# Flavor Issues

Xing Zhi-zhong
邢 志 忠
[IHEP, Beijing]

## Lecture on quarks

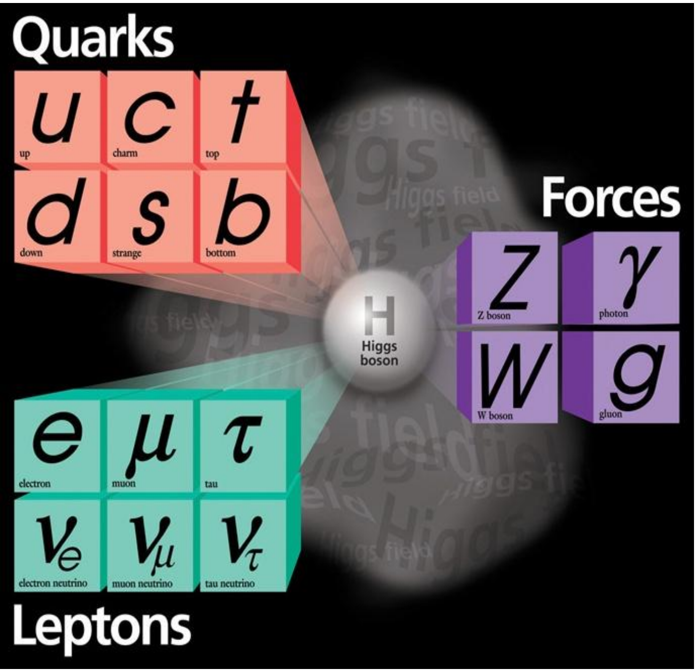
- ♣ Flavor puzzles
- ♣ The KM mechanism
- ♣ CP violation in the quark sector
- ♣ Cosmic baryon number asymmetry

@ 第六期理论物理前沿暑期讲习班——TeV 高能物理, 27/7— 8/8, 2015

# Matter & forces



**Two kinds of elementary particles: matter particles and force particles**

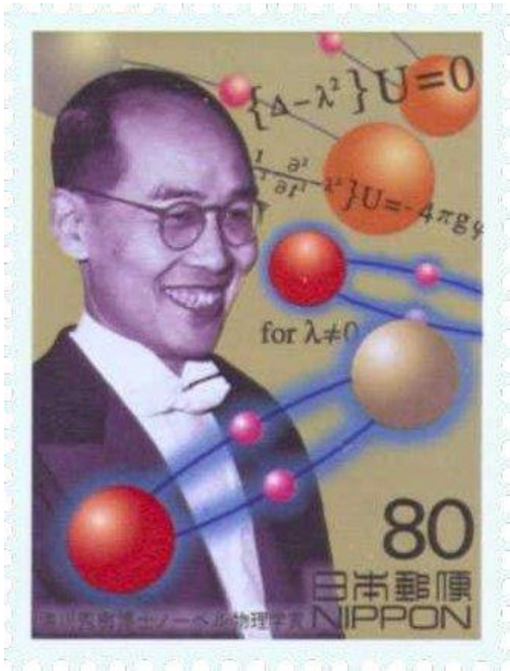



力的传播子质量$M$与力的作用范围$R$之间的关系:

$$\boxed{M \approx \frac{200\mathrm{MeV}\times 10^{-15}\mathrm{m}}{R}}$$

汤川秀树处男作: 一炮而红的诺贝尔物理学奖得主
1907/1935/1949/1981

| | 强度 | 范围 | 传播子 | 质量 |
|---|---|---|---|---|
| 强核力 | $1$ | $10^{-15}\mathrm{m}$ | 胶子/π介子 | $\sim 10^2\,\mathrm{MeV}$ |
| 电磁力 | $1/137$ | $\infty$ | 光子 | $=0$ |
| 弱核力 | $10^{-6}$ | $10^{-18}\mathrm{m}$ | W/Z/H? | $\sim 10^2\,\mathrm{GeV}$ |
| 引力 | $6\times 10^{-39}$ | $\infty$ | 引力子 | $=0$ |



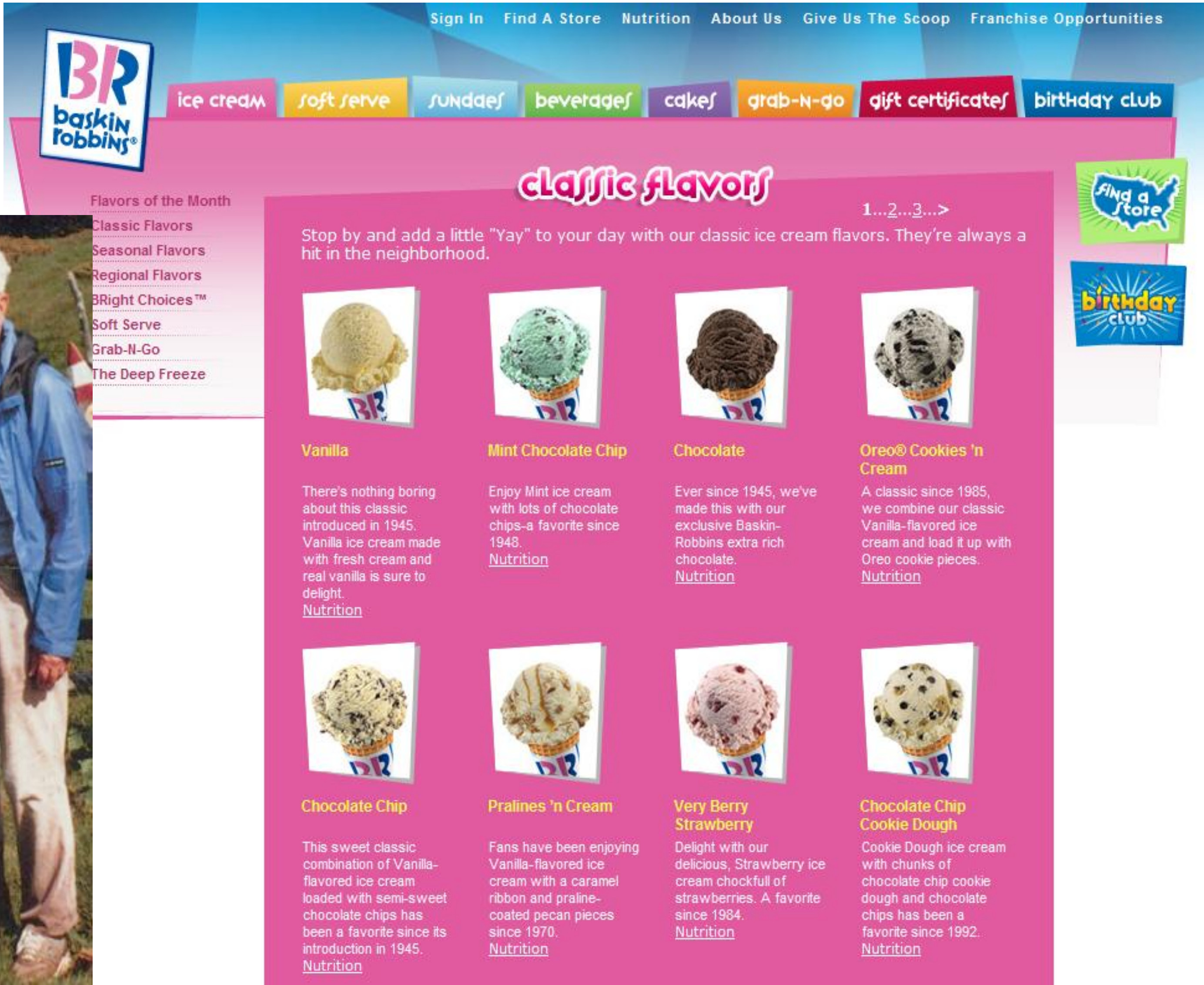
# Origin of "flavor"



**The term Flavor was coined by Harald Fritzsch and Murray Gell-Mann at a Baskin-Robbins ice-cream store in Pasadena in 1971.**

**Color & Flavor**

**QCD & QFD**



Part A

# Lepton flavors



- **1897:** **Discovery of electron (J.J. Thomson)**
- **1928:** **Prediction of positron (P.A.M. Dirac)**
- **1930:** **Postulation of neutrino (W. Pauli)**
- **1932:** **Discovery of positron (C.D. Anderson)**
- **1933:** **Effective theory of beta decay (E. Fermi)**
- **1936:** **Discovery of muon (J.C. Street *et al*; C.D. Anderson *et al*)**
- **1956:** **Discovery of electron anti-neutrino (C.L. Cowan *et al*)**
- **1956:** **Postulation of parity violation (T.D. Lee & C.N. Yang)**
- **1957:** **Discovery of parity violation (C.S. Wu *et al*)**
- **1962:** **Discovery of muon neutrino (G. Danby *et al*)**
- **1962:** **Postulation of neutrino conversion (S. Sakata *et al*)**
- **1975:** **Discovery of tau (M.L. Perl *et al*)**
- **2000:** **Discovery of tau neutrino (K. Kodama *et al*)**

