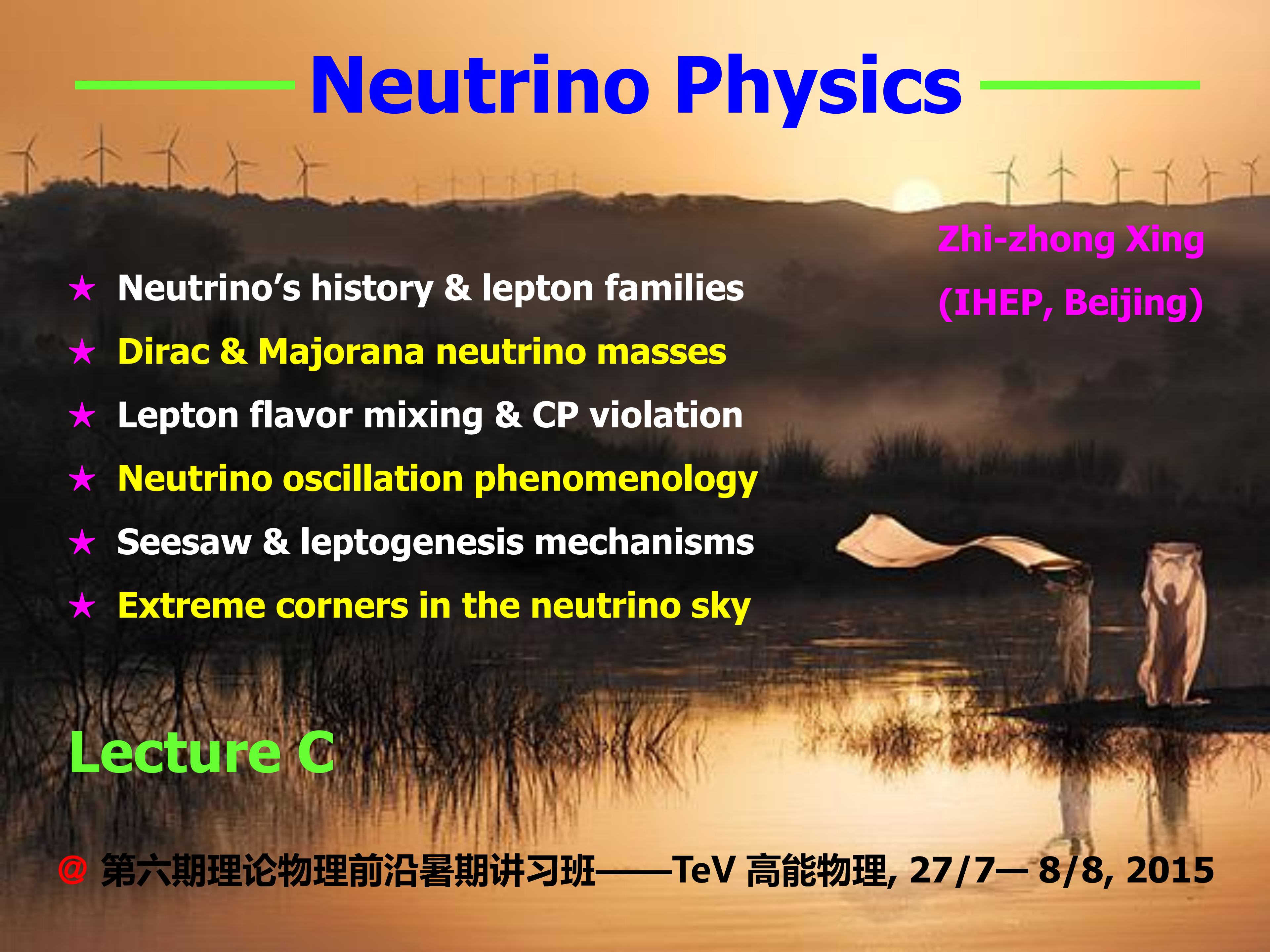


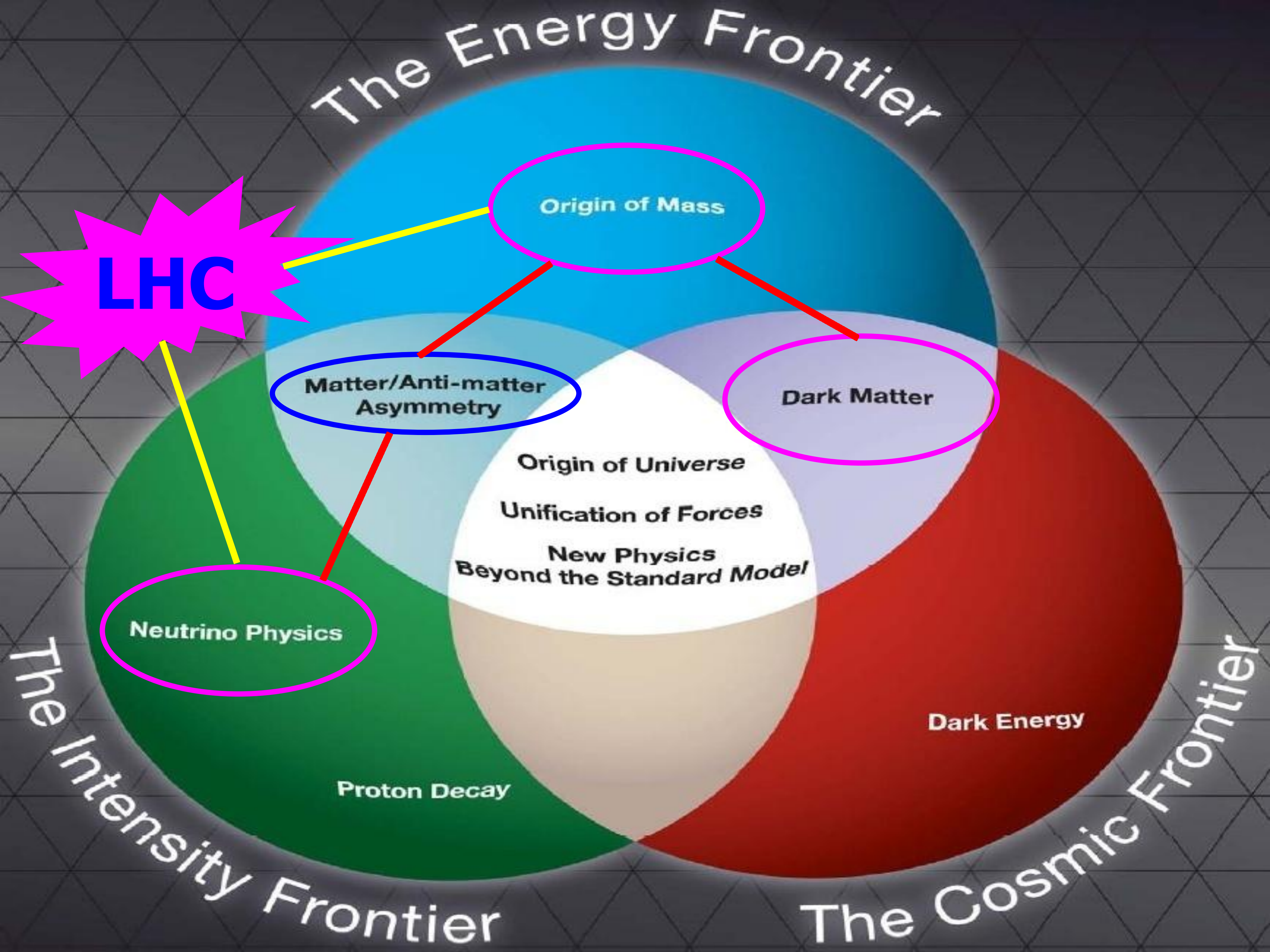
Neutrino Physics

Zhi-zhong Xing
(IHEP, Beijing)

★ Neutrino's history & lepton families
★ Dirac & Majorana neutrino masses
★ Lepton flavor mixing & CP violation
★ Neutrino oscillation phenomenology
★ Seesaw & leptogenesis mechanisms
★ Extreme corners in the neutrino sky

Lecture C

@ 第六期理论物理前沿暑期讲习班——TeV 高能物理, 27/7— 8/8, 2015



The Energy Frontier

LHC

Origin of Mass

Matter/Anti-matter Asymmetry

Dark Matter

Origin of Universe

Unification of Forces

New Physics Beyond the Standard Model

Neutrino Physics

Proton Decay

Dark Energy

The Intensity Frontier

The Cosmic Frontier

Lecture C1

- ★ How to Generate Neutrino Mass
- ★ 3 Typical Seesaw Mechanisms
- ★ The Leptogenesis Mechanism

Within the SM

All ν's are massless in the SM, a result of the model's simple structure:

---- SU(2)_L×U(1)_Y gauge symmetry and Lorentz invariance;

Fundamentals of the model, mandatory for consistency of a QFT.

---- Economical particle content:

No right-handed neutrinos --- a Dirac mass term is not allowed.

Only one Higgs doublet --- a Majorana mass term is not allowed.

---- Mandatory renormalizability:

No dimension ≥ 5 operators: a Majorana mass term is forbidden.

To generate ν-masses, one or more of the constraints must be relaxed

--- The gauge symmetry and Lorentz invariance cannot be abandoned

--- The particle content can be modified

--- The renormalizability can be abandoned

How many ways?

Beyond the SM (1)

Way 1: to relax the requirement of renormalizability (S. Weinberg 79)

$$\boxed{\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{\mathcal{L}_{\rm d=5}}{\Lambda} + \frac{\mathcal{L}_{\rm d=6}}{\Lambda^2} + \cdots}$$

Given the standard-model fields, the lowest-dimension operators that violate lepton and baryon numbers at the tree level are

$$\boxed{\frac{1}{M} HHLL}$$

neutrino mass

Seesaw: $m_{1,2,3} \sim \langle H \rangle^2 / M$

$$m_{1,2,3} < 1\,\mathrm{eV} \quad \Rightarrow \quad M > 10^{13}\ \mathrm{GeV}$$

$$\boxed{\frac{1}{M^2} QQQL}$$

proton decay

Example : $p \to \pi^0 + e^+$

$$\tau_p > 10^{33}\ \text{years} \quad \Rightarrow \quad M > 10^{15}\ \mathrm{GeV}$$

Neutrino masses and proton decays at the intensity frontier offer new windows onto physics at super-high energy scales.

Beyond the SM (2)

Way 2: to add **3 right-handed** neutrinos and demand the ***L*** symmetry.

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + {\rm h.c.}$$

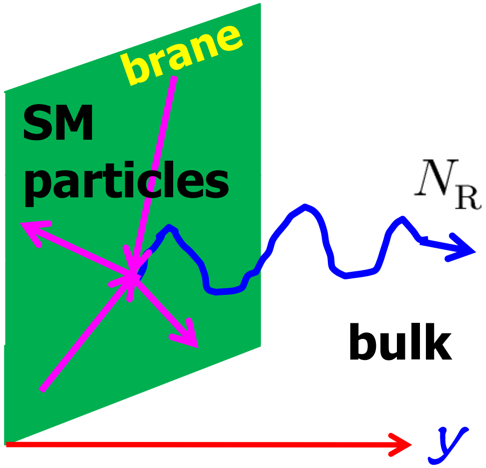
$$M_l = Y_l v/\sqrt{2}\,, \quad M_\nu = Y_\nu v/\sqrt{2}$$

But, such a pure **Dirac** mass term and lepton number conservation are not convincing, because non-perturbative quantum effects break both ***L*** and ***B*** symmetries and only preserve ***B* – *L*** (**G. 't Hooft**, 1976).

The flavor hierarchy puzzle:

$$y_i/y_e = m_i/m_e \lesssim 0.5\ {\rm eV}/0.5\ {\rm MeV} \sim 10^{-6}$$

A very speculative way out: the smallness of **Dirac** masses is ascribed to the assumption that ***N*_R** have access to an extra spatial dimension (Dienes, Dudas, Gherghetta 98; Arkani-Hamed, Dimopoulos, Dvali, March-Russell 98) :



The wavefunction of ***N*_R** spreads out over the extra dimension ***y***, giving rise to a suppressed Yukawa interaction at ***y* = 0**.

$$\left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R}\right]_{y=0} \sim \boxed{\frac{1}{\sqrt{L}}} \left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R}\right]_{y=L}$$

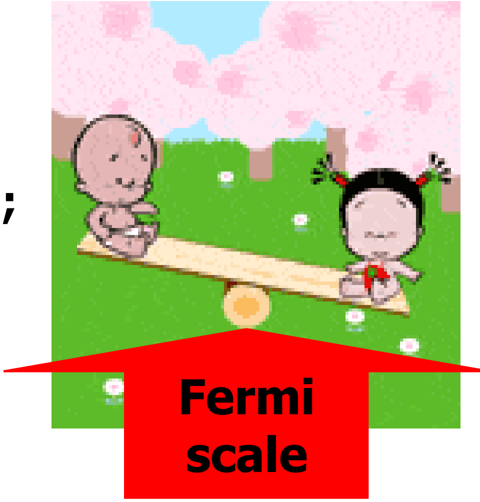
(e.g., King 08)

$$\Lambda_{\rm String}/\Lambda_{\rm Planck} \sim 10^{-12}$$

Beyond the SM (3)

Seesaw: add **new heavy degrees of freedom** and allow the *L* violation.

Seesaw—A Footnote Idea:
H. Fritzsch, **M. Gell-Mann**,
P. Minkowski, PLB 59 (1975) 256



Type (1): SM + 3 right-handed neutrinos (Minkowski 77; Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slanski 79; Mohapatra, Senjanovic 80)

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^c_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}}$$

Type (2): SM + 1 Higgs triplet (Konetschny, Kummer 77; Magg, Wetterich 80; Schechter, Valle 80; Cheng, Li 80; Lazarides et al 80; Mohapatra, Senjanovic 80)

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \frac{1}{2}\overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} - \lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + {\rm h.c.}}$$

variations

Type (3): SM + 3 triplet fermions (Foot, Lew, He, Joshi 89)

$$\boxed{-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} \sqrt{2} Y_\Sigma \Sigma^c \tilde{H} + \frac{1}{2}{\rm Tr}\left(\overline{\Sigma} M_\Sigma \Sigma^c\right) + {\rm h.c.}}$$

combinations

Seesaw mechanisms

Weinberg operator: **the unique** **dimension-five** **operator of ν-masses after integrating out heavy degrees of freedom.**

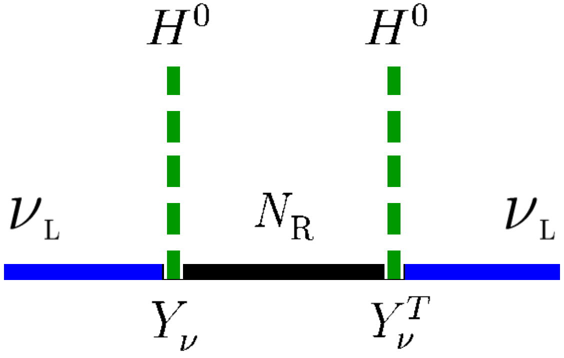
$$\frac{\mathcal{L}_{\rm d=5}}{\Lambda} = \begin{cases} \frac{1}{2}\left(Y_\nu M_{\rm R}^{-1} Y_\nu^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \\ -\frac{\lambda_\Delta}{M_\Delta} \left(Y_\Delta\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \\ \frac{1}{2}\left(Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \end{cases}$$

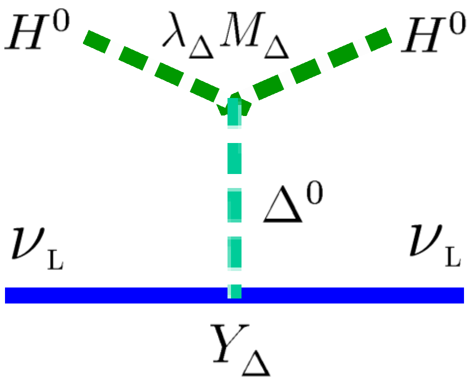
$$M_\nu = \begin{cases} -\frac{1}{2} Y_\nu \frac{v^2}{M_{\rm R}} Y_\nu^T & \text{(Type 1)} \\ \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta} & \text{(Type 2)} \\ -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T & \text{(Type 3)} \end{cases}$$

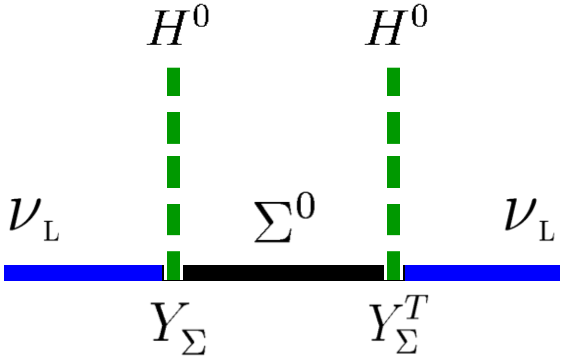
After SSB, a Majorana neutrino mass term is

$$-\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} M_\nu \nu^c_{\rm L} + {\rm h.c.}$$

$$\langle \tilde{H} \rangle = v/\sqrt{2}$$

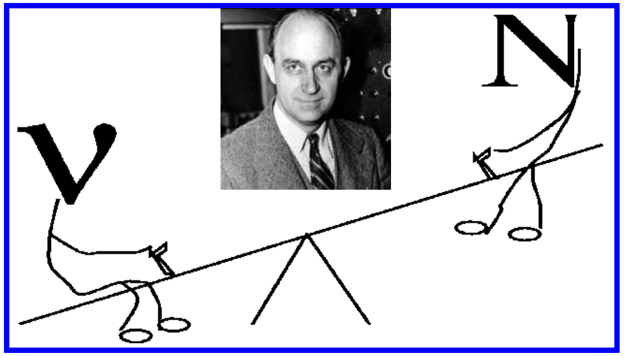






The seesaw scale (1)

What is the energy scale at which the seesaw mechanism works and new physics come in?



Planck

GUT to unify strong, weak & electromagnetic forces

Conventional Seesaws: heavy degrees of freedom near GUT

This appears to be rather reasonable, since one often expects new physics to appear around a fundamental scale

Fermi

Naturalness √	**Testability** ✗
Uniqueness ✗	**Hierarchy** ✗

The seesaw scale (2)

Planck scale $\Lambda \sim 10^{19}\,\mathrm{GeV}$

GUT scale? $\Lambda \sim 10^{16}\,\mathrm{GeV}$

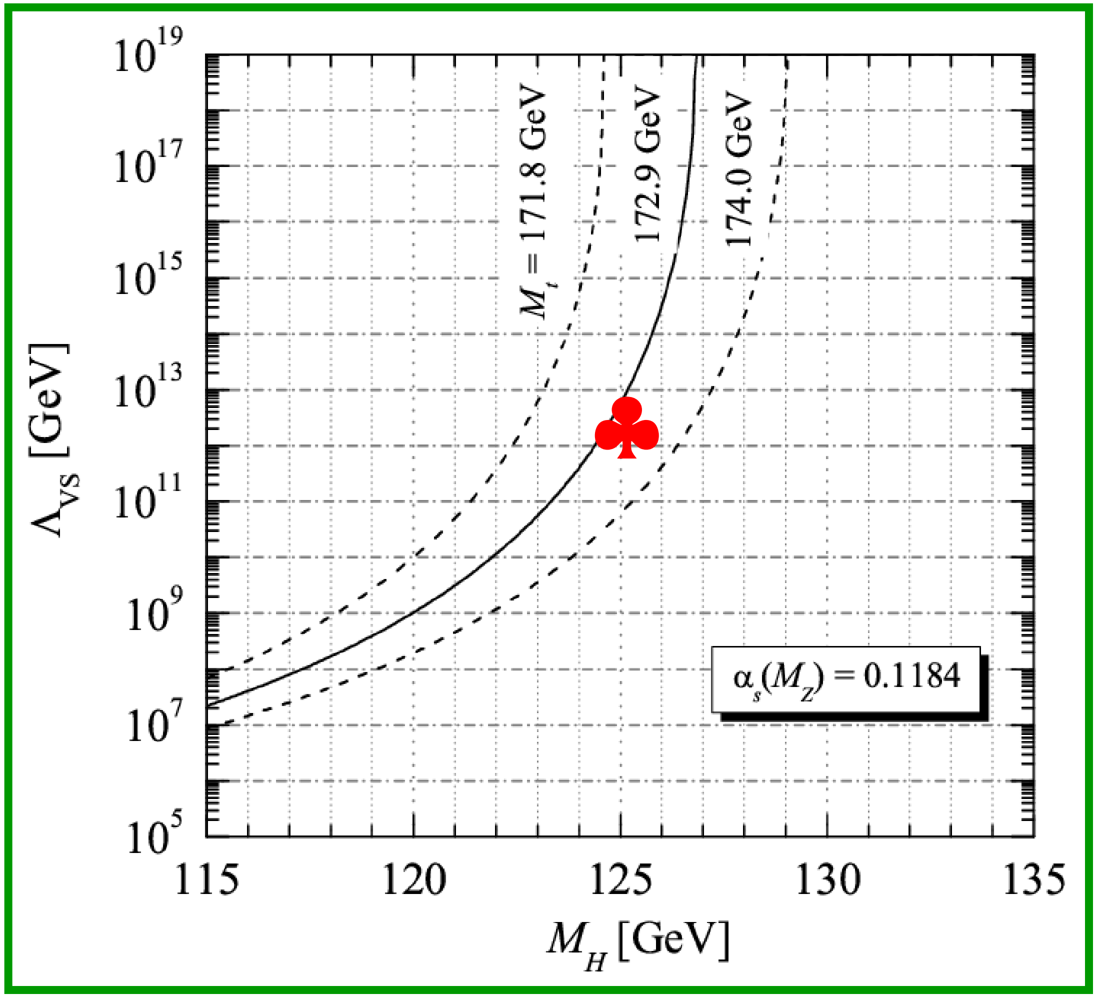
Seesaw scale? $\Lambda \sim 10^{12}\,\mathrm{GeV}$

TeV / SUSY? $\Lambda \sim 10^{3}\,\mathrm{GeV}$

Fermi scale $\Lambda \sim 10^{2}\,\mathrm{GeV}$

QCD scale $\Lambda \sim 10^{2}\,\mathrm{MeV}$

The SM vacuum stability for a light Higgs



Seesaws could make life easier?

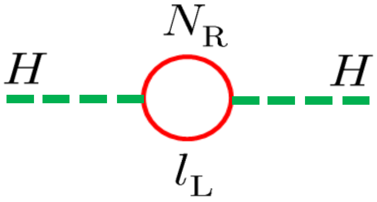
Elias-Miro et al., arXiv:1112.3022; Xing, Zhang, Zhou, arXiv:1112.3112; …….

