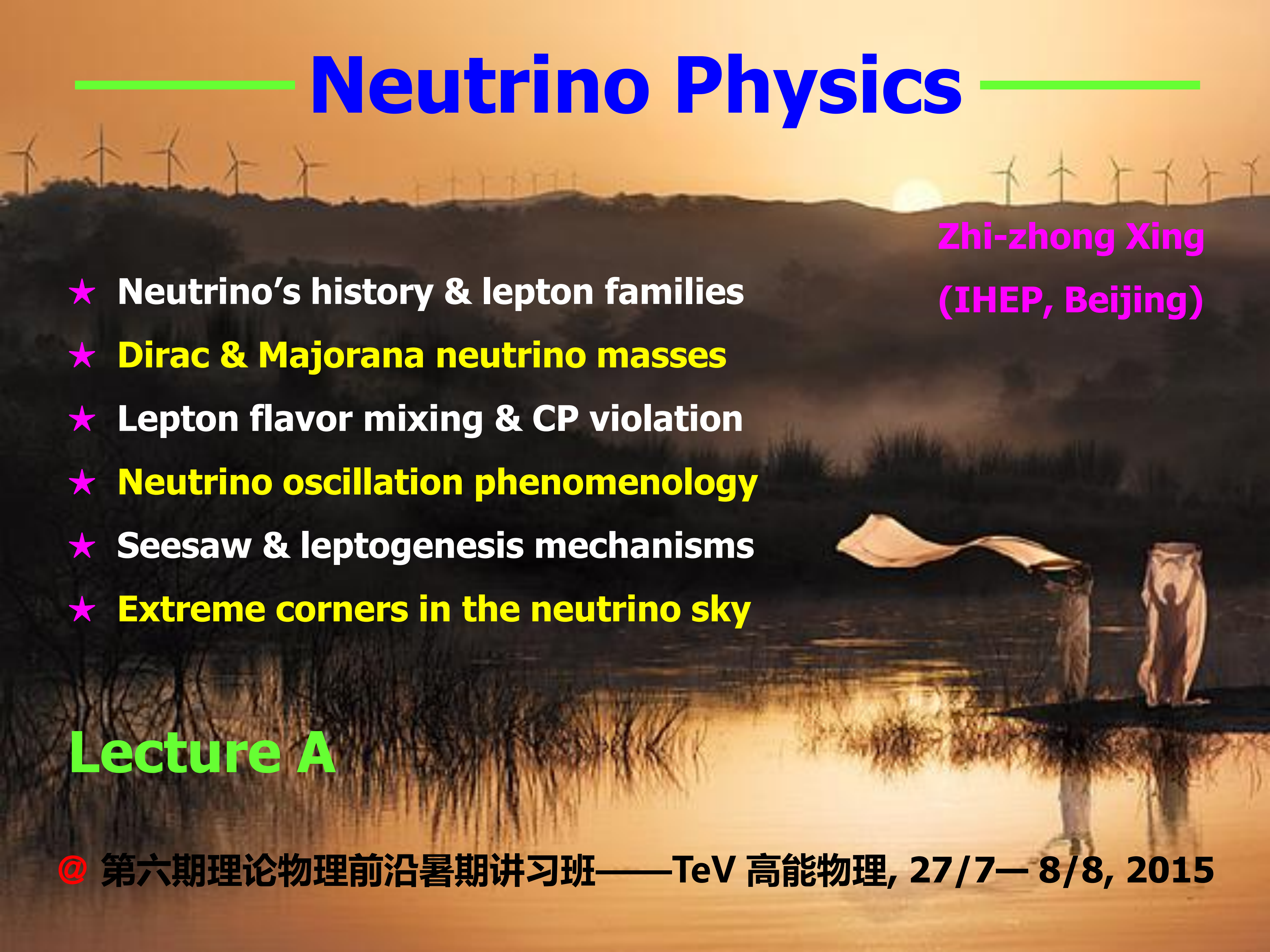


Neutrino Physics

Zhi-zhong Xing

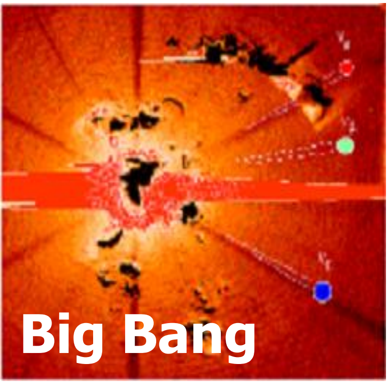
(IHEP, Beijing)

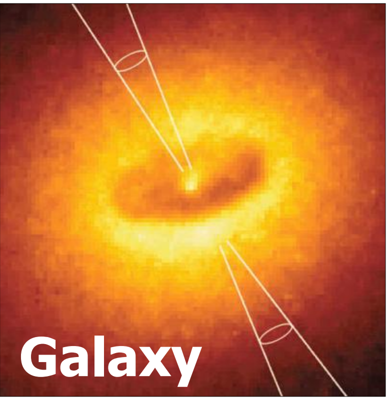
- ★ Neutrino's history & lepton families
- ★ Dirac & Majorana neutrino masses
- ★ Lepton flavor mixing & CP violation
- ★ Neutrino oscillation phenomenology
- ★ Seesaw & leptogenesis mechanisms
- ★ Extreme corners in the neutrino sky

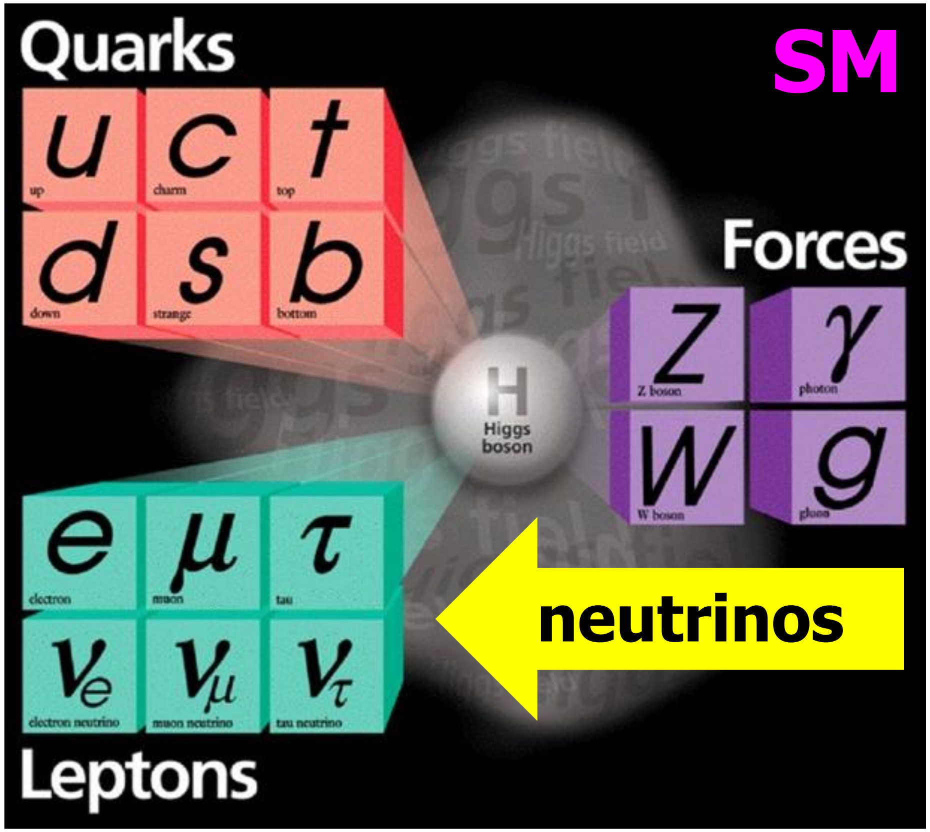
Lecture A

@ 第六期理论物理前沿暑期讲习班——TeV 高能物理, 27/7— 8/8, 2015

Neutrinos everywhere







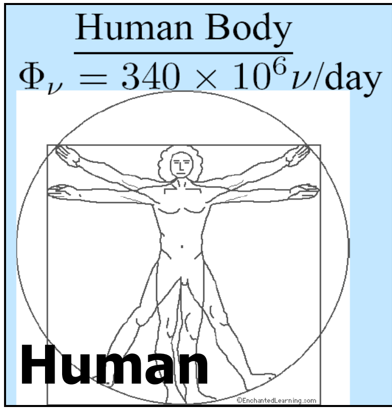
Properties:

charge = 0

spin = ½

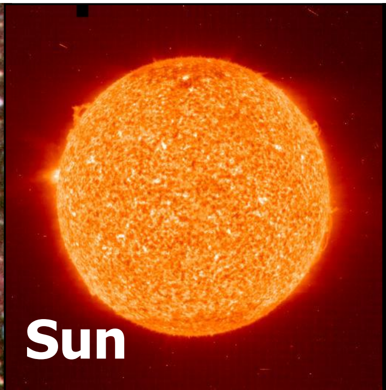
mass = 0

speed = c



Human Body

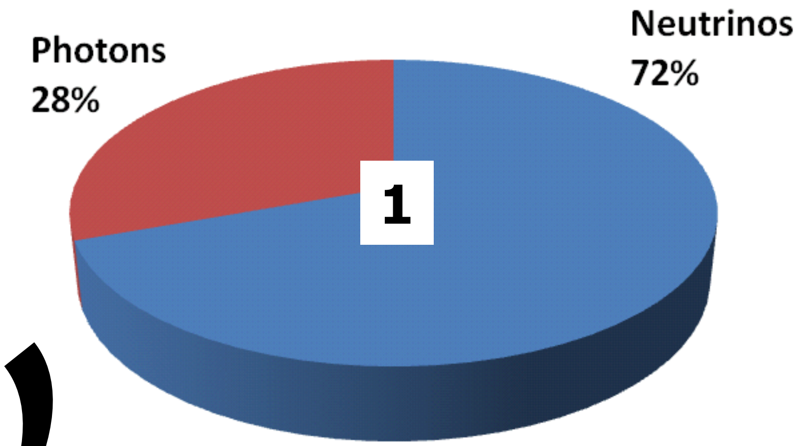
$\Phi_\nu = 340 \times 10^6 \nu/\text{day}$



Neutrinos:

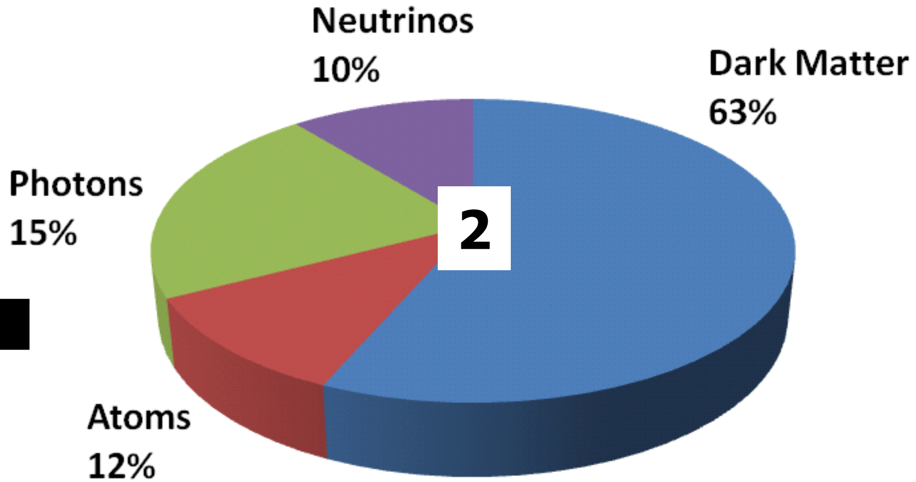
witness and participant in the evolution of the Universe

ν ν ν



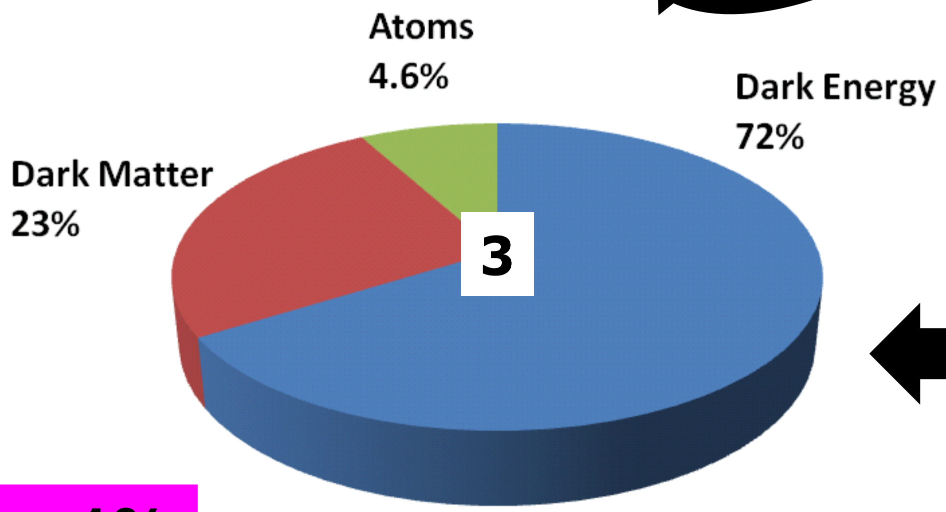
neutrino decoupling

t = 1 second



photon decoupling

t = 380 000 years



< 1%

Today

t = 13.7 billion years

Some open questions

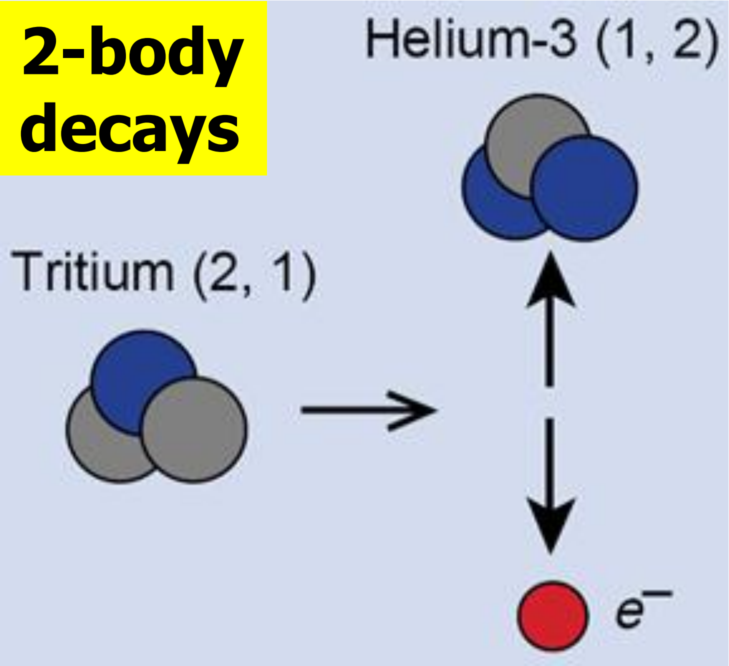
- the absolute ν mass scale?
- the mass hierarchy?
- the flavor desert?
- leptonic CP violation?
- the Majorana nature?
- How many species?
- cosmic ν background?
- supernova & stellar ν's?
- UHE cosmic ν's?
- warm dark matter?
- matter-antimatter asymmetry…

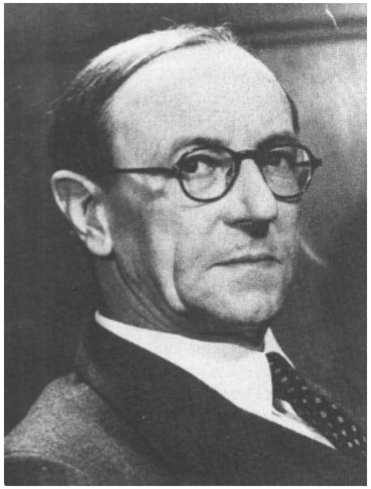
Lecture A1

- ★ Neutrinos from new physics
- ★ Interactions and discoveries
- ★ Flavors / families of leptons

