# Exotics on the Lattice

Ying Chen

Institute of High Energy Physics,
Chinese Academy of Sciences, China
For
CLQCD Collaboration: M. Gong, C. Liu, Z. Liu, Y.-B. Liu,
J.-P Ma, J.-B. Zhang et al

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# Outline

I. Introduction
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III. Glueballs on the lattice
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# I. Introduction

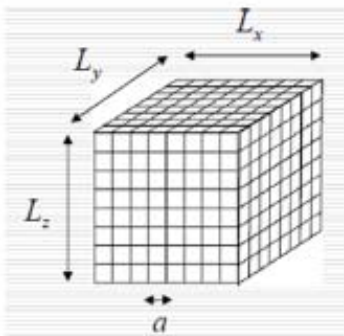
## 1. The lattice formulation of QCD---Lattice QCD

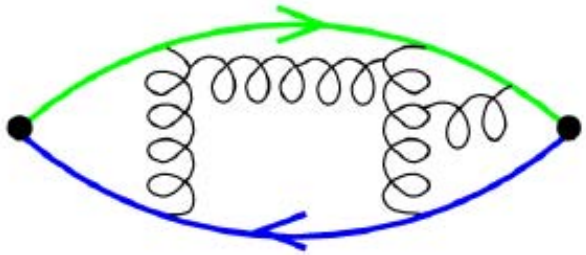
$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, e^{-S}$$

$$S = S_{gauge} + S_{quarks} = \int d^4x \left(\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) - \sum_i \log(\mathrm{Det}M_i)$$

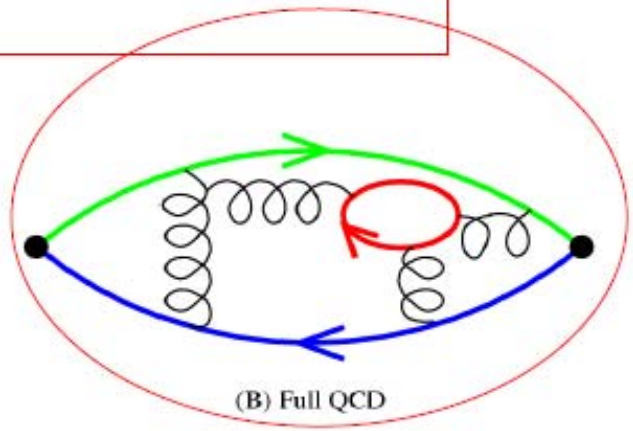
$$Z = \int \mathcal{D}A_\mu \, \det M \; e^{\int d^4x \, (-\frac{1}{4}F_{\mu\nu}F^{\mu\nu})}.$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \, \mathcal{O} \, e^{-S} \, .$$



(A) Quenched QCD: quark loops neglected



(B) Full QCD

Dominated in the present era

## 2. The methods for the hadron spectroscoapy in lattice QCD

- Interpolation field operators --- starting point for a meson (-like) system with given $J^{PC}$ and flavor quantrum numbers:

$$\mathcal{O}_i: \quad \bar{q}_1 \Gamma q_2 \quad [\bar{q}_1 \Gamma_1 q\,][\bar{q} \Gamma_2 q_2\,] \quad [q_1^T \Gamma_1 q][\bar{q} \Gamma_2 \bar{q}_2^T\,], \ldots\ldots$$

- Two-point functions --- Observables

$$\mathcal{C}_{ij}(t) = \left\langle 0 \middle| \mathcal{O}_i(t) \mathcal{O}_j^+(0) \middle| 0 \right\rangle$$
$$= \sum_n \langle 0 | \mathcal{O}_i | n \rangle \left\langle n \middle| \mathcal{O}_j^+ \middle| 0 \right\rangle e^{-E_n t}$$

In principle, all the physical states with the same quantum numbers $|n\rangle$ contribute to the two point functions $\mathcal{C}_{ij}(t)$ as the eigenstates of the QCD Hamiltonian with the energy eigenvalue $E_n$:

- " one-particle state" : $E_n = m_n$
- " two-particle state" : $E_n = \sqrt{m_1^2 + \vec{p}^2} + \sqrt{m_2^2 + \vec{p}^2} + \Delta E, \ \vec{p} = \frac{2\pi}{L} \vec{n}$
- ……

# Comparison of the hadron spectra

| Euclidean spacetime lattice | | Minkowski continuum spacetime |
|---|---|---|
| One particle states | ⟷ | Stable particles |
| Multiple particle states with discrete relative spatial Momentum (scattering States in a finite volume | ⟶ | Bound states of hadrons<br>Resonances |
| | ⟰ | Continuum scattering states |

All the energies are Discretized.