New hierarchy problem

Seesaw-induced fine-tuning problem: the Higgs mass is very sensitive to quantum corrections from the heavy degrees of freedom induced in the seesaw mechanisms (Vissani 98; Casas et al 04; Abada et al 07)

Type 1:

$$\delta m_H^2 = -\frac{y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)$$

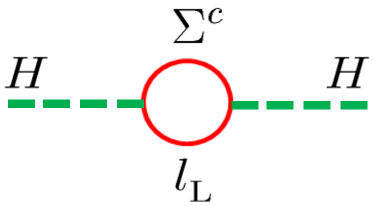


Type 2:

$$\delta m_H^2 = \frac{3}{16\pi^2}\left[\lambda_3\left(\Lambda^2 + M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right) + 4\lambda_\Delta^2 M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right]$$

Type 3:

$$\delta m_H^2 = -\frac{3y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)$$



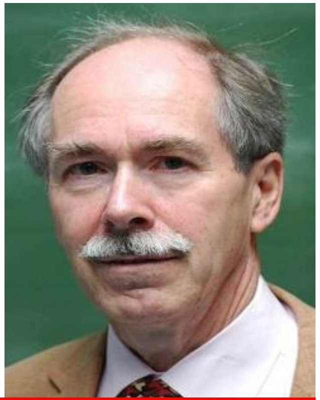
here y_i & M_i are eigenvalues of Y_ν (or Y_Σ) & M_R (or M_Σ), respectively.

An illustration of fine-tuning

$$M_i \sim \left[\frac{(2\pi v)^2 |\delta m_H^2|}{m_i}\right]^{1/3} \sim \underline{10^7 \mathrm{GeV}} \left[\frac{0.2\ \mathrm{eV}}{m_i}\right]^{1/3} \left[\frac{|\delta m_H^2|}{0.1\ \mathrm{TeV}^2}\right]^{1/3}$$

Possible way out: (1) Supersymmetric seesaw? (2) TeV-scale seesaw?

Lower scale seesaws?



There is no direct evidence for a large or extremely large seesaw scale. So eV-, keV-, MeV- or GeV-scale seesaws are all possible, at least in principle; they are technically natural according to 't Hooft's naturalness criterion.

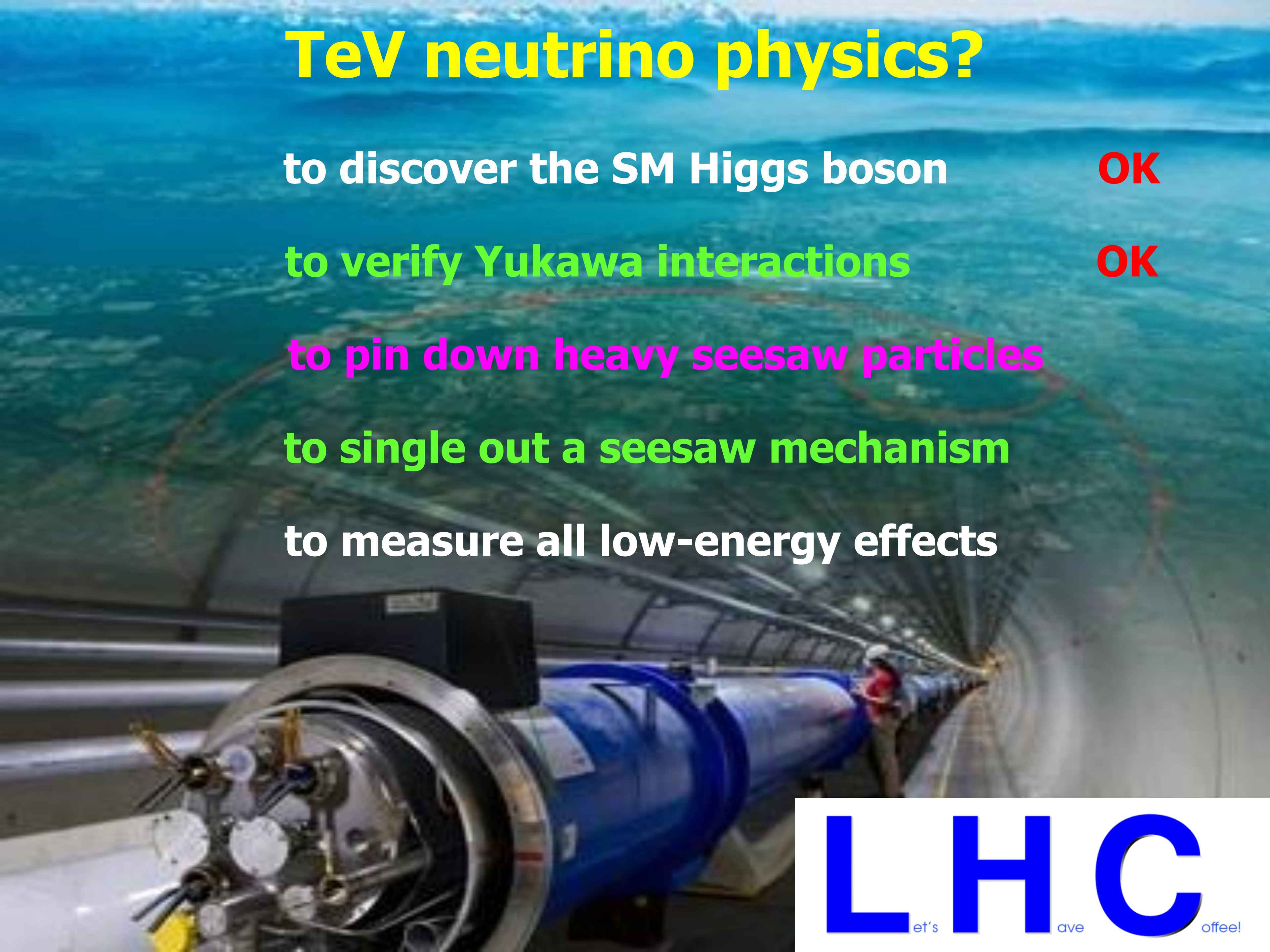
't Hooft's naturalness criterion (1980):

> *At any energy scale μ, a set of parameters, $\alpha_i(\mu)$ describing a system can be small, if and only if, in the limit $\alpha_i(\mu) \to 0$ for each of these parameters, the system exhibits an enhanced symmetery.*

Potential problems of low-scale seesaws:

- ♣ **No obvious connection to a theoretically well-justified fundamental scale (e.g., Fermi scale, TeV scale, GUT or Planck scale).**
- ♣ **The neutrino Yukawa couplings are simply tiny, no good reasons for the masses of three known neutrinos are so small.**
- ♣ **A very low seesaw scale is unable to allow the thermal leptogenesis to work, though there might be a very *contrived* way out.**

TeV neutrino physics?



to discover the SM Higgs boson OK

to verify Yukawa interactions OK

to pin down heavy seesaw particles

to single out a seesaw mechanism

to measure all low-energy effects

Let's **H**ave **C**offee!

Type-1 seesaw

Type-1 Seesaw: **add 3 *right-handed* Majorana neutrinos into the SM.**

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^{\rm c}_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

or

$$-\mathcal{L}_{\rm mass} = \overline{e_{\rm L}} M_l E_{\rm R} + \frac{1}{2}\overline{(\nu_{\rm L} \quad N^{\rm c}_{\rm R})} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}\begin{pmatrix}\nu^{\rm c}_{\rm L}\\ N_{\rm R}\end{pmatrix} + {\rm h.c.}$$

Diagonalization (flavor basis ⇒ mass basis):

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix}$$

$$V^\dagger V + \underline{S^\dagger S} = V V^\dagger + \underline{R R^\dagger} = 1$$

Hence *V* is not unitary

Seesaw:

$$M_\nu \equiv V \widehat{M}_\nu V^T \approx -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$$

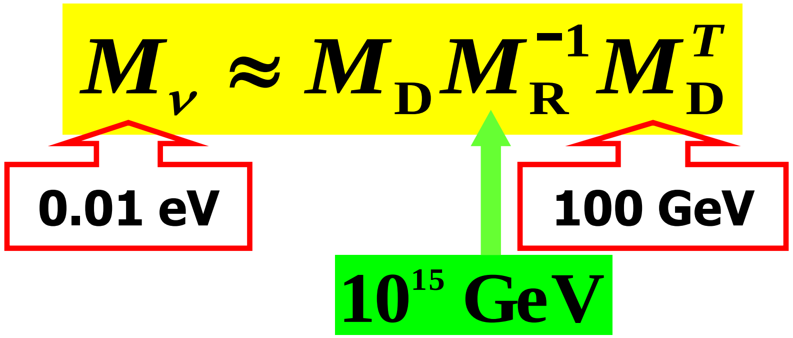
$$R \sim S \sim M_{\rm D} / M_{\rm R}$$

Strength of Unitarity Violation

$$V \approx \left(1 - \frac{1}{2} R R^\dagger\right) V_{\rm unitary}$$

Natural or unnatural?

Natural case: no large cancellation in the leading seesaw term.



$$M_\nu \approx M_D M_R^{-1} M_D^T$$

0.01 eV

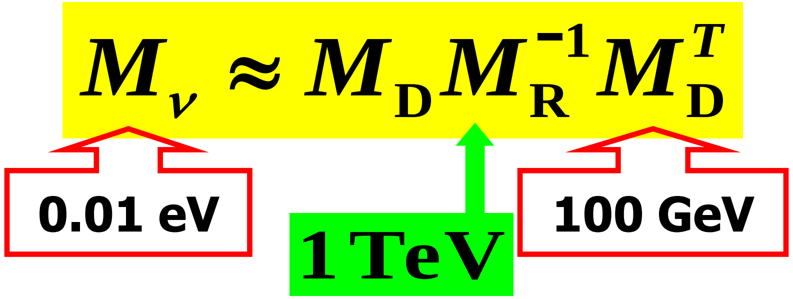
10^{15} GeV

100 GeV

$$R \sim S \sim M_D / M_R \sim 10^{-13}$$

$$\text{Unitarity Violation} \sim 10^{-26}$$

Unnatural case: large cancellation in the leading seesaw term.



$$M_\nu \approx M_D M_R^{-1} M_D^T$$

0.01 eV

1 TeV

100 GeV

$$R \sim S \sim M_D / M_R \sim 10^{-1}$$

$$\text{Unitarity Violation} \sim 10^{-2}$$

TeV-scale (right-handed) Majorana neutrinos: small masses of 3 light Majorana neutrinos come from sub-leading perturbations.

Structural cancellation

Given diagonal ***M*_R** with 3 mass igenvalues ***M*_1**, ***M*_2** and ***M*_3**, the leading (i.e., **type-I seesaw**) term of the active neutrino mass matrix vanishes, if and only if ***M*_D** has rank 1,

and if

$$M_{\rm D} = m \begin{pmatrix} y_1 & y_2 & y_3 \\ \alpha y_1 & \alpha y_2 & \alpha y_3 \\ \beta y_1 & \beta y_2 & \beta y_3 \end{pmatrix}$$

$$\frac{y_1^2}{M_1} + \frac{y_2^2}{M_2} + \frac{y_3^2}{M_3} = 0$$

$$M_\nu \approx M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T = 0$$

(Buchmueller, Greub 91; Ingelman, Rathsman 93; Heusch, Minkowski 94; ……; Kersten, Smirnov 07).

Tiny ν-masses can be generated from tiny corrections to this complete "**structural cancellation**", by deforming ***M*_D** or ***M*_R** .

Simple example:

$$M'_{\rm D} = M_{\rm D} + \epsilon X_{\rm D}$$

$$\begin{aligned} M'_\nu &= M'_{\rm D} M_{\rm R}^{-1} M'^{T}_{\rm D} \\ &\approx \epsilon \left(M_{\rm D} M_{\rm R}^{-1} X_{\rm D}^T + X_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T \right) + \mathcal{O}(\epsilon^2) \end{aligned}$$

Fast lessons

Lesson 1: two necessary conditions to test a seesaw model with heavy right-handed Majorana neutrinos at the LHC:

---Masses of heavy Majorana neutrinos must be of $O(1)$ TeV or below

---Light-heavy neutrino mixing (i.e. M_D/M_R) must be large enough

Lesson 2: A collider signature of the heavy Majorana ν's is essentially decoupled from masses and mixing parameters of light ν's.

Lesson 3: non-unitarity of the light ν flavor mixing matrix might lead to observable effects in ν oscillations and rare processes.

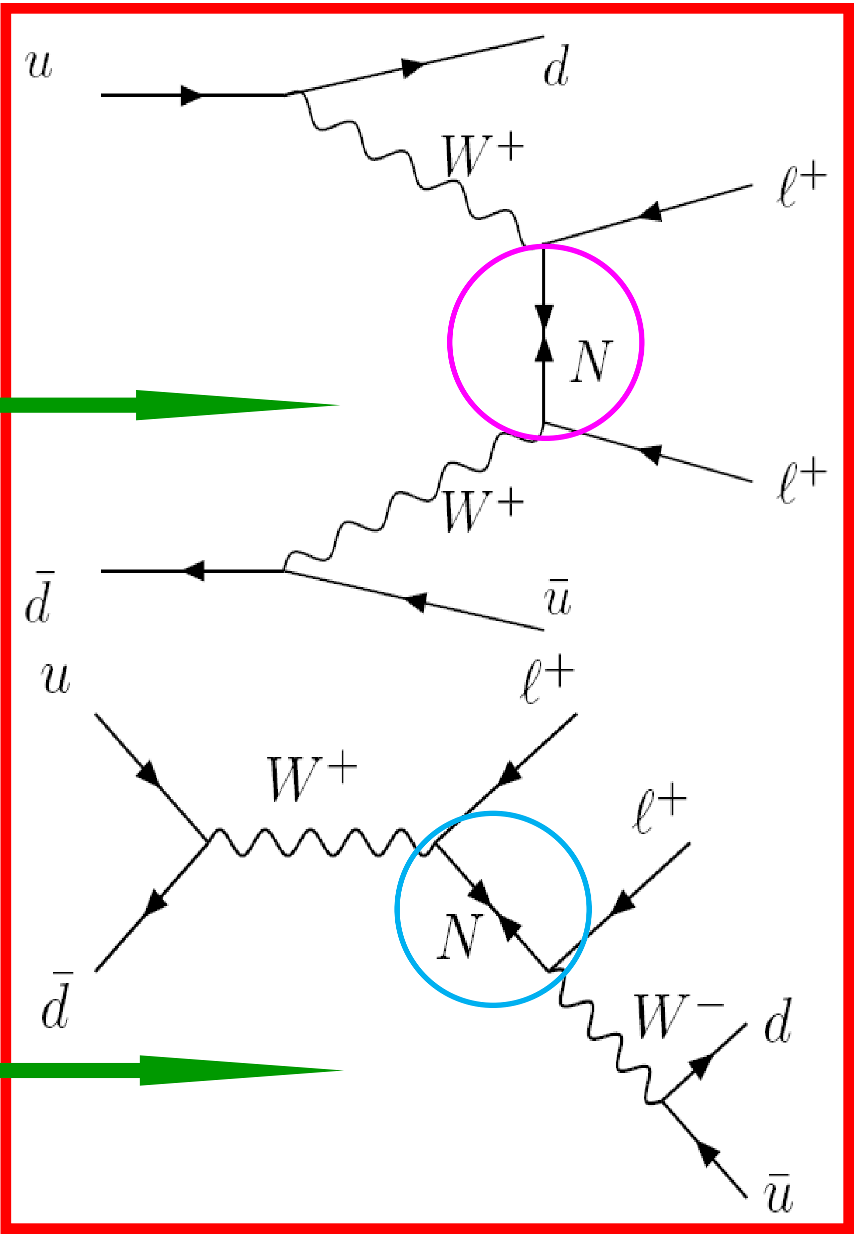
Lesson 4: nontrivial limits on heavy Majorana ν's could be derived at the LHC, if the SM backgrounds are small for a specific final state.

$\Delta L = 2$ like-sign dilepton events

$$\boxed{pp \to W^\pm W^\pm \to \mu^\pm \mu^\pm jj \text{ and } pp \to W^\pm \to \mu^\pm N \to \mu^\pm \mu^\pm jj}$$

Collider signature

Lepton number violation: like-sign dilepton events at hadron colliders, such as Tevatron (~2 TeV) and LHC (~14 TeV).



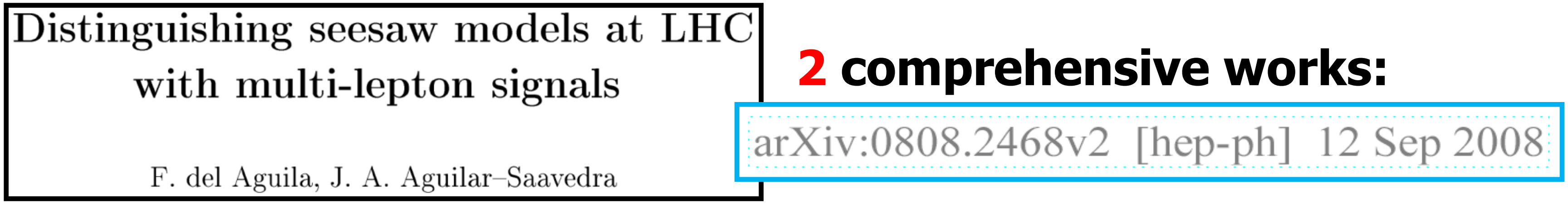
collider analogue to 0νββ decay

dominant channel

N can be produced on resonance

Testability at the LHC

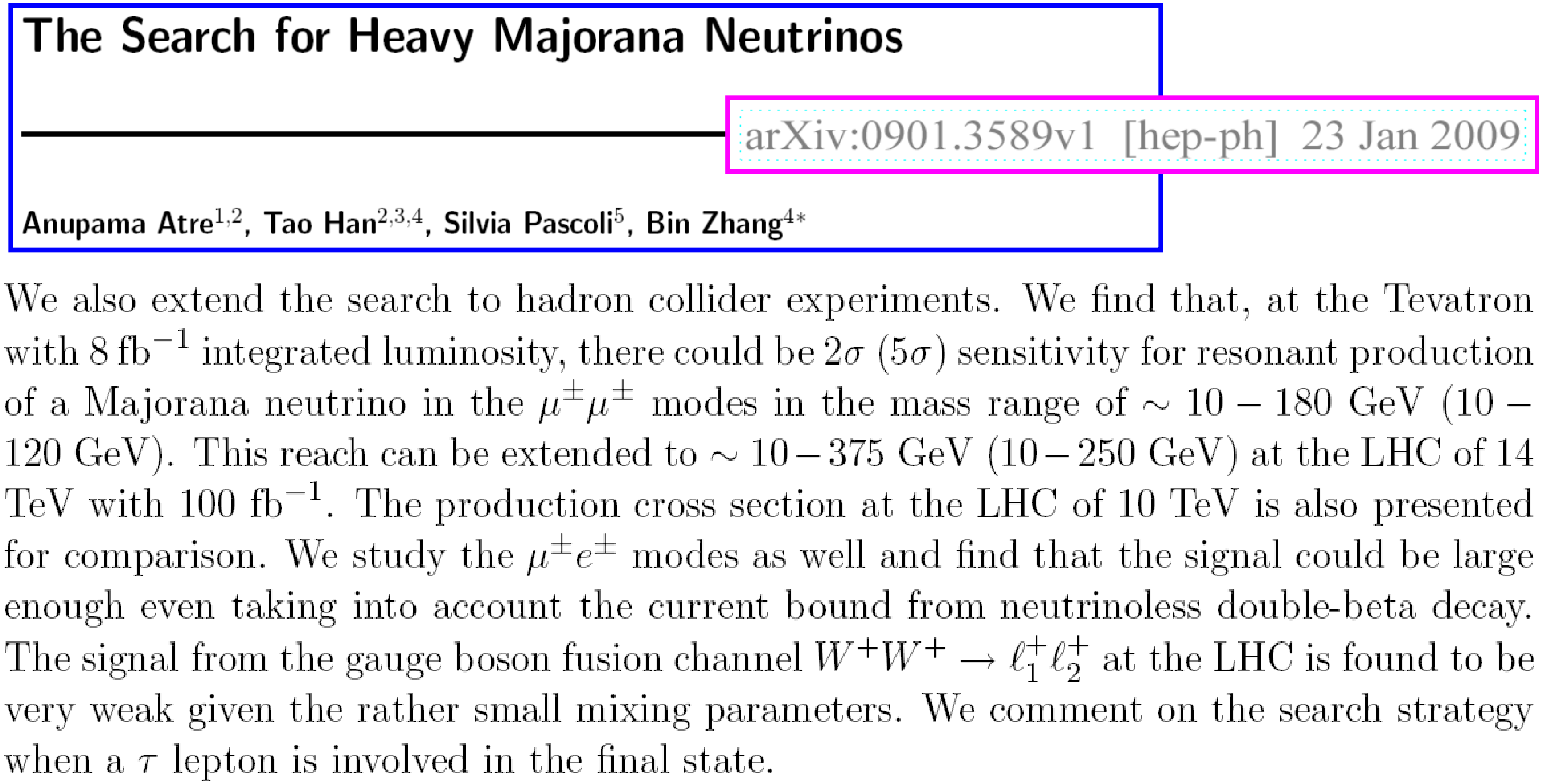
2 comprehensive works:



Distinguishing seesaw models at LHC with multi-lepton signals

F. del Aguila, J. A. Aguilar–Saavedra

arXiv:0808.2468v2 [hep-ph] 12 Sep 2008



The Search for Heavy Majorana Neutrinos

arXiv:0901.3589v1 [hep-ph] 23 Jan 2009

Anupama Atre[1,2], Tao Han[2,3,4], Silvia Pascoli[5], Bin Zhang[4*]

We also extend the search to hadron collider experiments. We find that, at the Tevatron with 8 fb^{-1} integrated luminosity, there could be 2σ (5σ) sensitivity for resonant production of a Majorana neutrino in the $\mu^\pm\mu^\pm$ modes in the mass range of $\sim 10-180$ GeV ($10-120$ GeV). This reach can be extended to $\sim 10-375$ GeV ($10-250$ GeV) at the LHC of 14 TeV with 100 fb^{-1}. The production cross section at the LHC of 10 TeV is also presented for comparison. We study the $\mu^\pm e^\pm$ modes as well and find that the signal could be large enough even taking into account the current bound from neutrinoless double-beta decay. The signal from the gauge boson fusion channel $W^+W^+ \to \ell_1^+\ell_2^+$ at the LHC is found to be very weak given the rather small mixing parameters. We comment on the search strategy when a τ lepton is involved in the final state.

Non-unitarity

Type-1 seesaw: a typical signature would be the unitarity violation of the 3×3 neutrino mixing matrix *V* in the charged-current interactions

Current experimental constraints at the 90% C.L. (Antusch *et al* 07):

$$|VV^\dagger| \approx \begin{pmatrix} 0.994 \pm 0.005 & < 7.0 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.0 \cdot 10^{-5} & 0.995 \pm 0.005 & < 1.0 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.0 \cdot 10^{-2} & 0.995 \pm 0.005 \end{pmatrix}$$

$\mu \to e + \gamma$ etc, *W* / *Z* decays, *universality*, ν-oscillation.

$$|V^\dagger V| \approx \begin{pmatrix} 1.00 \pm 0.032 & < 0.032 & < 0.032 \\ < 0.032 & 1.00 \pm 0.032 & < 0.032 \\ < 0.032 & < 0.032 & 1.00 \pm 0.032 \end{pmatrix}$$

accuracy of a few percent!

Extra CP-violating phases exist in a non-unitary ν mixing matrix may lead to observable *CP-violating effects* in short- or medium-baseline ν oscillations (Fernandez-Martinez *et al* 07; Xing 08).

Typical example: non-unitary CP violation in the ν_μ → ν_τ oscillation, an effect probably at the percent level.

Type-2 seesaw

Type-2 (Triplet) Seesaw: add one SU(2)_L Higgs triplet into the SM.