# Quark flavors



**1932:** Discovery of neutron (J. Chadwick) **up** and **down**

**1947:** Discovery of Kaon (G. Rochester, C. Butler) **strange**

**1963:** The Cabibbo angle of quark mixing (N. Cabibbo)

**1964:** The quark model (M. Gell-Mann; G. Zweig)

**1964:** Discovery of CP violation (J.W. Cronin, V.L. Fitch)

**1964:** The Higgs mechanism (P. Higgs; F. Englert, R. Brout; …)

**1967:** The standard model (S. Weinberg)

**1970:** The GIM mechanism (S. Glashow *et al*)

**1972:** Quantum Chromodynamics (H. Fritzsch, M. Gell-Mann, …)

**1973:** The origin of CP violation (M. Kobayashi, T. Maskawa)

**1974:** Discovery of **charm** (C.C. Ting; B. Richter)

**1977:** Discovery of **bottom** (L. Lederman *et al*)

**1995:** Discovery of **top** (F. Abe *et al*)

# Quark masses



**Quark masses (Higgs mass = 125 GeV. Xing, Zhang, Zhou, arXiv:1112.3112)**

| $\mu$ | $m_u(\mu)$ (MeV) | $m_d(\mu)$ (MeV) | $m_s(\mu)$ (MeV) | $m_c(\mu)$ (GeV) | $m_b(\mu)$ (GeV) | $m_t(\mu)$ (GeV) |
|---|---|---|---|---|---|---|
| $m_c(m_c)$ | $2.79^{+0.83}_{-0.82}$ | $5.69^{+0.96}_{-0.95}$ | $116^{+36}_{-24}$ | $1.29^{+0.05}_{-0.11}$ | $5.95^{+0.37}_{-0.15}$ | $385.7^{+8.1}_{-7.8}$ |
| 2 GeV | $2.4^{+0.7}_{-0.7}$ | $4.9 \pm 0.8$ | $100^{+30}_{-20}$ | $1.11^{+0.07}_{-0.14}$ | $5.06^{+0.29}_{-0.11}$ | $322.2^{+5.0}_{-4.9}$ |
| $m_b(m_b)$ | $2.02^{+0.60}_{-0.60}$ | $4.12^{+0.69}_{-0.68}$ | $84^{+26}_{-17}$ | $0.934^{+0.058}_{-0.120}$ | $4.19^{+0.18}_{-0.16}$ | $261.8^{+3.0}_{-2.9}$ |
| $M_W$ | $1.39^{+0.42}_{-0.41}$ | $2.85^{+0.49}_{-0.48}$ | $58^{+18}_{-12}$ | $0.645^{+0.043}_{-0.085}$ | $2.90^{+0.16}_{-0.06}$ | $174.2 \pm 1.2$ |
| $M_Z$ | $1.38^{+0.42}_{-0.41}$ | $2.82 \pm 0.48$ | $57^{+18}_{-12}$ | $0.638^{+0.043}_{-0.084}$ | $2.86^{+0.16}_{-0.06}$ | $172.1 \pm 1.2$ |
| $M_H$ | $1.34^{+0.40}_{-0.40}$ | $2.74^{+0.47}_{-0.47}$ | $56^{+17}_{-12}$ | $0.621^{+0.041}_{-0.082}$ | $2.79^{+0.15}_{-0.06}$ | $167.0^{+1.2}_{-1.2}$ |
| $m_t(m_t)$ | $1.31^{+0.40}_{-0.39}$ | $2.68 \pm 0.46$ | $55^{+17}_{-11}$ | $0.608^{+0.041}_{-0.080}$ | $2.73^{+0.15}_{-0.06}$ | $163.3 \pm 1.1$ |
| 1 TeV | $1.17 \pm 0.35$ | $2.40^{+0.42}_{-0.41}$ | $49^{+15}_{-10}$ | $0.543^{+0.037}_{-0.072}$ | $2.41^{+0.14}_{-0.05}$ | $148.1 \pm 1.3$ |
| $\Lambda_{\rm VS}$ | $0.61^{+0.19}_{-0.18}$ | $1.27 \pm 0.22$ | $26^{+8}_{-5}$ | $0.281^{+0.02}_{-0.04}$ | $1.16^{+0.07}_{-0.02}$ | $82.6 \pm 1.4$ |

$$\frac{m_u}{m_c} \sim \frac{m_c}{m_t} \sim \lambda^4 , \qquad \frac{m_d}{m_s} \sim \frac{m_s}{m_b} \sim \lambda^2 , \qquad \lambda \simeq 0.22$$

**Two useful working symmetries based on QCD:**

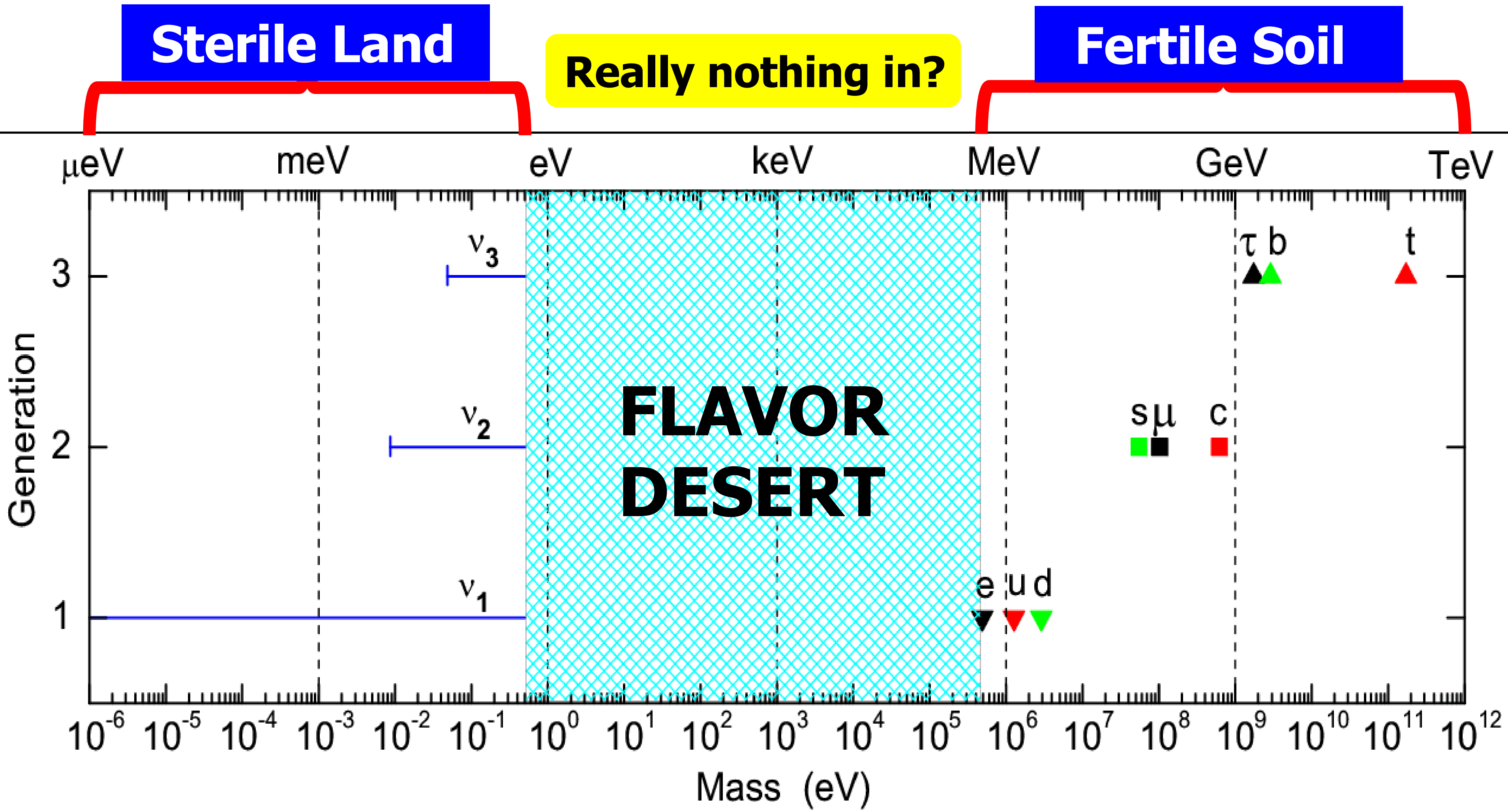
- **The chiral symmetry:** $m_u, m_d, m_s \to 0$ ;
- **The heavy quark symmetry:** $m_c, m_b, m_t \to \infty$ .

$\Lambda_{\rm QCD} \sim 200\,\mathrm{MeV}$

Part A

# Flavor puzzles (I)






**Gauge Hierarchy & Desert Puzzles / Flavor Hierarchy & Desert Puzzles**

**Implications of electron mass < u quark mass < d quark mass on .......**

# Flavor mixing



$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[ \overline{(u \quad c \quad t)_{\rm L}} \, \gamma^\mu \, \underset{\text{CKM}}{V} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W^+_\mu + \overline{(e \quad \mu \quad \tau)_{\rm L}} \, \gamma^\mu \, \underset{\text{PMNS}}{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu \right] + \text{h.c.}$$

## Quark flavor mixing matrix:

$$V_{\rm CKM} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

## Lepton flavor mixing matrix:

$$|U| = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix}$$

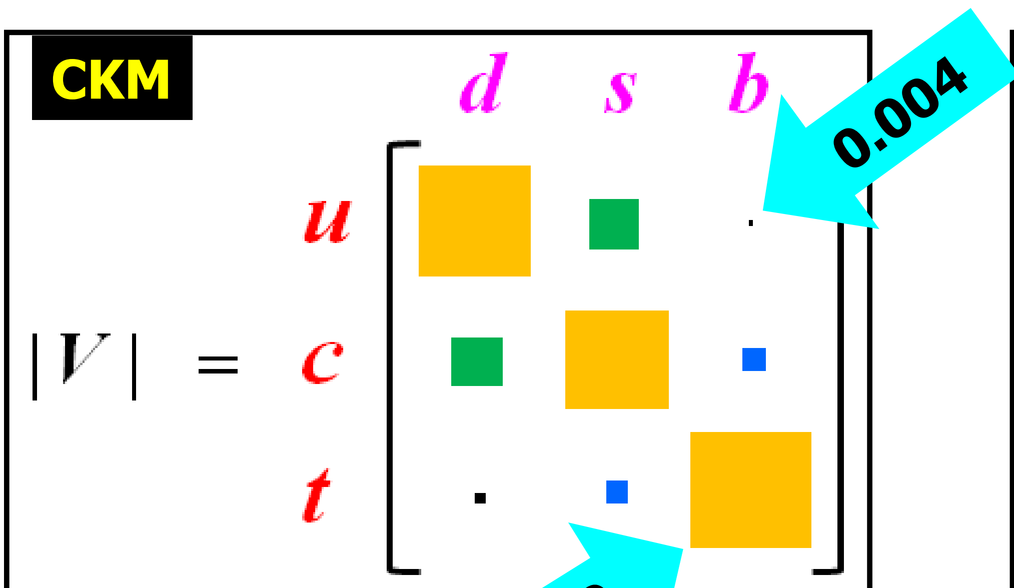
# Flavor puzzles (II)



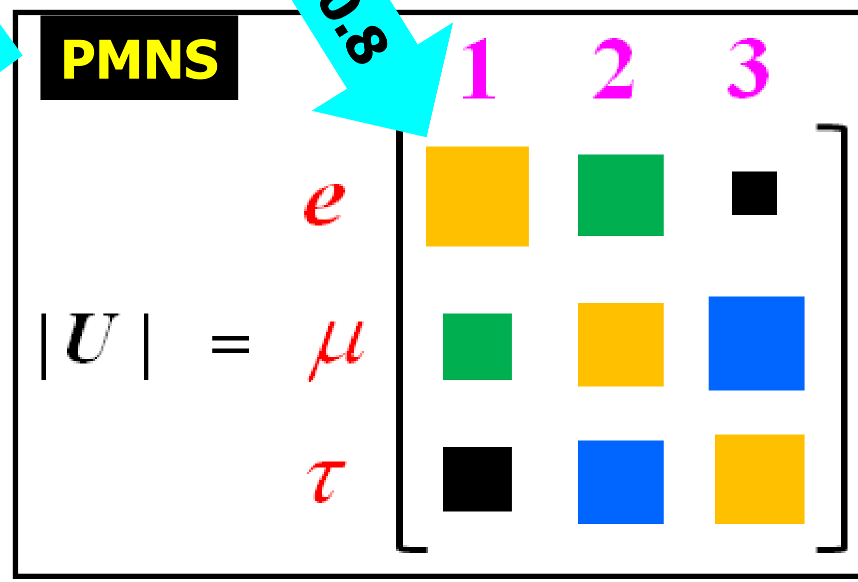
$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{(u\ \ c\ \ t)_{\mathrm{L}}}\,\gamma^{\mu}\,\underset{\text{CKM}}{V}\begin{pmatrix}d\\ s\\ b\end{pmatrix}_{\mathrm{L}} W^{+}_{\mu} + \overline{(e\ \ \mu\ \ \tau)_{\mathrm{L}}}\,\gamma^{\mu}\,\underset{\text{PMNS}}{U}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}_{\mathrm{L}} W^{-}_{\mu}\right] + \mathrm{h.c.}$$