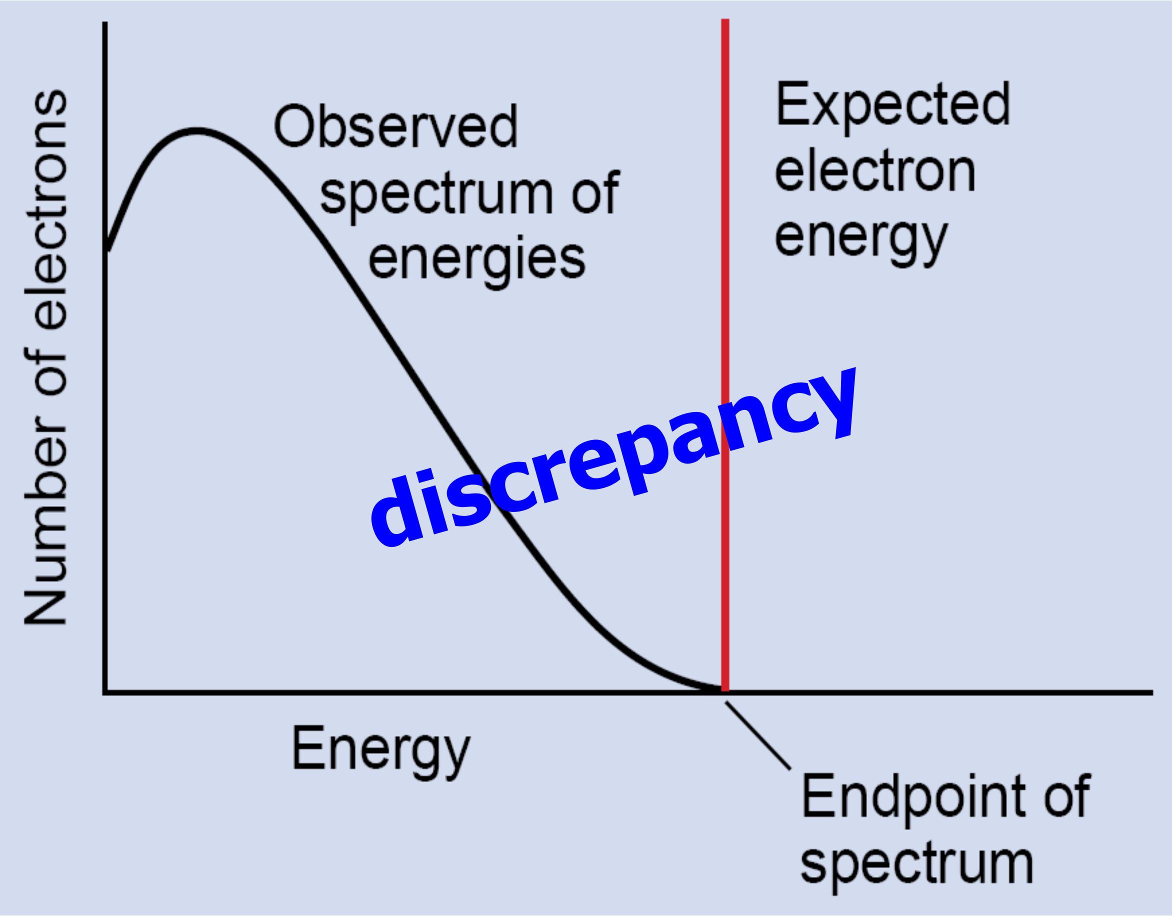
Beta decays in 1930

2-body decays





Energy crisis = New physics ?



J. Chadwick 1914/C. Ellis 1920-1927

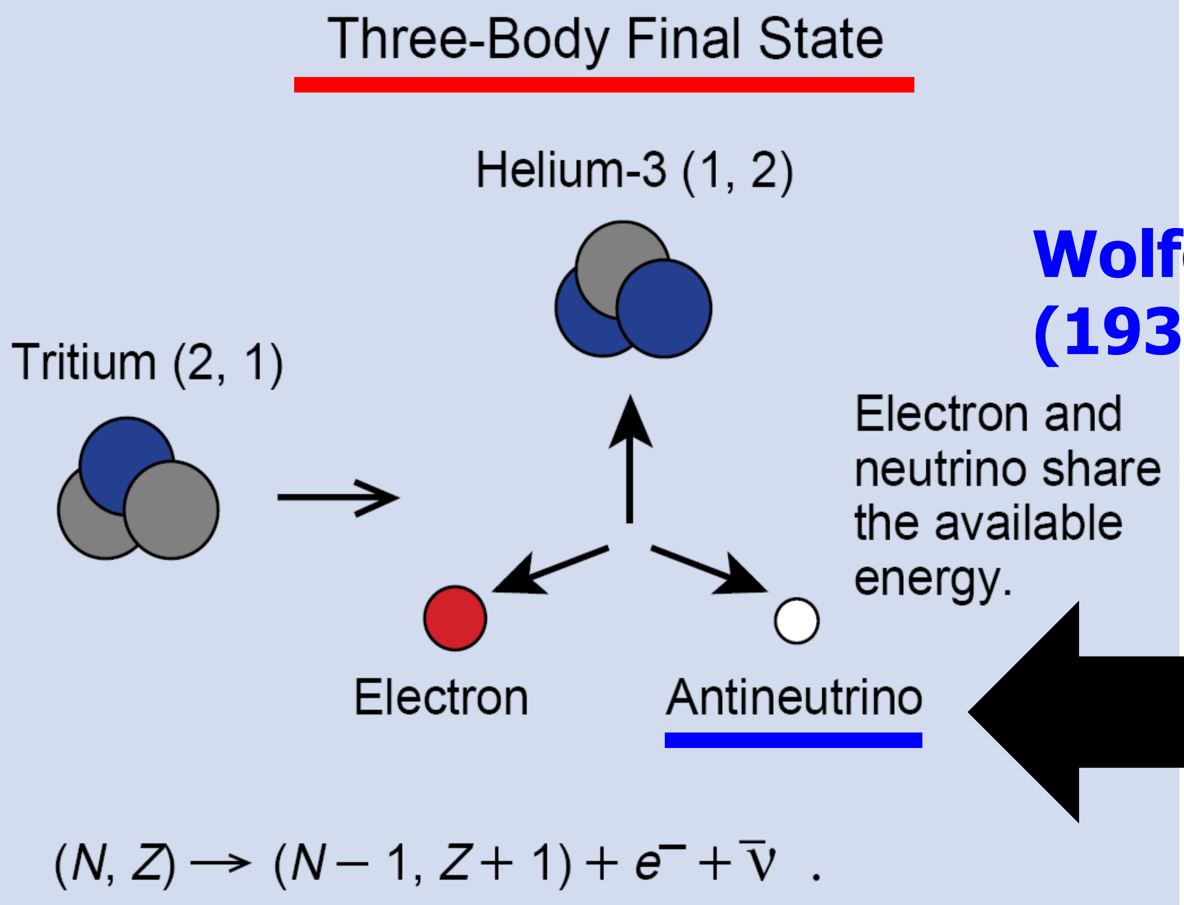
What to do?

Two ways out?

Niels Bohr

- **giving up sth**
- **adding in sth**

Wolfgang Pauli (1930)

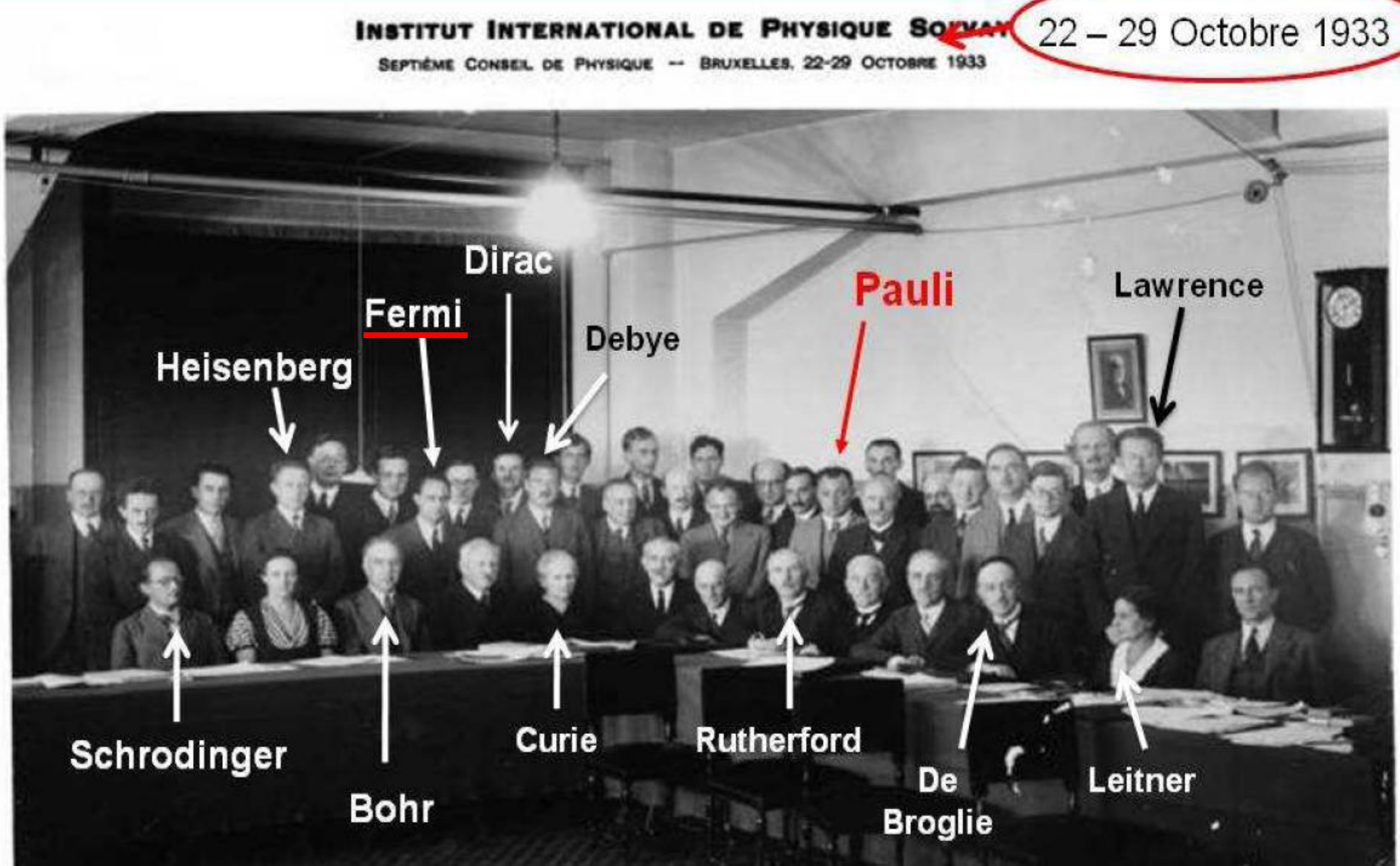


$$(N, Z) \rightarrow (N-1, Z+1) + e^- + \bar{\nu} \quad .$$

Pauli put forward this idea in a letter instead of a paper.....

Solvay 1933

Pauli gave a talk on his neutrino proposal in this congress.



22 – 29 Octobre 1933

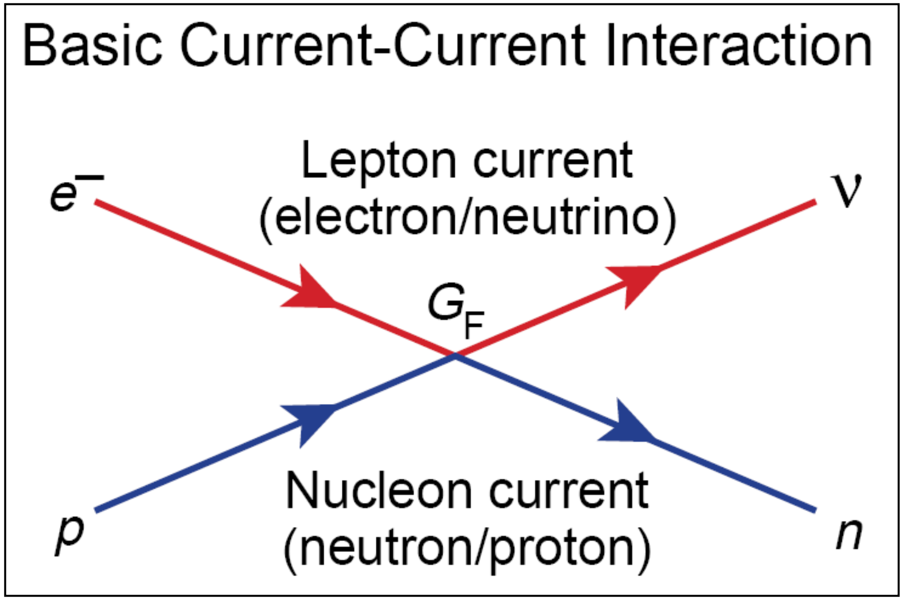
Fermi's theory

Enrico Fermi assumed a new force for β decay by combining 3 new concepts:

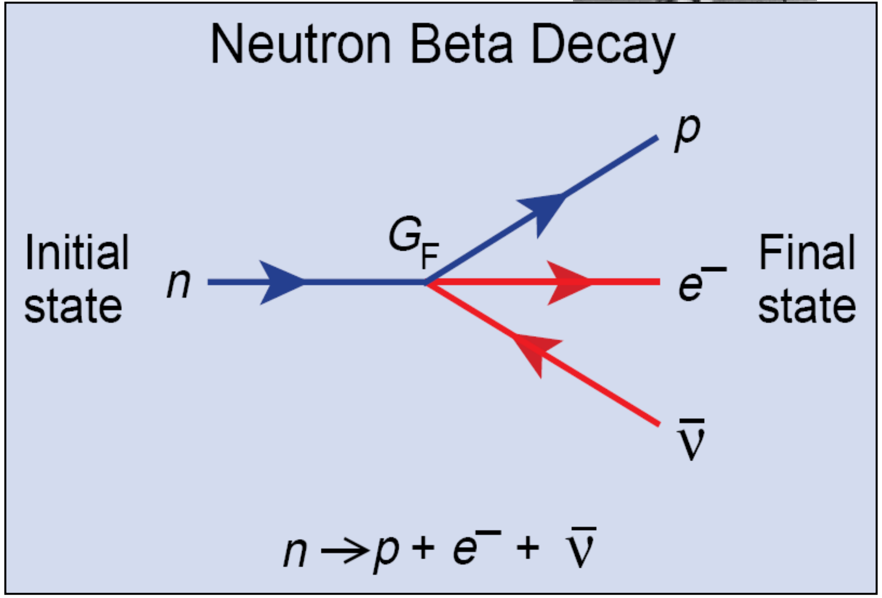
I will be remembered for this paper.