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \frac{1}{2}\overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} + {\rm h.c.}$$

$$\Delta \equiv \begin{pmatrix} H^- & -\sqrt{2}\ H^0 \\ \sqrt{2}\ H^{--} & -H^- \end{pmatrix}$$

or

$$-\mathcal{L}_{\rm mass} = \overline{e_{\rm L}} M_l E_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} M_{\rm L} \nu^c_{\rm L} + {\rm h.c.}$$

$$M_{\rm L} \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$

Potential:

$$V(H,\Delta) = -\mu^2 H^\dagger H + \lambda\left(H^\dagger H\right)^2 + \frac{1}{2}M_\Delta^2 {\rm Tr}\left(\Delta^\dagger\Delta\right) - \underline{\left[\lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + {\rm h.c.}\right]}$$

L and *B–L* violation

Naturalness? (t' Hooft 79, …, Giudice 08)

(1) *M_Δ* is O(1) TeV or close to the scale of gauge symmetry breaking.
(2) *λ_Δ* must be tiny, and *λ_Δ* =0 enhances the symmetry of the model.

$$M_{\rm L} \approx \lambda_\Delta Y_\Delta \frac{{\rm v}^2}{M_\Delta}$$

0.01 eV

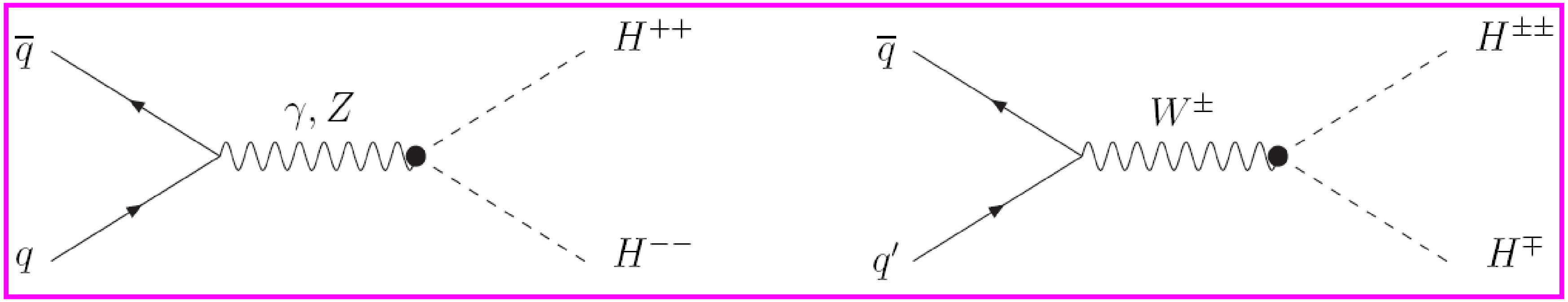
1 TeV

$$\lambda_\Delta Y_\Delta \sim 10^{-12} \Rightarrow \begin{cases} Y_\Delta \sim 1, \lambda_\Delta \sim 10^{-12} \\ \lambda_\Delta \sim Y_\Delta \sim 10^{-6} \\ \ldots\ldots \end{cases}$$

Collider signature

From a viewpoint of direct tests, the triplet seesaw has an advantage:

The SU(2)_L Higgs triplet contains a doubly-charged scalar which can be produced at colliders: it is dependent on its mass but independent of the (small) Yukawa coupling.



Typical LNV signatures: $\boxed{H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta}}$ $\boxed{H^{+} \to l^{+}_{\alpha} \bar{\nu}_{\beta}}$ $\boxed{H^{-} \to l^{-}_{\alpha} \nu}$

$$\mathcal{B}(H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta}) = \frac{(2-\delta_{\alpha\beta})|(M_{\rm L})_{\alpha\beta}|^2}{\sum\limits_{\rho,\sigma} |(M_{\rm L})_{\rho\sigma}|^2} , \qquad \mathcal{B}(H^{+} \to l^{+}_{\alpha} \overline{\nu}) = \frac{\sum\limits_{\beta} |(M_{\rm L})_{\alpha\beta}|^2}{\sum\limits_{\rho,\sigma} |(M_{\rm L})_{\rho\sigma}|^2}$$

Testability at the LHC

Lesson one: the above branching ratios **purely** depend on 3 neutrino masses, 3 flavor mixing angles and the CP-violating phases.

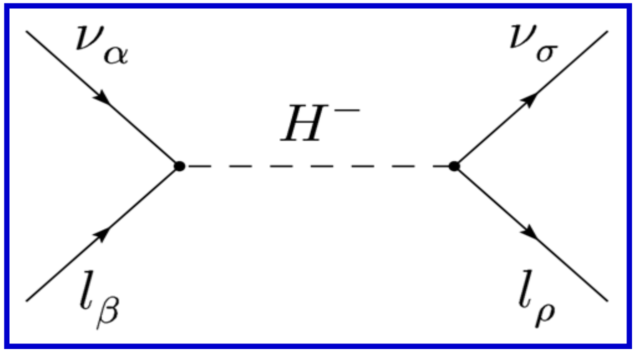
Lesson two: the **Majorana phases** may affect LNV $H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta}$ decay modes, but they do not enter $H^{+} \to l^{+}_{\alpha} \bar{\nu}_{\beta}$ and $H^{-} \to l^{-}_{\alpha} \nu$ processes.

$$\left|(M_{\rm L})_{\alpha\beta}\right|^2 = \left|\sum_{i=1}^{3}\left(m_i V_{\alpha i} V_{\beta i}\right)\right|^2 , \qquad \sum_{\beta}\left|(M_{\rm L})_{\alpha\beta}\right|^2 = \sum_{i=1}^{3}\left(m_i^2 \left|V_{\alpha i}\right|^2\right)$$

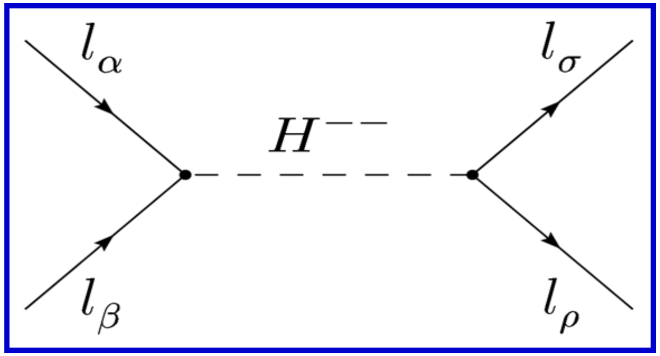
Dimension-6 operator:
(2 low-energy effects)

$$\frac{\mathcal{L}_{\rm d=6}}{\Lambda^2} = -\frac{(Y_\Delta)_{\alpha\beta}\,(Y_\Delta)^{\dagger}_{\rho\sigma}}{4M_\Delta^2}\left(\overline{l_{\alpha\rm L}}\gamma^{\mu} l_{\sigma\rm L}\right)\left(\overline{l_{\beta\rm L}}\gamma_{\mu} l_{\rho\rm L}\right)$$

1) **NSIs** of 3 neutrinos



2) **LFV** of 4 charged leptons



Seesaw trivialization

Linear trivialization: use three types of seesaws to make a family tree.

Type 1 + Type 2

Type 1 + Type 3

Type 2 + Type 3

Type 1 + Type 2 + Type 3

Weinberg's ***3rd law of progress in theoretical physics*** **(83):**

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry .... What could be better?

Linearly trivialized seesaws usually work at super-high energies.

Multiple trivialization: well motivated to lower the seesaw scale.

Example: inverse seesaw

The Inverse Seesaw: SM + 3 heavy right-handed neutrinos + 3 gauge singlet neutrinos + one Higgs singlet (Wyler, Wolfenstein 83; Mohapatra, Valle 86; Ma 87).

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \overline{N^{\rm c}_{\rm R}} Y_S \Phi S_{\rm R} + \frac{1}{2}\overline{S^{\rm c}_{\rm R}} \mu S_{\rm R} + {\rm h.c.}$$

LNV: tiny

ν-mass matrix:

$$\overline{\left(\nu_{\rm L} \quad N^{\rm c}_{\rm R} \quad S^{\rm c}_{\rm R}\right)} \begin{pmatrix} \mathbf{0} & M_{\rm D} & \mathbf{0} \\ M_{\rm D}^T & \mathbf{0} & M_S \\ \mathbf{0} & M_S^T & \mu \end{pmatrix} \begin{pmatrix} \nu^{\rm c}_{\rm L} \\ N_{\rm R} \\ S_{\rm R} \end{pmatrix}$$

$$\begin{aligned} M_{\rm D} &= Y_\nu \langle H \rangle \\ M_S &= Y_S \langle \Phi \rangle \end{aligned}$$

Effective light ν-mass matrix

$$M_\nu \approx M_{\rm D} \frac{1}{M_S^T} \mu \frac{1}{M_S} M_{\rm D}^T \quad \Longleftrightarrow \quad -\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} M_\nu \nu^c_{\rm L} + {\rm h.c.}$$

Merit: **more natural tiny ν-masses and appreciable collider signatures;**
Fault: **some new degrees of freedom.** **Is Weinberg's 3rd law applicable?**

Multiple seesaw mechanisms: to *naturally* lower seesaw scales to TeV (Babu et al 09; Xing, Zhou 09; Bonnet et al 09, etc).

Appendix

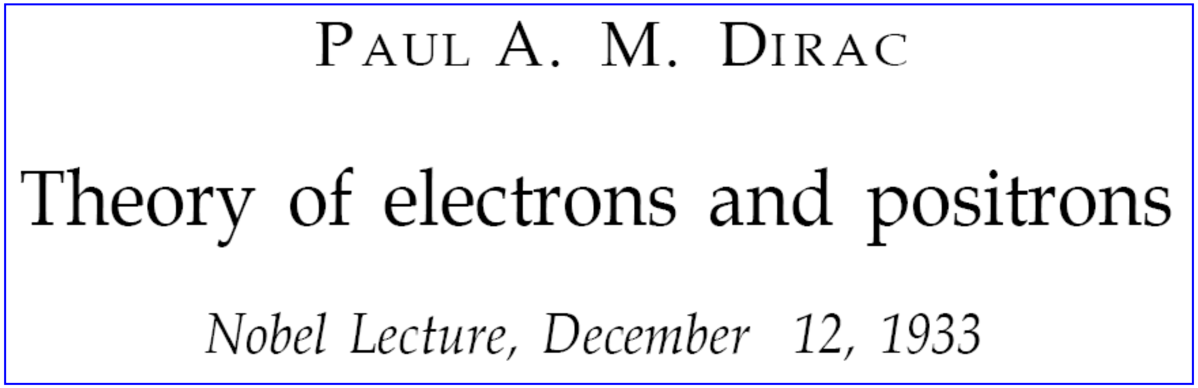
Misguiding principles for a theorist to go beyond the SM

(Schellekens 08: "The Emperor's Last Clothes?")

- **Agreement with observation**
- **Consistency**
- **Uniqueness**
- **Naturalness**
- **Simplicity**
- **Elegance**
- **Beauty**
- **……**

?

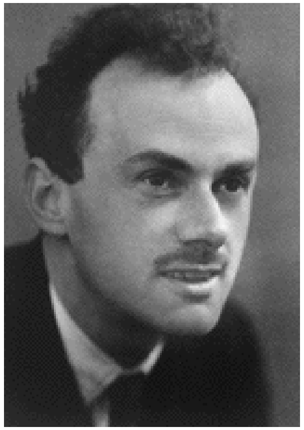
Dirac's Expectation

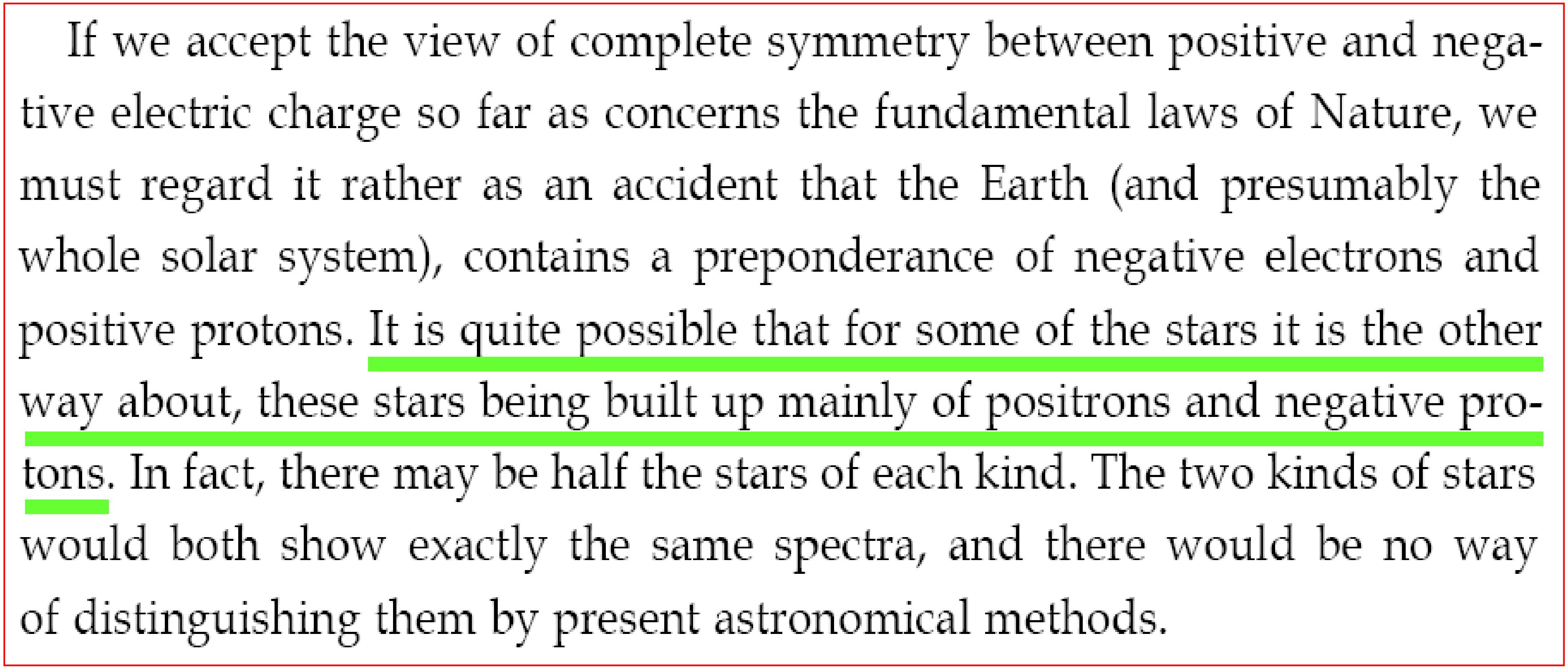


PAUL A. M. DIRAC

Theory of electrons and positrons

Nobel Lecture, December 12, 1933





If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of Nature, we must regard it rather as an accident that the Earth (and presumably the whole solar system), contains a preponderance of negative electrons and positive protons. It is quite possible that for some of the stars it is the other way about, these stars being built up mainly of positrons and negative protons. In fact, there may be half the stars of each kind. The two kinds of stars would both show exactly the same spectra, and there would be no way of distinguishing them by present astronomical methods.

The puzzle

Why we did **not** see an anti-universe expected by Dirac?

$$t = 10^{16}\,\mathrm{sec}$$

$$r = 10^{29}\,\mathrm{cm}$$

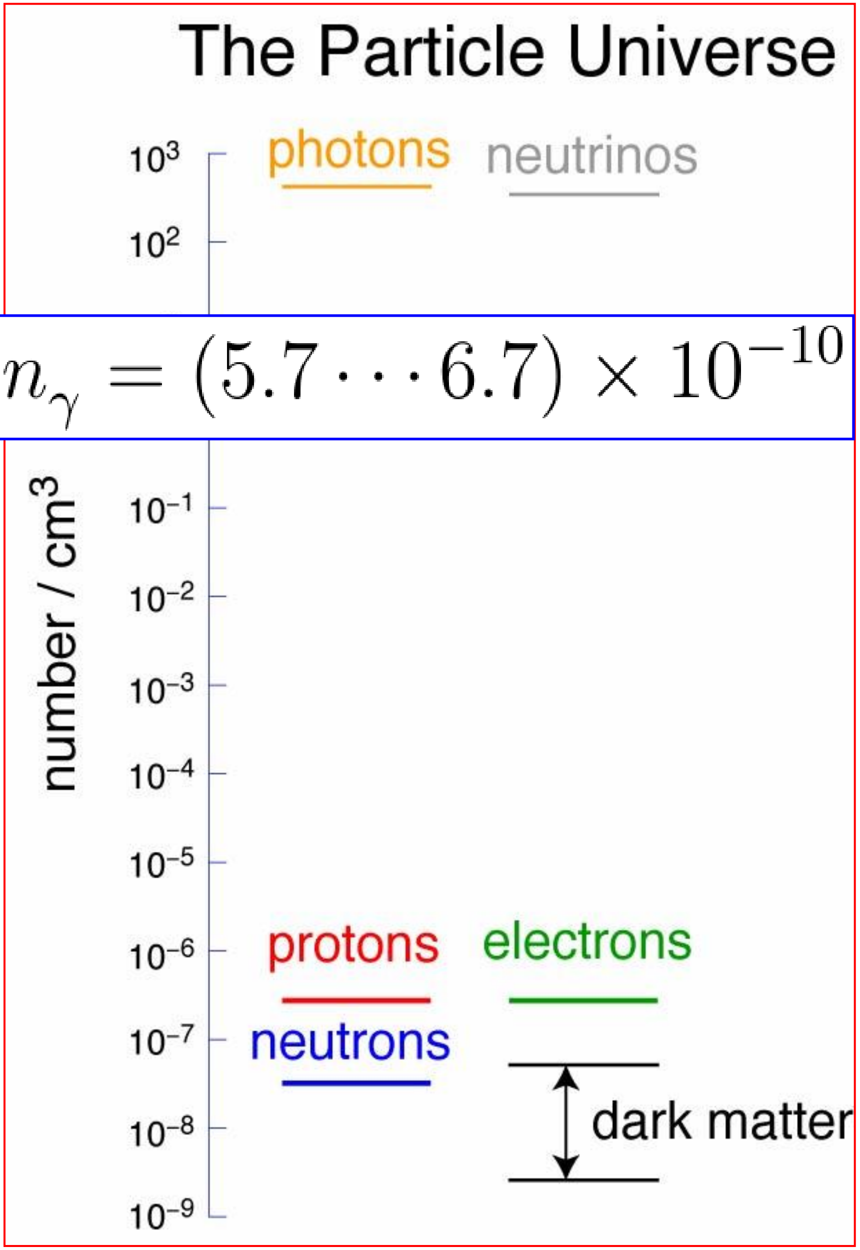
$$T = 2.7\ \mathrm{K}$$

$$400\gamma / \mathrm{cm}^3$$

$$10^{80}\ p, n$$

$$0\quad \bar{p}, \bar{n}$$

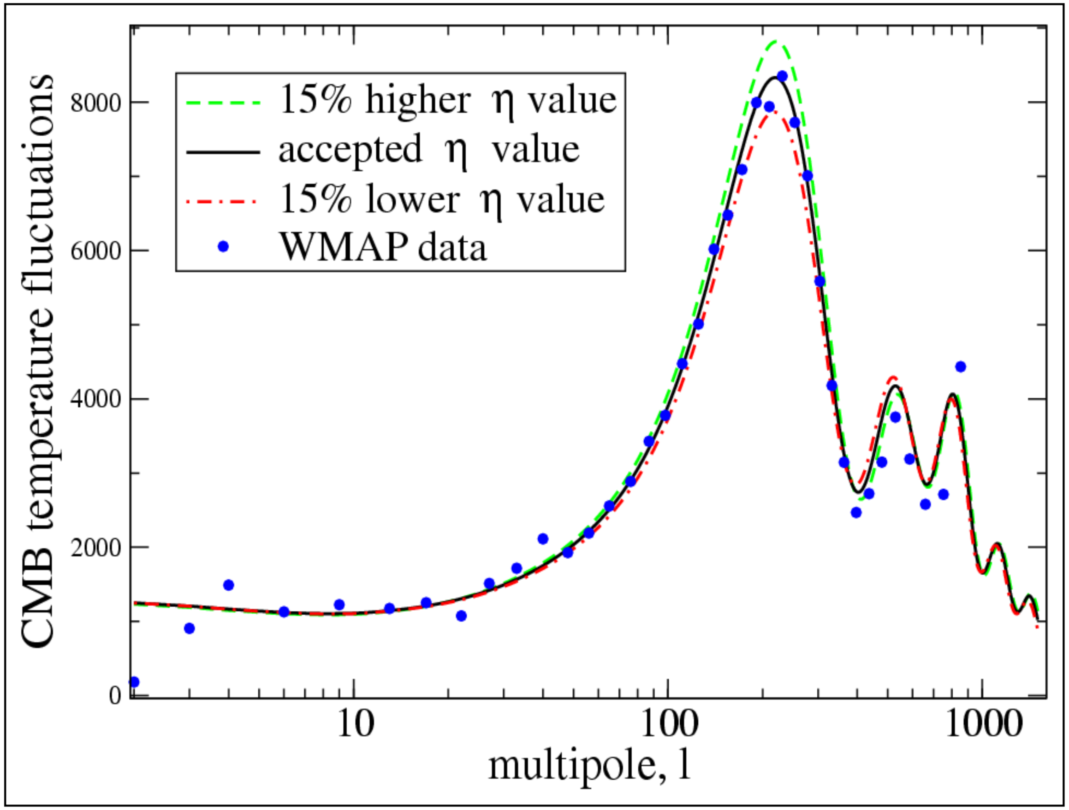
$$\eta \equiv n_{\mathrm{B}}/n_\gamma = (5.7\cdots6.7)\times 10^{-10}$$

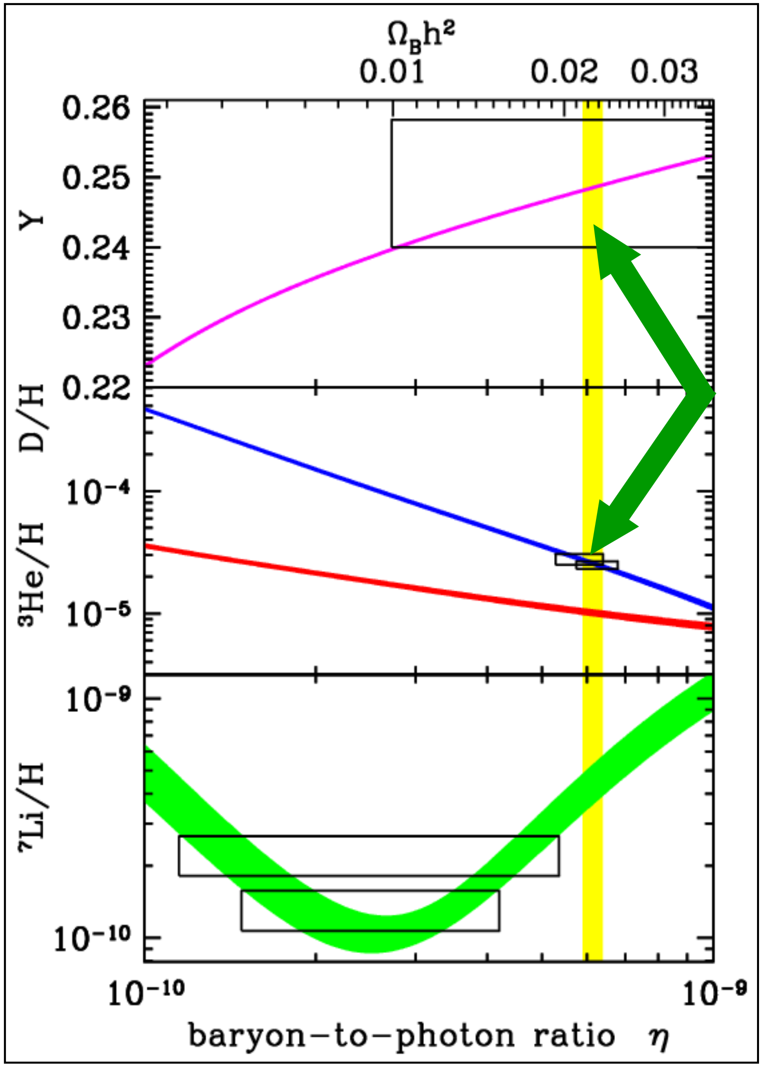


Evidence

η_B was historically determined from the Big Bang Nucleosynthesis: Primordial abundances of BBN light elements are sensitive to it.