**Quark mixing: hierarchy!**

**4/6 parameters**




**4 parameters**




**Lepton mixing: anarchy?**

# Why symmetries?



**Symmetries: crucial for understanding the laws of Nature.**

**Examples: they help simplify problems, classify complicated systems, fix conservation laws and even determine dynamics of interactions**

- SU(3) flavor symmetry ⇒ the quark model
- Continuous space-time (translational/rotational) symmetries ⇒ energy-momentum conservation laws
- Gauge symmetries ⇒ electroweak and strong interactions

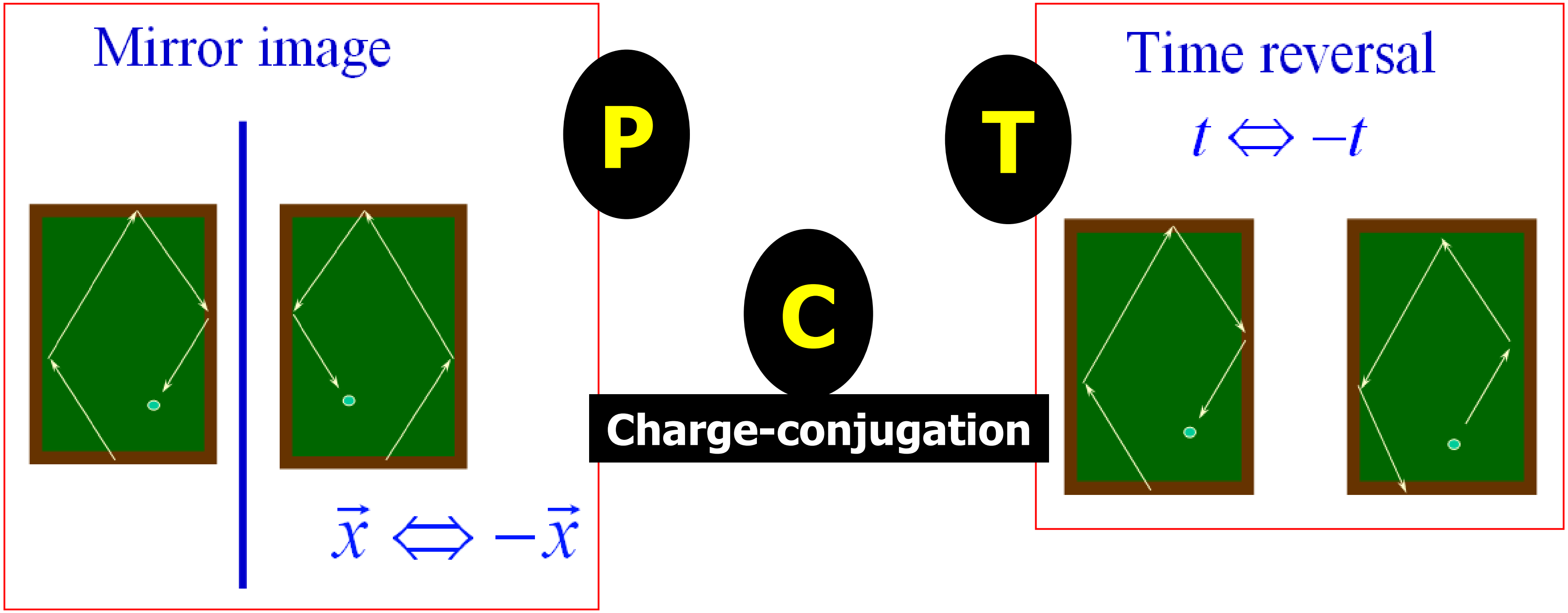
**Symmetries may keep exact or be broken: both important!**

- SU(3) flavor symmetry: broken
- Continuous space-time symmetries: exact
- U(1) electromagnetic gauge symmetry: exact (massless photon)
- SU(2) weak gauge symmetry: broken (massive *W*, *Z*, *etc*)
- SU(3) color gauge symmetry: exact (massless gluons)

# C/P/T



## Discrete space-time symmetries: C, P, T.



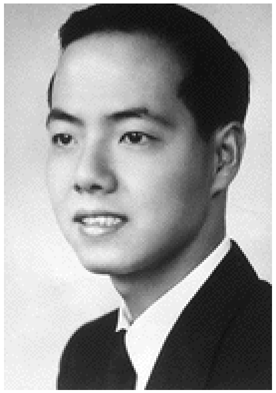

**For a long time it was believed that P/C/T should be exact**

**CPT theorem (G. Lueders 1954, W. Pauli 1955): CPT symmetry holds for all physical phenomena (or, any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry).**

# P and CP violation



**P violation:** **a new and revolutionary window in understanding the nature of symmetries, leading to the V-A theory of weak interactions.**





**1956** **1957** **1958**

**CP violation:** **one of the necessary conditions to explain why there is only matter rather than antimatter in our Universe (why we exist).**

**The first laboratory evidence for CP violation:**

$$|\eta_{+-}| = |A(K_L^0 \to \pi^+\pi^-)/A(K_S^0 \to \pi^+\pi^-)|$$
$$= (2.236 \pm 0.007) \times 10^{-3}.$$

**1964: CP violation (J.W. Cronin, Val L. Fitch)**


# Towards the KM paper

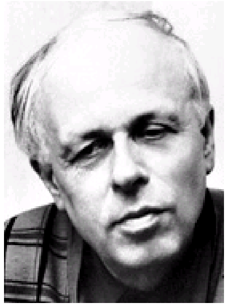


**1964: Discovery of CP violation in K decays (J.W. Cronin, Val L. Fitch)**

**NP 1980**

**1967: Sakharov conditions for cosmological matter-antimatter asymmetry (A. Sakharov)**

**NP 1975**



**1967: The standard model of electromagnetic and weak interactions without quarks (S. Weinberg)**

**NP 1979**

**0 citation for the first 4 yrs**

**1971: The first proof of the renormalizability of the standard model (G. 't Hooft)**

**NP 1999**

# KM in 1972 (1)



Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

## *CP*-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

*Department of Physics, Kyoto University, Kyoto*

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

**Japanese Archimedes**

**3 families allow for CP violation: Maskawa's bathtub idea!**

**"as I was getting out of the bathtub, an idea came to me"**

# KM in 1972 (2)



**In your research life, please try to read the original papers**

Next we consider a 6-plet model, another interesting model of $CP$-violation. Suppose that 6-plet with charges $(Q, Q, Q, Q-1, Q-1, Q-1)$ is decomposed into $SU_{\text{weak}}(2)$ multiplets as $2+2+2$ and $1+1+1+1+1+1$ for left and right components, respectively. Just as the case of $(A, C)$, we have a similar expression for the charged weak current with a $3\times3$ instead of $2\times2$ unitary matrix in Eq. (5). As was pointed out, in this case we cannot absorb all phases of matrix elements into the phase convention and can take, for example, the following expression:

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\underline{\sin\theta_3}e^{i\delta} \end{pmatrix}. \tag{13}$$

Then, we have $CP$-violating effects through the interference among these different current components. An interesting feature of this model is that the $CP$-violating effects of lowest order appear only in $\Delta S\neq0$ non-leptonic processes and in the

**Dear, have you seen a typo in the KM flavor mixing matrix**

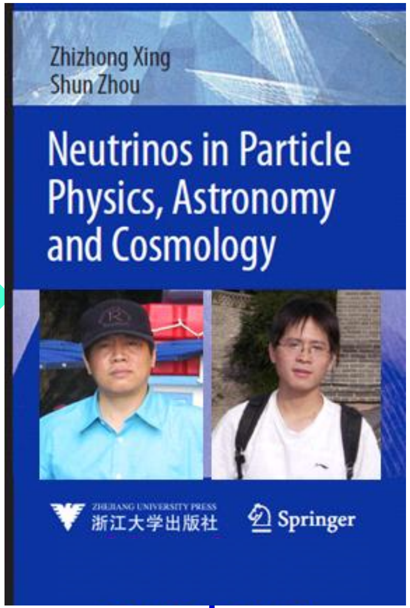
# Diagnosis of CP violation



**In the SM (+ 3 right-handed ν's) , the KM mechanism is responsible for CP violation.**

$$\boxed{\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{Y}}}$$

**Proof in**




$$\mathcal{L}_{\mathrm{G}} = -\frac{1}{4}\left(W^{i\mu\nu}W^{i}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right)$$

$$\mathcal{L}_{\mathrm{H}} = \left(D^{\mu}H\right)^{\dagger}\left(D_{\mu}H\right) - \mu^{2}H^{\dagger}H - \lambda\left(H^{\dagger}H\right)^{2}$$

$$\mathcal{L}_{\mathrm{F}} = \overline{Q_{\mathrm{L}}}\, i\not{D}\, Q_{\mathrm{L}} + \overline{\ell_{\mathrm{L}}}\, i\not{D}\, \ell_{\mathrm{L}} + \overline{U_{\mathrm{R}}}\, i\not{\partial}'\, U_{\mathrm{R}} + \overline{D_{\mathrm{R}}}\, i\not{\partial}'\, D_{\mathrm{R}} + \overline{E_{\mathrm{R}}}\, i\not{\partial}'\, E_{\mathrm{R}} + \underline{\overline{N_{\mathrm{R}}}\, i\not{\partial}'\, N_{\mathrm{R}}}$$

$$\mathcal{L}_{\mathrm{Y}} = -\overline{Q_{\mathrm{L}}}\, Y_{\mathrm{u}}\tilde{H}U_{\mathrm{R}} - \overline{Q_{\mathrm{L}}}\, Y_{\mathrm{d}}HD_{\mathrm{R}} - \overline{\ell_{\mathrm{L}}}\, Y_{l}HE_{\mathrm{R}} - \underline{\overline{\ell_{\mathrm{L}}}\, Y_{\nu}\tilde{H}N_{\mathrm{R}}} + \mathrm{h.c.}$$

**ν's Dirac mass**

**The strategy of diagnosis: given proper CP transformations of gauge, Higgs and fermion fields, we may prove that the 1st, 2nd and 3rd terms are formally invariant, and hence the 4th term can be invariant only if the corresponding Yukawa coupling matrices are real. (Note that the SM spontaneous gauge symmetry breaking itself doesn't affect CP.)**