------ Fermi in Italian Alps, Christmas 1933

- ★ **Pauli's idea: neutrinos**
- ★ **Dirac's idea: creation of particles**
- ★ **Heisenberg's idea: isospin symmetry**



Basic Current-Current Interaction



Neutron Beta Decay

$n \rightarrow p + e^- + \bar{\nu}$

Fermi’s paper

E. Fermi's publications on the Weak Interaction

REJECTED

E. Fermi, “Tentative Theory of Beta Rays”
Letter Submitted to Nature (1933)

31 Dec, 1933

ANNO IV - VOL. II - N. 12 QUINDICINALE 31 DICEMBRE 1933 - XII

LA RICERCA SCIENTIFICA

ED IL PROGRESSO TECNICO NELL' ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi “beta”

Nota del prof. ENRICO FERMI

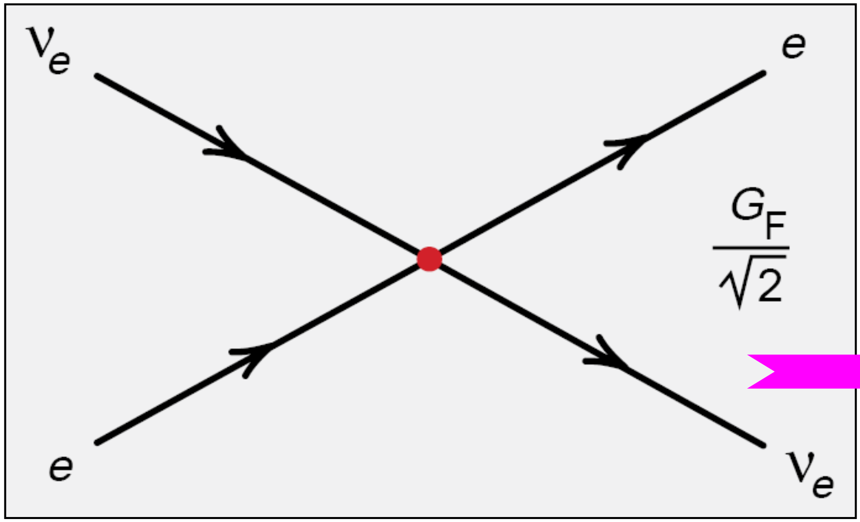
Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

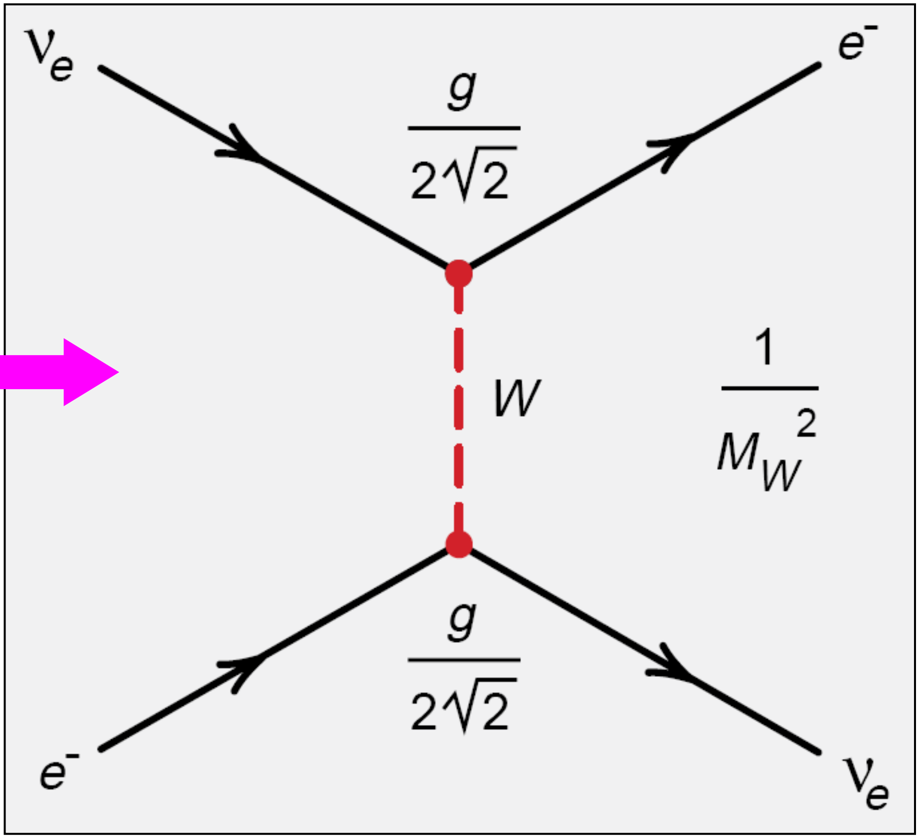
This is Fermi’s best theoretical work!
---- C.N. Yang

Published first in this journal and later in Z. Phys. in 1934

Weak interactions

From Fermi’s current-current interaction to weak charged-current gauge interactions





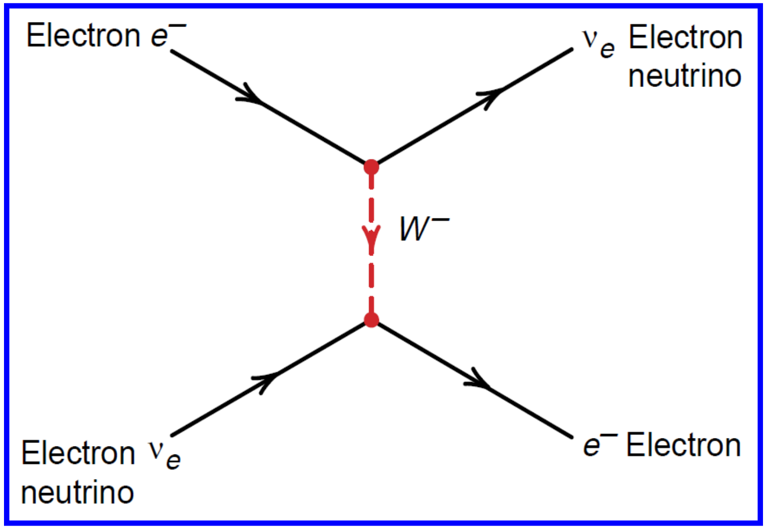
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

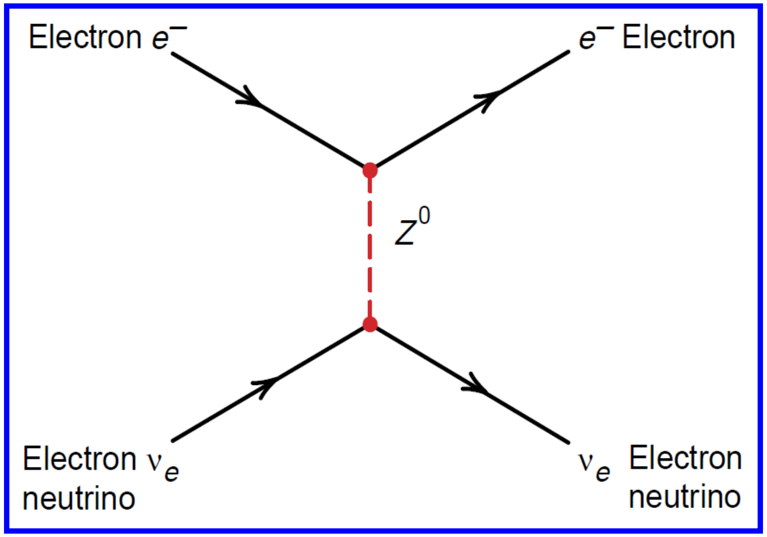
$$\boxed{M_W = 80.4 \mathrm{GeV}}$$

$$\boxed{G_F = 1.66 \times 10^{-5}\ \mathrm{GeV}^{-2}}$$

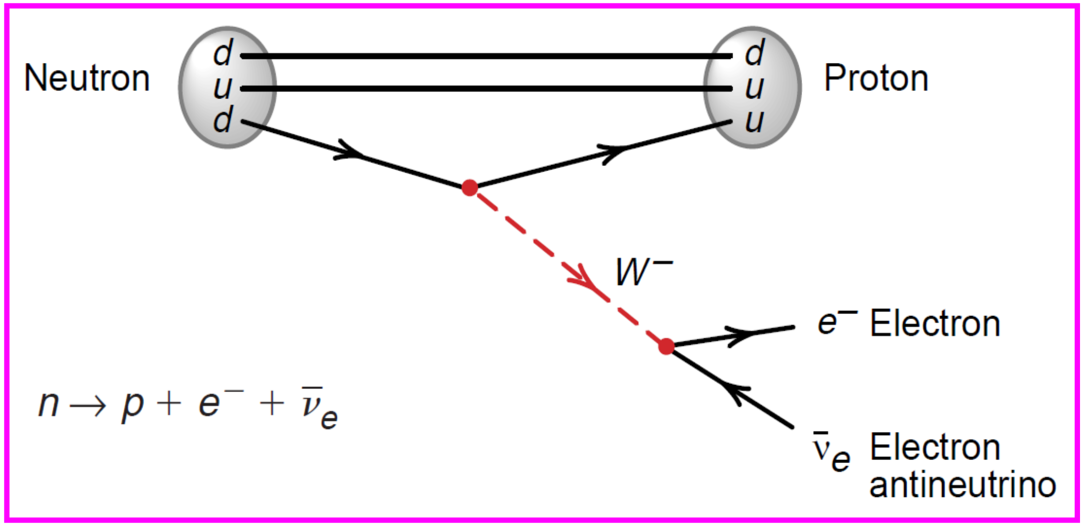
Weak interactions

Electron-neutrino scattering

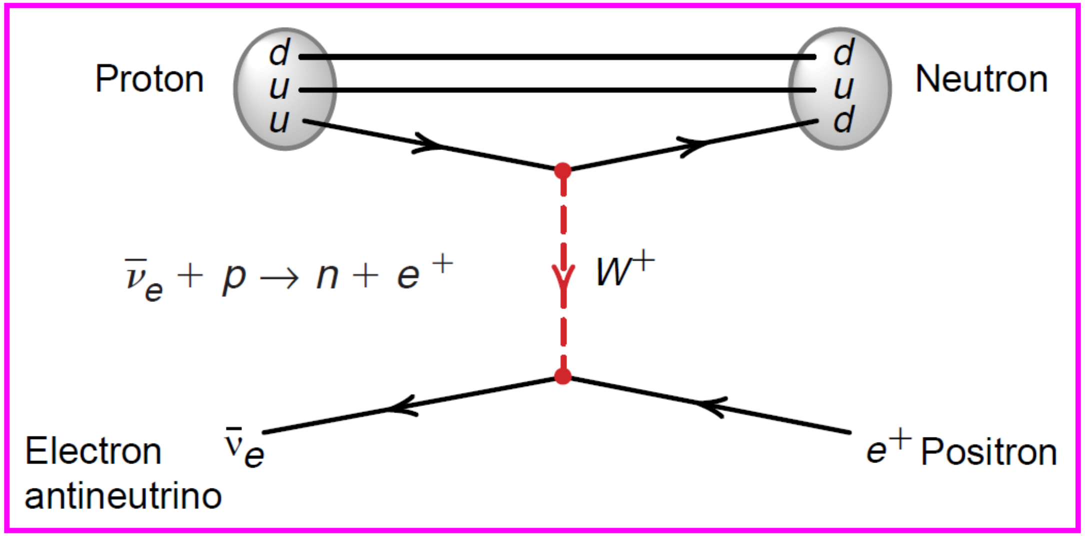




Neutron β decay / inverse β decay



$$n \rightarrow p + e^- + \bar{\nu}_e$$



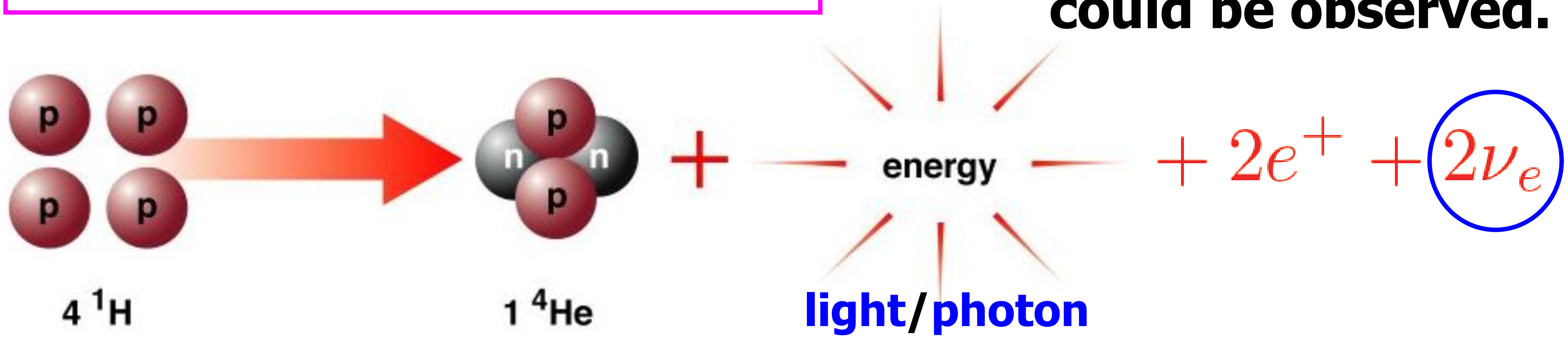
$$\bar{\nu}_e + p \rightarrow n + e^+$$

Exercise: **draw an electron-antineutrino scattering Feynman diagram.**

Why the sun shines?

$$\boxed{4p \rightarrow {}^{4}\mathrm{He} + 2\mathrm{e}^{+} + 2\nu_e + 26.73\ \mathrm{MeV}}$$

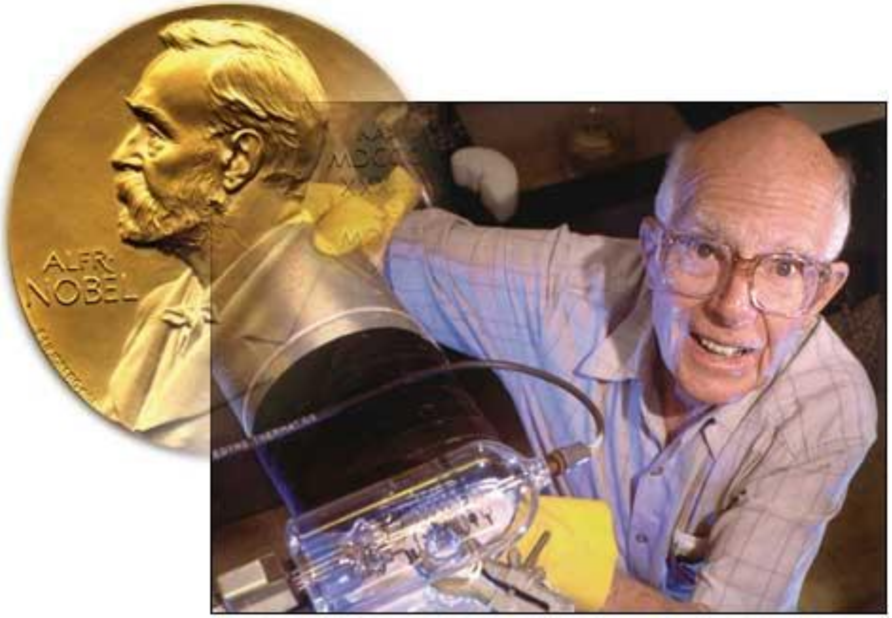
Only the neutrinos could be observed.



$+\ 2e^{+} + \boxed{2\nu_e}$

light/photon

Hans Bethe (1939), George Gamow & Mario Schoenberg (1940, 1941)

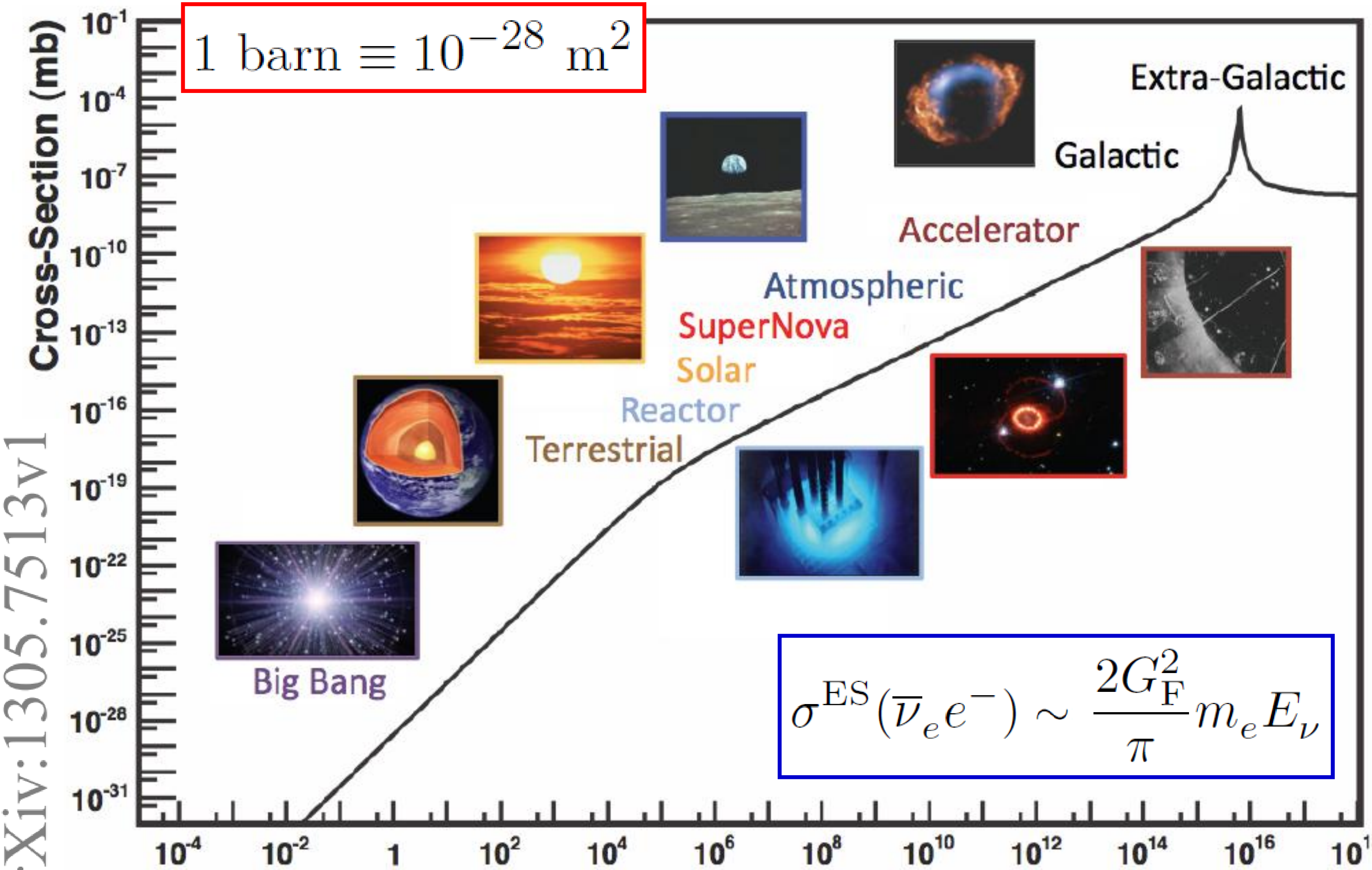


Raymond Davis: born in 1914, discovery in 1968 and Nobel Prize in 2002

Observed the solar neutrino and its anomaly in 1968

Too shy to be seen?

