boldsymbol{k}|) = \sum_{\lambda'} \int \frac{|\boldsymbol{k}'|^2 d|\boldsymbol{k}'|}{(2\pi)^3} V^J_{\lambda\lambda'}(|\boldsymbol{k}'|,|\boldsymbol{k}'|)\phi^J_{\lambda'}(|\boldsymbol{k}'|)$$

Helicity wave function and potential

$$\phi_\lambda = \phi_{-\lambda} \text{ for } 1^+, 0^-$$
$$\phi_\lambda = -\phi_{-\lambda} \text{ for } 1^-$$
$$V_{\lambda\lambda'} = V_{-\lambda-\lambda'}$$

so we choose
$\phi_1 = \sqrt{2}\phi_\pm$ and ϕ_0 for 1^+,
ϕ_0 for 0^+
$\phi_1 = \sqrt{2}\phi_\pm$ for 1^-.

$$(W - E_1 - E_2)\phi_i^J(|\boldsymbol{k}|) = \sum_j \int \frac{|\boldsymbol{k}'|^2 d|\boldsymbol{k}'|}{(2\pi)^3} V_{ij}^J(|\boldsymbol{k}'|, |\boldsymbol{k}'|)\phi_j^J(|\boldsymbol{k}'|)$$

$$V(1^+) = \begin{pmatrix} V_{11} + V_{1-1} & \sqrt{2}V_{10} \\ \frac{1}{\sqrt{2}}(V_{01} + V_{0-1}) & V_{00} \end{pmatrix}$$
$$V(1^-) = V_{11} - V_{1-1}$$
$$V(0^+) = V_{00}^0$$

POTENTIAL

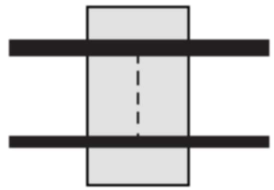
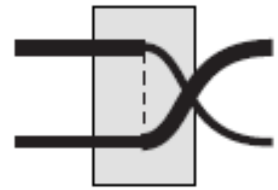
	Direct diagram			Crossed diagram			
							
Exchanged	$\mathbb{V}$		$(\mathbb{S})$	$(\mathbb{P})$		$(\mathbb{V})$	$(\mathbb{S})$
meson	ρ	ω	σ	π	η	ρ	ω
$Z^{(T)}_{P\bar{P}^*}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2}$
$Z^{(S)}_{P\bar{P}^*}$	$\frac{3}{2}$	$\frac{1}{2}$	1	$-\frac{3}{2}$	$\frac{1}{6}$	$\frac{3}{2}$	$\frac{1}{2}$

TABLE I: The flavor factors I_d^i and I_c^i for direct and cross diagrams and different exchange mesons. .