Part B

# CP transformations



**Gauge fields:**

$$\boxed{\left[B_\mu,\ W^1_\mu,\ W^2_\mu,\ W^3_\mu\right] \xrightarrow{\text{CP}} \left[-B^\mu,\ -W^{1\mu},\ +W^{2\mu},\ -W^{3\mu}\right]}$$

$$\boxed{\left[B_{\mu\nu},\ W^1_{\mu\nu},\ W^2_{\mu\nu},\ W^3_{\mu\nu}\right] \xrightarrow{\text{CP}} \left[-B^{\mu\nu},\ -W^{1\mu\nu},\ +W^{2\mu\nu},\ -W^{3\mu\nu}\right]}$$

**Higgs fields:**

$$\boxed{H(t,\mathbf{x}) = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \xrightarrow{\text{CP}} H^*(t,-\mathbf{x}) = \begin{pmatrix}\phi^-\\ \phi^{0*}\end{pmatrix}}$$

**Lepton or quark fields:**

$$\boxed{\overline{\psi_1}\gamma_\mu\left(1\pm\gamma_5\right)\psi_2 \xrightarrow{\text{CP}} -\overline{\psi_2}\gamma^\mu\left(1\pm\gamma_5\right)\psi_1}$$

$$\boxed{\overline{\psi_1}\gamma_\mu\left(1\pm\gamma_5\right)\partial^\mu\psi_2 \xrightarrow{\text{CP}} \overline{\psi_2}\gamma^\mu\left(1\pm\gamma_5\right)\partial_\mu\psi_1}$$

**Spinor bilinears:**

| | $\overline{\psi_1}\psi_2$ | $i\overline{\psi_1}\gamma_5\psi_2$ | $\overline{\psi_1}\gamma_\mu\psi_2$ | $\overline{\psi_1}\gamma_\mu\gamma_5\psi_2$ | $\overline{\psi_1}\sigma_{\mu\nu}\psi_2$ |
|---|---|---|---|---|---|
| C | $\overline{\psi_2}\psi_1$ | $i\overline{\psi_2}\gamma_5\psi_1$ | $-\overline{\psi_2}\gamma_\mu\psi_1$ | $\overline{\psi_2}\gamma_\mu\gamma_5\psi_1$ | $-\overline{\psi_2}\sigma_{\mu\nu}\psi_1$ |
| P | $\overline{\psi_1}\psi_2$ | $-i\overline{\psi_1}\gamma_5\psi_2$ | $\overline{\psi_1}\gamma^\mu\psi_2$ | $-\overline{\psi_1}\gamma^\mu\gamma_5\psi_2$ | $\overline{\psi_1}\sigma^{\mu\nu}\psi_2$ |
| T | $\overline{\psi_1}\psi_2$ | $-i\overline{\psi_1}\gamma_5\psi_2$ | $\overline{\psi_1}\gamma^\mu\psi_2$ | $\overline{\psi_1}\gamma^\mu\gamma_5\psi_2$ | $-\overline{\psi_1}\sigma^{\mu\nu}\psi_2$ |
| CP | $\overline{\psi_2}\psi_1$ | $-i\overline{\psi_2}\gamma_5\psi_1$ | $-\overline{\psi_2}\gamma^\mu\psi_1$ | $-\overline{\psi_2}\gamma^\mu\gamma_5\psi_1$ | $-\overline{\psi_2}\sigma^{\mu\nu}\psi_1$ |
| CPT | $\overline{\psi_2}\psi_1$ | $i\overline{\psi_2}\gamma_5\psi_1$ | $-\overline{\psi_2}\gamma_\mu\psi_1$ | $-\overline{\psi_2}\gamma_\mu\gamma_5\psi_1$ | $\overline{\psi_2}\sigma_{\mu\nu}\psi_1$ |

$\boxed{\mathcal{L}_\text{G}}$ $\boxed{\mathcal{L}_\text{H}}$ $\boxed{\mathcal{L}_\text{F}}$

**formally invariant under CP**

Part B

# CP violation



**The Yukawa interactions of fermions are formally invariant under CP if and only if**

$$Y_u = Y_u^* \,, \qquad Y_d = Y_d^*$$
$$Y_l = Y_l^* \,, \qquad Y_\nu = Y_\nu^*$$

**If the effective Majorana mass term is added into the SM, then the Yukawa interactions of leptons can be formally invariant under CP if**

$$M_L = M_L^* \,, \qquad Y_l = Y_l^*$$

**If the flavor states are transformed into the mass states, the source of flavor mixing and CP violation will show up in the *CC* interactions:**

**quarks**

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(u\ c\ t)_L}\ \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \text{h.c.}$$

**leptons**

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(e\ \mu\ \tau)_L}\ \gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W_\mu^- + \text{h.c.}$$

**Comment A: CP violation exists since fermions interact with both the gauge bosons and the Higgs boson.**

**Comment B: both the CC and Yukawa interactions have been verified.**

**Comment C: the CKM matrix *V* is unitary, the MNSP matrix *U* is too?**

# Parameter counting



**The 3×3 unitary matrix V can always be parametrized as a product of 3 unitary rotation matrices in the complex planes:**

$$O_1(\theta_1, \alpha_1, \beta_1, \gamma_1) = \begin{pmatrix} c_1 e^{i\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$O_2(\theta_2, \alpha_2, \beta_2, \gamma_2) = \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{i\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix}$$

$$O_3(\theta_3, \alpha_3, \beta_3, \gamma_3) = \begin{pmatrix} c_3 e^{i\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix}$$

where $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$ (for $i = 1, 2, 3$)

**Category A: 3 possibilities**

$$V = O_i O_j O_i \quad (i \neq j)$$

**Category B: 6 possibilities**

$$V = O_i O_j O_k \quad (i \neq j \neq k)$$

Part B

# Phases



**For instance, the standard parametrization is given below:**

**V**

$$= \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix} \begin{pmatrix} c_3 e^{\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix} \begin{pmatrix} c_1 e^{\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 c_3 e^{i(\alpha_1+\gamma_2+\alpha_3)} & s_1 c_3 e^{i(-\beta_1+\gamma_2+\alpha_3)} & s_3 e^{i(\gamma_1+\gamma_2-\beta_3)} \\ -s_1 c_2 e^{i(\beta_1+\alpha_2+\gamma_3)} - c_1 s_2 s_3 e^{i(\alpha_1-\beta_2+\beta_3)} & c_1 c_2 e^{i(-\alpha_1+\alpha_2+\gamma_3)} - s_1 s_2 s_3 e^{i(-\beta_1-\beta_2+\beta_3)} & s_2 c_3 e^{i(\gamma_1-\beta_2-\alpha_3)} \\ s_1 s_2 e^{i(\beta_1+\beta_2+\gamma_3)} - c_1 c_2 s_3 e^{i(\alpha_1-\alpha_2+\beta_3)} & -c_1 s_2 e^{i(-\alpha_1+\beta_2+\gamma_3)} - s_1 c_2 s_3 e^{i(-\beta_1-\alpha_2+\beta_3)} & c_2 c_3 e^{i(\gamma_1-\alpha_2-\alpha_3)} \end{pmatrix}$$

$$= \begin{pmatrix} e^{ia} & 0 & 0 \\ 0 & e^{ib} & 0 \\ 0 & 0 & e^{ic} \end{pmatrix} \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \begin{pmatrix} e^{ix} & 0 & 0 \\ 0 & e^{iy} & 0 \\ 0 & 0 & e^{iz} \end{pmatrix}$$

$$\boxed{\begin{aligned} &a = (\alpha_1 - \beta_1) - (\alpha_2 + \beta_2 - \gamma_2) - \gamma_3\,, \quad b = -\beta_2 - \alpha_3\,, \quad c = -\alpha_2 - \alpha_3\,; \\ &x = \beta_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3)\,, \quad y = -\alpha_1 + (\alpha_2 + \beta_2) + (\alpha_3 + \gamma_3)\,, \quad z = \gamma_1\,. \end{aligned}}$$

$$\delta = \beta_3 - \gamma_1 - \gamma_2$$

# Physical phases



**If fermions are the Dirac particles, then phases *x*, *y* and *z* can be removed. The quark or lepton flavor mixing matrix:**

**Quark or Dirac neutrino mixing matrix**

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

**If neutrinos are Majorana particles, left- and right-handed fields are correlated. Hence only a common phase of three left-handed fields can be redefined (e.g., *z* = 0). Then**

**Majorana neutrino mixing matrix**

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# Flavor mixing + CPV



## Quark mixing:


$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

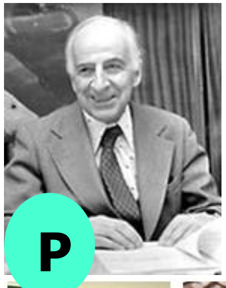
**Experiments:**

$\theta_{12}$ ➡ $\theta_{23}$ ➡ $\theta_{13}$ ➡ $\delta$ new physics ?

$\sim 13^\circ$ $\sim 2^\circ$ $\sim 0.2^\circ$ $\sim 65^\circ$ unitarity ?

turning point

## Lepton mixing: (in general, we consider three Majorana neutrinos)




$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$


$\theta_{23}$ ➡ $\theta_{12}$ ➡ $\theta_{13}$ ➡ $\delta/\rho/\sigma$ new physics ?

$\sim 45^\circ$ $\sim 34^\circ$ $\sim 9^\circ$ $\sim$? ? ? unitarity ?