## Luescher's Relation:

$$E_n = (m_1^2 + p^2)^{1/2} + (m_2^2 + p^2)^{1/2}$$

$$\tan\delta(p) = \frac{\sqrt{\pi}\; p\; L}{2\; \mathcal{Z}_{00}\left(1; (\frac{pL}{2\pi})^2\right)}$$

Resonances

$$T(p) = \frac{-\sqrt{s}\,\Gamma(p)}{s - m_R^2 + i\sqrt{s}\Gamma(p)} = \frac{1}{\cot\delta(p) - i}$$

$$\Gamma(p) = g^2 \frac{p^{2l+1}}{s}\,, \quad \frac{p^{2l+1}}{\sqrt{s}} \cot\delta(p) = \frac{1}{g^2}(m_R^2 - s)$$

Bound states

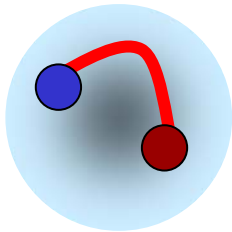
$$p\cot(\delta_0(p)) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2\,, \quad -|p_B| = \frac{1}{a_0} - \frac{1}{2} r_0 |p_B|^2$$

$$T = \frac{}{\cot(\delta_l(p_B)) - i} = \infty$$

$$m_B = E_{H_1}(p_B) + E_{H_2}(p_B)\,, \; p_B = i|p_B|$$

# 3. Exotic hadrons are also hot topics in lattice QCD

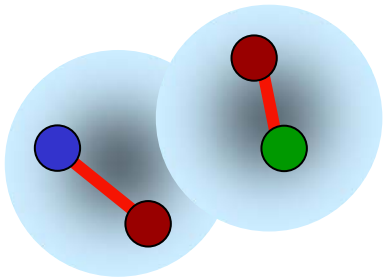
**Hybrid**



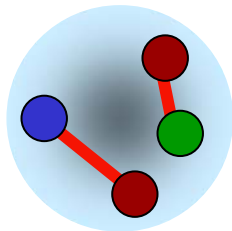
**Glueball**

**Multi-quark states**

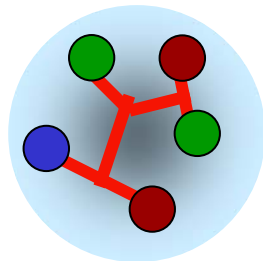
**Hadron molecule**



**tetraquark**



**Pentaquark**



# 3. Especially XYZ particle discovered in recent years

- Experimental hadron spectroscopy --- XYZ particles

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the $C$-parity is given for the neutral members of the corresponding isotriplets.

| State | $M$, MeV | $\Gamma$, MeV | $J^{PC}$ | Process (mode) | Experiment ($\#\sigma$) | Year | Status |
|---|---|---|---|---|---|---|---|
| $X(3872)$ | $3871.68 \pm 0.17$ | $< 1.2$ | $1^{++}$ | $B \to K(\pi^+\pi^- J/\psi)$ | Belle [772, 992] (>10), BaBar [993] (8.6) | 2003 | Ok |
| | | | | $p\bar{p} \to (\pi^+\pi^- J/\psi)\ldots$ | CDF [994, 995] (11.6), D0 [996] (5.2) | 2003 | Ok |
| | | | | $pp \to (\pi^+\pi^- J/\psi)\ldots$ | LHCb [997, 998] (np) | 2012 | Ok |
| | | | | $B \to K(\pi^+\pi^-\pi^0 J/\psi)$ | Belle [999] (4.3), BaBar [1000] (4.0) | 2005 | Ok |
| | | | | $B \to K(\gamma\, J/\psi)$ | Belle [1001] (5.5), BaBar [1002] (3.5) LHCb [1003] (> 10) | 2005 | Ok |
| | | | | $B \to K(\gamma\, \psi(2S))$ | BaBar [1002] (3.6), Belle [1001] (0.2) LHCb [1003] (4.4) | 2008 | NC! |
| | | | | $B \to K(D\bar{D}^*)$ | Belle [1004] (6.4), BaBar [1005] (4.9) | 2006 | Ok |
| $Z_c(3885)^+$ | $3883.9 \pm 4.5$ | $25 \pm 12$ | $1^{+-}$ | $Y(4260) \to \pi^-(D\bar{D}^*)^+$ | BES III [1006] (np) | 2013 | NC! |
| $Z_c(3900)^+$ | $3891.2 \pm 3.3$ | $40 \pm 8$ | $?^{?-}$ | $Y(4260) \to \pi^-(\pi^+ J/\psi)$ | BES III [1007] (8), Belle [1008] (5.2) T. Xiao *et al.* [CLEO data] [1009] (>5) | 2013 | Ok |
| $Z_c(4020)^+$ | $4022.9 \pm 2.8$ | $7.9 \pm 3.7$ | $?^{?-}$ | $Y(4260, 4360) \to \pi^-(\pi^+ h_c)$ | BES III [1010] (8.9) | 2013 | NC! |
| $Z_c(4025)^+$ | $4026.3 \pm 4.5$ | $24.8 \pm 9.5$ | $?^{?-}$ | $Y(4260) \to \pi^-(D^*\bar{D}^*)^+$ | BES III [1011] (10) | 2013 | NC! |
| $Z_b(10610)^+$ | $10607.2 \pm 2.0$ | $18.4 \pm 2.4$ | $1^{+-}$ | $\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$ | Belle [1012–1014] (>10) | 2011 | Ok |
| | | | | $\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$ | Belle [1013] (16) | 2011 | Ok |
| | | | | $\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$ | Belle [1015] (8) | 2012 | NC! |
| $Z_b(10650)^+$ | $10652.2 \pm 1.5$ | $11.5 \pm 2.2$ | $1^{+-}$ | $\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$ | Belle [1012, 1013] (>10) | 2011 | Ok |
| | | | | $\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$ | Belle [1013] (16) | 2011 | Ok |
| | | | | $\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$ | Belle [1015] (6.8) | 2012 | NC! |