η_B can now be measured from Cosmic Microwave Background: Relative sizes of those Doppler peaks of CMB temperature anisotropy are sensitive to it.





Sakharov conditions

Baryogenesis: 1) **Just-So** --- $B > 0$ from the very beginning up to now;
2) **Dynamical picture** --- $B > 0$ evolved from $B = 0$ after inflation.

Condition 1: baryon number (B) violation.
[GUT, SUSY & even SM allow it, but no direct experimental evidence]

Condition 2: breaking of C and CP symmetries.
[C & CP asymmetries are both needed to keep B violation survivable]

Condition 3: departure from thermal equilibrium.
[Thermal equilibrium might erase B asymmetry due to CPT symmetry]


(Sakharov, 67)

Baryogenesis Mechanisms

- Planck/GUT Baryogenesis;
- Electroweak Baryogenesis;
- Leptogenesis;
- Affleck-Dine Mechanism; ...

Sakharov's paper: almost no citation during 1967-1979

Now >1300 times

Neutrino Physics

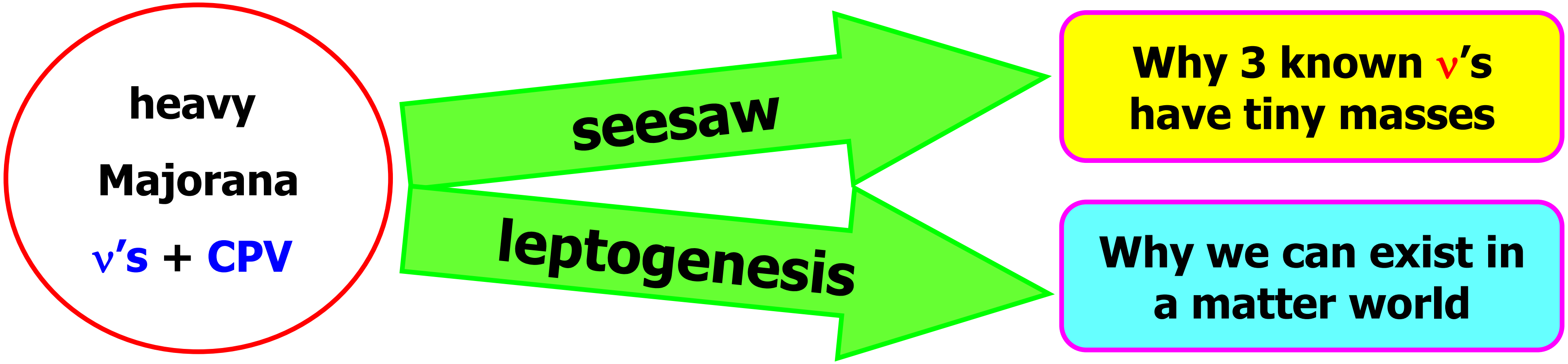
Remarks on CP violation

CP violation from the ***CKM*** quark mixing matrix is not the whole story to explain the **matter-antimatter asymmetry** of the visible Universe.

Two reasons for this in the SM:

- **CP** violation from the **SM**'s quark sector is highly suppressed;
- The electroweak phase transition is not strongly first order.

New sources of CP violation are necessarily required.



heavy
Majorana
ν's + CPV

seesaw

leptogenesis

Why 3 known ν's
have tiny masses

Why we can exist in
a matter world

Encouraging news: current ν data hint at $\delta \sim 270$ degrees.

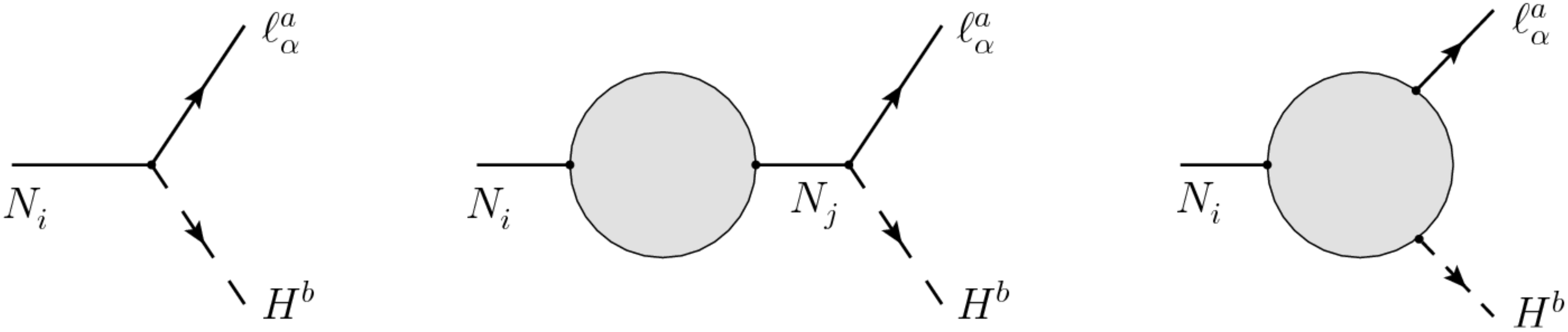
Thermal leptogenesis (1)

◆ **add 3 heavy right-handed Majorana neutrinos into SM & keep its SU(2)×U(1) gauge symmetry**

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}} Y_l H E_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2} \overline{N_{\rm R}^{\rm c}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

Fukugita, Yanagida 86

◆ **lepton-number-violating & CP-violating decays of heavy neutrinos:**



$$\varepsilon_i \equiv \frac{\sum\limits_\alpha \left[\Gamma(N_i \to \ell_\alpha + H) - \Gamma(N_i \to \overline{\ell}_\alpha + \overline{H})\right]}{\sum\limits_\alpha \left[\Gamma(N_i \to \ell_\alpha + H) + \Gamma(N_i \to \overline{\ell}_\alpha + \overline{H})\right]}$$

$$\approx \frac{1}{8\pi (Y_\nu^\dagger Y_\nu)_{ii}} \sum_j {\rm Im}\left[(Y_\nu^\dagger Y_\nu)_{ij}\right]^2 \left[f_{\rm V}\left(\frac{M_j^2}{M_i^2}\right) + f_{\rm S}\left(\frac{M_j^2}{M_i^2}\right)\right]$$

$$f_{\rm V}(x) = \begin{cases} \sqrt{x}\left[1 - (1+x)\ln\dfrac{1+x}{x}\right] & ({\rm SM}) \,, \\ -\sqrt{x}\ln\dfrac{1+x}{x} & ({\rm SUSY}) \,; \end{cases}$$

$$f_{\rm S}(x) = \begin{cases} \dfrac{\sqrt{x}}{1-x} & ({\rm SM}) \,, \\ \dfrac{2\sqrt{x}}{1-x} & ({\rm SUSY}) \,. \end{cases}$$

Thermal leptogenesis (2)

◆ **to prevent CP asymmetries from being washed out by the inverse decays and scattering processes, the decays of heavy neutrinos must be out of thermal equilibrium (their decay rates must be smaller than the expansion rate of the Universe.**

$$\boxed{\Gamma(N_i \to \ell_\alpha + H) < H(T = M_i)}$$

The net lepton number asymmetry:

$$\boxed{Y_{\mathrm{L}} \equiv \frac{n_{\mathrm{L}} - n_{\overline{\mathrm{L}}}}{s} = \frac{1}{g_*} \sum_i \kappa_i \varepsilon_i}$$

κ_i **: efficiency factors**

g_* **: number of relativistic d.o.f**

s **: entropy density**

(Boltzmann equations for time evolution of particle number densities)

◆ **non-perturbative but (*B*–*L*)-conserving weak sphaleron reactions convert a lepton number asymmetry to a baryon number asymmetry.**

$$\boxed{\partial_\mu J^\mu_{\mathrm{B}} = \partial_\mu J^\mu_{\mathrm{L}} = \frac{N_f}{32\pi^2}\left(-g^2 W^i_{\mu\nu}\tilde{W}^{i\mu\nu} + g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right)}$$

at the quantum level via triangle anomaly.

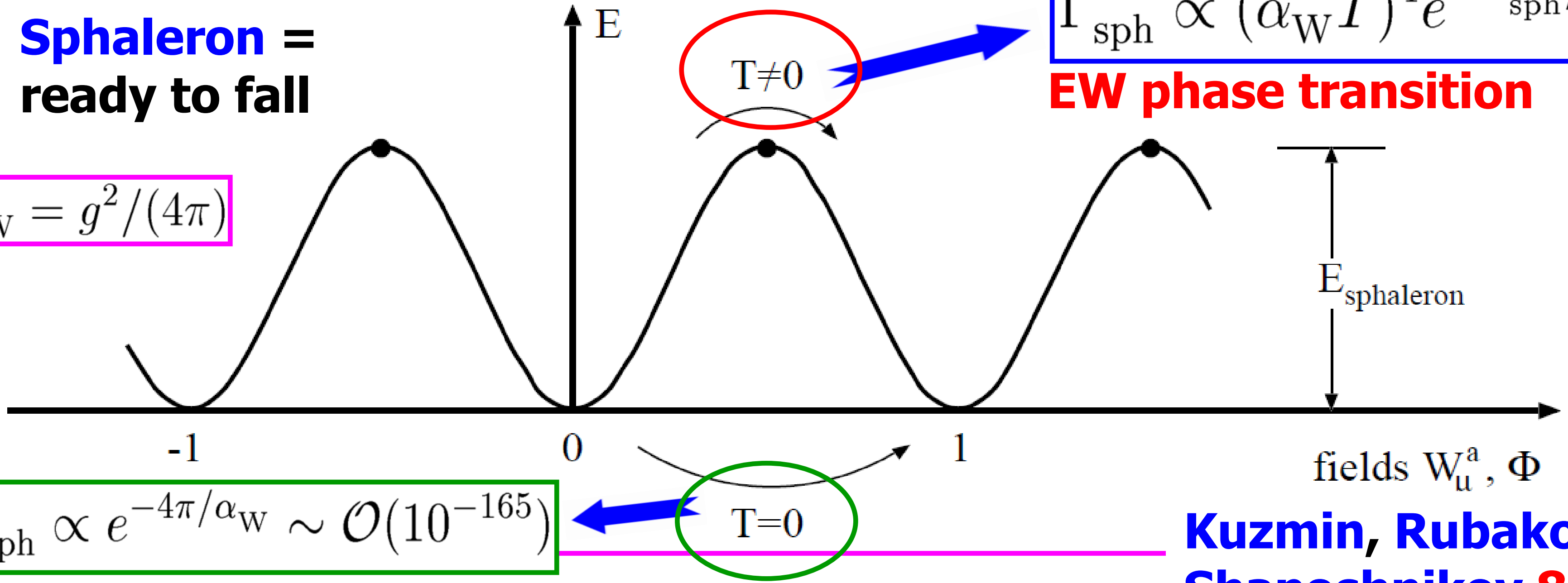
$$\boxed{B - L = \int \mathrm{d}^3x \left(J^0_{\mathrm{B}} - J^0_{\mathrm{L}}\right) = 0}$$

(*B*–*L*) is conserved in the SM ('t Hooft, 76)

Chern-Simons (CS) numbers = ±1, ±2, …

$$\boxed{\Delta B = \Delta L = N_f \Delta N_{\mathrm{CS}}}$$

Thermal leptogenesis (3)



Sphaleron = ready to fall

$\alpha_{\mathrm{W}} = g^2/(4\pi)$

$\Gamma_{\mathrm{sph}} \propto (\alpha_{\mathrm{W}} T)^4 e^{-E_{\mathrm{sph}}/T}$

EW phase transition

$\Gamma_{\mathrm{sph}} \propto e^{-4\pi/\alpha_{\mathrm{W}}} \sim \mathcal{O}(10^{-165})$

Kuzmin, Rubakov, Shaposhnikov 85.

Sphaleron-induced (*B*+*L*)-violating process is in thermal equilibrium when the temperature:

10^2 GeV < *T* < 10^{12} GeV

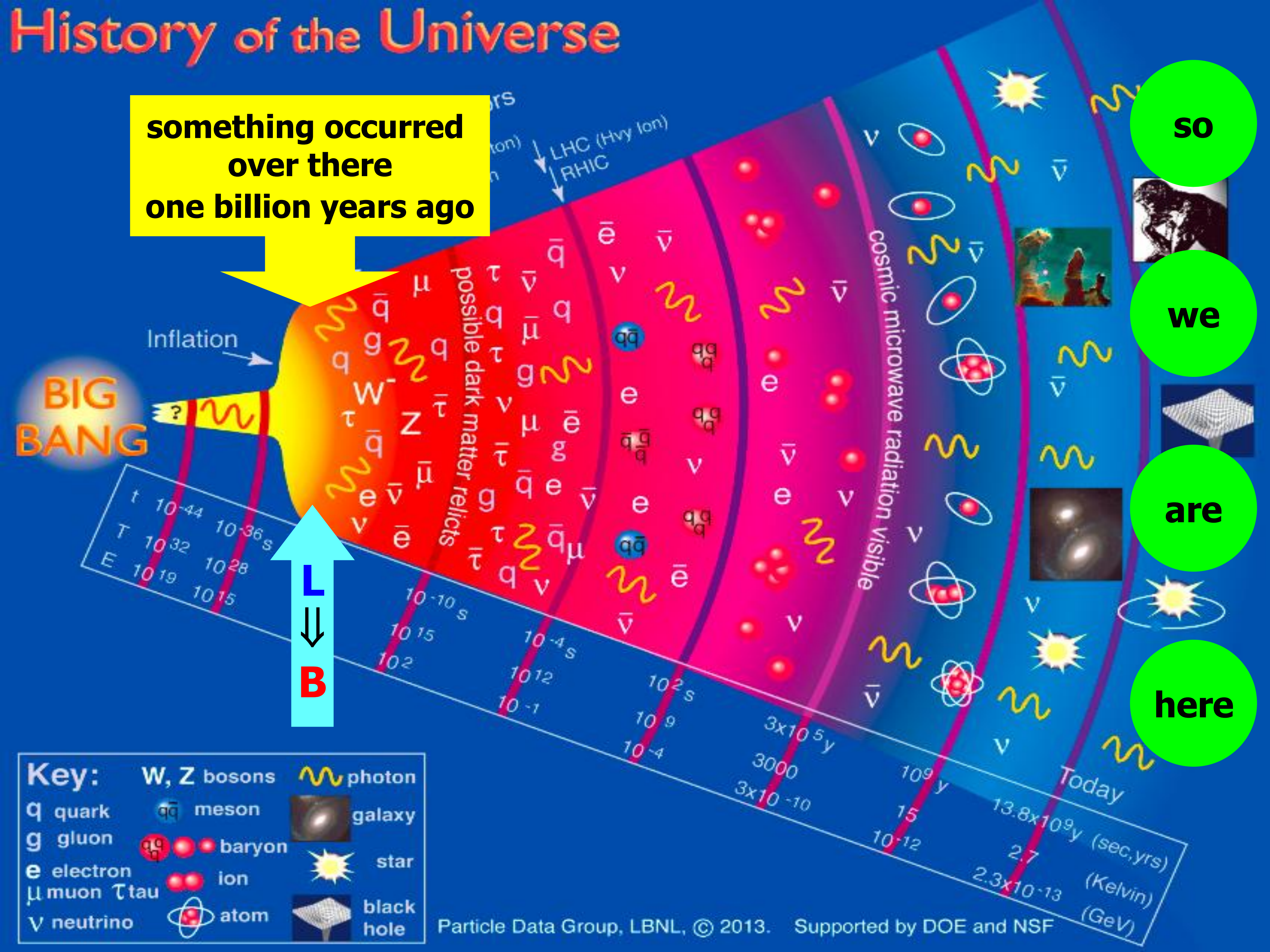
$$Y_{\mathrm{B}} \equiv \frac{n_{\mathrm{B}} - n_{\overline{\mathrm{B}}}}{s} = -C Y_{\mathrm{L}}$$

Baryogenesis via leptogenesis is realized:

$$\left.\frac{n_{\mathrm{B}}}{s}\right|_{\mathrm{equilibrium}} = C \left.\frac{n_{\mathrm{B}} - n_{\mathrm{L}}}{s}\right|_{\mathrm{equilibrium}} = -C \left.\frac{n_{\mathrm{L}}}{s}\right|_{\mathrm{initial}}$$

$$\left.\frac{n_{\overline{\mathrm{B}}}}{s}\right|_{\mathrm{equilibrium}} = C \left.\frac{n_{\overline{\mathrm{B}}} - n_{\overline{\mathrm{L}}}}{s}\right|_{\mathrm{equilibrium}} = -C \left.\frac{n_{\overline{\mathrm{L}}}}{s}\right|_{\mathrm{initial}}$$

$$\begin{aligned} C &= \frac{8N_f + 4N_\Phi}{22N_f + 13N_\Phi} \\ &= \begin{cases} 28/79 & (\mathrm{SM}) \\ 8/23 & (\mathrm{MSSM}) \end{cases} \end{aligned}$$



**something occurred
over there
one billion years ago**

**L
⇓
B**

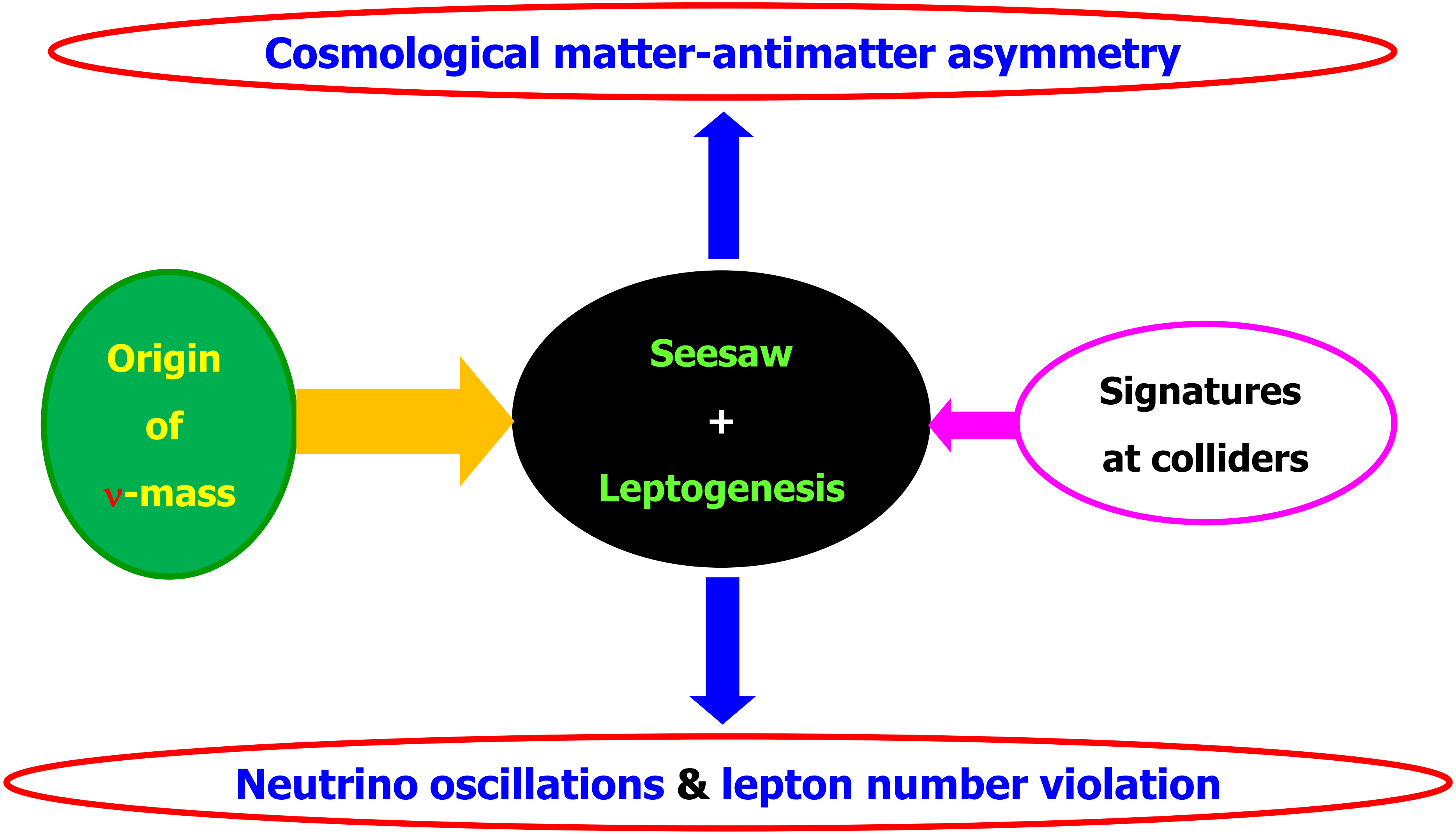
so

we

are

here

A grand picture?