# K/D/B decays (1)



**There are 3 types of CP-violating effects in K/D/B decays:**

**Type A: direct CP violation**

$$A(i \to f) = A_1 e^{i\theta_1} e^{+i\phi_1} + A_2 e^{i\theta_2} e^{+i\phi_2}$$

$$A(\bar{i} \to \bar{f}) = A_1 e^{i\theta_1} e^{-i\phi_1} + A_2 e^{i\theta_2} e^{-i\phi_2}$$

**Strong phases:** $\theta_{1,2}$

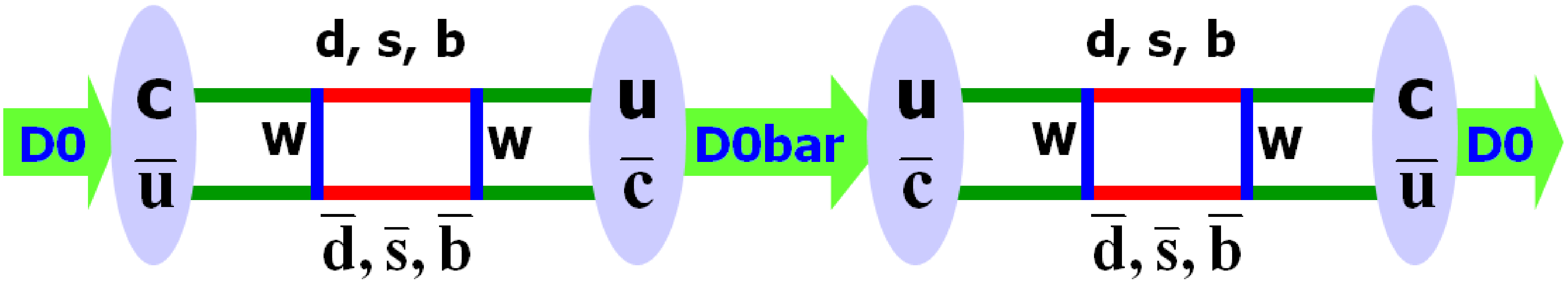
**Weak phases:** $\phi_{1,2}$

**CP asymmetry:**

$$\mathcal{A}_{\rm CP} = |A(i \to f)|^2 - |A(\bar{i} \to \bar{f})|^2$$

$$= 4A_1 A_2 \sin(\theta_1 - \theta_2) \sin(\phi_1 - \phi_2)$$

**Type B: CP violation from $K^0$-$\bar{K}^0$, $D^0$-$\bar{D}^0$, $B^0_{d,s}$-$\bar{B}^0_{d,s}$ mixing**



Part C

# K/D/B decays (2)



**Taking neutral D-meson system for example, the mixing is**

$$\begin{aligned}|D_1\rangle &= p|D^0\rangle + q|\bar{D}^0\rangle \\ |D_2\rangle &= p|D^0\rangle - q|\bar{D}^0\rangle\end{aligned}$$

$$\begin{aligned}|D_+\rangle &= \frac{1}{\sqrt{2}}\left(|D^0\rangle + |\overline{D}^0\rangle\right) \\ |D_-\rangle &= \frac{1}{\sqrt{2}}\left(|D^0\rangle - |\overline{D}^0\rangle\right)\end{aligned}$$

$$\frac{q}{p} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right)^{1/2}$$

**mass state** | **flavor states** | **CP state**

convention $CP|D^0\rangle = |\overline{D}^0\rangle$

**An expansion of the off-diagonal terms of the Hamiltonian for D-meson mixing to 2nd order in perturbation theory is given by**

$$\left(M - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2M_D}\left[\langle D^0|\mathcal{H}_{\text{weak}}^{\Delta C=2}|\overline{D}^0\rangle + \sum_n \frac{\langle D^0|\mathcal{H}_{\text{weak}}^{\Delta C=1}|n\rangle\langle n|\mathcal{H}_{\text{weak}}^{\Delta C=1}|\overline{D}^0\rangle}{M_D - E_n + i\epsilon}\right]$$

**contributes only to M_12**
**sensitive to new physics**

**contributes both to M_12 & Γ_12**
**dominated by the SM contribution**

**CP violation in D0-D0bar mixing:**

$$\Delta_D \equiv \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4}$$

**(SM prediction: < 10^-4 with large uncertainties)**

# K/D/B decays (3)



**Type C: CP violation from the interplay of decay & mixing [indirect CP violation] (SM: ≤ 10^-3 for D-meson system)**

$$\boxed{\lambda_f \equiv \frac{q}{p} \cdot \frac{\langle f|\mathcal{H}_{\rm eff}|\bar{D}^0\rangle}{\langle f|\mathcal{H}_{\rm eff}|D^0\rangle}} \quad \boxed{\mathrm{Im}\lambda_f - \mathrm{Im}\bar{\lambda}_{\bar{f}} \neq 0} \quad \boxed{\bar{\lambda}_{\bar{f}} \equiv \frac{p}{q} \cdot \frac{\langle \bar{f}|\mathcal{H}_{\rm eff}|D^0\rangle}{\langle \bar{f}|\mathcal{H}_{\rm eff}|\bar{D}^0\rangle}}$$

**CP violation at the resonance:**

$$\boxed{(D^0_{\rm phys}\bar{D}^0_{\rm phys})_\pm \longrightarrow (f_1 f_2)_\mp}$$

**initial CP** **final CP**

**The observed phenomena of CP violation (signature > 5σ):**

**(Particle Data Group 2014)**

- Indirect $CP$ violation in $K \to \pi\pi$ and $K \to \pi\ell\nu$ decays, and in the $K_L \to \pi^+\pi^- e^+ e^-$ decay, is given by

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}\,.$$

- Direct $CP$ violation in $K \to \pi\pi$ decays is given by

$$\mathcal{R}e(\epsilon'/\epsilon) = (1.65 \pm 0.26) \times 10^{-3}\,.$$

# K/D/B decays (4)



- $CP$ violation in the interference of mixing and decay in the tree-dominated $b \to c\bar{c}s$ transitions, such as $B^0 \to \psi K^0$, is given by (we use $K^0$ throughout to denote results that combine $K_S$ and $K_L$ modes, but use the sign appropriate to $K_S$):

**A. Carter, A.I. Sanda**
**1980, 1981**

$$S_{\psi K^0} = +0.682 \pm 0.019\,.$$

**I.I. Bigi, A.I. Sanda**
**1981, ….**

- $CP$ violation in the interference of mixing and decay in various modes related to $b \to q\bar{q}s$ (penguin) transitions is given by

$$S_{\eta' K^0} = +\,0.63 \pm 0.06\,,$$

$$S_{\phi K^0} = +\,0.74\,{}^{+0.11}_{-0.13}\,,$$

$$S_{f_0 K^0} = +\,0.69\,{}^{+0.10}_{-0.12}\,,$$

$$S_{K^+K^-K_S} = +\,0.68\,{}^{+0.09}_{-0.10}\,,$$

- $CP$ violation in the interference of mixing and decay in the $B^0 \to \pi^+\pi^-$ mode is given by

$$S_{\pi^+\pi^-} = -0.66 \pm 0.06\,.$$

- Direct $CP$ violation in the $B^0 \to \pi^+\pi^-$ mode is given by

$$C_{\pi^+\pi^-} = -0.31 \pm 0.05\,.$$

# K/D/B decays (5)



- $CP$ violation in the interference of mixing and decay in various modes related to $b \to c\bar{c}d$ transitions is given by

$$\begin{aligned} S_{\psi\pi^0} &= -0.93 \pm 0.15\,, \\ S_{D^+D^-} &= -0.98 \pm 0.17\,. \\ S_{D^{*+}D^{*-}} &= -0.71 \pm 0.09\,. \end{aligned}$$

- Direct $CP$ violation in the $\overline{B}^0 \to K^-\pi^+$ mode is given by

$$\mathcal{A}_{\overline{B}^0 \to K^-\pi^+} = -0.082 \pm 0.006\,.$$

- Direct $CP$ violation in $B^\pm \to D_+K^\pm$ decays ($D_+$ is the $CP$-even neutral $D$ state) is given by

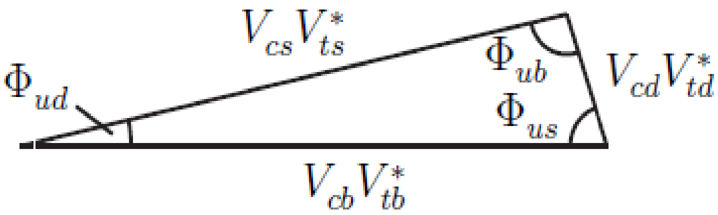
$$\mathcal{A}_{B^+ \to D_+K^+} = +0.19 \pm 0.03\,.$$

- Direct $CP$ violation in the $\overline{B}^0_s \to K^+\pi^-$ mode is given by

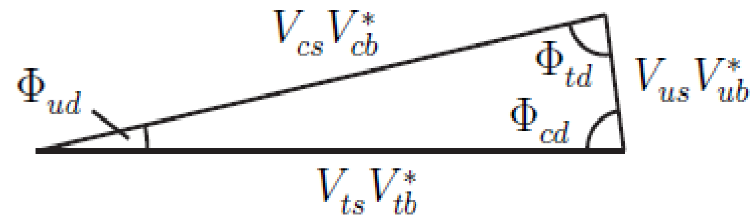
$$\mathcal{A}_{\overline{B}^0_s \to K^+\pi^-} = +0.26 \pm 0.04\,.$$