Brambilla et al., arXiv:1404.2723

TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the $C$-parity is given for the neutral members of the corresponding isotriplets.

| State | $M$, MeV | $\Gamma$, MeV | $J^{PC}$ | Process (mode) | Experiment (#$\sigma$) | Year | Status |
|---|---|---|---|---|---|---|---|
| $Y(3915)$ | $3918.4 \pm 1.9$ | $20 \pm 5$ | $0/2^{?+}$ | $B \to K(\omega J/\psi)$ | Belle [1050] (8), BaBar [1000, 1051] (19) | 2004 | Ok |
| | | | | $e^+e^- \to e^+e^-(\omega J/\psi)$ | Belle [1052] (7.7), BaBar [1053] (7.6) | 2009 | Ok |
| $\chi_{c2}(2P)$ | $3927.2 \pm 2.6$ | $24 \pm 6$ | $2^{++}$ | $e^+e^- \to e^+e^-(D\bar{D})$ | Belle [1054] (5.3), BaBar [1055] (5.8) | 2005 | Ok |
| $X(3940)$ | $3942^{+9}_{-8}$ | $37^{+27}_{-17}$ | $?^{?+}$ | $e^+e^- \to J/\psi\,(D\bar{D}^*)$ | Belle [1048, 1049] (6) | 2005 | NC! |
| $Y(4008)$ | $3891 \pm 42$ | $255 \pm 42$ | $1^{--}$ | $e^+e^- \to (\pi^+\pi^- J/\psi)$ | Belle [1008, 1056] (7.4) | 2007 | NC! |
| $\psi(4040)$ | $4039 \pm 1$ | $80 \pm 10$ | $1^{--}$ | $e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$ | PDG [1] | 1978 | Ok |
| | | | | $e^+e^- \to (\eta J/\psi)$ | Belle [1057] (6.0) | 2013 | NC! |
| $Z(4050)^+$ | $4051^{+24}_{-43}$ | $82^{+51}_{-55}$ | $?^{?+}$ | $\bar{B}^0 \to K^-(\pi^+\chi_{c1})$ | Belle [1058] (5.0), BaBar [1059] (1.1) | 2008 | NC! |
| $Y(4140)$ | $4145.8 \pm 2.6$ | $18 \pm 8$ | $?^{?+}$ | $B^+ \to K^+(\phi J/\psi)$ | CDF [1060] (5.0), Belle [1061] (1.9), LHCb [1062] (1.4), CMS [1063] (>5) D0 [1064] (3.1) | 2009 | NC! |
| $\psi(4160)$ | $4153 \pm 3$ | $103 \pm 8$ | $1^{--}$ | $e^+e^- \to (D^{(*)}\bar{D}^{(*)})$ | PDG [1] | 1978 | Ok |
| | | | | $e^+e^- \to (\eta J/\psi)$ | Belle [1057] (6.5) | 2013 | NC! |
| $X(4160)$ | $4156^{+29}_{-25}$ | $139^{+113}_{-65}$ | $?^{?+}$ | $e^+e^- \to J/\psi\,(D^*\bar{D}^*)$ | Belle [1049] (5.5) | 2007 | NC! |
| $Z(4200)^+$ | $4196^{+35}_{-30}$ | $370^{+99}_{-110}$ | $1^{+-}$ | $\bar{B}^0 \to K^-(\pi^+ J/\psi)$ | Belle [1065] (7.2) | 2014 | NC! |
| $Z(4250)^+$ | $4248^{+185}_{-45}$ | $177^{+321}_{-72}$ | $?^{?+}$ | $\bar{B}^0 \to K^-(\pi^+\chi_{c1})$ | Belle [1058] (5.0), BaBar [1059] (2.0) | 2008 | NC! |
| $Y(4260)$ | $4250 \pm 9$ | $108 \pm 12$ | $1^{--}$ | $e^+e^- \to (\pi\pi J/\psi)$ | BaBar [1066, 1067] (8), CLEO [1068, 1069] (11) Belle [1008, 1056] (15), BES III [1007] (np) | 2005 | Ok |