Cosmological matter-antimatter asymmetry

Origin
of
ν-mass

Seesaw
+
Leptogenesis

Signatures
at colliders

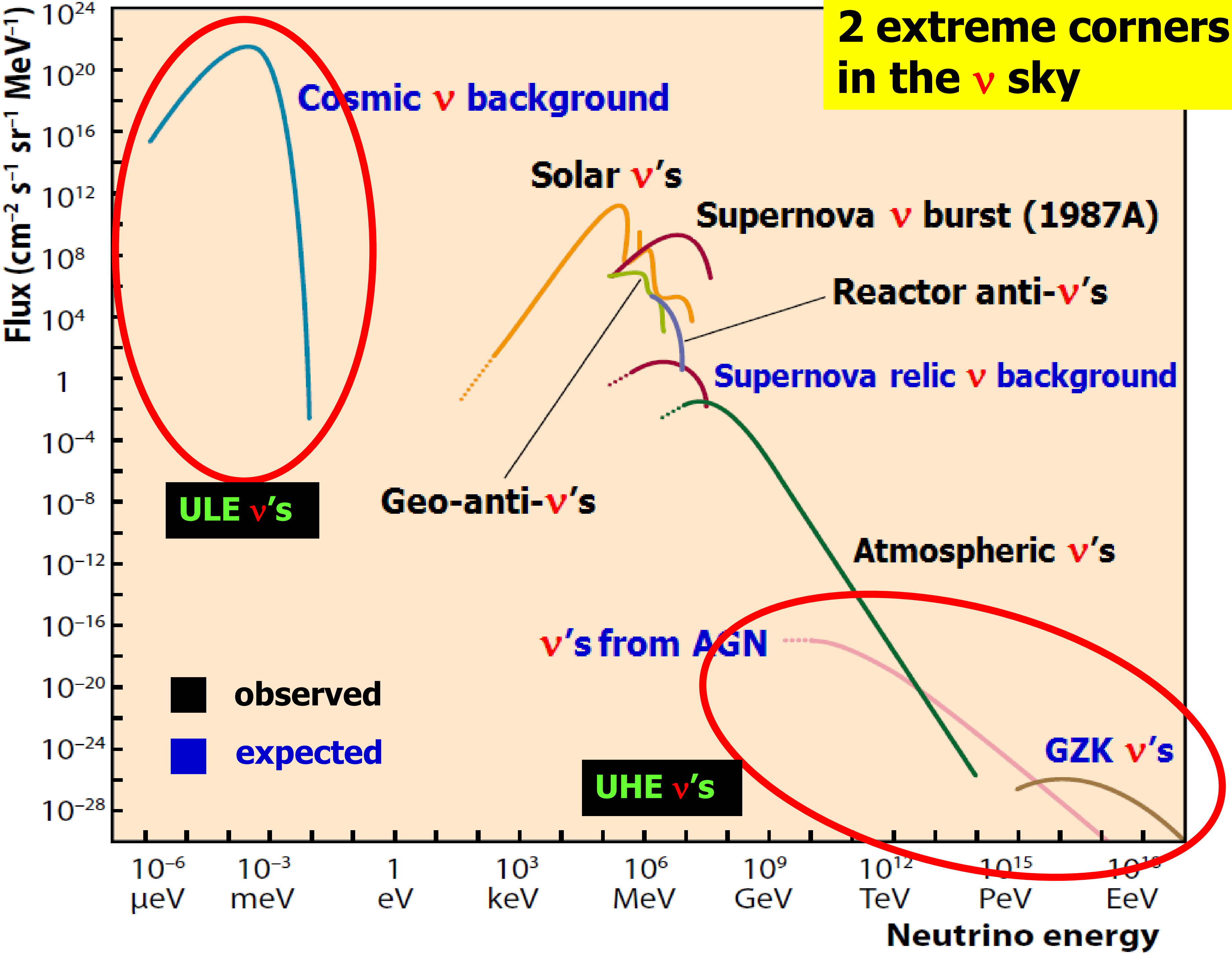
Neutrino oscillations & lepton number violation

Cosmic messenger: both neutrino astronomy and neutrino cosmology.
Surprise maker: history of neutrino physics is always full of surprises.

Lecture C2

- ★ Cosmic neutrino background
- ★ keV sterile neutrino dark matter
- ★ Ultrahigh-energy cosmic neutrinos

2 extreme corners in the ν sky



Formation of CνB

When *T* ~ a few MeV after Big Bang, the survival particles: photons, electrons, positrons, neutrinos and antineutrinos

Electroweak reactions: $\boxed{\gamma + \gamma \rightleftharpoons e^+ + e^- \rightleftharpoons \nu_\alpha + \overline{\nu}_\alpha \ (\text{for } \alpha = e, \mu, \tau)}$

$\boxed{\nu_e + n \rightleftharpoons e^- + p, \overline{\nu}_e + p \rightleftharpoons e^+ + n}$ $\boxed{\overline{\nu}_e + e^- + p \rightleftharpoons n}$

Neutrinos decoupled from matter:

Weak interactions $\Gamma \sim G_F^2 T^5$

Hubble expansion $H \sim \dfrac{\sqrt{g_*}\,T^2}{M_{\rm Pl}}$

Number density of 6 relic ν's:

$$n_\nu = \frac{9}{11} n_\gamma \approx 336 \left(\frac{T_\gamma}{2.725\ \text{K}}\right)^3 \text{cm}^{-3}$$

$\boxed{\Gamma > H}$ $\boxed{\Gamma \sim H}$ $\boxed{\Gamma < H}$

ν's in thermal contact with plasma

ν's not in interaction with matter

arrow of time

neutrino decoupling

$\boxed{T < m_e}$ $\boxed{e^+ + e^- \to \gamma + \gamma}$

neutrino and photon temperatures:

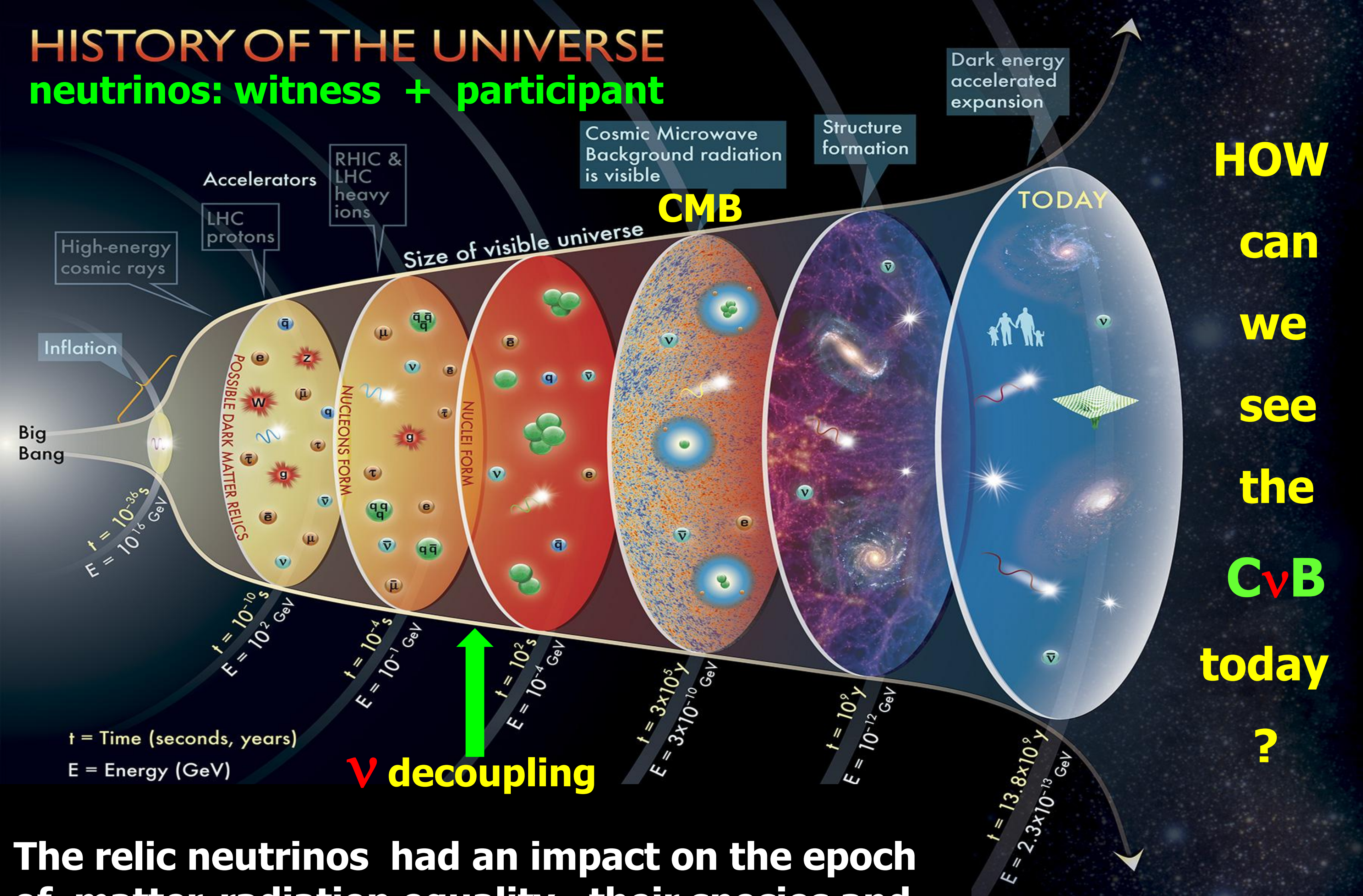
$\boxed{T_\nu = T_\gamma}$

$$\boxed{T_{\rm fr} \sim \left(\frac{\sqrt{g_*}}{G_F^2 M_{\rm Pl}}\right)^{1/3} \sim 1\ \text{MeV}}$$

$$\boxed{T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma}$$

HISTORY OF THE UNIVERSE

neutrinos: witness + participant



CMB

ν decoupling

HOW can we see the CνB today ?

The relic neutrinos had an impact on the epoch of matter-radiation equality, their species and masses left an imprint on the CMB anisotropies and large scale structures.

$$\Omega_\nu \simeq \frac{1}{94\ h^2\ \mathrm{eV}} \sum_i m_i$$

Detection of CνB

- ★ **CνB-induced mechanical effects on Cavendish-type torsion balance;**
- ★ **Capture of relic ν's on radioactive β-decaying nuclei (Weinberg 62);**
- ★ **Z-resonance annihilation of UHE cosmic ν's and relic ν's (Weiler 82).**

Temperature today

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \simeq 1.945\ \mathrm{K}$$

Mean momentum today

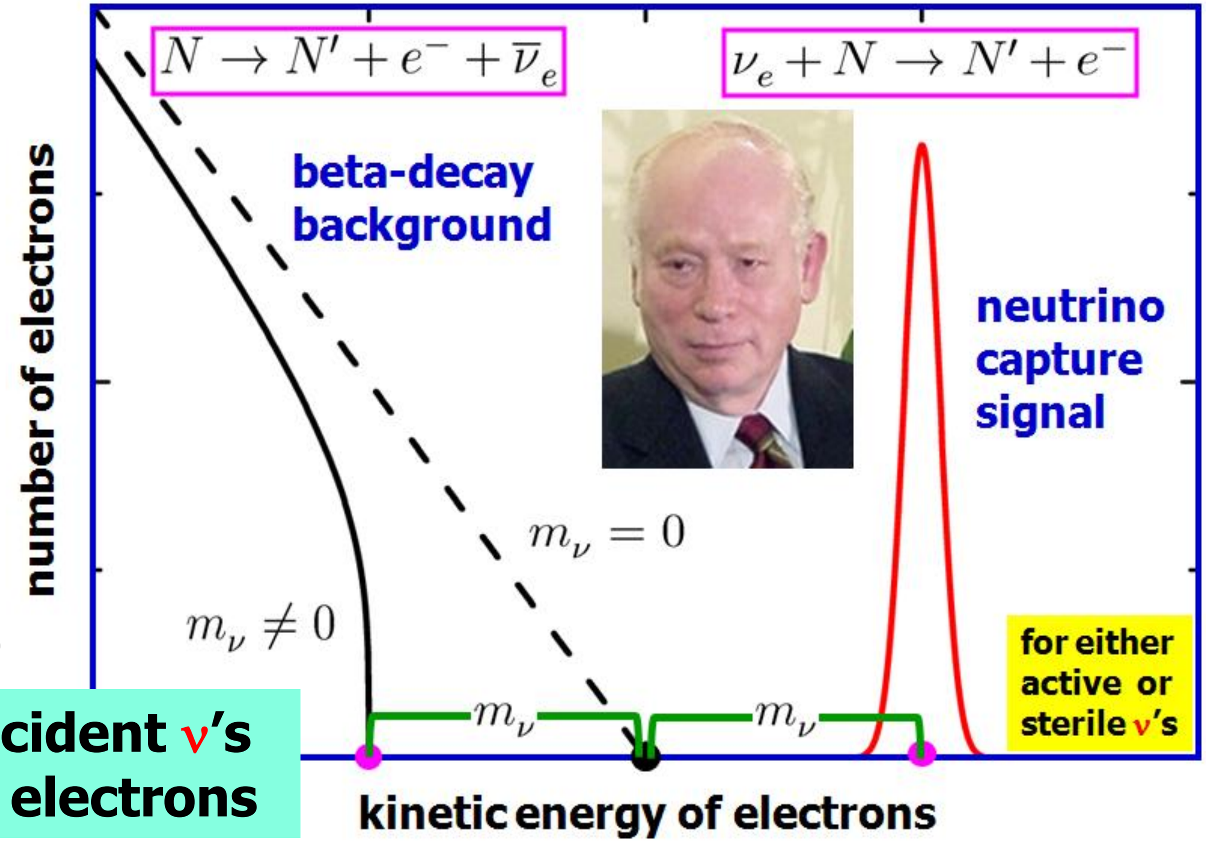
$$\langle p_\nu \rangle \simeq 3.151 T_\nu \simeq 5.281 \times 10^{-4}\ \mathrm{eV}$$

At least 2 ν's cold today

Non-relativistic ν's!

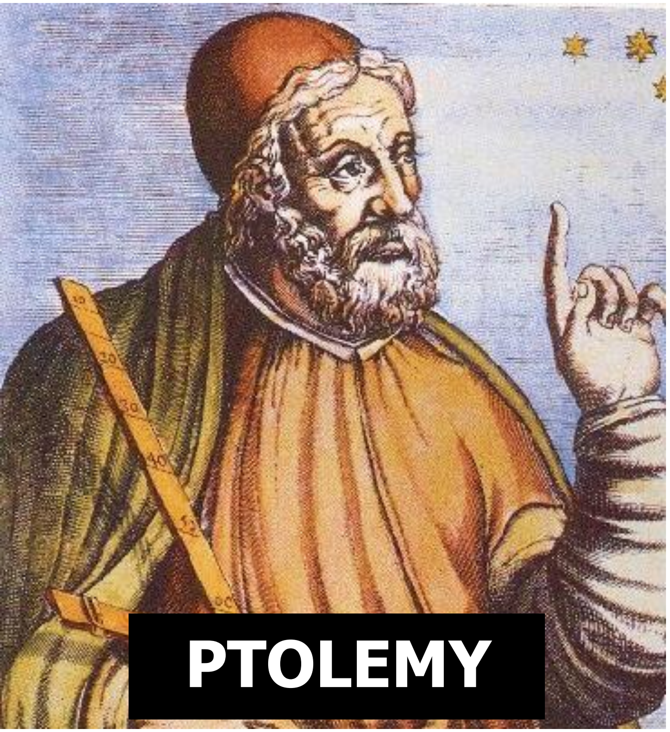
(Irvine & Humphreys, 83)

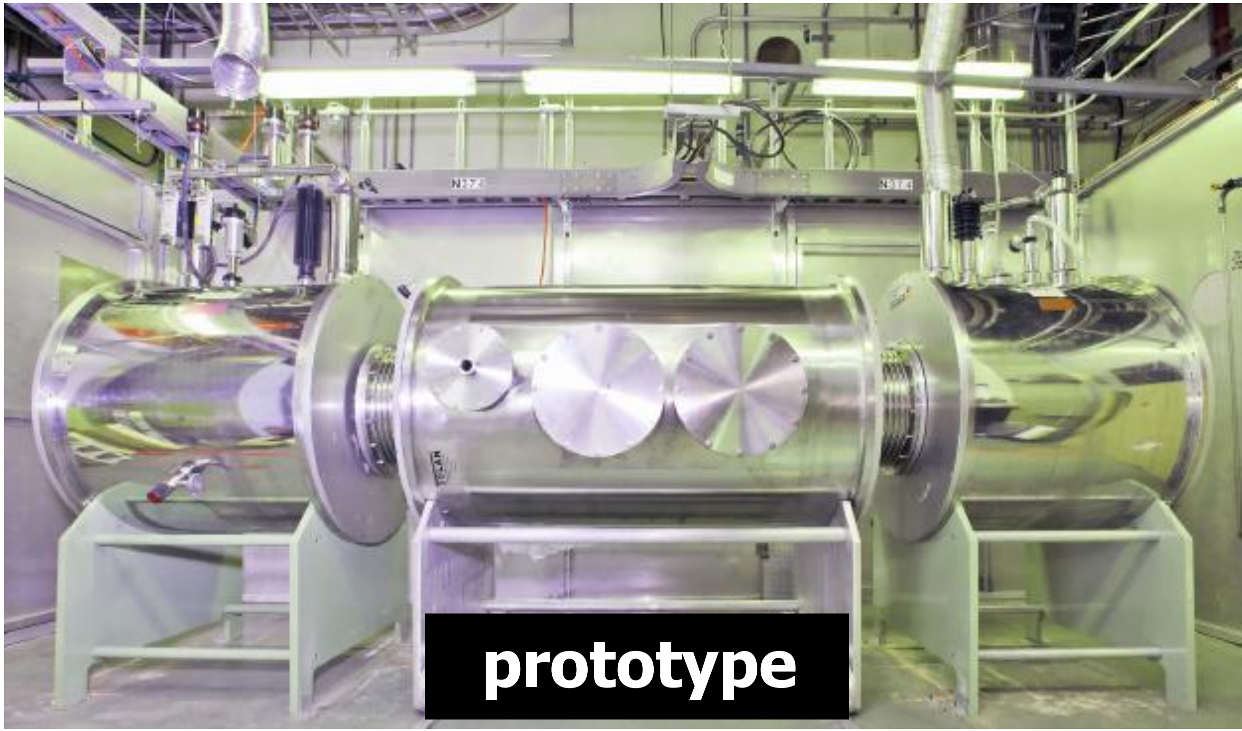
Relic neutrino capture on β-decaying nuclei



no energy threshold on incident ν's
mono-energetic outgoing electrons

Towards a real experiment?





- ★ **first experiment**
- ★ **100 g of tritium**
- ★ **graphene target**
- ★ **planned energy resolution 0.15 eV**

★ **CνB capture rate**

$$\Gamma^{\mathrm{D}}_{\mathrm{C}\nu\mathrm{B}} \sim 4\ \mathrm{yr}^{-1}$$

$$\Gamma^{\mathrm{M}}_{\mathrm{C}\nu\mathrm{B}} \sim 8\ \mathrm{yr}^{-1}$$

D = Dirac

M = Majorana

PTOLEMY
Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield (Betts et al, arXiv:1307.4738)

Signal + Background

Salient feature: **the cross section of a capture reaction scales with** $\frac{c}{v_\nu}$ **so that the number of events converges to a constant for** $v_\nu \to 0$ **:**

$$\sigma(\nu_e N)\cdot \frac{v_\nu}{c}\bigg|_{v_\nu \to 0} = \text{const.} \quad \textbf{e.g.} \quad \sigma(\nu_e{}^3\mathrm{H})\cdot \frac{v_\nu}{c}\bigg|_{v_\nu \to 0} \simeq (7.84 \pm 0.03)\times 10^{-45}\,\mathrm{cm}^2$$

(Cocco et al 07; Lazauskas et al 08; Li , Xing 11; Long, Lunardini, Sabancilar 14).

$$\boxed{\nu_e + {}^3\mathrm{H} \to {}^3\mathrm{He} + e^-}$$

Capture rate: **(1 MCi = 100 g =** $N_\mathrm{T} \approx 2.1\times 10^{25}$ **tritium atoms)**

$$\frac{\mathrm{d}\mathcal{N}_{\mathrm{C}\nu\mathrm{B}}}{\mathrm{d}T_e} \approx 6.5 \sum_i |V_{ei}|^2 \frac{n_{\nu_i}}{\langle n_{\nu_i}\rangle}\cdot \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(T_e - T_e^i)^2}{2\sigma^2}\right]\mathrm{yr}^{-1}\,\mathrm{MCi}^{-1} \qquad \boxed{T_e^i = Q_\beta + E_{\nu_i}}$$

Background: **(tritium β-decay)** $E_e = T_e^\prime + m_e$ $\qquad \langle n_{\nu_i}\rangle \approx \langle n_{\overline{\nu}_i}\rangle \approx 56\ \mathrm{cm}^{-3}$

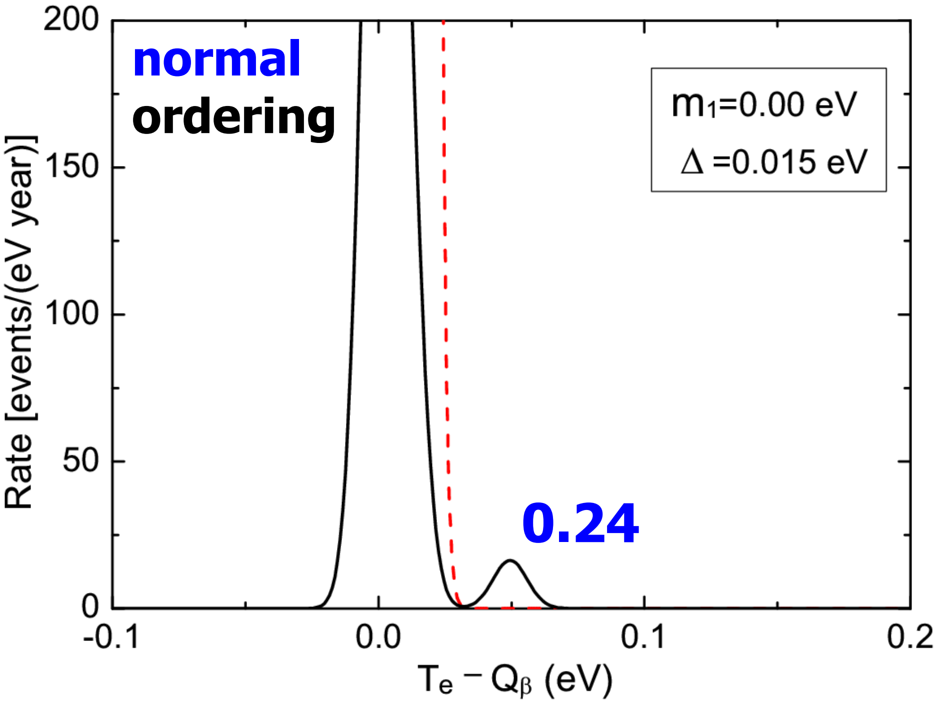
$$\begin{aligned}\frac{\mathrm{d}\mathcal{N}_\beta}{\mathrm{d}T_e} \approx 5.55 \int_0^{Q_\beta - \min(m_i)} \mathrm{d}T_e^\prime \left\{ N_\mathrm{T}\frac{G_\mathrm{F}^2\,\cos^2\theta_\mathrm{C}}{2\pi^3} F(Z,\,E_e)\sqrt{E_e^2 - m_e^2}\,E_e(Q_\beta - T_e^\prime)\right.\\ \left.\times \sum_i \left[|V_{ei}|^2 \sqrt{\left(Q_\beta - T_e^\prime\right)^2 - m_i^2}\;\Theta(Q_\beta - T_e^\prime - m_i)\right]\frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(T_e - T_e^\prime)^2}{2\sigma^2}\right]\right\}\end{aligned}$$