HQET Lagrangian

$$
\begin{aligned}
\mathcal{L}_{P^*P\mathbb{P}} &= -\frac{2g\sqrt{m_P m_{P^*}}}{f_\pi}(P_b P^{*\dagger}_{a\lambda} + P^*_{b\lambda}P^\dagger_a)\partial^\lambda \mathbb{P}_{ba} + \frac{2g\sqrt{m_P m_{P^*}}}{f_\pi}(\tilde{P}^{*\dagger}_{a\lambda}\tilde{P}_b + \tilde{P}^\dagger_a \tilde{P}^*_{b\lambda})\partial^\lambda \mathbb{P}_{ab}, \\
\mathcal{L}_{P^*P\mathbb{V}} &= -i\sqrt{2}\lambda g_V \varepsilon_{\lambda\alpha\beta\mu}(P^{*\mu\dagger}_a \overleftrightarrow{\partial}^\lambda P_b + P^\dagger_a \overleftrightarrow{\partial}^\lambda P^{*\mu}_b)(\partial^\alpha \mathbb{V}^\beta)_{ba} \\
&- i\sqrt{2}\lambda g_V \varepsilon_{\lambda\alpha\beta\mu}(\tilde{P}^{*\mu\dagger}_a \overleftrightarrow{\partial}^\lambda \tilde{P}_b + \tilde{P}^\dagger_a \overleftrightarrow{\partial}^\lambda \tilde{P}^{*\mu}_b)(\partial^\alpha \mathbb{V}^\beta)_{ab}, \\
\mathcal{L}_{PP\mathbb{V}} &= -i\frac{\beta g_V}{\sqrt{2}} P^\dagger_a \overleftrightarrow{\partial}_\mu P_b \mathbb{V}^\mu_{ba} + i\frac{\beta g_V}{\sqrt{2}} \tilde{P}^\dagger_a \overleftrightarrow{\partial}_\mu \tilde{P}_b \mathbb{V}^\mu_{ab}, \\
\mathcal{L}_{P^*P^*\mathbb{V}} &= i\frac{\beta g_V}{\sqrt{2}} P^{*\dagger}_a \overleftrightarrow{\partial}_\mu P^*_b \mathbb{V}^\mu_{ba} - i2\sqrt{2}\lambda g_V m_{P^*} P^{*\mu}_b P^{*\nu\dagger}_a(\partial_\mu \mathbb{V}_\nu - \partial_\nu \mathbb{V}_\mu)_{ba} - i\frac{\beta g_V}{\sqrt{2}} \tilde{P}^{*\dagger}_a \overleftrightarrow{\partial}_\mu \tilde{P}^*_b \mathbb{V}^\mu_{ab} \\
&- i2\sqrt{2}\lambda g_V m_{P^*} \tilde{P}^{*\mu\dagger}_a \tilde{P}^{*\nu}_b(\partial_\mu \mathbb{V}_\nu - \partial_\nu \mathbb{V}_\mu)_{ab}, \\
\mathcal{L}_{PP\sigma} &= -2g_\sigma m_P P^\dagger_a P_a \sigma - 2g_\sigma m_P \tilde{P}^\dagger_a \tilde{P}_a \sigma, \\
\mathcal{L}_{P^*P^*\sigma} &= 2g_\sigma m_{P^*} P^{*\dagger}_a P^*_a \sigma + 2g_\sigma m_{P^*} \tilde{P}^{*\dagger}_a \tilde{P}^*_a \sigma.
\end{aligned}
$$

$$
\mathbb{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \quad \mathbb{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}.
$$

With above Lagrangians, the potential kernel $\mathcal{V}$ can be written as

$$\begin{aligned}
\mathcal{V}_{\mathbb{V}}^{Direct} &= i\frac{\beta^2 g_V^2}{2}[\frac{(k_1+k_1')\cdot(k_2+k_2')}{q^2-m_{\mathbb{V}}^2}]\epsilon_1\cdot\epsilon_1' \\
\mathcal{V}_{\sigma}^{Direct} &= i4g_\sigma^2 m_P m_{P^*}\frac{\epsilon_1\cdot\epsilon_1'}{q^2-m_\sigma^2} \\
\mathcal{V}_{\mathbb{P}}^{Cross} &= i\frac{4g^2 m_P m_{P^*}}{f_\pi^2}\frac{q\cdot\epsilon_1 q\cdot\epsilon_1'}{q^2-m_{\mathbb{P}}^2} \\
\mathcal{V}_{\mathbb{V}}^{Cross} &= i8\lambda^2 g_V^2\frac{1}{q^2-m_{\mathbb{V}}^2}(q\cdot\epsilon_1 q\cdot\epsilon_1' k_2\cdot k_2' \\
&+ \epsilon_1\cdot\epsilon_1'(k_2\cdot q k_2'\cdot q - k_2\cdot k_2' q^2))
\end{aligned}$$

where $q=k_1'-k_1$ for direct diagram and $q=k_2'-k_1=k_2-k_1'$ for cross diagram.

Form factor

heavy meson: $h(k^2)=\frac{\Lambda^4}{(m^2-k^2)^2+\Lambda^4}$.

light meson: $f(q^2)=\frac{\Lambda^2-m^2}{\Lambda^2+|q^2|}$.