| | | | | $e^+e^- \to (f_0(980)J/\psi)$ | BaBar [1067] (np), Belle [1008] (np) | 2012 | Ok |
| | | | | $e^+e^- \to (\pi^- Z_c(3900)^+)$ | BES III [1007] (8), Belle [1008] (5.2) | 2013 | Ok |
| | | | | $e^+e^- \to (\gamma X(3872))$ | BES III [1070] (5.3) | 2013 | NC! |
| $Y(4274)$ | $4293 \pm 20$ | $35 \pm 16$ | $?^{?+}$ | $B^+ \to K^+(\phi J/\psi)$ | CDF [1060] (3.1), LHCb [1062] (1.0), CMS [1063] (>3), D0 [1064] (np) | 2011 | NC! |
| $X(4350)$ | $4350.6^{+4.6}_{-5.1}$ | $13^{+18}_{-10}$ | $0/2^{?+}$ | $e^+e^- \to e^+e^-(\phi J/\psi)$ | Belle [1071] (3.2) | 2009 | NC! |
| $Y(4360)$ | $4354 \pm 11$ | $78 \pm 16$ | $1^{--}$ | $e^+e^- \to (\pi^+\pi^-\psi(2S))$ | Belle [1072] (8), BaBar [1073] (np) | 2007 | Ok |
| $Z(4430)^+$ | $4458 \pm 15$ | $166^{+37}_{-32}$ | $1^{+-}$ | $\bar{B}^0 \to K^-(\pi^+\psi(2S))$ | Belle [1074, 1075] (6.4), BaBar [1076] (2.4) LHCb [1077] (13.9) | 2007 | Ok |
| | | | | $\bar{B}^0 \to K^-(\pi^+ J/\psi)$ | Belle [1065] (4.0) | 2014 | NC! |
| $X(4630)$ | $4634^{+9}_{-11}$ | $92^{+41}_{-32}$ | $1^{--}$ | $e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$ | Belle [1078] (8.2) | 2007 | NC! |
| $Y(4660)$ | $4665 \pm 10$ | $53 \pm 14$ | $1^{--}$ | $e^+e^- \to (\pi^+\pi^-\psi(2S))$ | Belle [1072] (5.8), BaBar [1073] (5) | 2007 | Ok |
| $\Upsilon(10860)$ | $10876 \pm 11$ | $55 \pm 28$ | $1^{--}$ | $e^+e^- \to (B^{(*)}_{(s)}B^{(*)}_{(s)}(\pi))$ | PDG [1] | 1985 | Ok |
| | | | | $e^+e^- \to (\pi\pi\Upsilon(1S, 2S, 3S))$ | Belle [1013, 1014, 1079] (>10) | 2007 | Ok |
| | | | | $e^+e^- \to (f_0(980)\Upsilon(1S))$ | Belle [1013, 1014] (>5) | 2011 | Ok |
| | | | | $e^+e^- \to (\pi Z_b(10610, 10650))$ | Belle [1013, 1014] (>10) | 2011 | Ok |
| | | | | $e^+e^- \to (\eta\Upsilon(1S, 2S))$ | Belle [948] (10) | 2012 | Ok |
| | | | | $e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$ | Belle [948] (9) | 2012 | Ok |
| $Y_b(10888)$ | $10888.4 \pm 3.0$ | $30.7^{+8.9}_{-7.7}$ | $1^{--}$ | $e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$ | Belle [1080] (2.3) | 2008 | NC! |

# II. XYZ particles on the lattice

## 1. Zc(3900)

Since Zc(3900) has definitely a ccbar component and is close to DD*bar threshold, it is sometimes assigned to be a DD*bar molecule. So we carry out a lattice study on the D-D*bar scattering, from which we hope to infer some information on the interaction between D and D*bar.