$$1\ \mathrm{barn} \equiv 10^{-28}\ \mathrm{m}^2$$



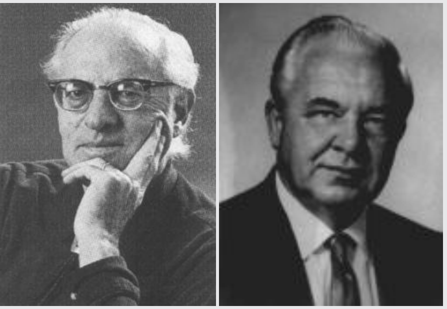
$$\sigma^{\mathrm{ES}}(\overline{\nu}_e e^-) \sim \frac{2G_\mathrm{F}^2}{\pi} m_e E_\nu$$

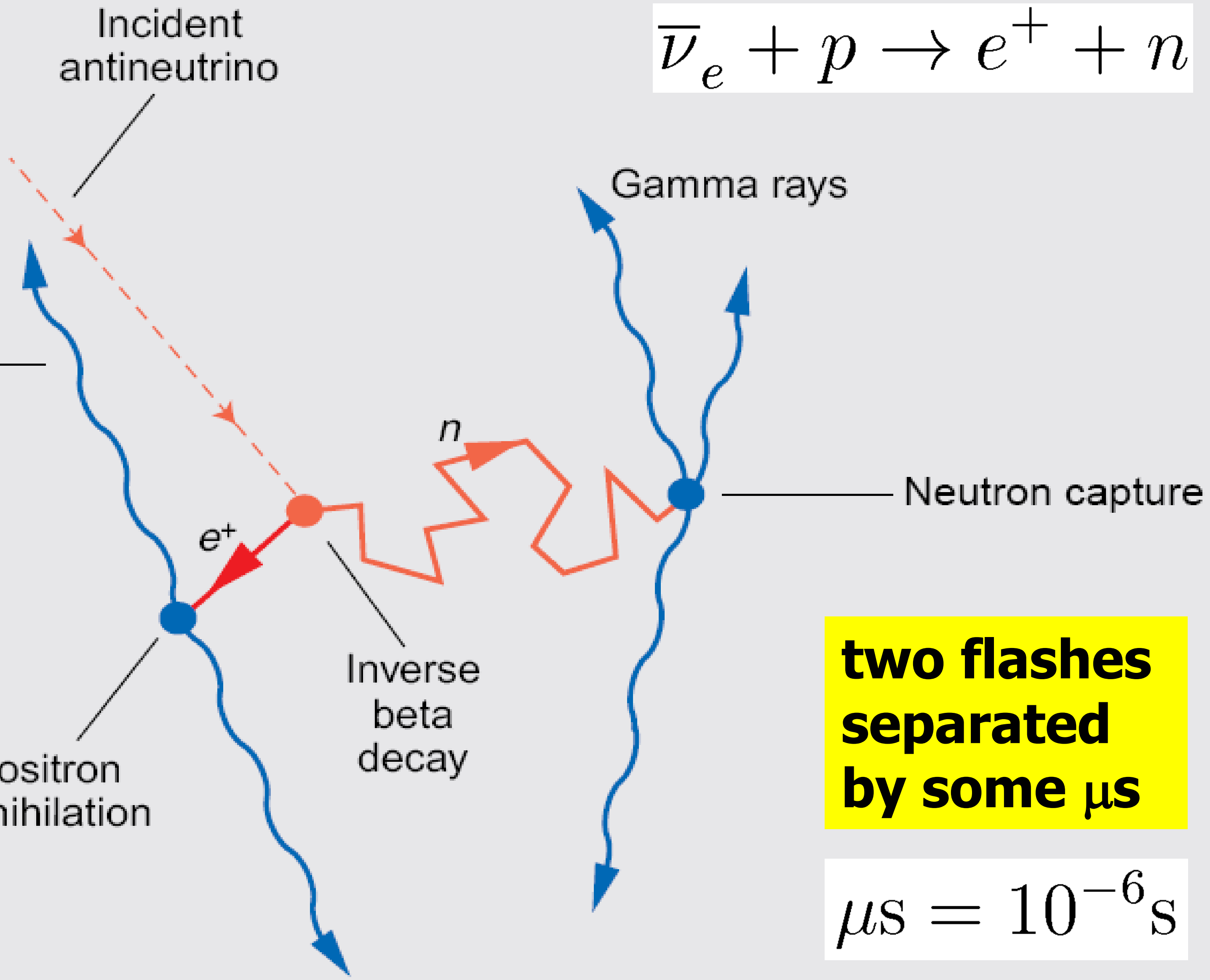
J.A. Formaggic and G.P. Zeller: a review.

Neutrinos in 1956

F. Reines and C. Cowan detected reactor antineutrinos via

$$\overline{\nu}_e + p \rightarrow e^+ + n$$





two flashes separated by some μs

$$\mu\mathrm{s} = 10^{-6}\mathrm{s}$$

Positive result?

Reines and Cowan's telegram to Pauli on 14/06/1956:

We're happy to inform you that we've definitely detected neutrinos from fission fragments by observing inverse β decay of protons. Observed cross section agrees well with expected $6\times10^{-44}\,\mathrm{cm}^2$. **(Pauli didn't reply, a case of champagne)**

Such a theoretical value was based on a parity-conserving formulation of the β decay with 4 independent degrees of freedom for ν's.

$$\boxed{\sigma(\overline{\nu}_e p) = \sigma(\nu_e n) \approx 9.1\times10^{-44}\left(\frac{E_\nu}{\mathrm{MeV}}\right)^2\ \mathrm{cm}^2}$$

This value is at least doubled after the discovery of parity violation in 1957, leading to the two-component neutrino theory in 1957 and the V–A weak theory in 1958.

Reines' excuse

A new paper on this experiment published in Phys. Rev. in 1960 reported a cross section twice as large as that given in 1956.

Reines (1979): our initial analysis grossly overestimated the detection efficiency with the result that the measured cross section was at first thought to be in good agreement with [the pre-parity violation] prediction.

Skepticism

{over 39 years}

The Nobel Prize finally came to Frederick Reines in 1995!

Pontecorvo’s idea

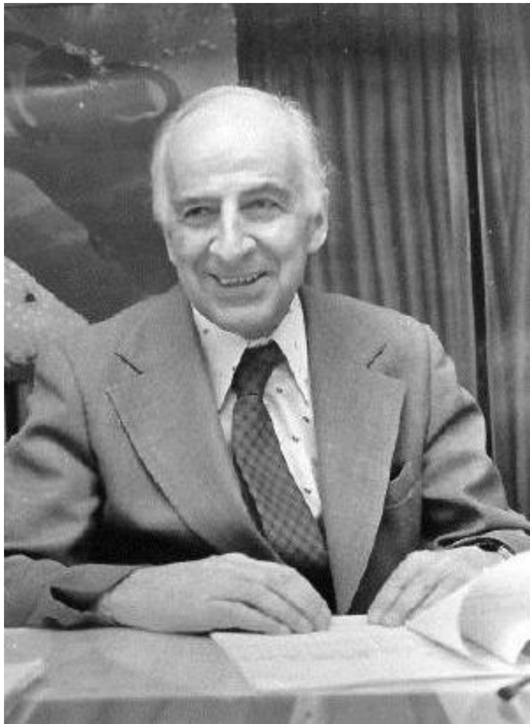
★ Theory of the Symmetry of Electrons and Positrons

Ettore Majorana

Nuovo Cim. 14 (1937) 171

Are massive neutrinos and antineutrinos identical or different —— a fundamental puzzling question in particle physics.

★ Mesonium and Anti-mesonium



Bruno Pontecorvo

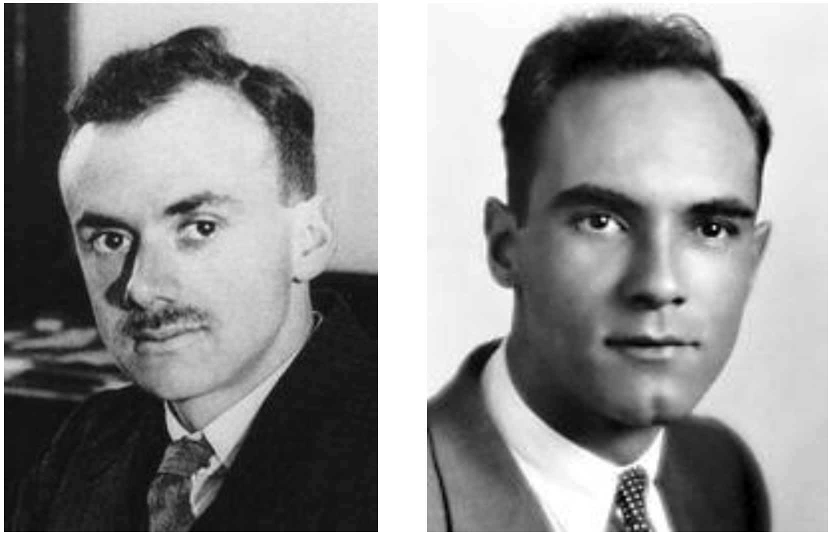
Zh. Eksp. Teor. Fiz. 33 (1957) 549
Sov. Phys. JETP 6 (1957) 429

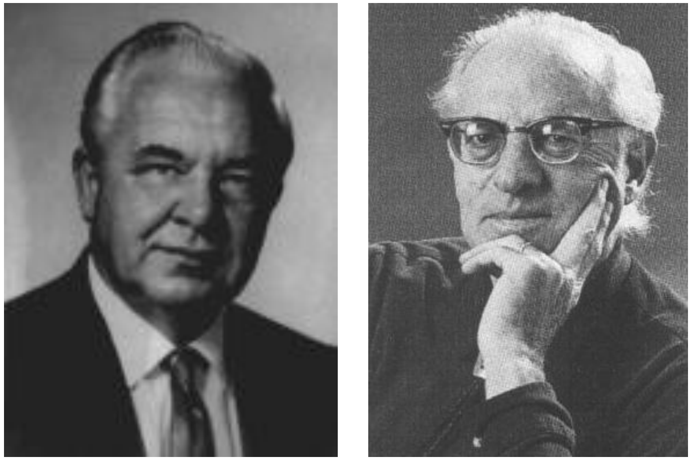
If the two-component neutrino theory turned out to be incorrect and if the conservation law of neutrino charge didn’t apply, then neutrino-antineutrino transitions would in principle be possible to take place in vacuum.

Electron and its neutrino

The electron was discovered in 1897, by Joseph Thomson.

The electron's anti-particle, positron, was predicted by Paul Dirac in 1928, and discovered by Carl Anderson in 1932.

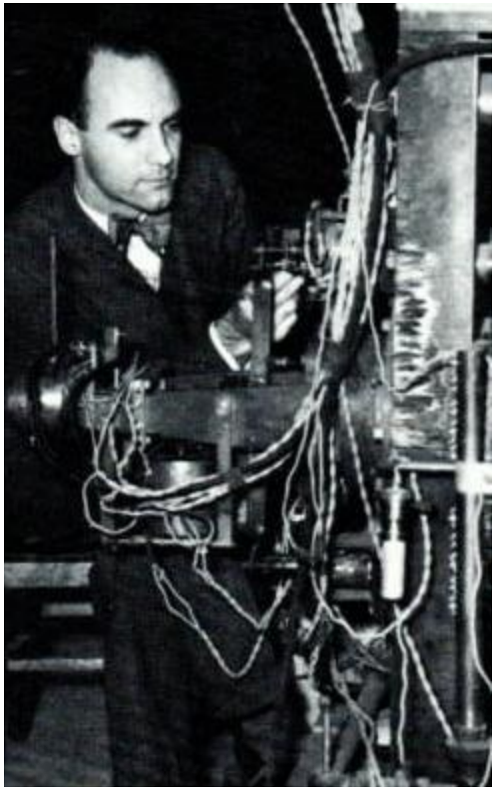




In 1956 Clyde Cowan and Frederick Reines discovered the positron's partner, electron antineutrino.

Muon

The **muon** particle, a sister of the electron, was discovered in 1936 by **Carl Anderson** and his first student **S. Neddermeyer**; and independently by **J. Street** *et al*.



It was not **Hideki Yukawa**'s "pion". And it was the first flavor puzzle.

Isidor Rabi famously asked:

Who ordered that?

Isidor Isaac Rabi

FAMILY

Muon neutrino

The **muon neutrino**, the muon's neutral counterpart, was discovered by **Leon Lederman**, **Melvin Schwartz** and **Jack Steinberger** in 1962.

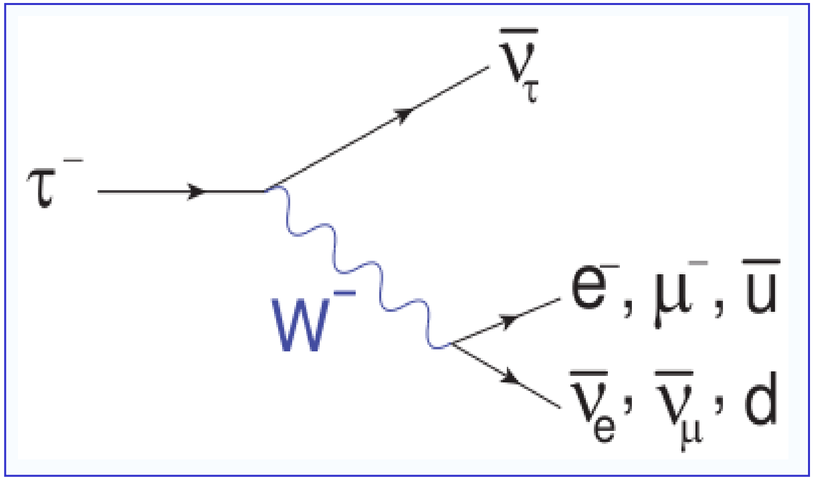
Neutrino flavor conversion was proposed by **Z. Maki**, **M. Nakagawa** and **S. Sakata** in 1962.

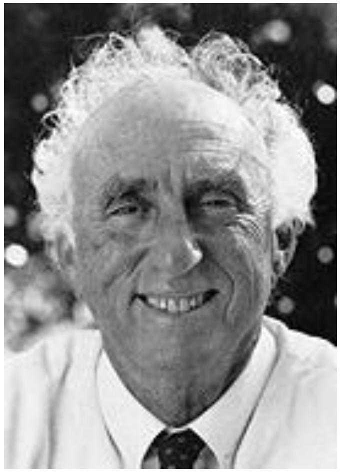
Neutrinos convert into **antineutrinos** first proposed by **Bruno Pontecorvo** in 1957.