# Unitarity triangles



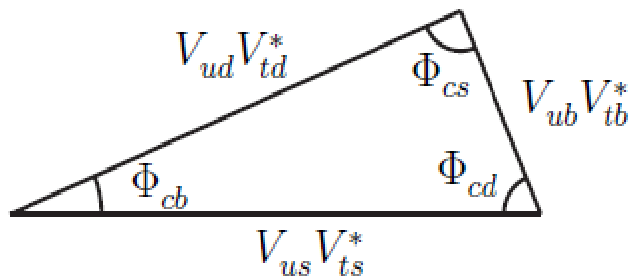



$(\triangle_u)$

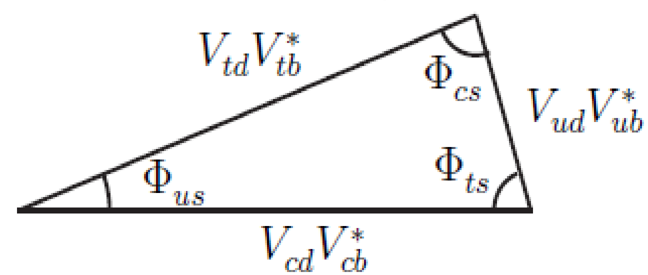



$(\triangle_d)$

$\boxed{\alpha = 90^\circ}$



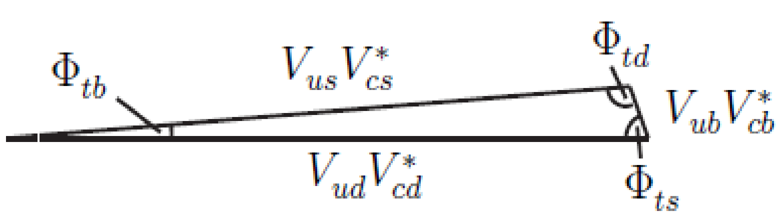

**bottom**

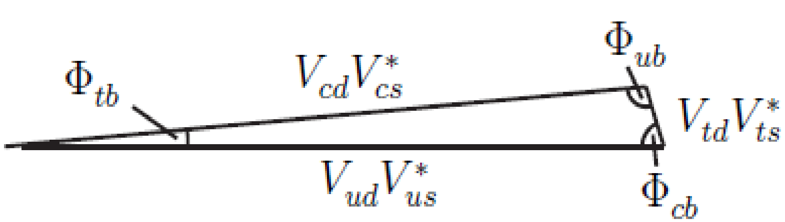



$(\triangle_c)$

$(\triangle_s)$

**charm**

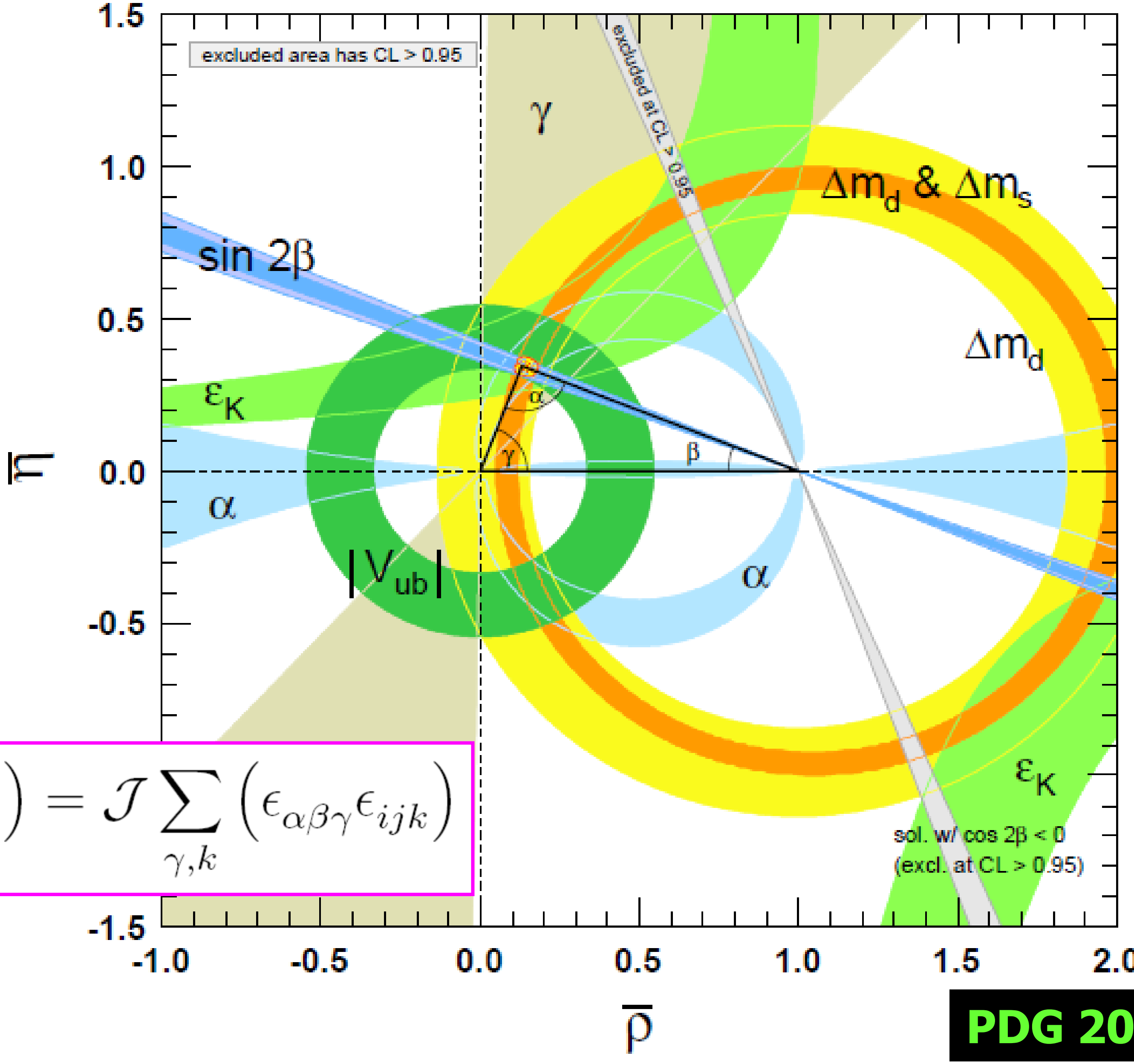


# Current data



$$\alpha = \left(89.0^{+4.4}_{-4.2}\right)^\circ$$




**Jarlskog invariant:**

$$\mathrm{Im}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right) = \mathcal{J}\sum_{\gamma,k}\left(\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\right)$$

$$\mathcal{J}_q \simeq 3\times 10^{-5}$$

**PDG 2012**

# Bjorken's talk in Hawaii



## CP AND B PHYSICS: PROGRESS AND PROSPECTS

J. D. Bjorken
*Stanford Linear Accelerator Center*
*Stanford University, Stanford, California 94309*

This summary of the 2nd International Conference on *B* Physics and CP Violation (Honolulu, 24–27 March, 1997) contains, in addition to what is implied in the title,

Maybe there is a right angle in the unitarity triangle, in particular maybe $\gamma = 90°$. This is not in the same class of dramatic surprises as the previous two categories, but nevertheless an observed regularity of shape of the unitarity triangle might send a rather strong message. There is a small, elite right-angle club, consisting to the best of my knowledge of Berthold Stech and myself. Harald Fritsch qualifies as a corresponding member (e-mail only), having also advocated a right angle[8], but the wrong one ($\alpha$).

**H. Fritzsch & ZZX in 1995, first predicting $\alpha = 90$ degrees!**

# Strong CP violation



♠ A P- and T-violating θ-term in QCD, coming from the instanton solution to the U(1)A problem:

$$\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

♠ The mass term of quarks:

$$\mathcal{L}_\mathrm{m} = -\overline{\begin{pmatrix} u & c & t & d & s & b \end{pmatrix}}_\mathrm{L} \mathcal{M} \begin{pmatrix} u \\ c \\ t \\ d \\ s \\ b \end{pmatrix}_\mathrm{R} + \mathrm{h.c.}$$

The chiral transformation of the quark fields

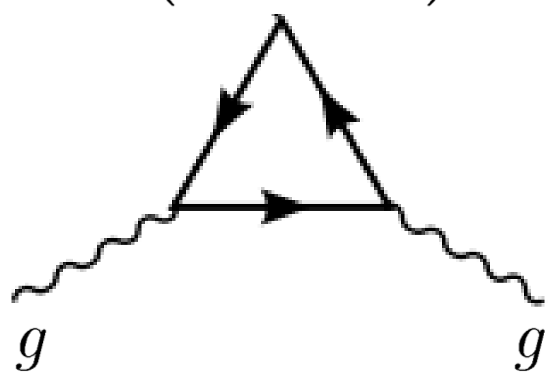
$$\psi_q \to \exp\left(i\alpha_q \gamma_5\right) \psi_q$$

leads to the changes:

$$\theta \to \theta - 2\sum_q \alpha_q$$

$$\arg\left(\det \mathcal{M}\right) \to \arg\left(\det \mathcal{M}\right) + 2\sum_q \alpha_q$$

$$\partial_\mu \left(\bar{\psi}_q \gamma^\mu \gamma_5 \psi_q\right)$$




The change of the θ-term due to the anomaly:

$$\partial_\mu \left(\bar{\psi}_q \gamma^\mu \gamma_5 \psi_q\right) = 2im_q \bar{\psi}_q \gamma_5 \psi_q + \frac{\alpha_s}{4\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

# Why a problem?



Then the effective CP-violating θ-term in QCD turns out to be:

$$\mathcal{L}_{\bar\theta} = \bar\theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

$$\bar\theta = \theta + \arg(\det \mathcal{M})$$

QCD

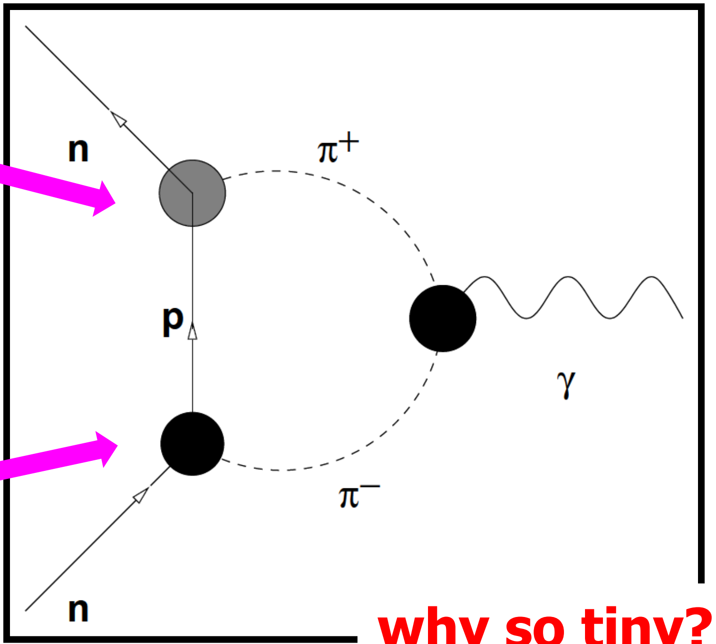
QFD

It is a sum of the QCD contribution (the vacuum angle θ) and the electroweak one (related to the phase structure of the quark mass matrix).

CP-violating

Best bound on the effective CP-violating $\bar\theta$ term is given by the experimental upper limit on the neutron electric dipole moment

CP-conserving




why so tiny?

$$d_n \sim 5 \times 10^{-16} \bar\theta \; e \text{ cm} < 2.9 \times 10^{-26} e \text{ cm} \quad \Longrightarrow \quad \bar\theta < 10^{-10}$$

Part C

# Possible solutions



**There are 3 distinct approaches to the strong CP problem (Peccei 98):**

**----The QCD vacuum dynamics itself selects $\bar{\theta}$ to be vanishing.**

**----Impose an additional chiral symmetry to dynamically drive $\bar{\theta} \to 0$.**

**----CP symmetry is spontaneously broken, with a naturally small $\bar{\theta}$.**

**Additional chiral symmetry: 1) $m_u = 0$ (Kaplan, Manohar, 86);**

**2) Peccei-Quinn U(1) symmetry (77).**

**A phenomenological measure of weak or strong CP-violating effects?**

$$\mathrm{CP_{weak}} \sim \frac{1}{\Lambda_{\mathrm{EW}}^6}(m_u - m_c)(m_c - m_t)(m_t - m_u)(m_d - m_s)(m_s - m_b)(m_b - m_d)\mathcal{J}_q \sim 10^{-13}$$

$$\mathrm{CP_{strong}} \sim \frac{1}{\Lambda_{\mathrm{QCD}}^6} m_u m_c m_t m_d m_s m_b \sin\bar{\theta} \sim 10^2 \sin\bar{\theta} < 10^{-8}$$

$$\boxed{\bar{\theta} = \theta + \arg(\det \mathcal{M})}$$

**In any case, the CP-violating effects in the quark sector are not large enough to interpret the cosmological matter-antimatter asymmetry.**

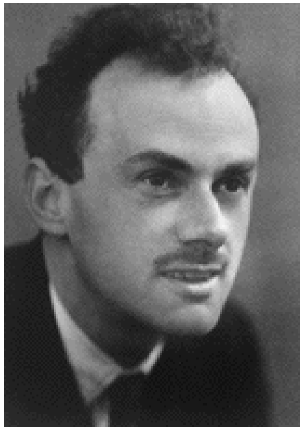
# Dirac's Expectation



PAUL A. M. DIRAC

Theory of electrons and positrons

*Nobel Lecture, December 12, 1933*



If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of Nature, we must regard it rather as an accident that the Earth (and presumably the whole solar system), contains a preponderance of negative electrons and positive protons. It is quite possible that for some of the stars it is the other way about, these stars being built up mainly of positrons and negative protons. In fact, there may be half the stars of each kind. The two kinds of stars would both show exactly the same spectra, and there would be no way of distinguishing them by present astronomical methods.