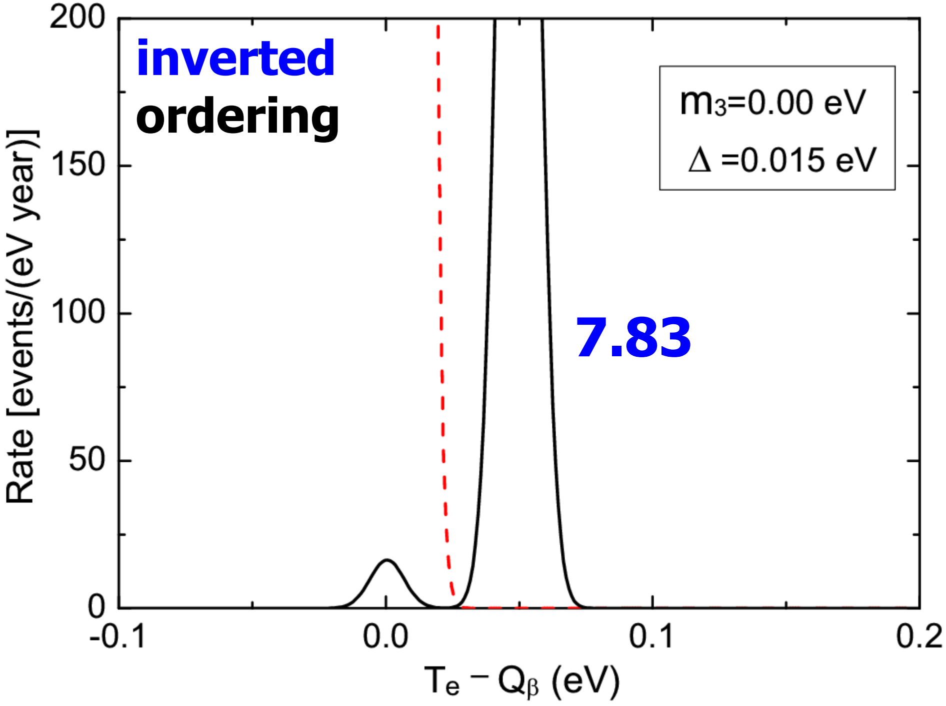
Energy resolution (Gaussian function) : $\Delta = 2\sqrt{2\ln 2}\,\sigma \approx 2.35482\,\sigma$ **.**

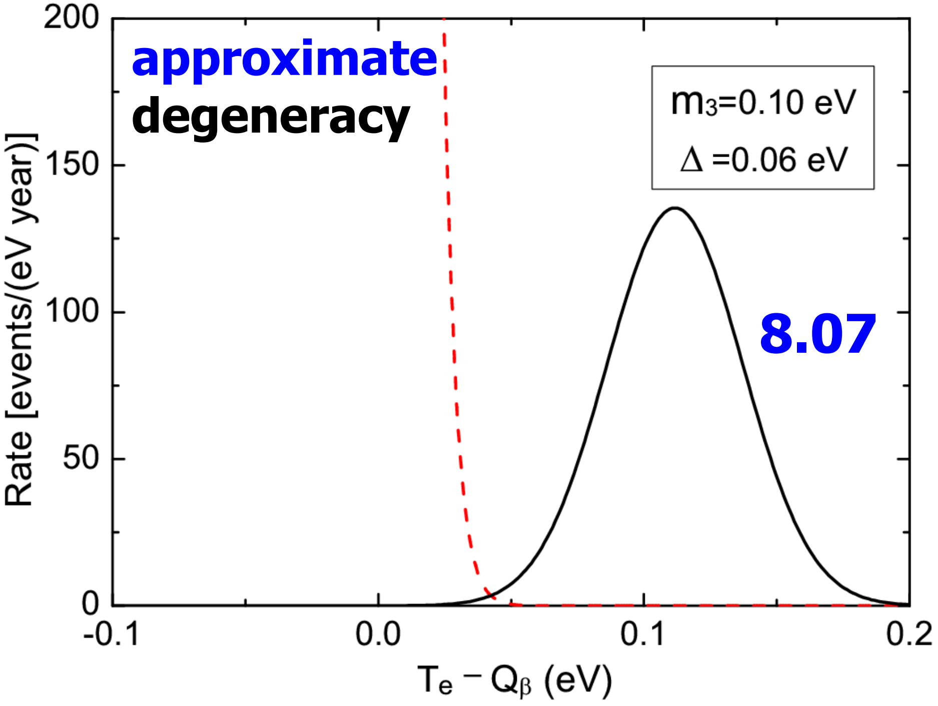
Illustration

Target mass: **100 g** **tritium atoms**
Input θ(13) : **10 degrees**
Number of events per year: **~ 8**
(Li, Xing, 2011).

The gravitational clustering effect may help enhance the signal rates (Ringwald, Wong 2004).







A Naïve (Why Not) Picture

How dark is dark?

$\Omega_{\mathrm{CDM}}/\Omega_{\mathrm{m}} \approx 83\%$

$\Omega_{\mathrm{B}}/\Omega_{\mathrm{m}} \approx 17\%$

Hot dark matter: CνB is guaranteed but not significant.

Cold dark matter: most likely? At present most popular.

Warm dark matter: suppress the small-scale structures.

If you think so,

Do not put all your eggs in one basket

dark

cold

matter

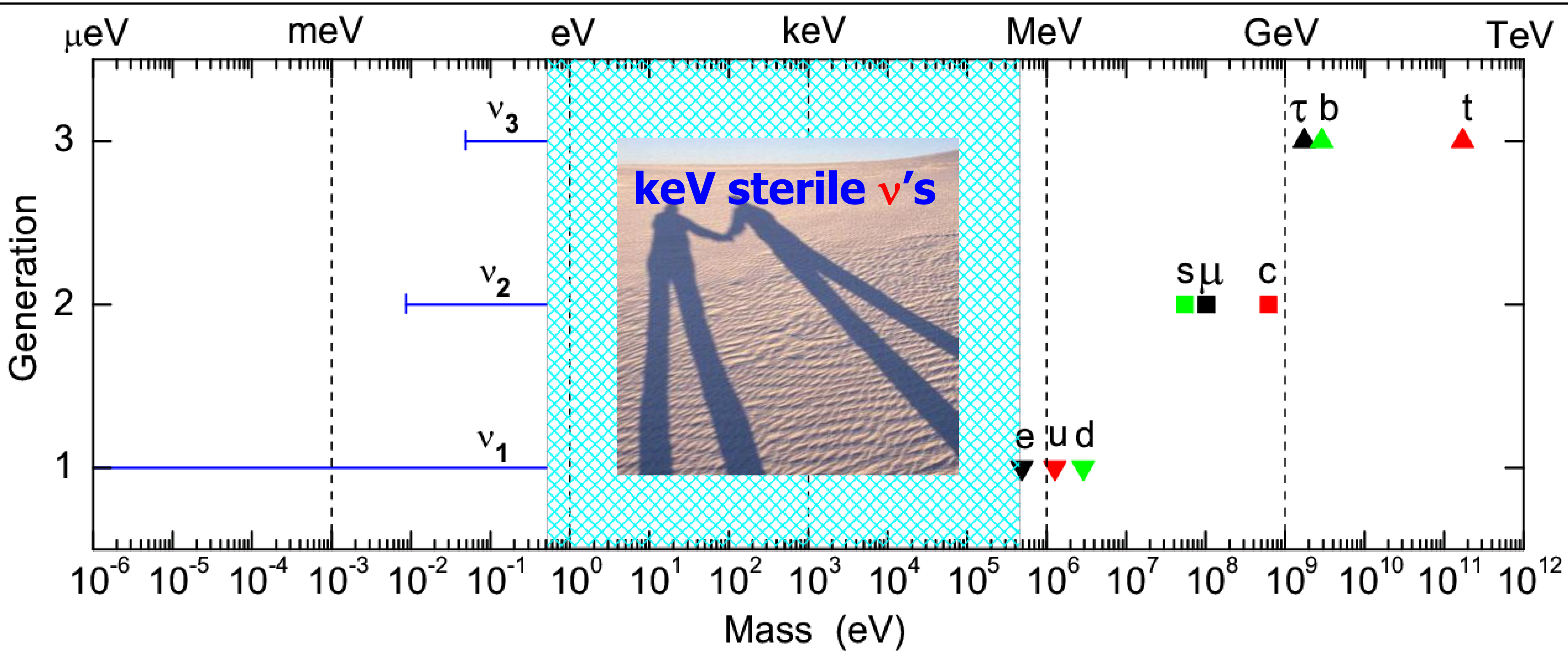
hot
dark
matter

warm
dark
matter

keV sterile neutrinos?

NO strong prior theoretical motivation for the existence of keV sterile ν's. Typical models: Asaka et al, 05; Kusenko et al, 10; Lindner et al, 11….

A purely phenomenological argument to support keV sterile ν's in the FLAVOR DESERT of the standard model (Xing, 09).

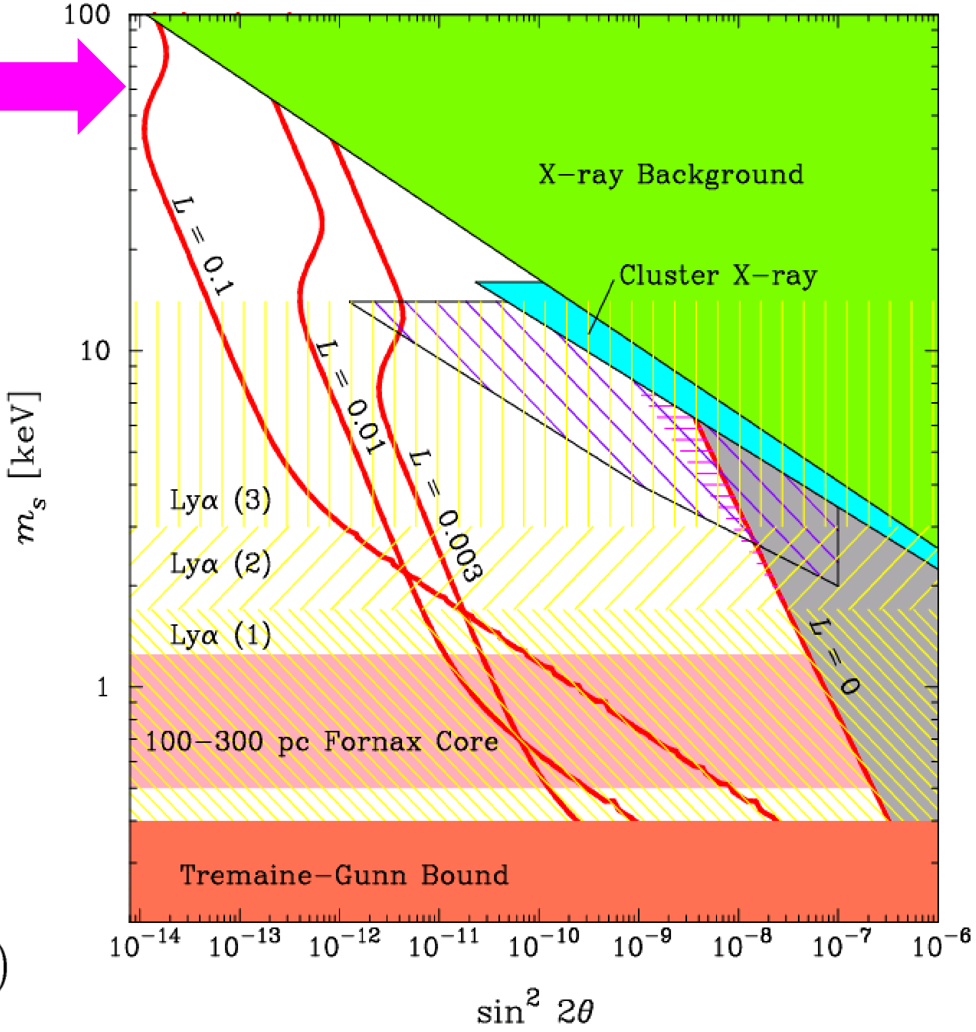


3.5 keV X-ray line? (Bulbul et al, 1402.2301; Boyarsky et al, 1402.4119)

Mixing

Production: via active-sterile ν oscillations in the early Universe, etc; **Salient feature:** warm DM in the form of keV sterile ν's can suppress the formation of dwarf galaxies and other small-scale structures.

Bounds on 2-flavor parameters: (Abazajian, Koushiappas, 2006)



For simplicity, we assume only one type of keV sterile neutrinos:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} & V_{e4} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} & V_{\mu 4} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} & V_{\tau 4} \\ V_{s1} & V_{s2} & V_{s3} & V_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

Standard parameterization of V: 6 mixing angles & 3 (Dirac) or 6 (Majorana) CP-violating phases.

$$V_{s1} \simeq s_{14}\ e^{-i\delta_{14}} \ , \qquad V_{s2} \simeq s_{24}\ e^{-i\delta_{24}}$$

$$V_{s3} \simeq s_{34}\ e^{-i\delta_{34}} \ , \qquad V_{s4} \simeq 1$$

$$V_{e4} \simeq -c_{12}c_{13}s_{14}e^{i\delta_{14}} - s_{12}c_{13}s_{24}e^{i\left(\delta_{24}-\delta_{12}\right)}$$

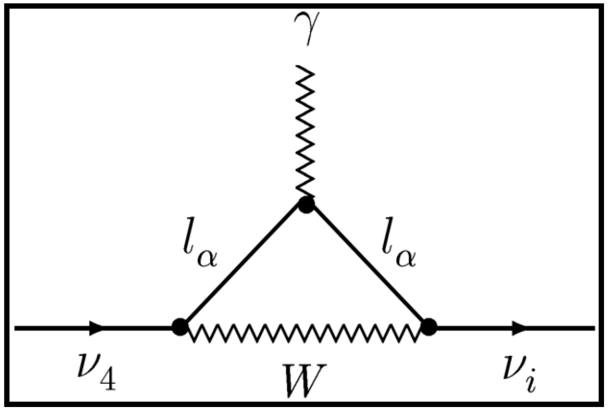
Decay rates

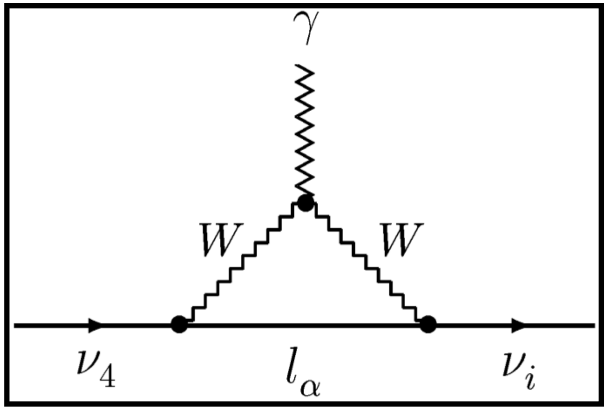
Dominant decay mode **[Cν = 1 (Dirac) or 2 (Majorana)]:**

$$\sum_{\alpha=e}^{\tau}\sum_{\beta=e}^{\tau}\Gamma(\nu_4\to\nu_\alpha+\nu_\beta+\overline{\nu}_\beta) \;=\; \frac{C_\nu G_{\rm F}^2 m_4^5}{192\pi^3}\sum_{\alpha=e}^{\tau}|V_{\alpha4}|^2 \;=\; \frac{C_\nu G_{\rm F}^2 m_4^5}{192\pi^3}\sum_{i=1}^{3}|V_{si}|^2$$

Lifetime **(the Universe's age ~ 10^17 s):**

$$\tau_{\nu_4} \;\simeq\; \frac{2.88\times10^{27}}{C_\nu}\left(\frac{m_4}{1\ {\rm keV}}\right)^{-5}\left(\frac{s_{14}^2+s_{24}^2+s_{34}^2}{10^{-8}}\right)^{-1}\ {\rm s}$$





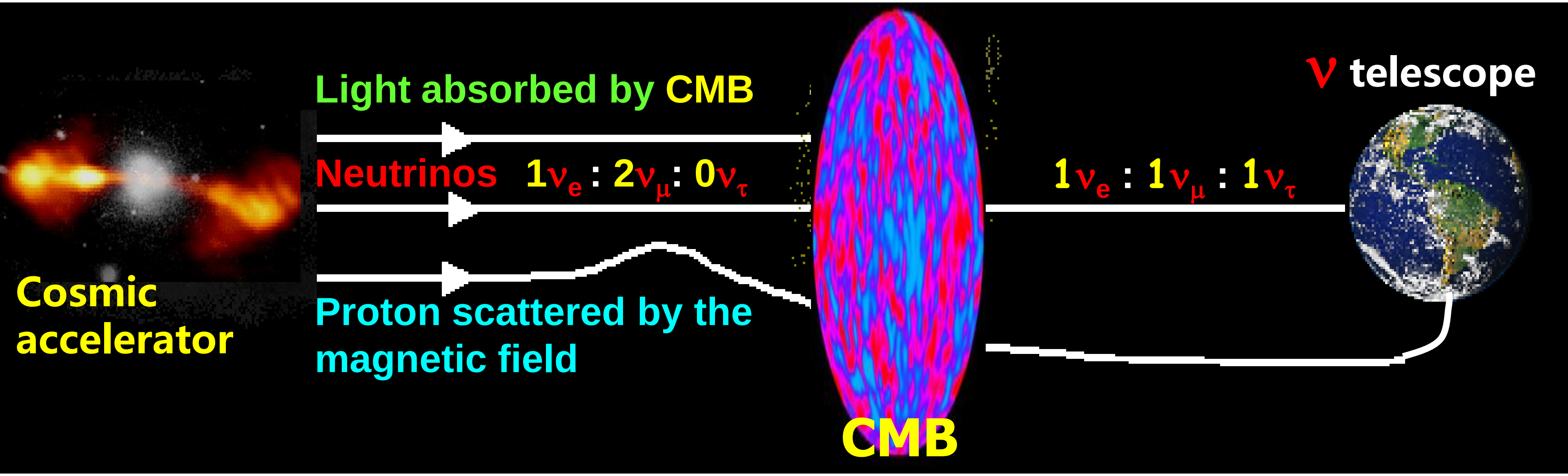
Radiative decay: **X-ray and Lyman-α forest observations.**

$$\sum_{i=1}^{3}\Gamma(\nu_4\to\nu_i+\gamma) \simeq \frac{9\alpha_{\rm em}C_\nu G_{\rm F}^2 m_4^5}{512\pi^4}\sum_{i=1}^{3}\left|\sum_{\alpha=e}^{\tau}V_{\alpha4}V_{\alpha i}^*\right|^2$$

$$= \frac{9\alpha_{\rm em}C_\nu G_{\rm F}^2 m_4^5}{512\pi^4}\sum_{i=1}^{3}|V_{s4}V_{si}^*|^2$$

$$\simeq \frac{9\alpha_{\rm em}C_\nu G_{\rm F}^2 m_4^5}{512\pi^4}\left(s_{14}^2+s_{24}^2+s_{34}^2\right)$$

UHE cosmic messenger



Cosmic accelerator

Light absorbed by CMB

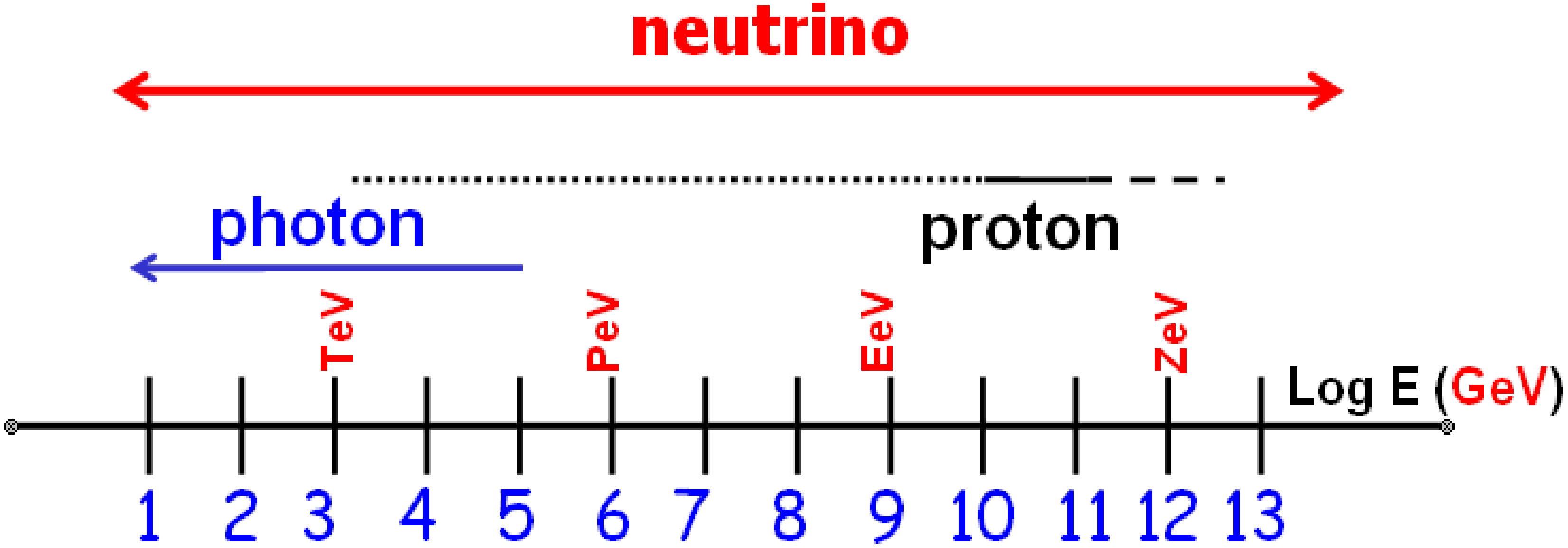
Neutrinos $1\nu_e : 2\nu_\mu : 0\nu_\tau$