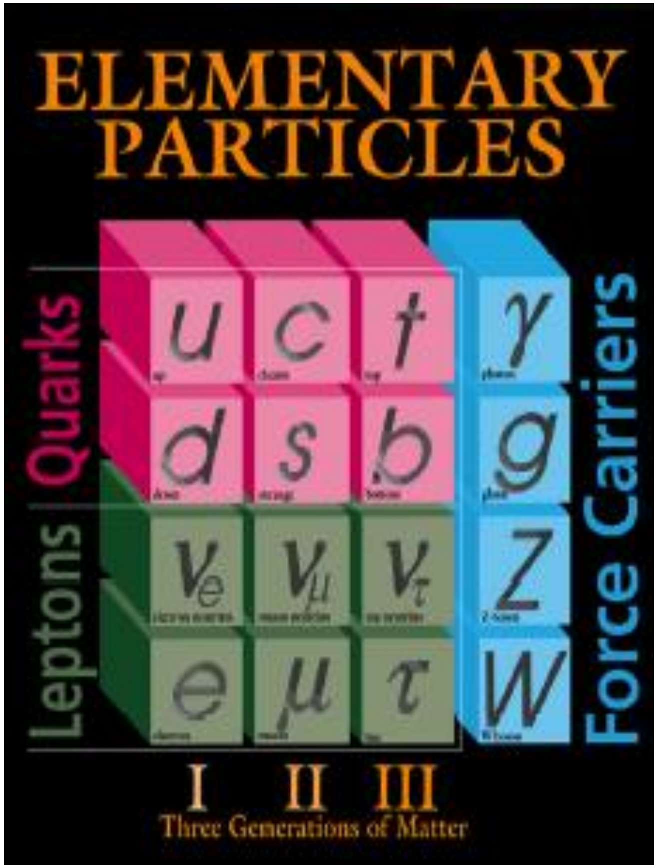
Tau and its neutrino

The tau particle was discovered by Martin Perl in 1975 via:

$$e^{+} + e^{-} \to e^{\pm} + \mu^{\mp} + \text{undetected particles}$$







In 2000, the tau neutrino was finally discovered at the Fermilab.

The lepton family is complete!

Leptons and Nobel Prizes

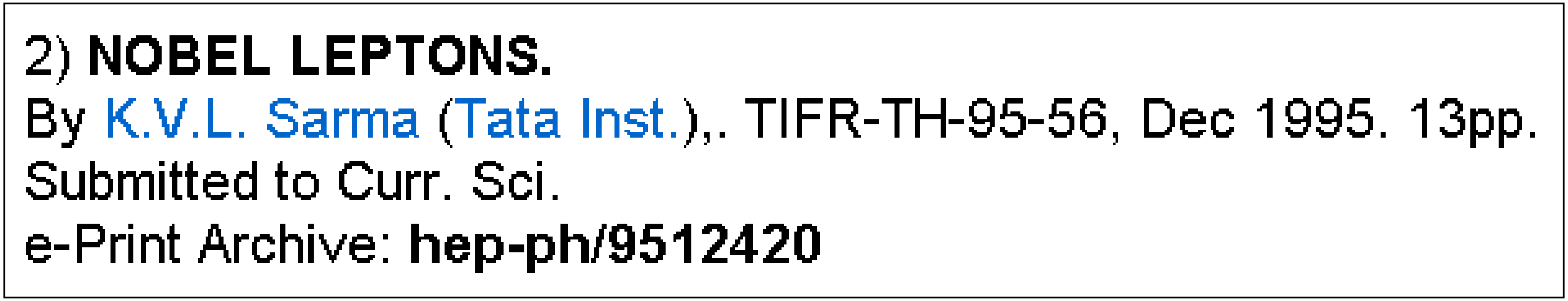
e	**J.J. Thomson 1897** ☺	**J.J. Thomson 1906 (NP)**
ν_e	**C.L. Cowan et al. 1956**	**F.J. Reines 1995 (NP)**
μ	**J.C. Street et al.** **C.D. Anderson 1936** ☺	**1975 – 1936 = 1936 – 1897 = 39**
ν_μ	**G. Danby et al. 1962**	**M. Schwartz, L.M. Lederman,** **J. Steinberger 1988 (NP)**
τ	**M.L. Perl et al. 1975** ☺	**M.L. Perl 1995 (NP)**
ν_τ	**K. Kodama et al. 2000**	

Antimatter: Positron.

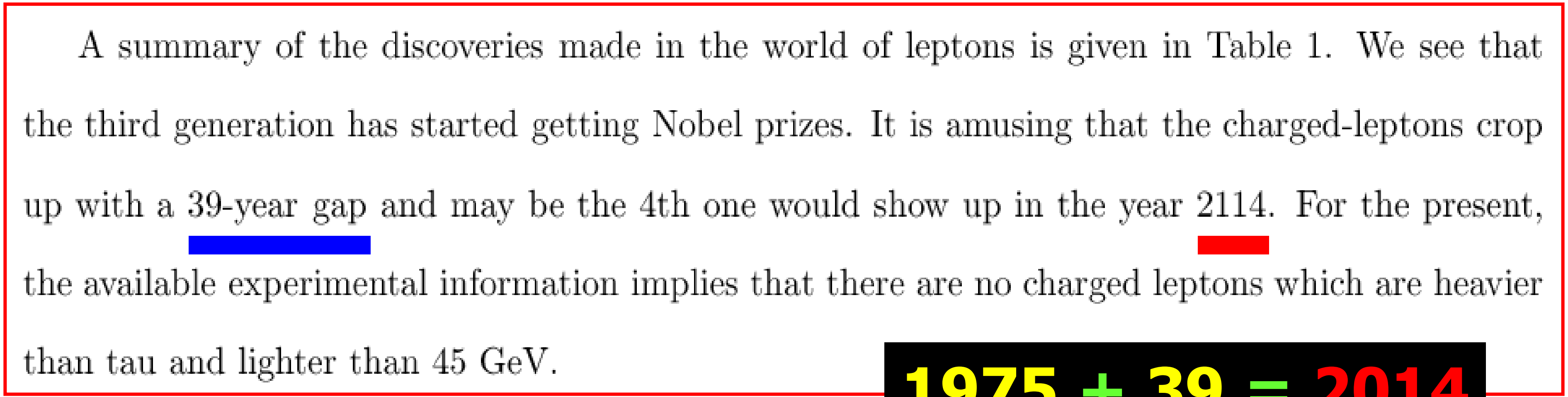
Predicted by P.A.M. Dirac in 1928.
Discovered by C.D. Anderson in 1932; Nobel Prize in 1936.

Sarma-Xing theorem

In 1995 it was an Indian theorist who first discovered the 39-year gap of charged leptons.



2) **NOBEL LEPTONS.**
By K.V.L. Sarma (Tata Inst.),. TIFR-TH-95-56, Dec 1995. 13pp.
Submitted to Curr. Sci.
e-Print Archive: **hep-ph/9512420**



A summary of the discoveries made in the world of leptons is given in Table 1. We see that the third generation has started getting Nobel prizes. It is amusing that the charged-leptons crop up with a 39-year gap and may be the 4th one would show up in the year 2114. For the present, the available experimental information implies that there are no charged leptons which are heavier than tau and lighter than 45 GeV.

1975 + 39 = 2014

My contribution: corrected 2114 to 2014, so the discovery would be possible 100 years earlier (just this year)!

Lecture A2

- ★ **The Dirac mass term**
- ★ **The Majorana mass term**
- ★ **On the neutrinoless ββ decay**

What is mass?

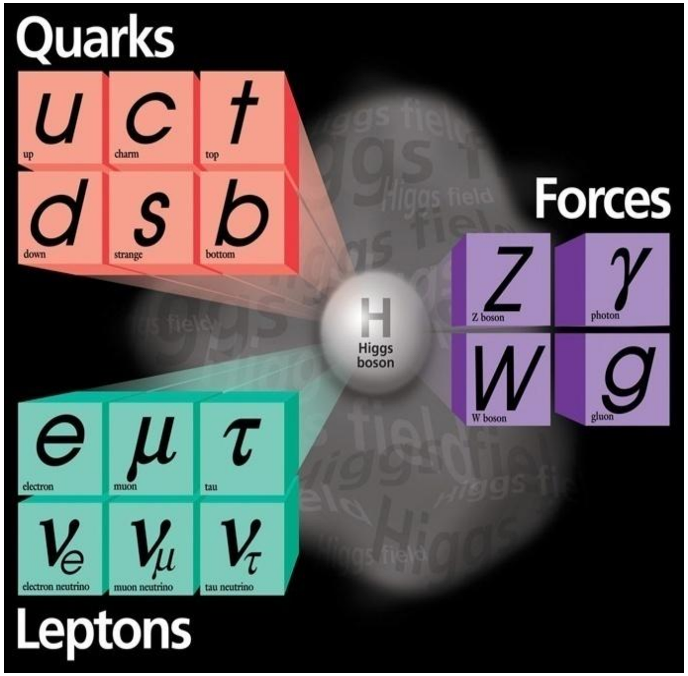
Mass is the inertial energy of a particle existing at rest.

- A **massless** particle has no way to exist at rest. It must always move at the speed of light.
- A **massive** fermion (lepton or quark) must exist in both the left- and right-handed states.

The **Brout-Englert-Higgs** mechanism is responsible for the origin of *W* / *Z* and fermion masses in the SM.

$$L_{\mathrm{SM}} = L(f,G) + \underline{L(f,H)} + \underline{L(G,H)} + L(G) - V(H)$$

All the **bosons** were discovered in **Europe**, and most of the **fermions** were discovered in **America**.



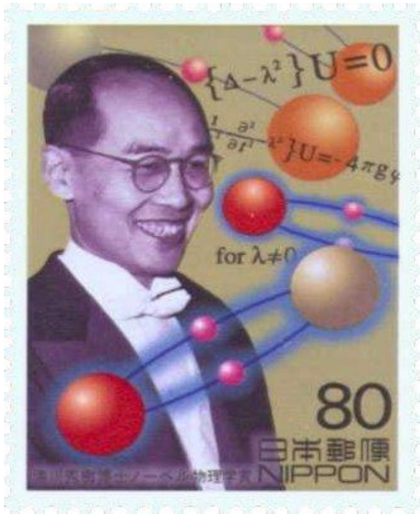
Four forces

force	strength	range	mediator	mass
strong	1	$10^{-15}\,\mathrm{m}$	**gluon/π**	$\sim 10^{2}\,\mathrm{MeV}$
EM	$1/137$	∞	**photon**	$=0$
weak	10^{-6}	$10^{-18}\,\mathrm{m}$	**W/Z/H**	$\sim 10^{2}\,\mathrm{GeV}$
gravitation	6×10^{-39}	∞	**graviton**	$=0$

Yukawa relation for the mediator's mass *M* and the force's range *R*:

$$\boxed{M \approx \frac{200\,\mathrm{MeV}\times 10^{-15}\,\mathrm{m}}{R}}$$

汤川秀树 (Hideki Yukawa): His first paper in 1935 made him get the Nobel Prize in 1949.



In the SM

All ν's are massless due to the model's simple structure:

---- SU(2)×U(1) gauge symmetry and Lorentz invariance

Fundamentals of a quantum field theory

---- Economical particle content:

No right-handed neutrino; only a single Higgs doublet

---- Mandatory renormalizability:

No dimension ≥ 5 operator (*B*-*L* conserved in the SM)

Neutrinos are massless in the SM: Natural or not?

YES: It's toooooooo light and almost left-handed;

NO: No fundamental symmetry/conservation law.

Some notations

Define the left- and right-handed neutrino fields:

$$\nu_{\rm L} = \begin{pmatrix} \nu_{e{\rm L}} \\ \nu_{\mu{\rm L}} \\ \nu_{\tau{\rm L}} \end{pmatrix}$$

[redacted]

$$N_{\rm R} = \begin{pmatrix} N_{1{\rm R}} \\ N_{2{\rm R}} \\ N_{3{\rm R}} \end{pmatrix}$$

Extend the SM's particle content

$$\begin{aligned} \psi_{\rm L} &\equiv \frac{1-\gamma_5}{2}\psi \\ \psi_{\rm R} &\equiv \frac{1+\gamma_5}{2}\psi \end{aligned}$$

The charge-conjugate counterparts are defined below and transform as right- and left-handed fields, respectively:

$$(\nu_{\rm L})^c \equiv \mathcal{C}\overline{\nu_{\rm L}}^T \; , \qquad (N_{\rm R})^c \equiv \mathcal{C}\overline{N_{\rm R}}^T$$

$$\overline{(\nu_{\rm L})^c} = (\nu_{\rm L})^T\mathcal{C} \; , \qquad \overline{(N_{\rm R})^c} = (N_{\rm R})^T\mathcal{C}$$

$$(\nu_{\rm L})^c = (\nu^c)_{\rm R} \text{ and } (N_{\rm R})^c = (N^c)_{\rm L} \text{ hold}$$

Properties of the charge-conjugation matrix:

$$\mathcal{C}\gamma_\mu^T\mathcal{C}^{-1} = -\gamma_\mu \; , \qquad \mathcal{C}\gamma_5^T\mathcal{C}^{-1} = \gamma_5 \; , \qquad \mathcal{C}^{-1} = \mathcal{C}^\dagger = \mathcal{C}^T = -\mathcal{C}$$

They are from the requirement that the charge-conjugated field must satisfy the same Dirac equation ($\mathcal{C} = i\gamma^2\gamma^0$ in the Dirac representation)

Dirac mass term

A Dirac neutrino is described by a 4-component spinor:

$$\nu = \nu_{\rm L} + N_{\rm R}$$

Step 1: **the gauge-invariant Dirac mass term and SSB:**

$$-\mathcal{L}_{\rm Dirac} = \overline{\ell_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + {\rm h.c.} \quad \Longrightarrow \quad -\mathcal{L}'_{\rm Dirac} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + {\rm h.c.}$$

$$M_{\rm D} = Y_\nu \langle H \rangle \text{ with } \langle H \rangle \simeq 174\ {\rm GeV}$$

Step 2: **basis transformation:**

$$V^\dagger M_{\rm D} U = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$$

$$-\mathcal{L}'_{\rm Dirac} = \overline{\nu'_{\rm L}} \widehat{M}_\nu N'_{\rm R} + {\rm h.c.}$$

Mass states link to flavor states:

$$\nu'_{\rm L} = V^\dagger \nu_{\rm L} \ \text{ and } \ N'_{\rm R} = U^\dagger N_{\rm R}$$

$$\nu' = \nu'_{\rm L} + N'_{\rm R} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Step 3: **physical mass term and kinetic term:**

$$-\mathcal{L}'_{\rm Dirac} = \overline{\nu'} \widehat{M}_\nu \nu' = \sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i$$

$$\mathcal{L}_{\rm kinetic} = i\overline{\nu_{\rm L}} \gamma_\mu \partial^\mu \nu_{\rm L} + i\overline{N_{\rm R}} \gamma_\mu \partial^\mu N_{\rm R} = i\overline{\nu'} \gamma_\mu \partial^\mu \nu' = i \sum_{k=1}^{3} \overline{\nu_k} \gamma_\mu \partial^\mu \nu_k$$

Dirac neutrino mixing

Weak charged-current interactions of leptons:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)_{\rm L}} \ \gamma^{\mu} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)_{\rm L}} \ \gamma^{\mu} V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the mass basis

One may take mass states = flavor states for the charged leptons. So *V* is just the MNSP matrix of neutrino mixing.

Both the mass and CC terms are invariant with respect to a global phase transformation, and thus lepton number is conserved. However, lepton flavors are violated.