# The puzzle



**Why we did not see an anti-universe expected by Dirac?**

$$t = 10^{16}\,\text{sec}$$

$$r = 10^{29}\,\text{cm}$$

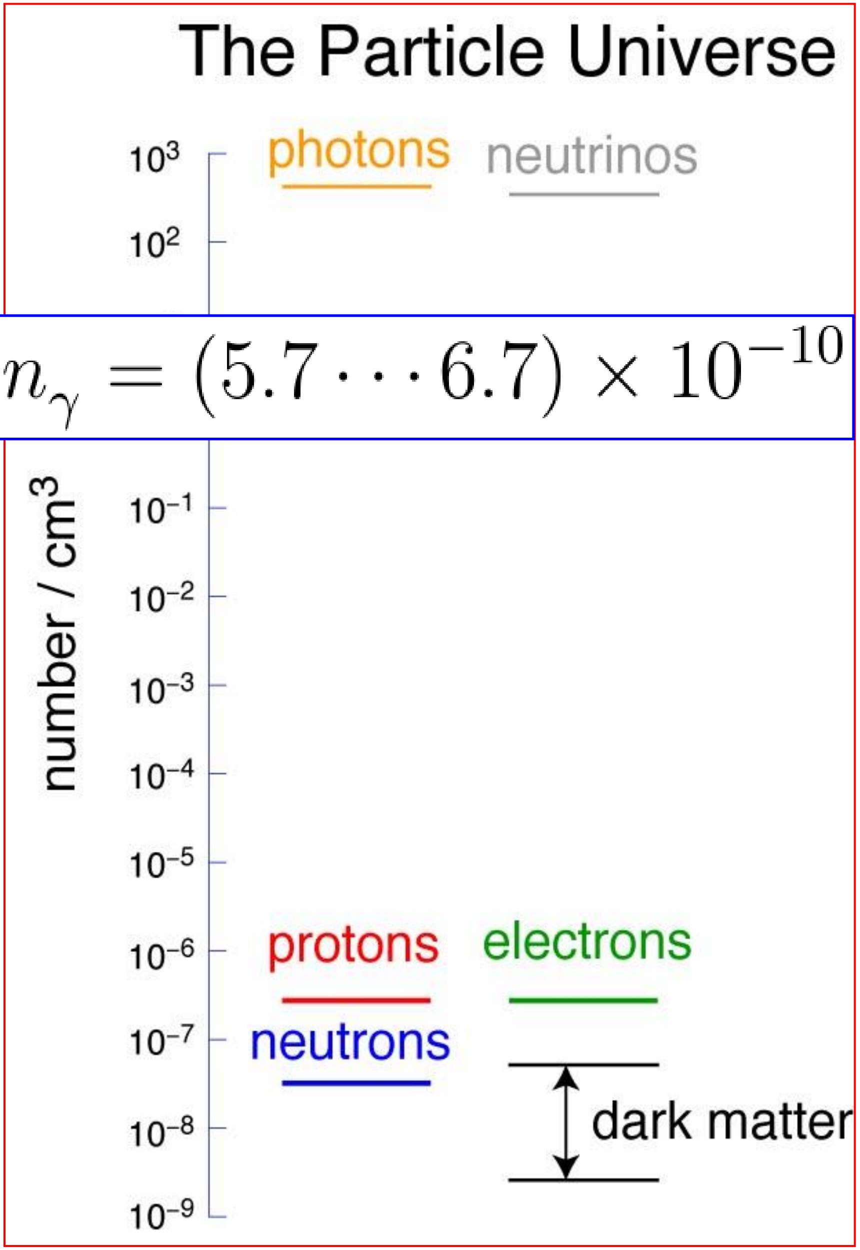
$$T = 2.7\ \text{K}$$

$$400\gamma\,/\,\text{cm}^3$$

$$10^{80}\quad p, n$$

$$0\qquad \bar{p}, \bar{n}$$

$$\eta \equiv n_{\mathrm{B}}/n_{\gamma} = (5.7\cdots6.7)\times10^{-10}$$

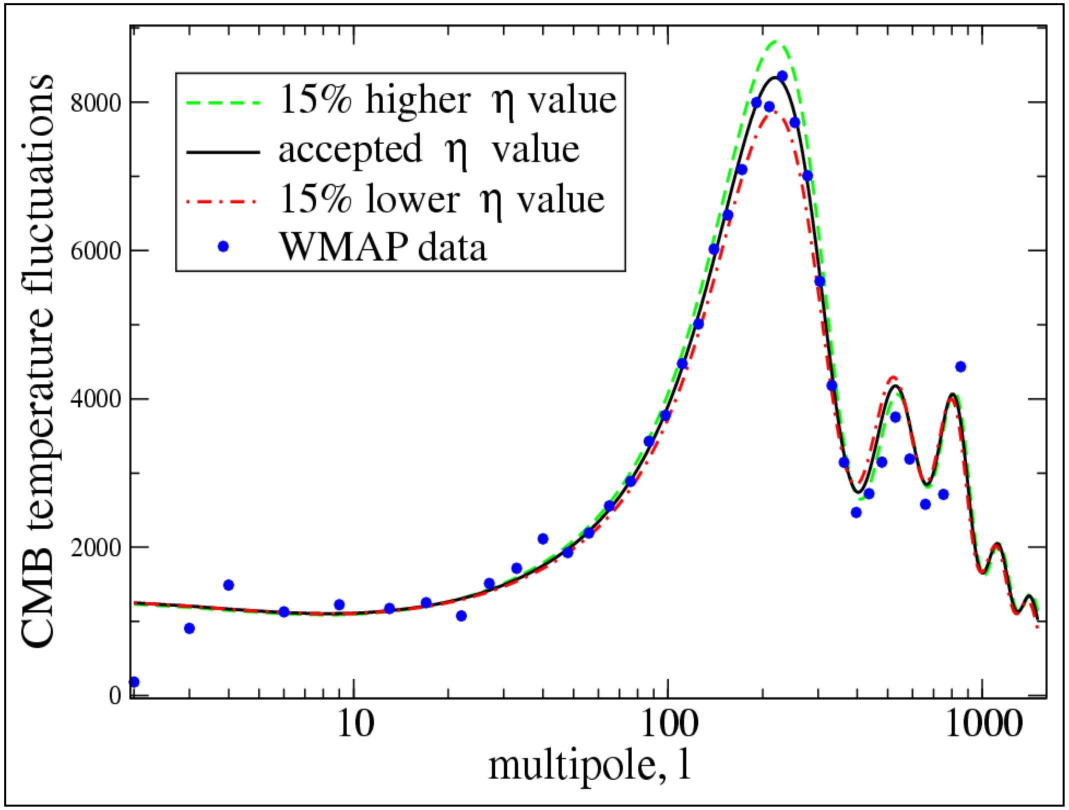


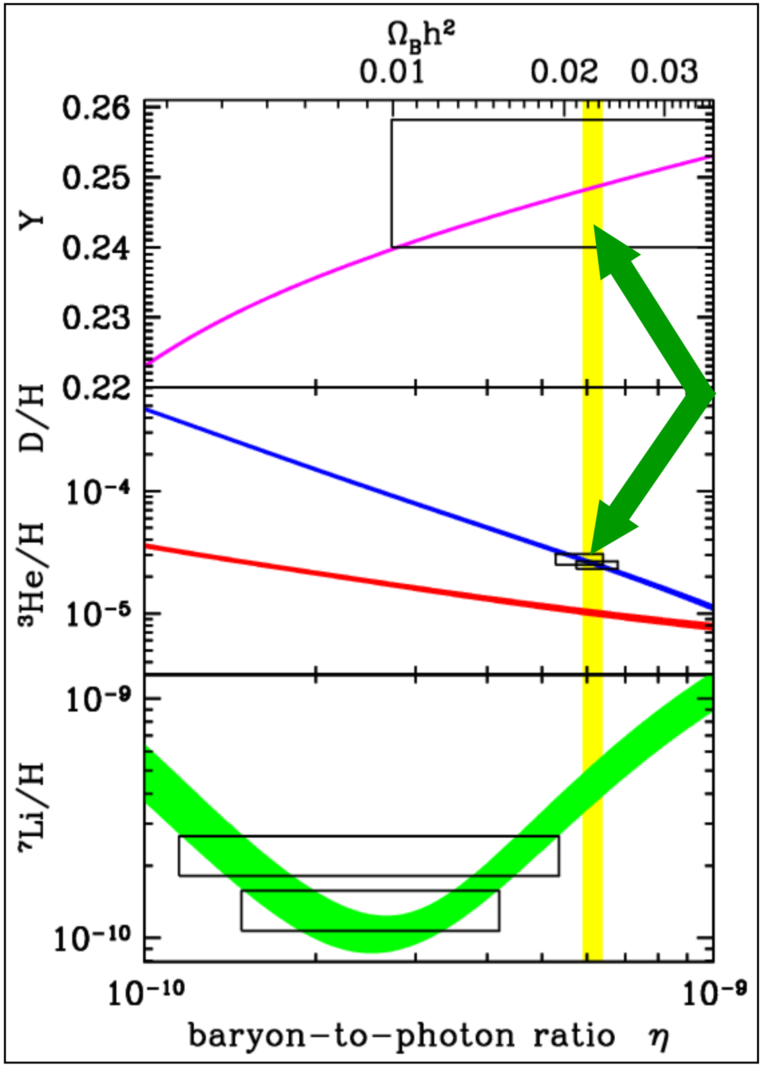
# Evidence



**η_B was historically determined from the Big Bang Nucleosynthesis: Primordial abundances of BBN light elements are sensitive to it.**

**η_B can now be measured from Cosmic Microwave Background: Relative sizes of those Doppler peaks of CMB temperature anisotropy are sensitive to it.**



# Sakharov conditions



**Baryogenesis:** 1) **Just-So** --- **B > 0** from the very beginning up to now;
2) **Dynamical picture** --- **B > 0** evolved from **B = 0** after inflation.

**Condition 1:** baryon number (B) violation.
[GUT, SUSY & even SM allow it, but no direct experimental evidence]

**Condition 2:** breaking of C and CP symmetries.
[C & CP asymmetries are both needed to keep B violation survivable]

**Condition 3:** departure from thermal equilibrium.
[Thermal equilibrium might erase B asymmetry due to CPT symmetry]


**Baryogenesis Mechanisms**

- Planck/GUT Baryogenesis;
- Electroweak Baryogenesis;
- Leptogenesis;
- Affleck-Dine Mechanism; ...