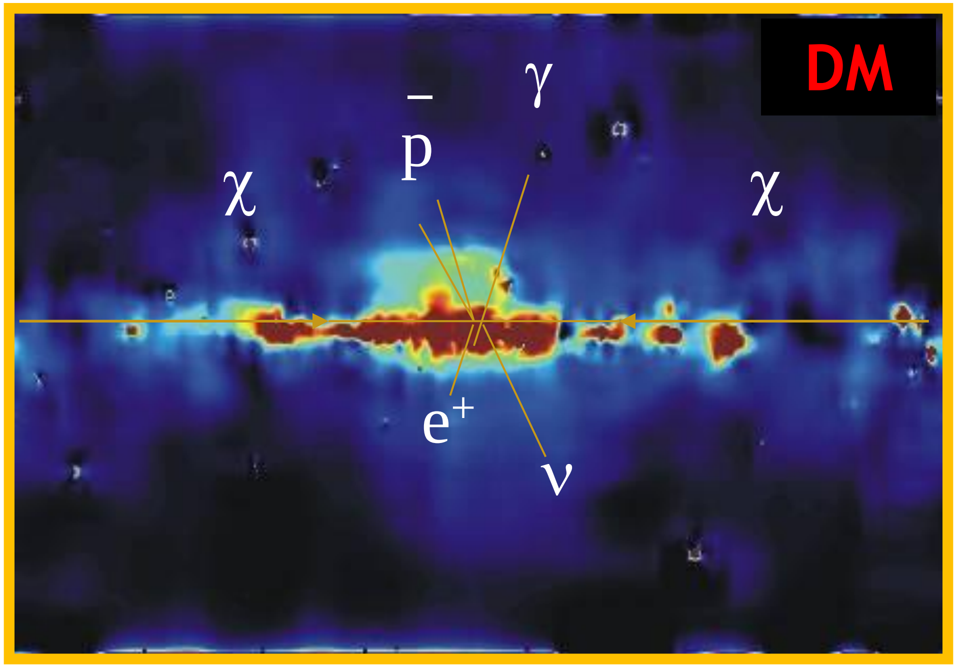
Proton scattered by the magnetic field

CMB

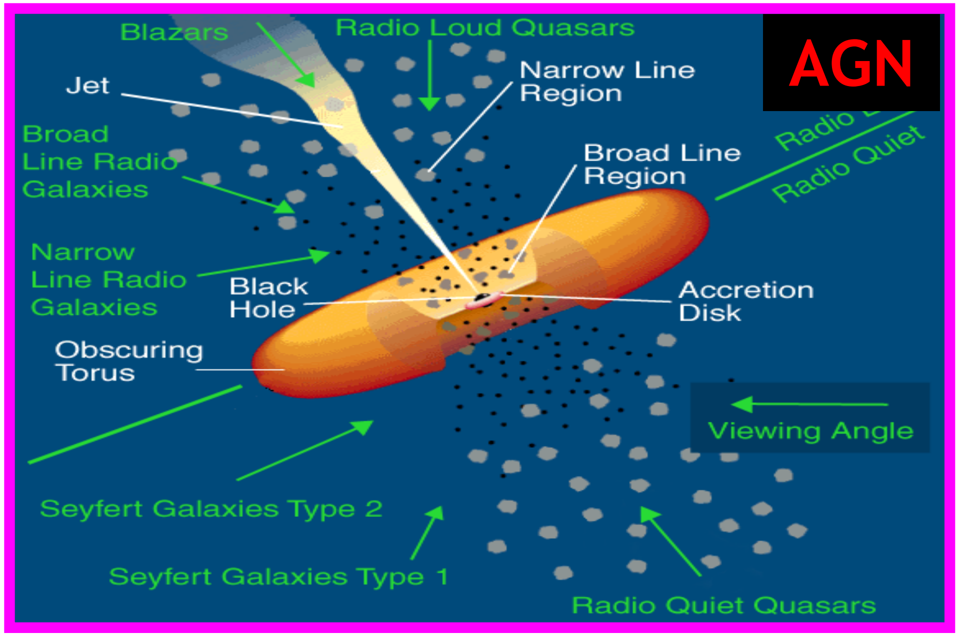
$1\nu_e : 1\nu_\mu : 1\nu_\tau$

ν telescope

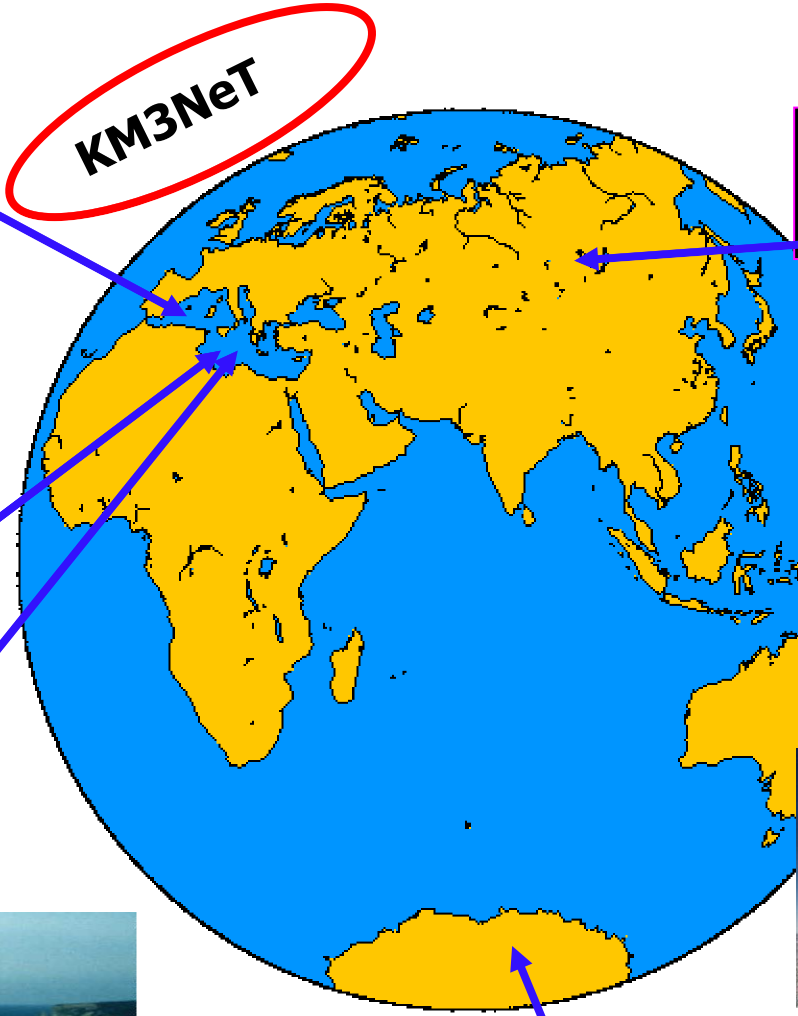




Possible astrophysical sources of UHE cosmic neutrinos ...



Optical Cherencov NTs



ANTARES
La-Seyne-sur-Mer,
France

NEMO
Catania, Italy

NESTOR
Pylos, Greece

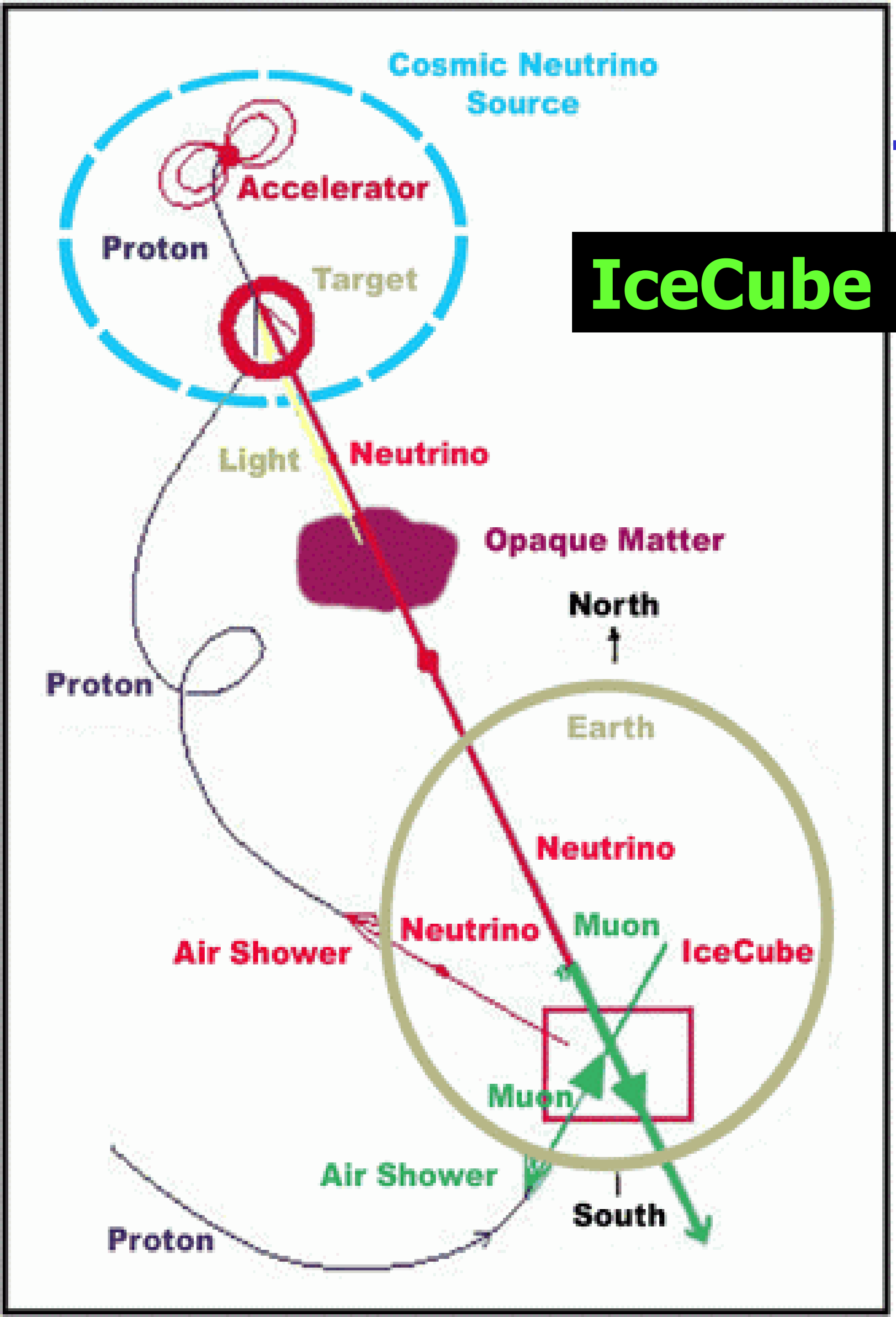
[illegible]

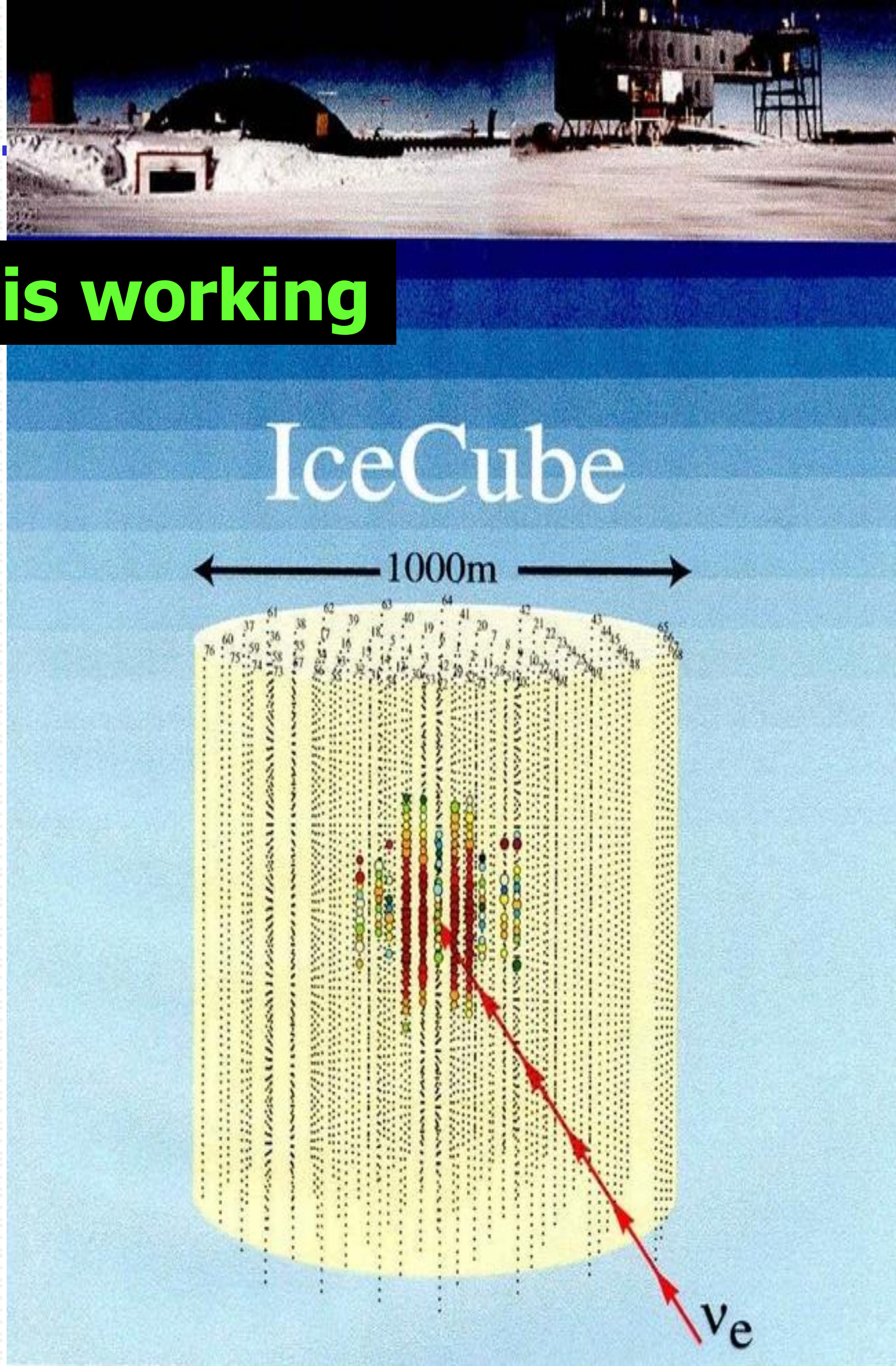
BAIKAL
Russia

AMANDA and **IceCube**

South Pole, Antarctica

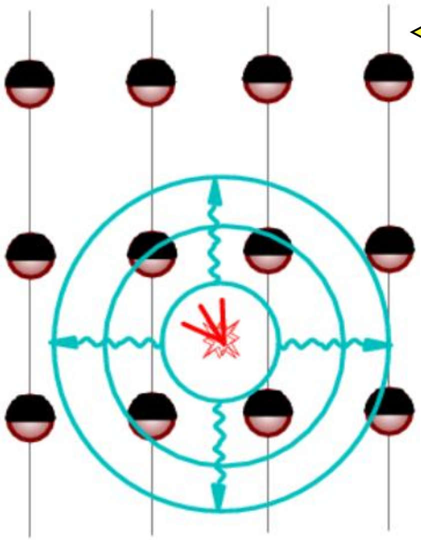
IceCube is working

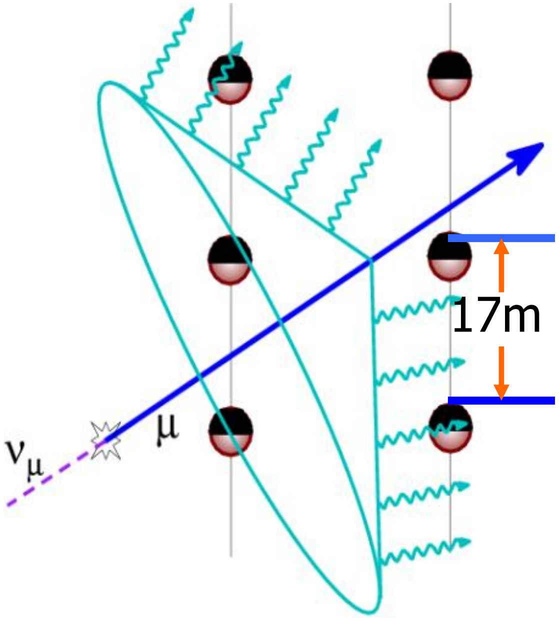




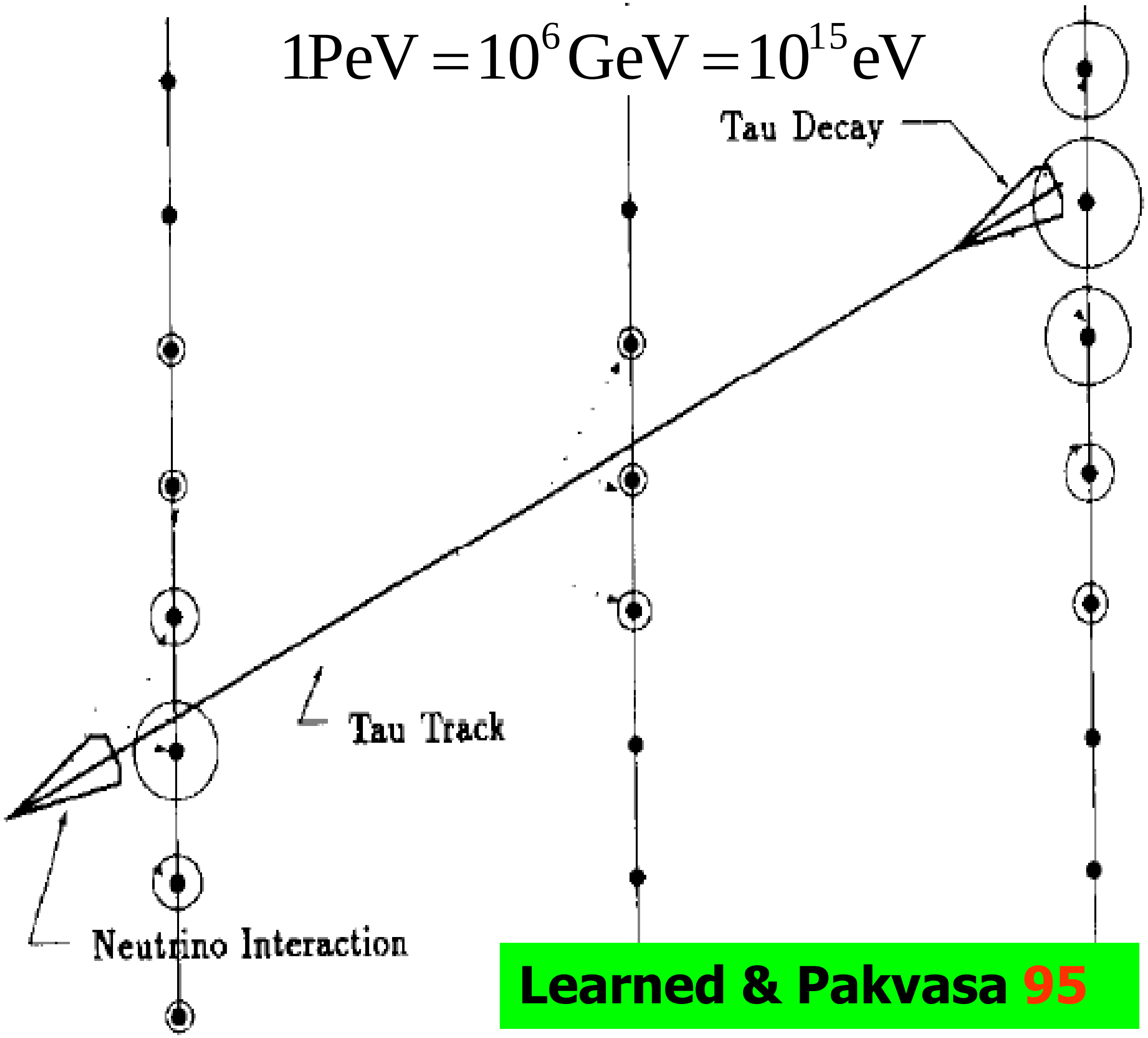
Flavor identification

Halzen, astro-ph/0602132





$$1\mathrm{PeV} = 10^{6}\,\mathrm{GeV} = 10^{15}\,\mathrm{eV}$$



Learned & Pakvasa 95

2 PeV Events

IceCube:

arXiv:1304.5356 (PRL)

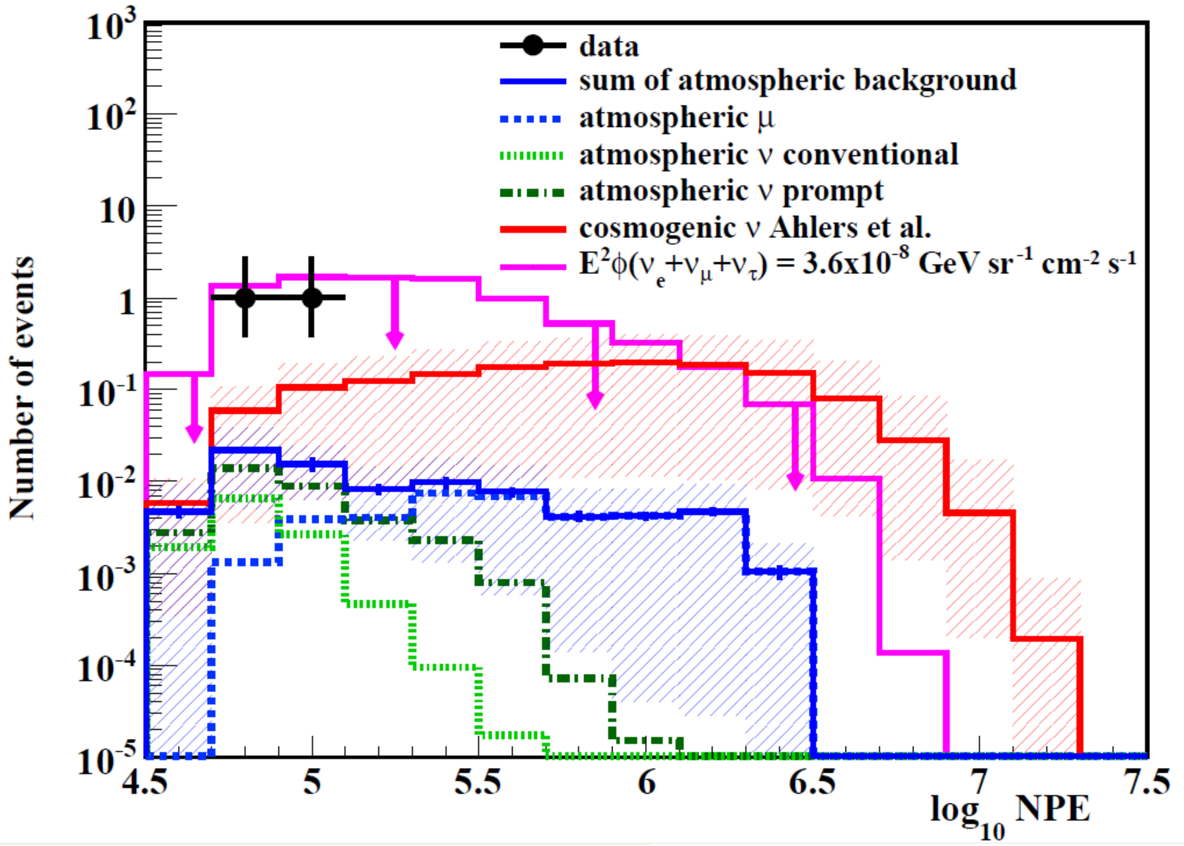
Event 1: 1.04 ± 0.16 PeV

Event 2: 1.14 ± 0.17 PeV

Very unlikely

--- ATM conventional ν's

--- Cosmogenic ν's



neutral-current $\nu_{e,\mu,\tau}$ $(\bar{\nu}_{e,\mu,\tau})$ or charged-current ν_e $(\bar{\nu}_e)$ interactions

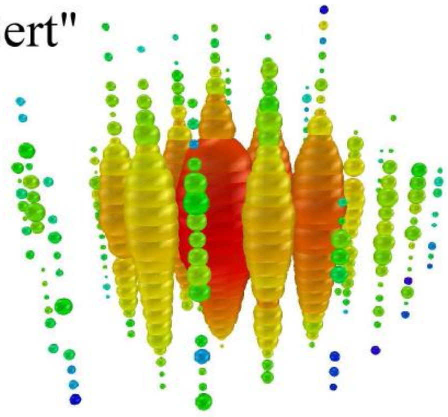
Disfavored

--- ATM prompt ν's

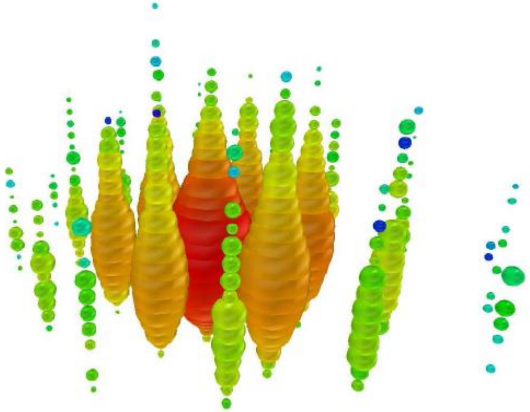
Plausible (2.8σ)