$l(x) \to e^{i\Phi} l(x)$ √

$\nu'_{\rm L}(x) \to e^{i\Phi} \nu'_{\rm L}(x)$ ×

$N'_{\rm R}(x) \to e^{i\Phi} N'_{\rm R}(x)$ ×

×

	e^-	ν_e	e^+	$\overline{\nu}_e$	μ^-	ν_μ	μ^+	$\overline{\nu}_\mu$	τ^-	ν_τ	τ^+	$\overline{\nu}_\tau$
L	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1
L_e	$+1$	$+1$	-1	-1	0	0	0	0	0	0	0	0
L_μ	0	0	0	0	$+1$	$+1$	-1	-1	0	0	0	0
L_τ	0	0	0	0	0	0	0	0	$+1$	$+1$	-1	-1

Majorana mass term (1)

A Majorana mass term can be obtained by introducing the Higgs triplet into the SM, writing out the gauge-invariant Yukawa interactions and Higgs potentials, integrating out heavy degrees of freedom (type-II seesaw mechanism):

$$-\mathcal{L}'_{\mathrm{Majorana}} = \frac{1}{2}\overline{\nu_{\mathrm{L}}} M_{\mathrm{L}} (\nu_{\mathrm{L}})^c + \mathrm{h.c.}$$

The Majorana mass matrix must be a symmetric matrix. It can be diagonalized by a unitary matrix

$$\overline{\nu_L} M_{\mathrm{L}} (\nu_L)^c = [\overline{\nu_L} \underline{M_{\mathrm{L}}} (\nu_L)^c]^T = -\overline{\nu_L} \mathcal{C}^T M_{\mathrm{L}}^T \overline{\nu_L}^T = \overline{\nu_L} \underline{M_{\mathrm{L}}^T} (\nu_L)^c$$

Diagonalization:

$$V^\dagger M_{\mathrm{L}} V^* = \widehat{M}_\nu \equiv \mathrm{Diag}\{m_1, m_2, m_3\}$$

$$-\mathcal{L}'_{\mathrm{Majorana}} = \frac{1}{2}\overline{\nu'_{\mathrm{L}}} \widehat{M}_\nu (\nu'_{\mathrm{L}})^c + \mathrm{h.c.}$$

$$\nu'_{\mathrm{L}} = V^\dagger \nu_{\mathrm{L}} \ \text{and} \ (\nu'_{\mathrm{L}})^c = \mathcal{C}\overline{\nu'_{\mathrm{L}}}^T$$

Physical mass term:

$$\nu' = \nu'_{\mathrm{L}} + (\nu'_{\mathrm{L}})^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$-\mathcal{L}'_{\mathrm{Majorana}} = \frac{1}{2}\overline{\nu'} \widehat{M}_\nu \nu' = \frac{1}{2}\sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i$$

$$\text{Majorana condition } (\nu')^c = \nu'$$

Majorana mass term (2)

Kinetic term (you may prove $\overline{(\psi_{\rm L})^c}\gamma_\mu\partial^\mu(\psi_{\rm L})^c = \overline{\psi_{\rm L}}\gamma_\mu\partial^\mu\psi_{\rm L}$)

$$\mathcal{L}_{\rm kinetic} = i\overline{\nu_{\rm L}}\gamma_\mu\partial^\mu\nu_{\rm L} = i\overline{\nu'_{\rm L}}\gamma_\mu\partial^\mu\nu'_{\rm L} = \frac{i}{2}\overline{\nu'}\gamma_\mu\partial^\mu\nu' = \frac{i}{2}\sum_{k=1}^{3}\overline{\nu_k}\gamma_\mu\partial^\mu\nu_k$$

Question: why is there a factor **1/2** in the **Majorana** mass term?

Answer: it allows us to get the correct **Dirac** equation of motion.

A proof: write out the Lagrangian of free massive **Majorana** neutrinos

$$\begin{aligned}\mathcal{L}_\nu &= i\overline{\nu_{\rm L}}\gamma_\mu\partial^\mu\nu_{\rm L} - \left[\frac{1}{2}\overline{\nu_{\rm L}}M_{\rm L}(\nu_{\rm L})^c + {\rm h.c.}\right] \\ &= i\overline{\nu'_{\rm L}}\gamma_\mu\partial^\mu\nu'_{\rm L} - \left[\frac{1}{2}\overline{\nu'_{\rm L}}\widehat{M}_\nu(\nu'_{\rm L})^c + {\rm h.c.}\right] \\ &= \frac{1}{2}\left(i\overline{\nu'}\gamma_\mu\partial^\mu\nu' - \overline{\nu'}\widehat{M}_\nu\nu'\right) = -\frac{1}{2}\left(i\partial^\mu\overline{\nu'}\gamma_\mu\nu' + \overline{\nu'}\widehat{M}_\nu\nu'\right)\end{aligned}$$

Euler-Lagrange equation:

$$\partial^\mu\frac{\partial\mathcal{L}_\nu}{\partial\left(\partial^\mu\overline{\nu'}\right)} - \frac{\partial\mathcal{L}_\nu}{\partial\overline{\nu'}} = 0 \Longrightarrow$$

$$i\gamma_\mu\partial^\mu\nu' - \widehat{M}_\nu\nu' = 0$$

$$i\gamma_\mu\partial^\mu\nu_k - m_k\nu_k = 0$$

Majorana neutrino mixing

Weak charged-current interactions of leptons:

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\,\overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the flavor basis

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\,\overline{(e\ \mu\ \tau)_{\rm L}}\ \gamma^\mu V \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

In the mass basis

The MNSP matrix *V* contains 2 extra CP-violating phases.

Mass and CC terms are not simultaneously invariant under a global phase transformation --- Lepton number violation

$$l(x) \to e^{i\Phi} l(x)$$

$$\nu'_{\rm L}(x) \to e^{i\Phi} \nu'_{\rm L}(x)$$

$$-\mathcal{L}'_{\rm Majorana} = \frac{1}{2}\overline{\nu'_{\rm L}}\,\widehat{M}_\nu (\nu'_{\rm L})^c + {\rm h.c.}$$

$$\overline{\nu'_{\rm L}} \to e^{-i\Phi}\,\overline{\nu'_{\rm L}} \text{ and } (\nu'_{\rm L})^c \to e^{-i\Phi}(\nu'_{\rm L})^c$$

$$e^{-2i\Phi}$$

The ββ decay

ββ decay: certain even-even nuclei have a chance to decay into the second nearest neighbors via two simultaneous β decays (equivalent to the decays of two neutrons).

necessary conditions:

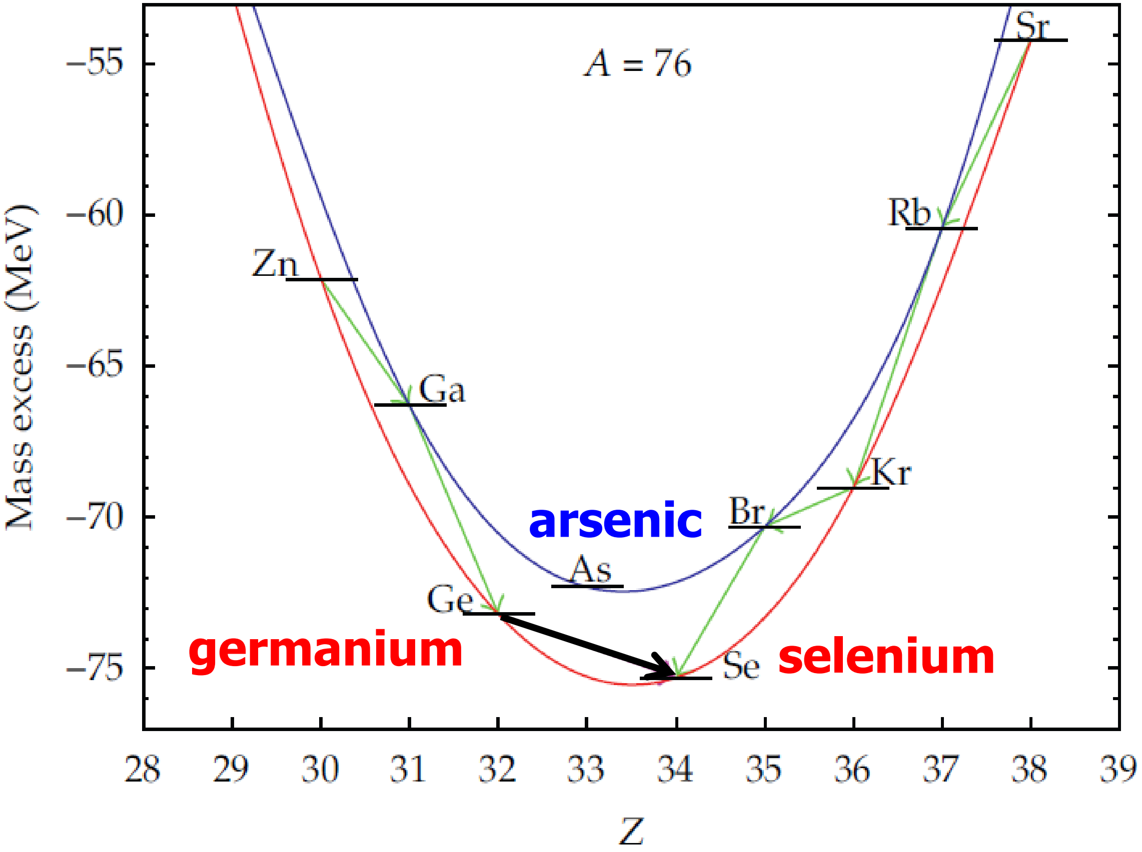
$$m(Z,A) > m(Z+2,A)$$

$$m(Z,A) < m(Z+1,A)$$

$$(Z, A) \rightarrow (Z+2, A) + 2e^{-} + 2\bar{\nu}_e .$$

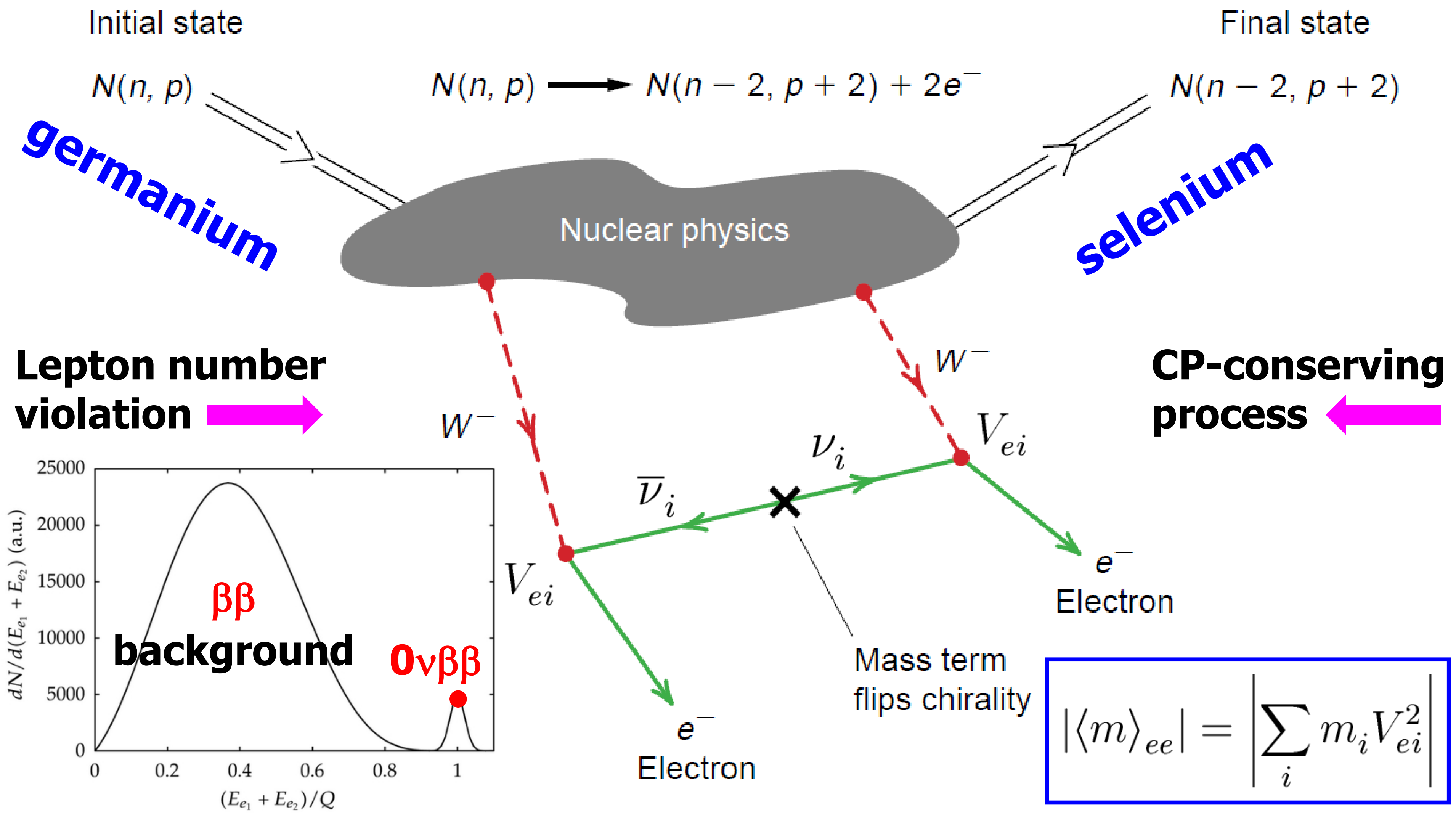
1935

Maria Goeppert Mayer



The 0νββ decay

The neutrinoless double beta decay can happen if massive neutrinos are the Majorana particles (W.H. Furry 1939):



$$N(n, p) \longrightarrow N(n-2, p+2) + 2e^-$$

germanium

selenium

Lepton number violation ➡

CP-conserving process ⬅

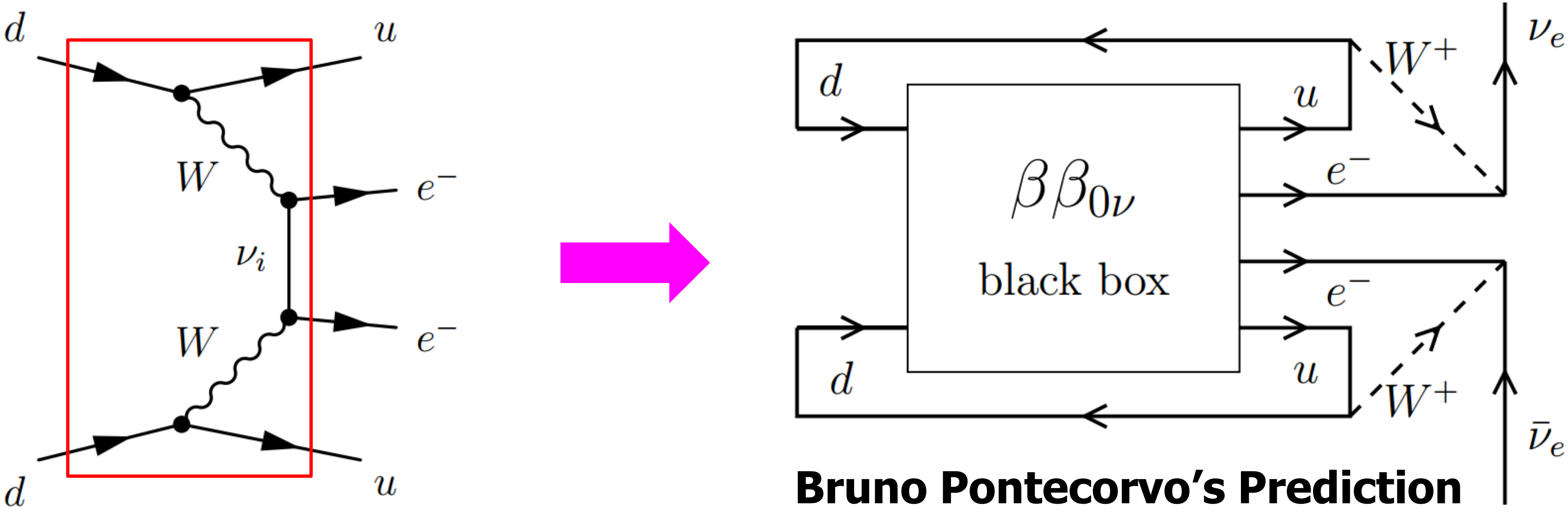
ββ background

0νββ

$$\boxed{|\langle m \rangle_{ee}| = \left| \sum_i m_i V_{ei}^2 \right|}$$

Schechter-Valle theorem

THEOREM (1982): if a 0νββ decay happens, there must be an effective Majorana mass term.



Bruno Pontecorvo's Prediction

That is why we want to see 0νββ

Four-loop ν mass:

$$\boxed{\delta m_\nu = \mathcal{O}(10^{-24}\,\mathrm{eV})}$$

(Duerr, Lindner, Merle, 2011)

Note: The black box can in principle have many different processes (new physics). Only in the simplest case, which is most interesting, it's likely to constrain neutrino masses

Current limits on 0νββ

GERDA has killed the Heidelberg-Moscow's claim on 0νββ.