Sakharov's paper: almost no citation during 1967-1979

Now >1300 times

**Neutrino Physics**

# Remarks on CP violation

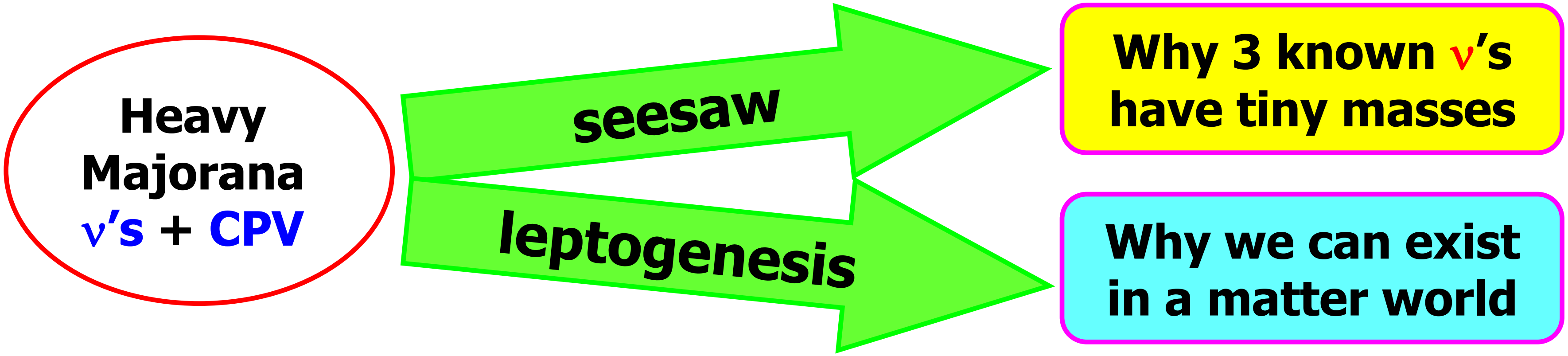


**CP** violation from the ***CKM*** quark mixing matrix is not the whole story to explain the **matter-antimatter asymmetry** of the visible Universe.


## Two reasons for this in the SM:

- **CP** violation from the **SM**'s quark sector is highly suppressed;
- The electroweak phase transition is not strongly first order.

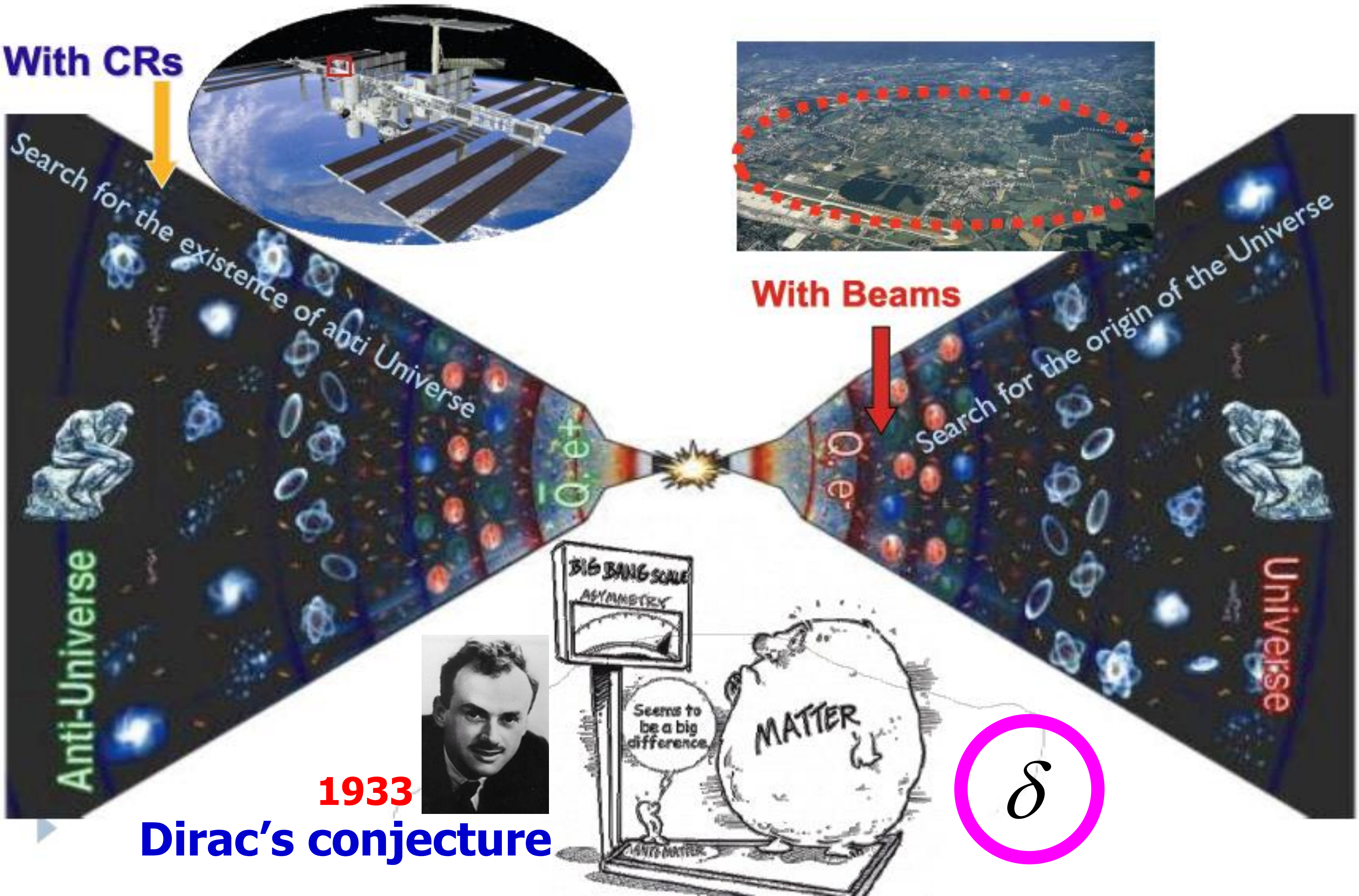
## New sources of CP violation are necessarily required.



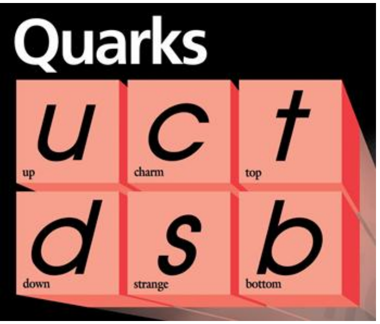

**Encouraging news: current $\nu$ data hint at $\delta \sim 270$ degrees.**

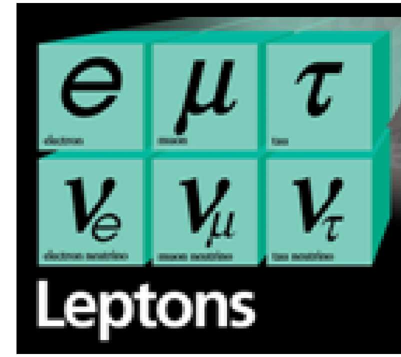
# Cosmic CP violation

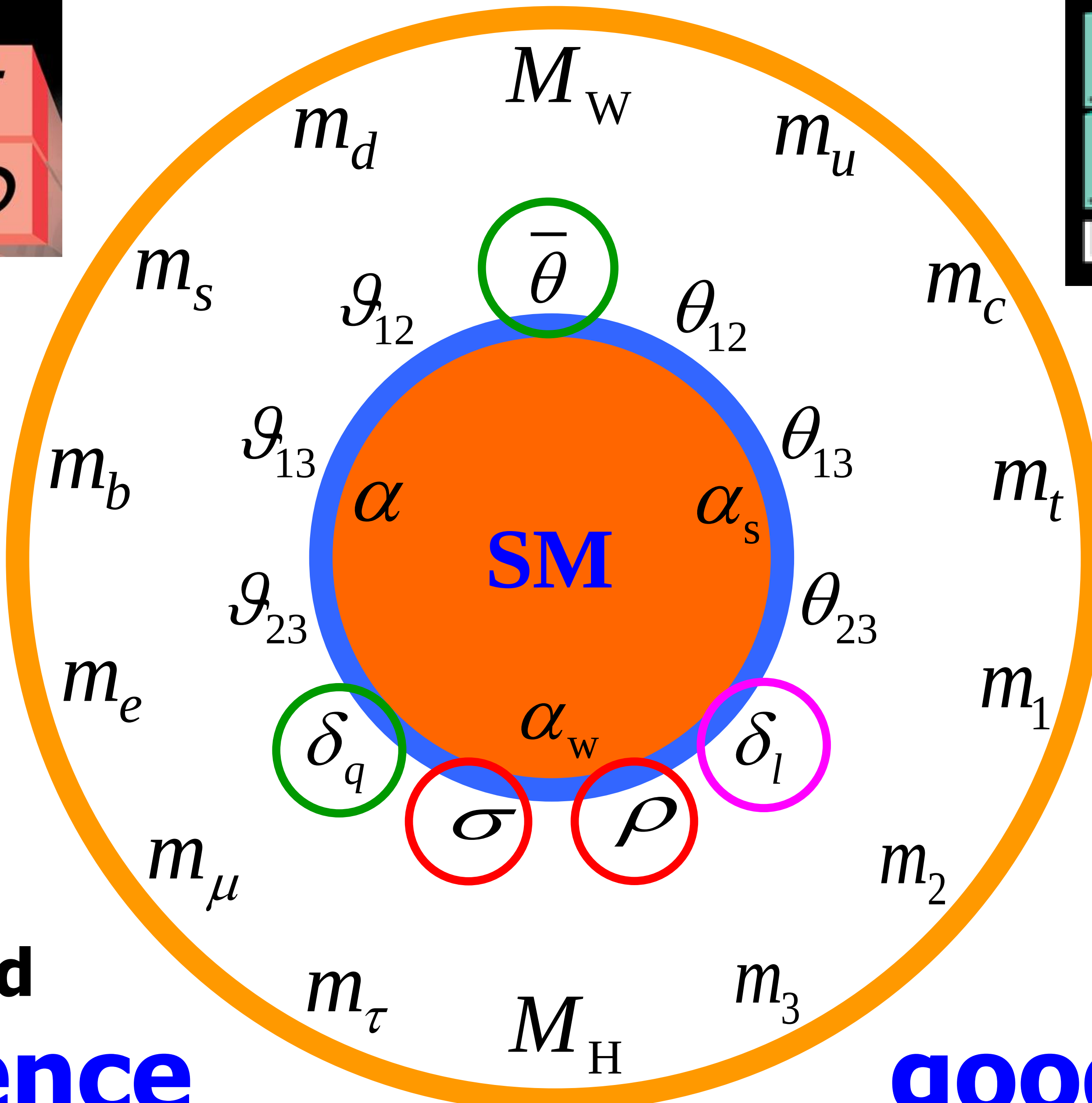




# Standard Flavors + Massive Neutrinos in a Pizza




**1/5**

**OK!**

**4/5**

**We need**

**patience**

**and**

**good idea**