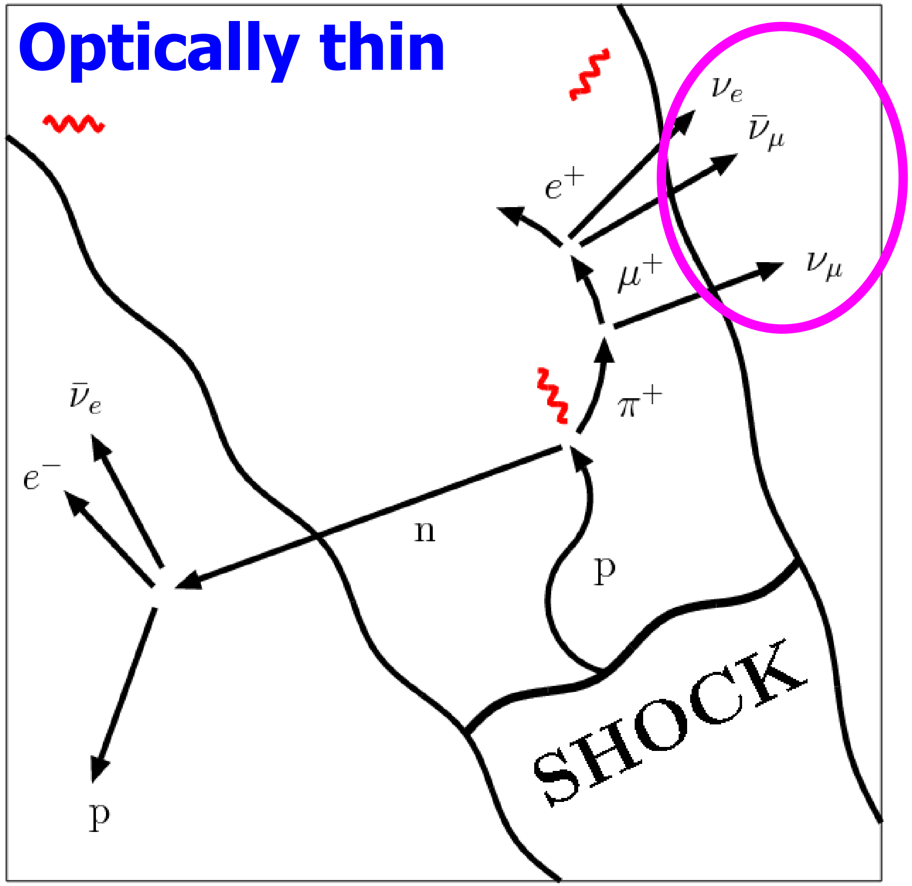
--- Astrophysical ν's

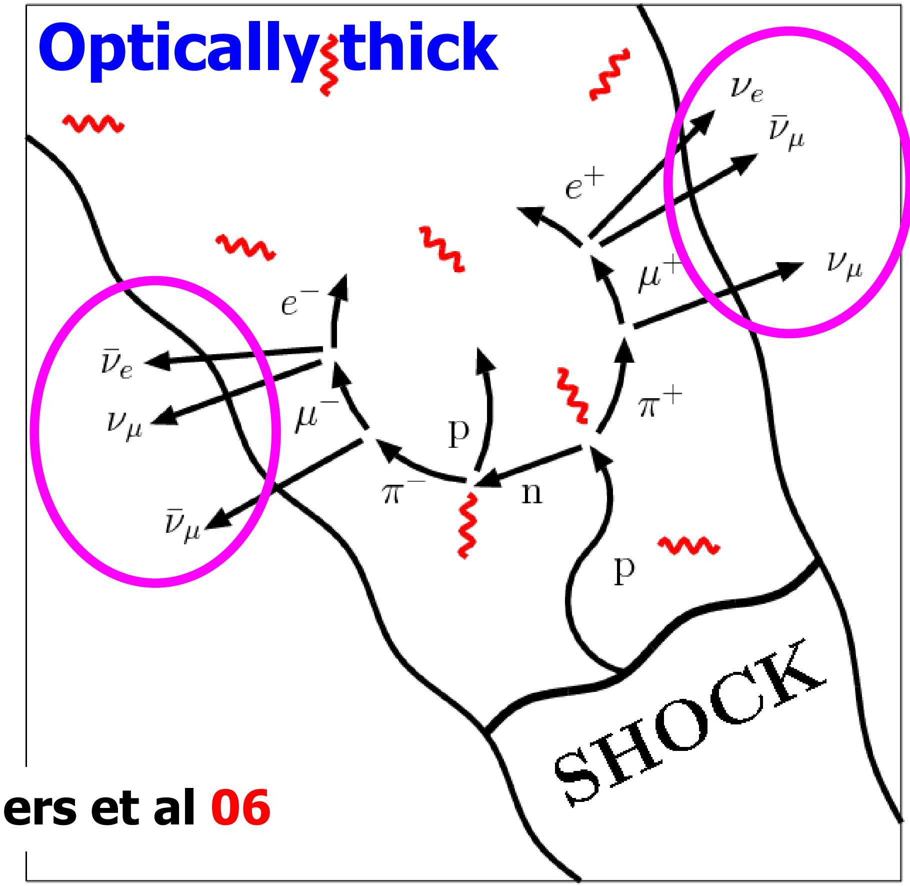
"Bert"



"Ernie"




Optically thin


Optically thick

Ahlers et al 06

Conventional mechanism:

$$p + \gamma \to \Delta^+ \to \pi^+ + n$$

$$p + p \to \pi^\pm + X$$

$$\boxed{\Phi_e^S : \Phi_\mu^S : \Phi_\tau^S = 1 : 2 : 0}$$

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$$

$$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

Oscillations

The transition probability: $\alpha,\beta = e,\mu,\tau \qquad j,k = 1,2,3$

$$P(\nu_\alpha \to \nu_\beta) = \sum_{j=1}^{3} |V_{\alpha j}|^2 |V_{\beta j}|^2 + 2\mathrm{Re}\sum_{j<k} V_{\alpha j} V_{\beta k} V^*_{\alpha k} V^*_{\beta j} \exp\left\{-\mathrm{i}\frac{\Delta m^2_{kj}}{2E} L\right\}$$

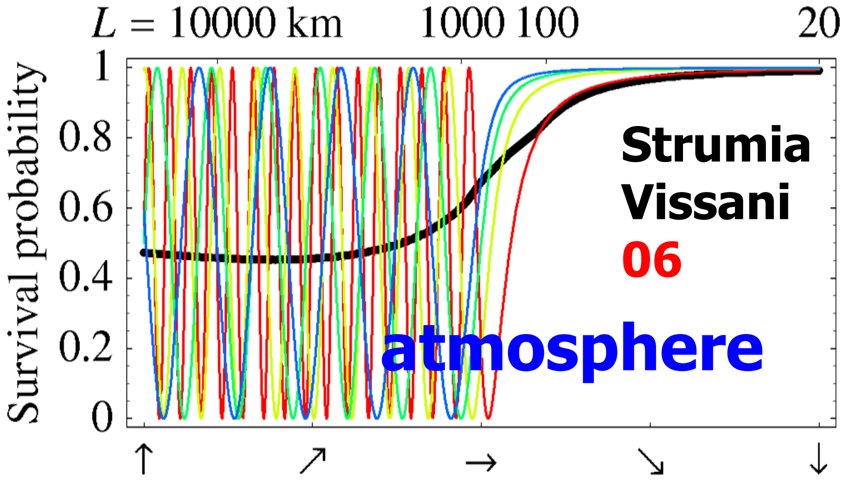
Expected sources (AGN) at a typical distance: ~100 Mpc.

For $|\Delta m^2| \sim 10^{-4}\ \mathrm{eV}^2$ **, the oscillation length in vacuum:**

$$L_{\mathrm{OSC}} \equiv \frac{4\pi E_\nu}{|\Delta m^2|} \sim 8\times10^{-25}\mathrm{Mpc}\left(\frac{E_\nu}{1\ \mathrm{eV}}\right) \qquad 1\ \mathrm{Mpc} \approx 3.1\times10^{22}\ \mathrm{m}.$$

After many oscillations, the averaged probability of UHE cosmic neutrinos is

$$P(\nu_\alpha \to \nu_\beta) = \sum_{j=1}^{3} |V_{\alpha j}|^2 |V_{\beta j}|^2$$



Strumia Vissani 06

atmosphere

Flavor democracy

At an astrophysical source: $\Phi_e^{\rm S} : \Phi_\mu^{\rm S} : \Phi_\tau^{\rm S} = 1 : 2 : 0$

At a ν-telescope:

$$\Phi_\beta^{\rm T} = \sum_\alpha \Phi_\alpha^{\rm S} P(\nu_\alpha \to \nu_\beta) = \sum_\alpha \sum_{i=1}^{3} \Phi_\alpha^{\rm S} |V_{\alpha i}|^2 |V_{\beta i}|^2$$

If there is a μ-τ symmetry for *V*: $\boxed{|V_{\mu i}| = |V_{\tau i}|}$ $\quad (i = 1, 2, 3)$

Then the unitarity of *V* leads to: $\Phi_e^{\rm T} : \Phi_\mu^{\rm T} : \Phi_\tau^{\rm T} = 1 : 1 : 1$

In the PDG parametrization (Xing, Zhou 08):

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & +c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ +s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

→

CPC: $\begin{cases} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{cases}$

or

CPV: $\begin{cases} \delta = \pm\pi/2 \\ \theta_{23} = \pi/4 \end{cases}$

Near flavor democracy (Learned, Pakvasa 95)

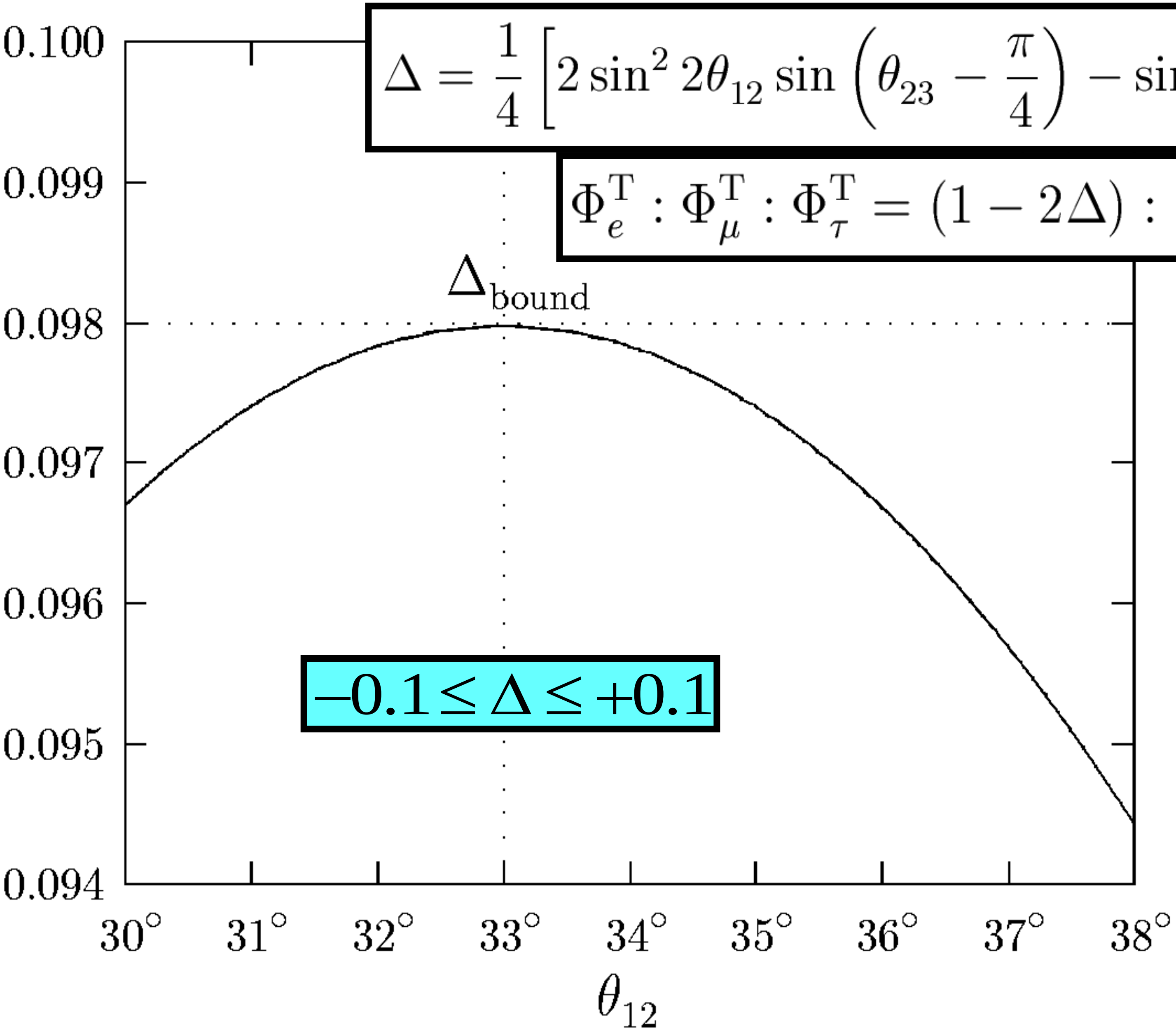
The μ-τ symmetry breaking (Xing 06)

$$\Phi_e^{\rm T} : \Phi_\mu^{\rm T} : \Phi_\tau^{\rm T} = (1 - 2\Delta) : (1 + \Delta) : (1 + \Delta)$$

μ-τ symmetry breaking

$$\Delta = \frac{1}{4}\left[2\sin^2 2\theta_{12}\sin\left(\theta_{23}-\frac{\pi}{4}\right) - \sin 4\theta_{12}\sin\theta_{13}\cos\delta\right]$$

$$\Phi_e^{\rm T} : \Phi_\mu^{\rm T} : \Phi_\tau^{\rm T} = (1-2\Delta) : (1+\Delta) : (1+\Delta)$$



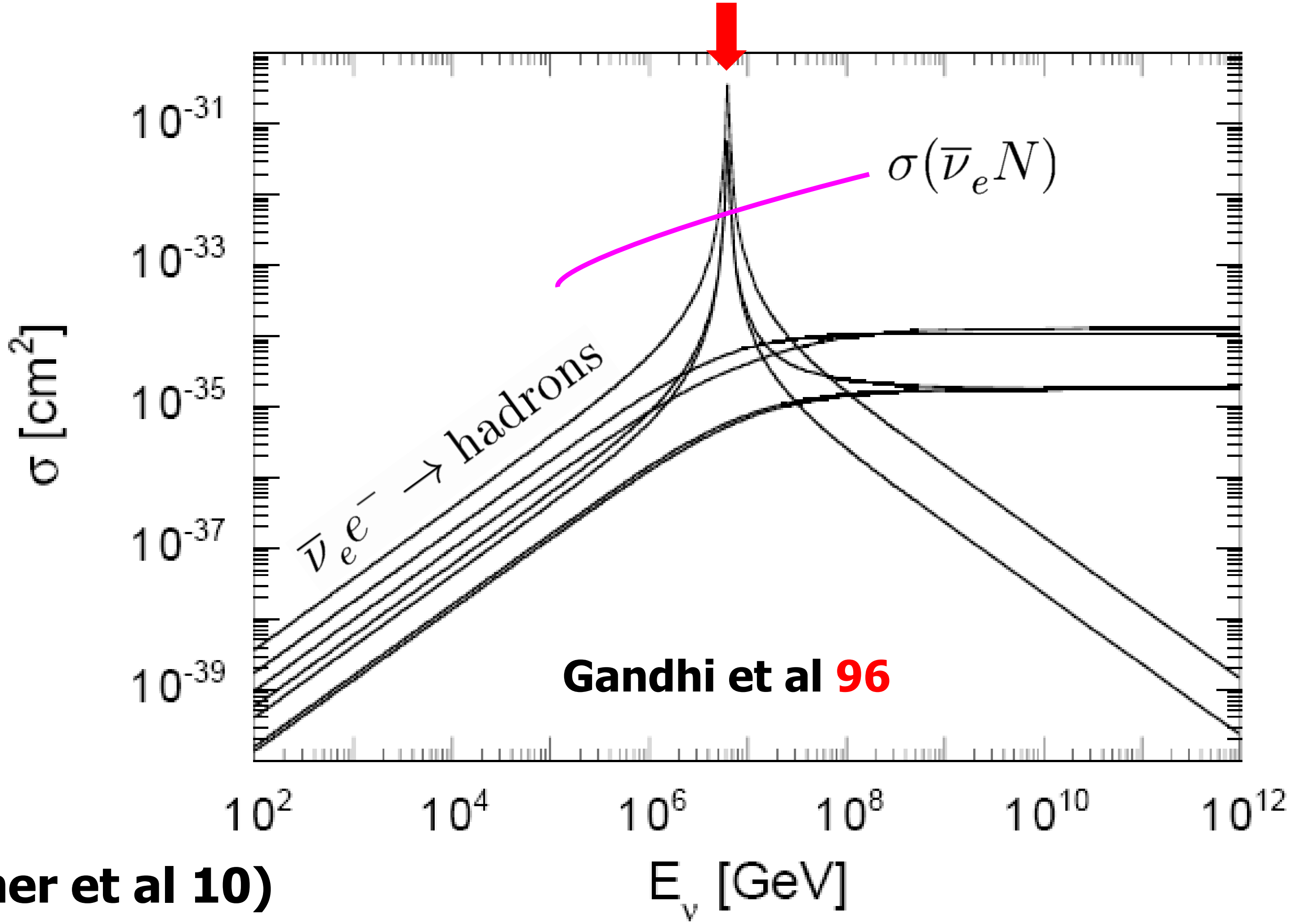
A Question:

Even if such a nice flavor distribution is detected, we still do not know if UHE cosmic ν's originate from pγ or pp collisions

Glashow resonance

$\overline{\nu}_e + e^- \rightarrow W^- \rightarrow \text{anything}$ **Unique for electron anti-ν's!**

(Glashow 60) $E_{\overline{\nu}_e} \simeq M_W^2/(2m_e) \simeq 6.3\ \text{PeV}$



Gandhi et al 96

An interesting discriminator between pγ & pp collisions at an optically thin source of cosmic rays. (Anchordoqui et al 05, Hummer et al 10)

Cosmic Flavor Physics

CνB

Hot DM

keV ν's

Warm DM

Baryogenesis

Leptogenesis

Energetic ν's

from cold DM

UHE

Cosmic ν's

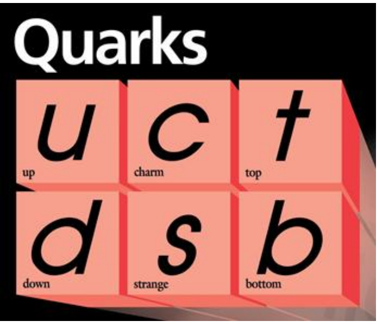
Supernova ν's

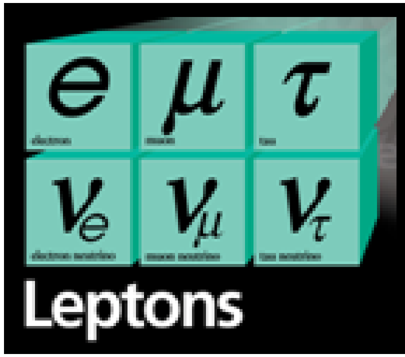
(relic background)

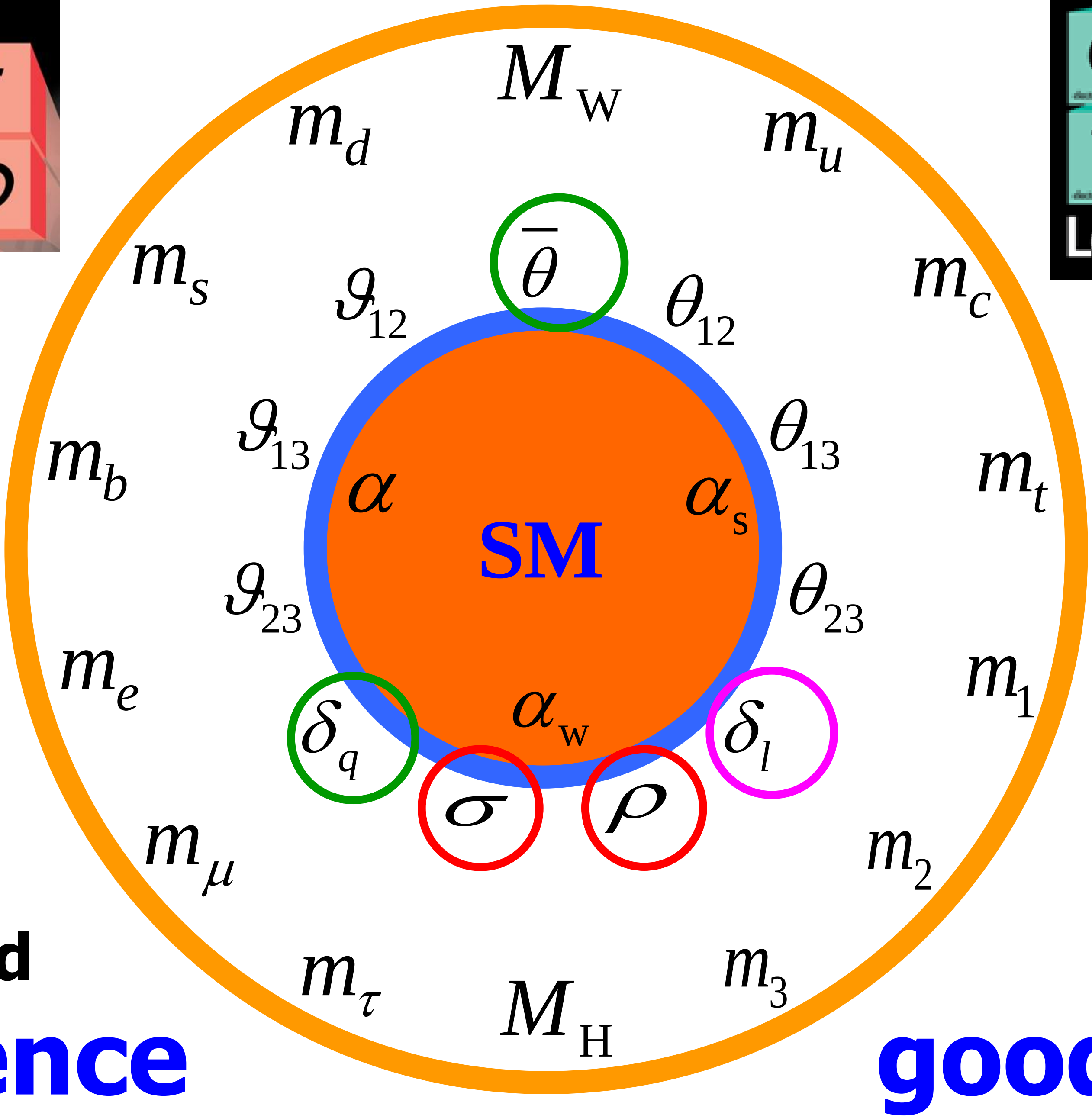
……

A New Road Ahead?

Standard Flavors + Massive Neutrinos in a Pizza







1/5

OK!

4/5

NO!

We need
patience

and
good idea