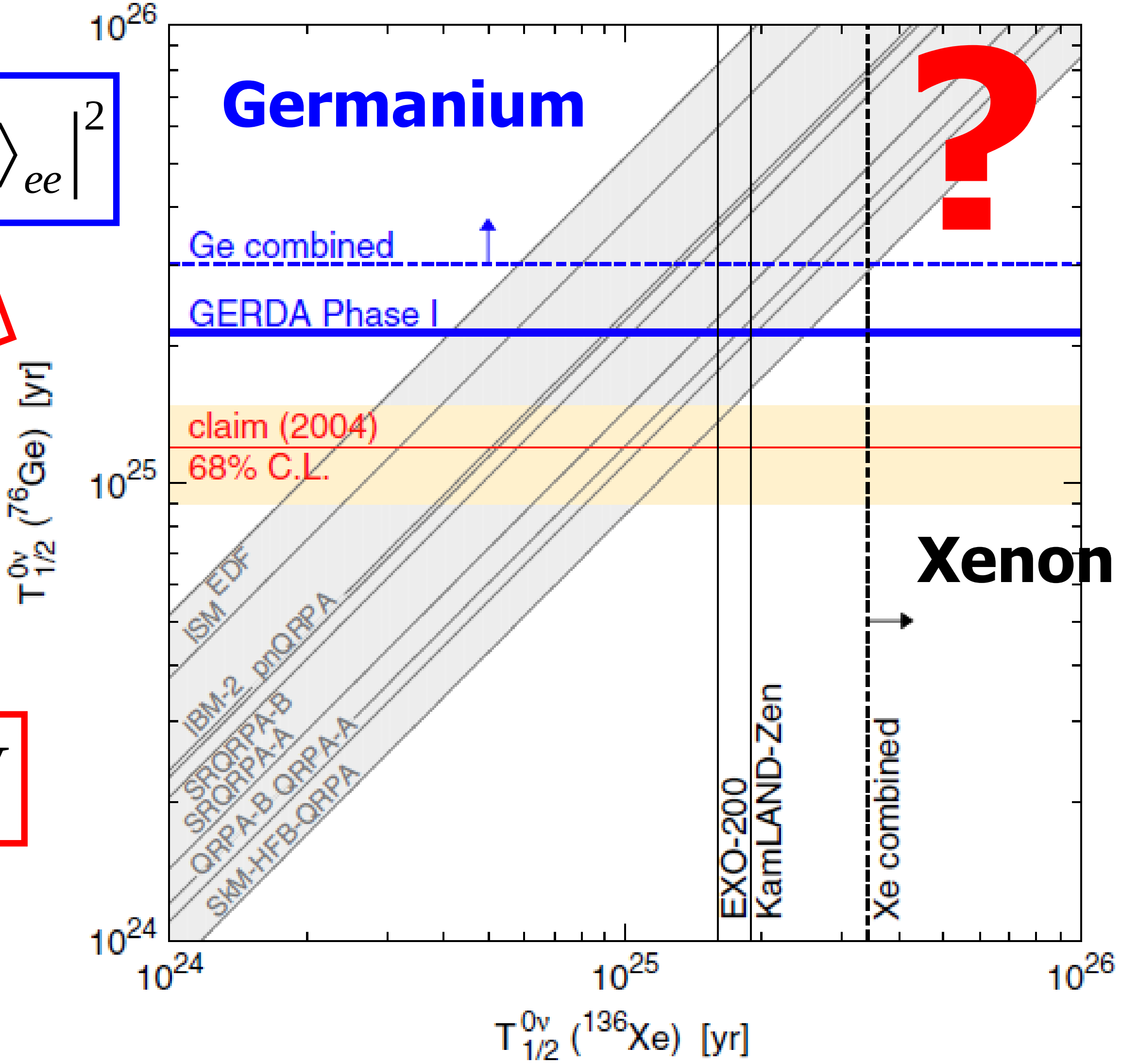
PRL **111,** 122503 (2013)

$$\left(T^{0\nu}_{1/2}\right)^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2\left|\langle m\rangle_{ee}\right|^2$$

$$T^{0\nu}_{1/2} > 3.0\times10^{25}\ \text{yr (90\% C.L.)}.$$

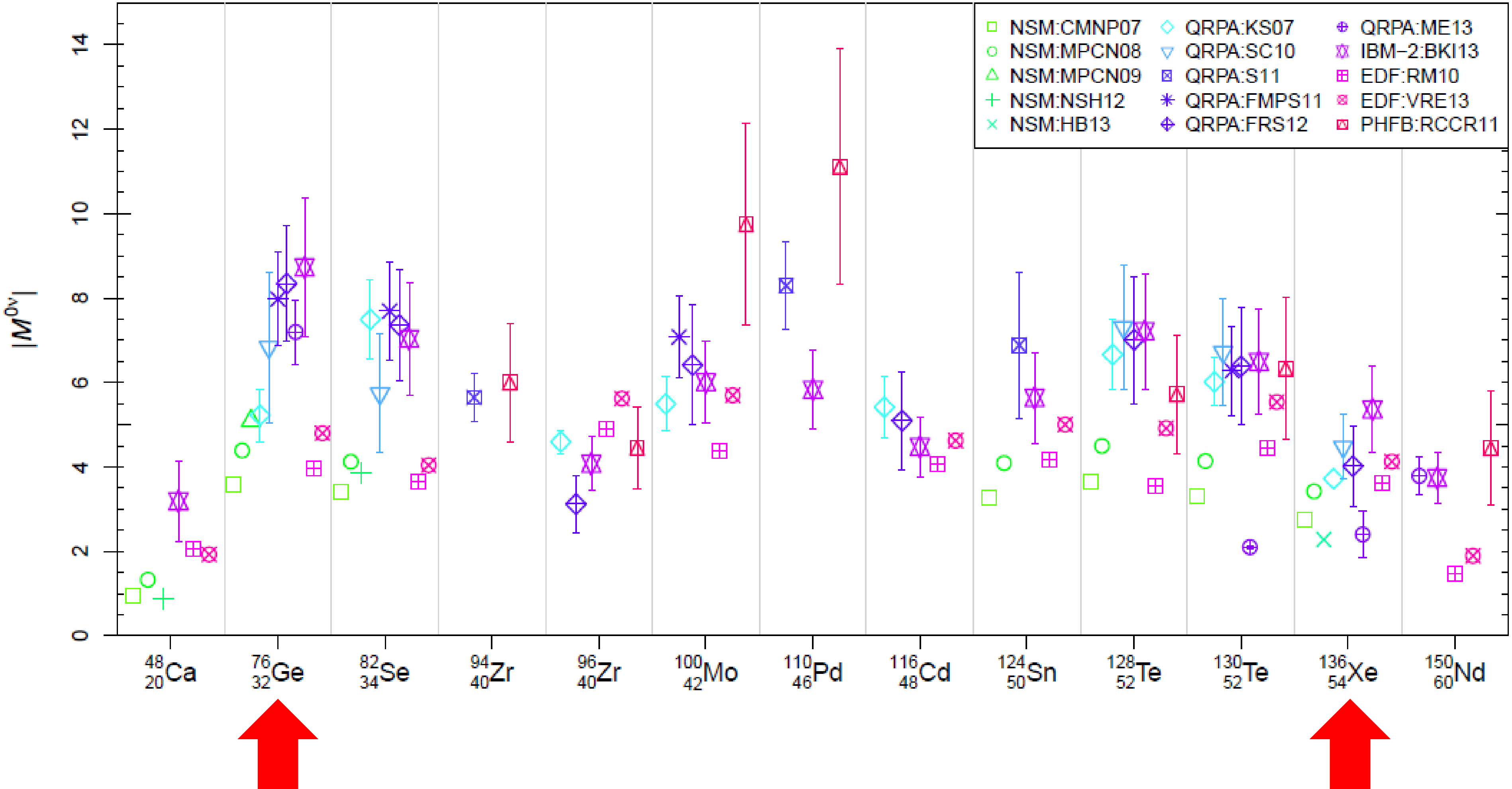
$$\left|\langle m\rangle_{ee}\right| < 0.2 \rightarrow 0.4\ \text{eV}$$

$$|\langle m\rangle_{ee}| = \left|\sum_i m_i V_{ei}^2\right|$$



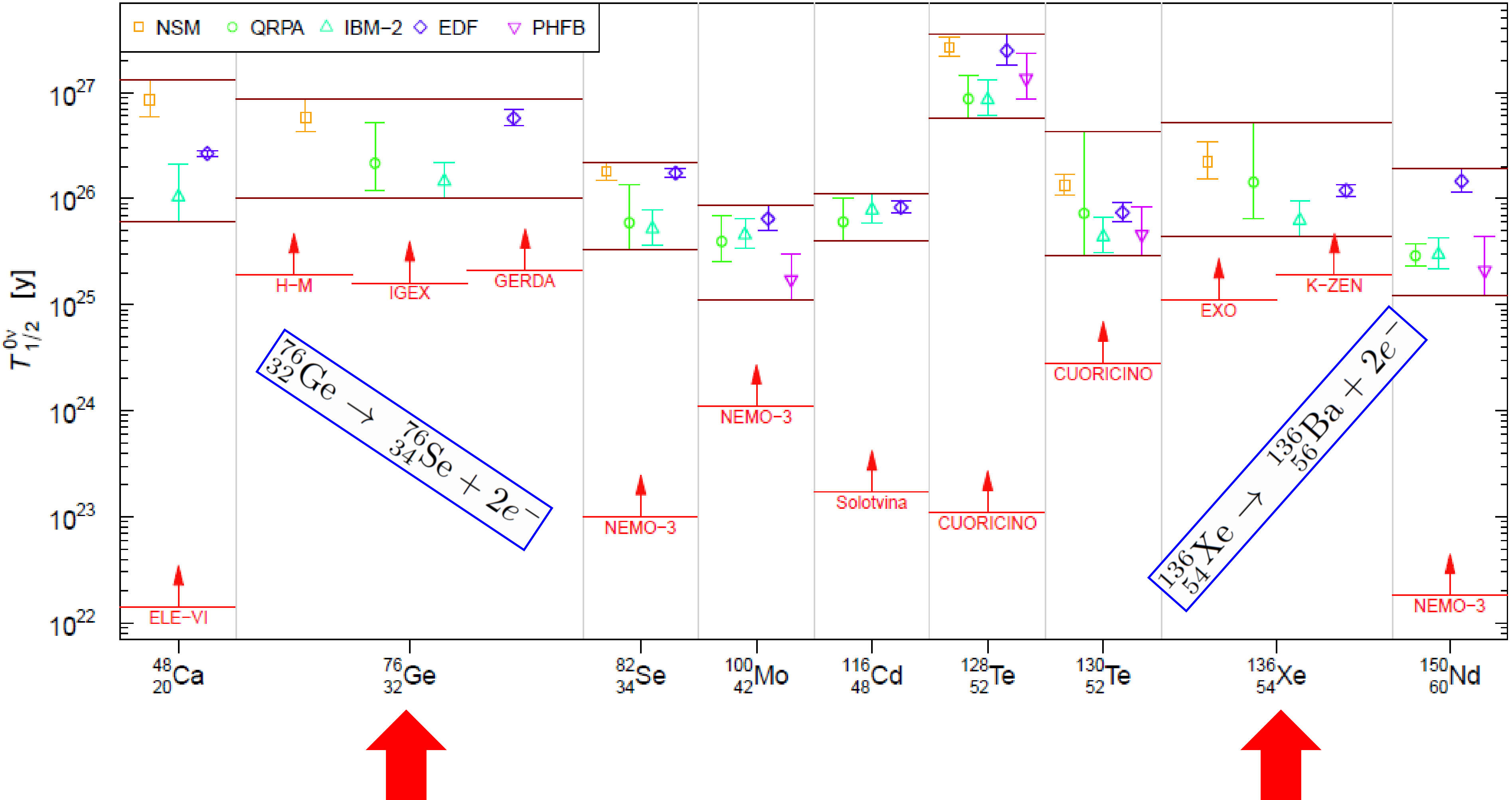
Nuclear matrix elements

Unfortunately, nuclear matrix elements can be calculated only based on some models which describe many-body interactions of nucleons in nuclei. Since different models focus on different aspects of nuclear physics, large uncertainties (a factor of 2 or 3) are unavoidable.

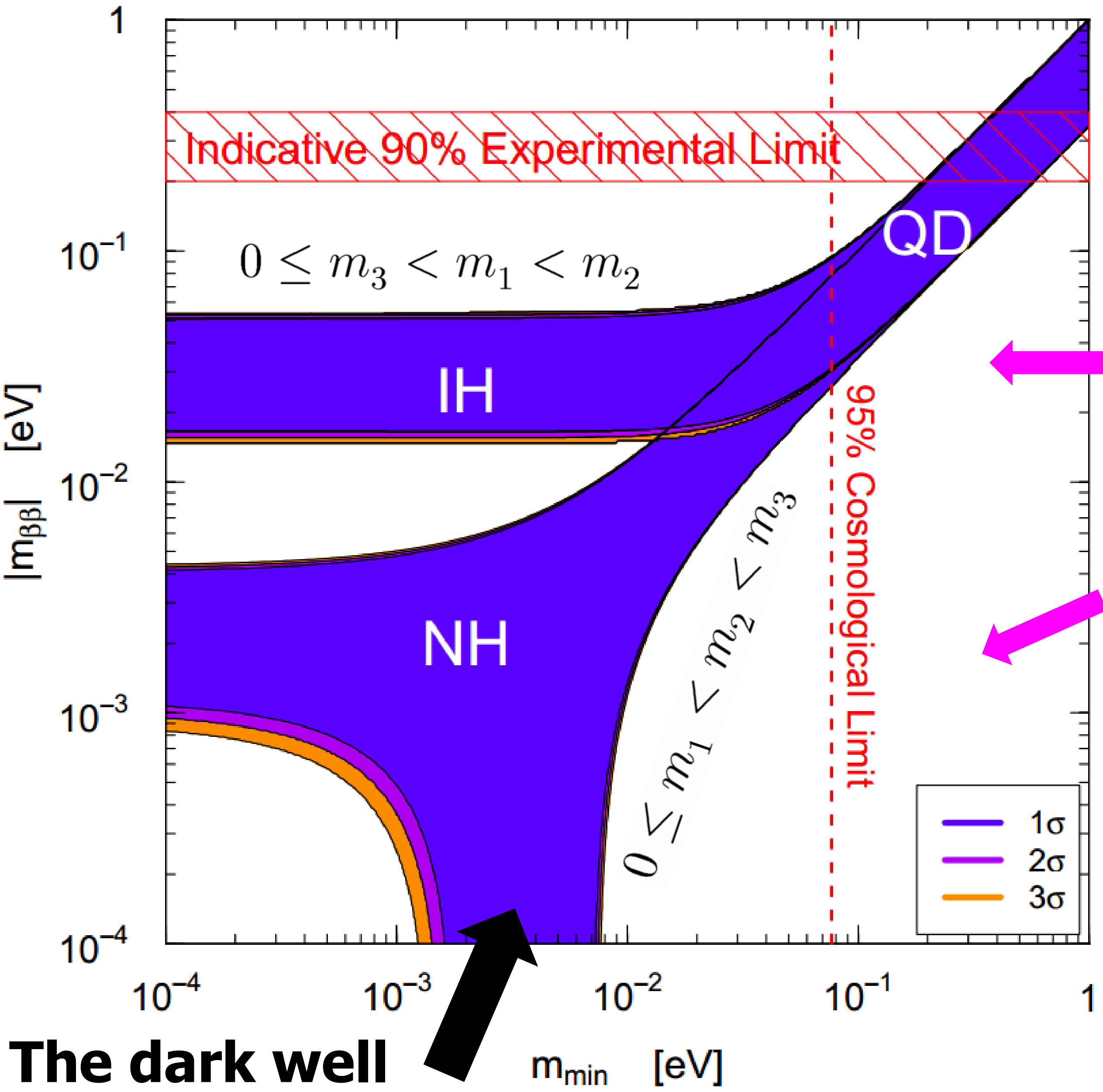


Half-life

Comparing the 90% C.L. experimental lower limits on the half-life of a 0νββ-decaying nuclide with the corresponding range of theoretical prediction, given a value of 0.1 eV for the effective Majorana neutrino mass term (Bilenky and Giunti, 1411.4791).



Effective 0νββ mass



The dark well

The effective mass

$$\left|\langle m\rangle_{ee}\right| = \left|\sum_i m_i V_{ei}^2\right|$$

Maury Goodman asks

An intelligent design?