Sigularity
In cross diagram

For exchanged meson we make a replacement $q^2\to-|q^2|$ to remove the singularities as Gross et al..

CONNECTION TO OBE MODEL

The BSE can be related to the nonrelativistic OBE model by nonrelativization and Fourier transformation. The reduced equation is

$$[\frac{\nabla^2}{2\mu} - E]\phi(\boldsymbol{r}) = \int d\boldsymbol{r} V(\boldsymbol{r},\boldsymbol{r}')\phi(\boldsymbol{r}') \tag{1}$$

where μ is reduced mass and $E = m_1 + m_2 - W$ is the binding energy. The potential in coordinate space can be defined as

$$V(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{(2\pi)^3}\frac{1}{2}\int d\boldsymbol{q}d\boldsymbol{q}' e^{i[\boldsymbol{q}'\cdot\frac{\boldsymbol{r}-\boldsymbol{r}'}{2} - \boldsymbol{q}\cdot\frac{\boldsymbol{r}+\boldsymbol{r}'}{2}]} V(\boldsymbol{k},\boldsymbol{k}'), \tag{2}$$

where $\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$ and $\boldsymbol{q}' = \boldsymbol{k}' + \boldsymbol{k}$.

For direct diagram

$$V(\boldsymbol{q}) \to V(\boldsymbol{r})\delta(\boldsymbol{r}-\boldsymbol{r}')$$

The Schrödinger equation can be obtained.

For cross diagram

$$V(\boldsymbol{q}') \to V(\boldsymbol{r})\delta(\boldsymbol{r}+\boldsymbol{r}')$$

so the wave functions ϕ in the two sides are about $\boldsymbol{r}$ and -$\boldsymbol{r}$ It is no longer feasible to treat this issue with the Shrödinger equation.

RESULTS

		$D\bar{D}^*$				$B\bar{B}^*$				
		BS		OBE		BS		OBE		
	$I^G(J^{PC})$	Λ	E	Λ	E	Λ	E	Λ	E	
	$0^-(0^{--})$	–	–			1.5	1.6			
		–	–			1.7	4.1			
		–	–			1.9	6.7			
	$0^+(0^{-+})$	–	–			–	–			
	$0^-(1^{--})$	–	–			1.6	1.4			
		–	–			1.7	3.7			
		–	–			1.8	6.4			
	$0^+(1^{-+})$	–	–			–	–			
	$0^-(1^{+-})$	1.3	0.2	1.4	3.44	1.1	0.6	1.4	1.56	
		1.4	6.0	1.5	16.57	1.2	7.8	1.5	12.95	
	$0^+(1^{++})$	2.0	0.2	1.1	0.61	1.3	0.2	1.1	0.61	
X(3872)		2.2 ✔	1.4	1.2 ✔	4.42	1.5	3.0	1.2	4.42	
		2.4	4.1	1.3	11.78	1.7	7.4	1.3	11.78	
	$1^+(0^-)$	–	–			–	–			
	$1^-(0^-)$	–	–			–	–			
	$1^+(1^-)$	–	–			–	–			
	$1^-(1^-)$	–	–			–	–			
	$1^+(1^+)$	–	–	–	–	–	–	2.1	0.22	
Zc(3900)		– ✗	–	– ✗	–	– ✗	–	2.2 ✔	1.64	**Zb(10610)**
		–	–	–	–	–	–	2.5	4.74	
	$1^-(1^+)$	–	–	–	–	–	–	4.9	0.14	
		–	–	–	–	–	–	5.0	0.41	

SUMMARY

The BB* and DD* system are studied in a BSE approach with quasipotential potential approximation by adopting the covariant spectator theory

Both direct and cross diagrams are considered in the one-boson-exchange potential

Isoscalar DD* bound state with JPC=1++ : X(3872).

Isovector DD* bound state: none

Isovector BB* bound state: none

Disfavors the molecular state explanations for Zb(10610) and Zc(3900).

Thank you!