I asked myself in 2003

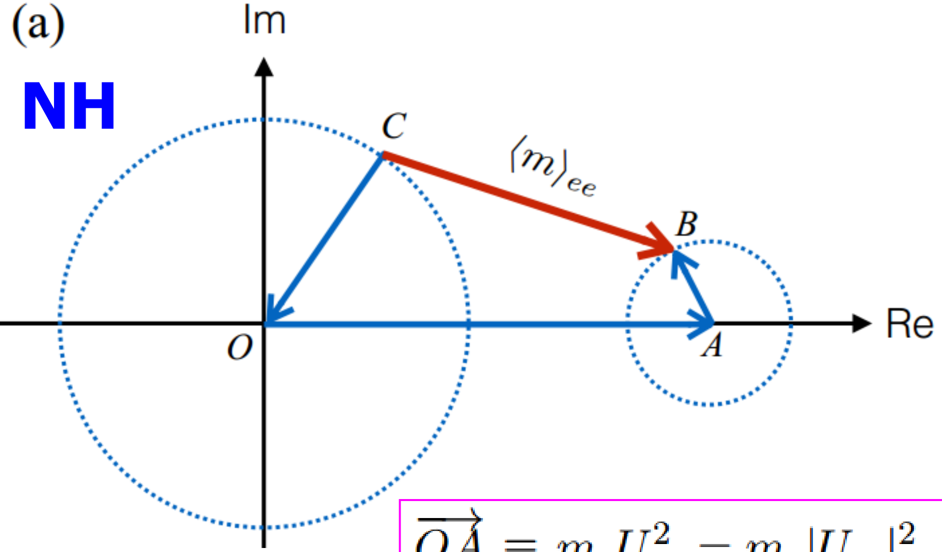
Vanishing 0νββ mass?

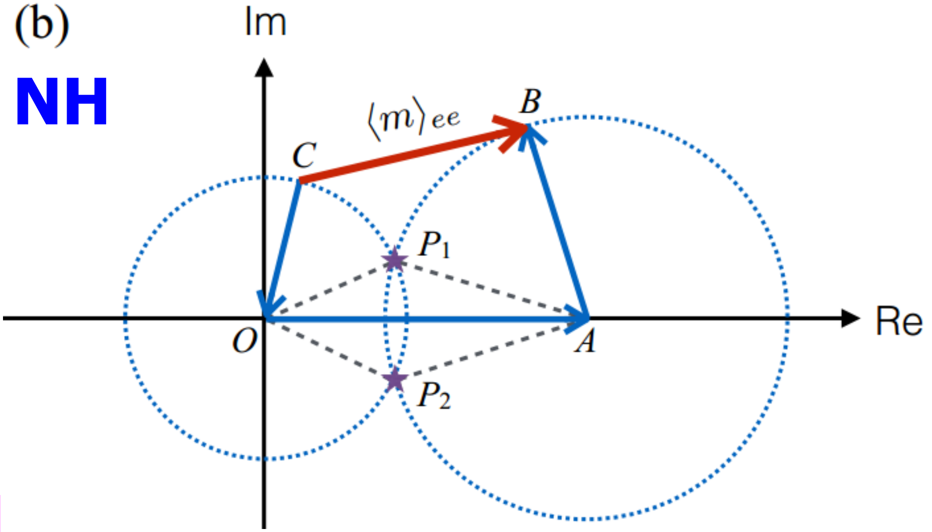
hep-ph/0305195, PRD

CP phases also matter

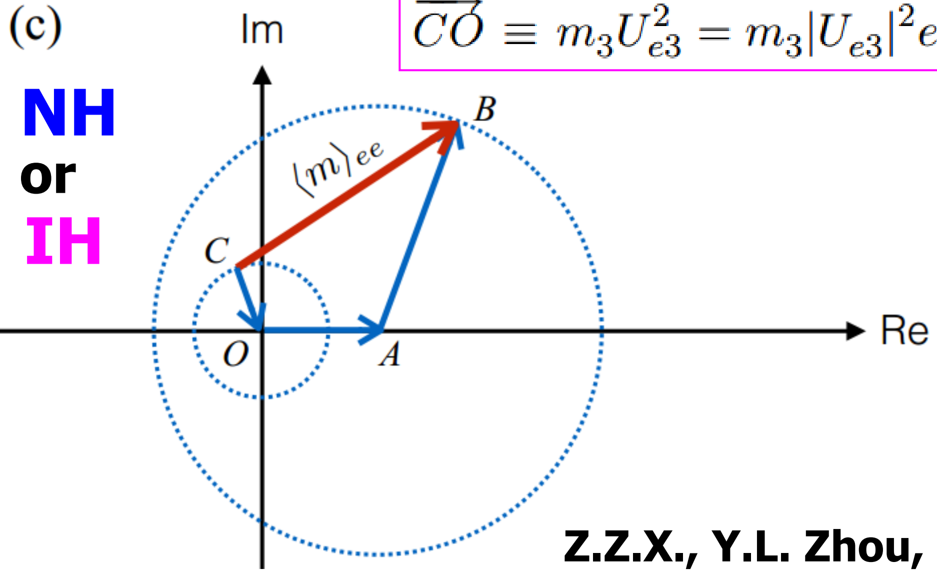
In case of new physics, is it destructive or constructive?

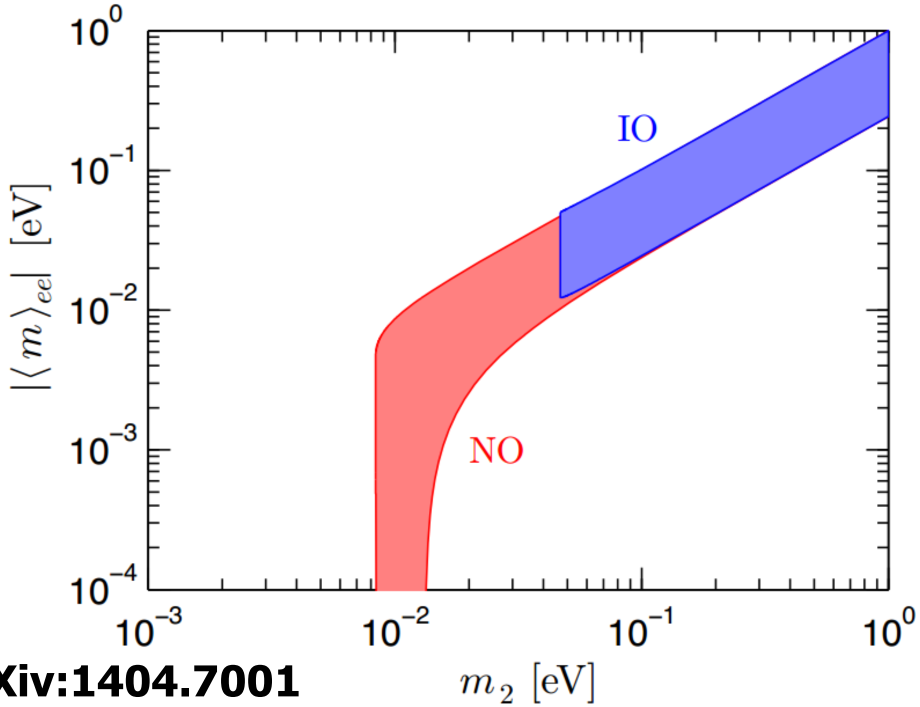
Coupling-rod diagram





$$\overrightarrow{OA} \equiv m_2 U_{e2}^2 = m_2 |U_{e2}|^2 \; ,$$
$$\overrightarrow{AB} \equiv m_1 U_{e1}^2 = m_1 |U_{e1}|^2 e^{{\rm i}\rho}$$
$$\overrightarrow{CO} \equiv m_3 U_{e3}^2 = m_3 |U_{e3}|^2 e^{{\rm i}\sigma}$$





Z.Z.X., Y.L. Zhou, arXiv:1404.7001

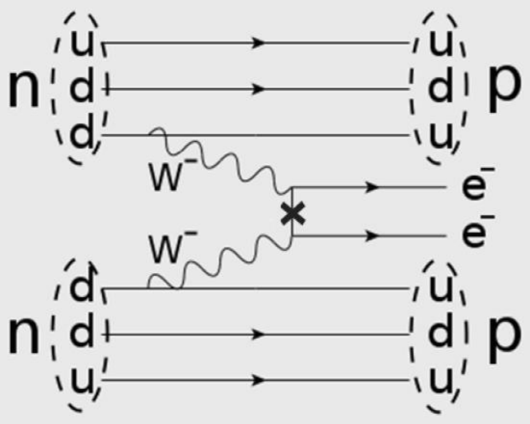
New physics

Type (A): **NP directly related to extra species of neutrinos.**

Example 1: heavy Majorana neutrinos from type-I seesaw

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2} \overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}}$$

$$\Gamma_{0\nu\beta\beta} \propto \left| \sum_{i=1}^{3} V_{ei}^2 m_i \; - \; \sum_{k=1}^{n} \frac{R_{ek}^2}{M_k} M_A^2 \mathcal{F}(A, M_k) \right|^2$$



In most cases the heavy contribution is negligible

Example 2: light sterile neutrinos from LSND etc

$$\langle m \rangle'_{ee} \equiv \sum_{i=1}^{6} m_i V_{ei}^2 = \underline{\langle m \rangle_{ee}} \left(c_{14} c_{15} c_{16}\right)^2 + \underline{m_4 \left(\hat{s}^*_{14} c_{15} c_{16}\right)^2 + m_5 \left(\hat{s}^*_{15} c_{16}\right)^2 + m_6 \left(\hat{s}^*_{16}\right)^2}$$

In this case the new contribution might be constructive or destructive

Type (B): **NP has little to do with the neutrino mass issue.**

SUSY, Left-right, and some others that I don’t understand

YES or **NO**?

QUESTION: are massive neutrinos the **Majorana** particles?

One might be able to answer **YES** through a measurement of the **$0\nu\beta\beta$** decay or other **LNV** processes someday, but how to answer with **NO**?

YES
or
I don't know!

The same question: how to distinguish between **Dirac** and **Majorana** neutrinos in a realistic experiment?

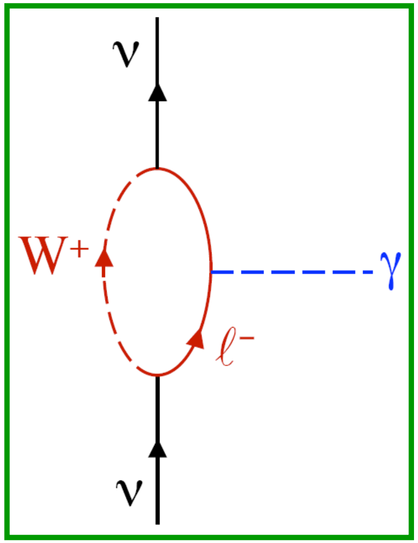
Answer 1: The **$0\nu\beta\beta$** decay is currently the only possibility.

Answer 2: In principle their dipole moments are different.

Answer 3: They show different behavior if nonrelativistic.

Electromagnetic properties

Without electric charges, neutrinos have electromagnetic interactions with the photon via quantum loops.



Given the SM interactions, a massive Dirac neutrino can only have a tiny magnetic dipole moment:

$$\boxed{\mu_\nu \sim \frac{3eG_{\rm F}}{8\sqrt{2}\pi^2} m_\nu = 3\times10^{-20}\,\frac{m_\nu}{0.1\,{\rm eV}}\,\mu_{\rm B}}$$

A massive Majorana neutrino can not have magnetic & electric dipole moments, as its antiparticle is itself.

Proof: Dirac neutrino's electromagnetic vertex can be parametrized as

$$\boxed{\Gamma_\mu(p,p') = f_{\rm Q}(q^2)\gamma_\mu + f_{\rm M}(q^2) i\sigma_{\mu\nu}q^\nu + f_{\rm E}(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_{\rm A}(q^2)\left(q^2\gamma_\mu - q_\mu q^\nu\gamma_\nu\right)\gamma_5}$$

Majorana neutrinos

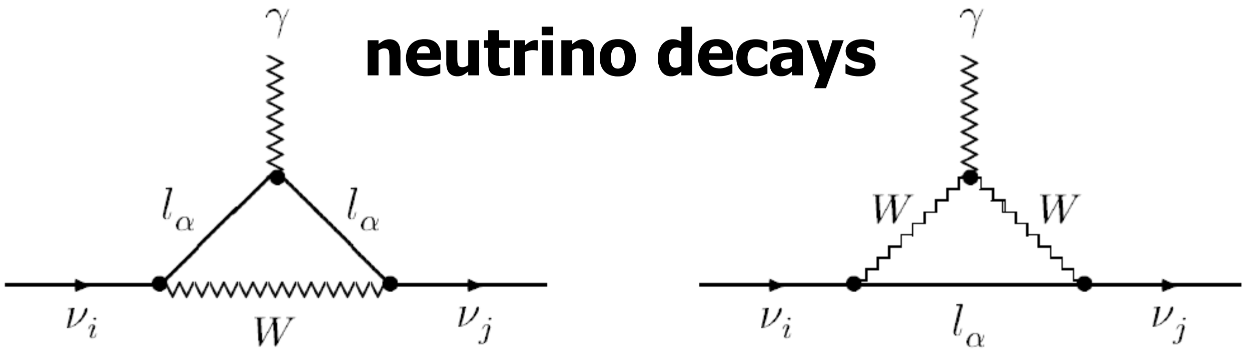
$$\boxed{\underline{\overline{\psi}\Gamma_\mu\psi} = \overline{\psi^c}\Gamma_\mu\psi^c = \psi^T\mathcal{C}\Gamma_\mu\mathcal{C}\overline{\psi}^T = \left(\psi^T\mathcal{C}\Gamma_\mu\mathcal{C}\overline{\psi}^T\right)^T = -\overline{\psi}\mathcal{C}^T\Gamma_\mu^T\mathcal{C}^T\psi = \underline{\overline{\psi}\mathcal{C}\Gamma_\mu^T\mathcal{C}^{-1}\psi}}$$

$\Longrightarrow$ $\boxed{f_{\rm Q}(q^2) = f_{\rm M}(q^2) = f_{\rm E}(q^2) = 0}$ **intrinsic property of Majorana ν's.**

Transition dipole moments

Both Dirac & Majorana neutrinos can have *transition* dipole moments (of a size comparable with μ_ν) that may give rise to neutrino decays, scattering with electrons, interactions with external magnetic field & contributions to ν masses. (Data: < a few × 10^-11 Bohr magneton).

neutrino decays

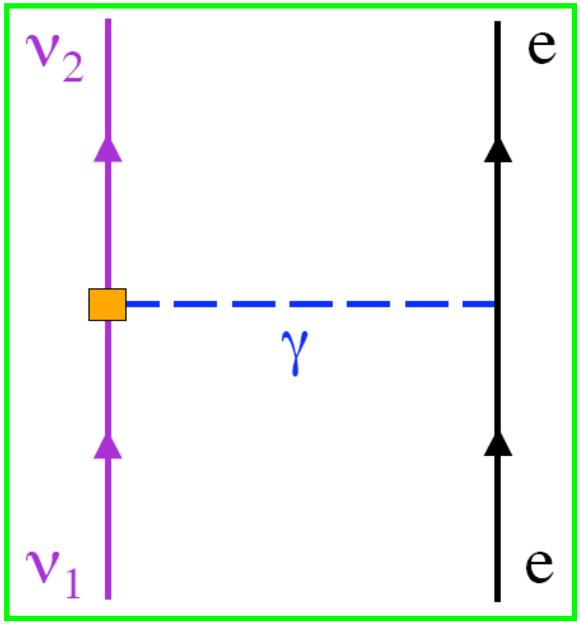


$$\mu_{\text{eff}} \equiv \sqrt{|\mu_{ij}|^2 + |\epsilon_{ij}|^2}$$

$$\Gamma_{\nu_i \to \nu_j + \gamma} = 5.3 \times \left(1 - \frac{m_j^2}{m_i^2}\right)^3 \left(\frac{m_i}{1\ \text{eV}}\right)^3 \left(\frac{\mu_{\text{eff}}}{\mu_B}\right)^2 \text{s}^{-1}$$

$$\frac{d\sigma'_\mu}{dT} = \frac{\alpha^2 \pi}{m_e^2} \sum_{k=1}^{3} \left| \sum_{j=1}^{3} e^{iq_j L} V_{ej} \left(i\frac{\mu_{jk}}{\mu_B} + \frac{\epsilon_{jk}}{\mu_B} \right) \right|^2 \left(\frac{1}{T} - \frac{1}{E_\nu} \right)$$

scattering



Summary

(A) Three reasons for neutrinos to be massless in the SM.

(B) The Dirac mass term and lepton number conservation.

(C) The Majorana mass term and lepton number violation.
---- the Majorana mass matrix must be symmetric;
---- factor 1/2 in front of the mass term makes sense.

(D) The $0\nu\beta\beta$ decay can determine the nature of neutrinos.
---- if a signal is seen, neutrinos must be of Majorana;
---- if a signal is not seen, then there is no conclusion.

(E) Electromagnetic dipole moment of massive neutrinos.
---- Dirac neutrinos have magnetic dipole moments;
---- Majorana neutrinos have no dipole moments;
---- Dirac & Majorana neutrinos: transition moments.

The phenomenology of massive neutrinos will be explored