Y. Chen et al., (CLQCD Collaboration), Phys. Rev. D 89, 094506(2014)

Lattice parameters:

- N_f=2 gauge configurations with the twisted mass fermions generated by ETMC.
- $L^3 \times T = 32^3 \times 64, \quad a = 0.067\, fm$
- three pion masses 485, 420, 300 MeV
- single-particle interpolation field operators for D mesons

$$[D^+]: \ \mathcal{P}^{(d)}(\mathbf{x},t) = [\bar{d}\gamma_5 c](\mathbf{x},t)$$

$$\mathcal{P}^{(u/d)}(\mathbf{k},t) = \sum_{\mathbf{x}} \mathcal{P}^{(u/d)}(\mathbf{x},t) e^{-i\mathbf{k}\cdot\mathbf{x}}.$$

Two-particle operators:

$$I^G(J^P) = 1^c(1^+) \qquad \begin{cases} D^{*+}D^0 + cD^{*0}D^+ \\ D^{*-}\bar{D}^0 + c\bar{D}^{*0}D^- \\ [D^{*0}D^0 - D^{*+}D^-] + c[D^{*0}D^0 - D^{*-}D^+] \end{cases}$$

The G-parity of Zc(3900) is positive, so we choose c=1

## The calculation procedure:

a) The masses of D mesons

b) The energies of DD*bar system

c) Define the scattering momenta of D mesons in the DD*bar system

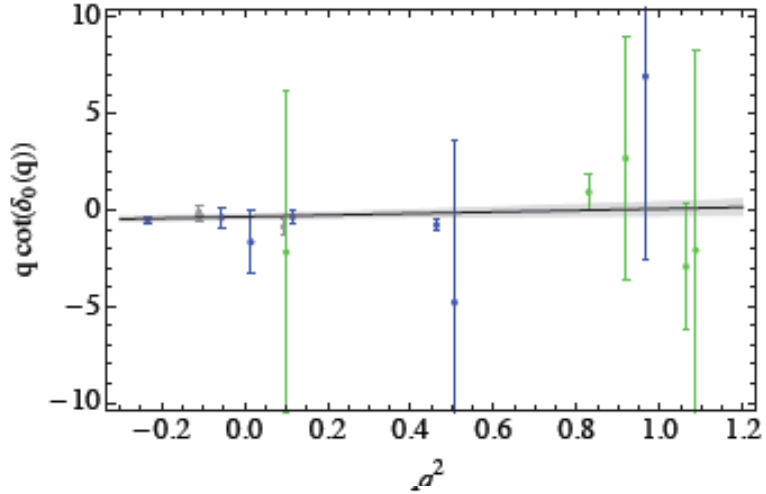
$$E_{1\cdot 2}(\mathbf{k}) = \sqrt{m_1^2 + \bar{\mathbf{k}}^2} + \sqrt{m_2^2 + \bar{\mathbf{k}}^2} \qquad q^2 = \bar{\mathbf{k}}^2 L^2/(2\pi)^2$$

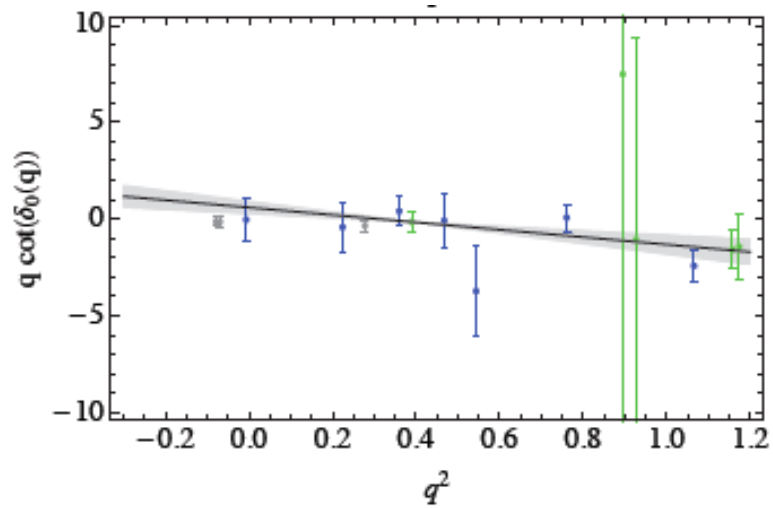
d) Use the Leuscher formular to get the scattering phase shift

$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} \mathcal{Z}_{00}(1; q^2)$$

e) For near threshold scattering, one can use the effective range expansion to parameterize the phase shift versus q.

$$k^{2l+1} \cot \delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \cdots$$



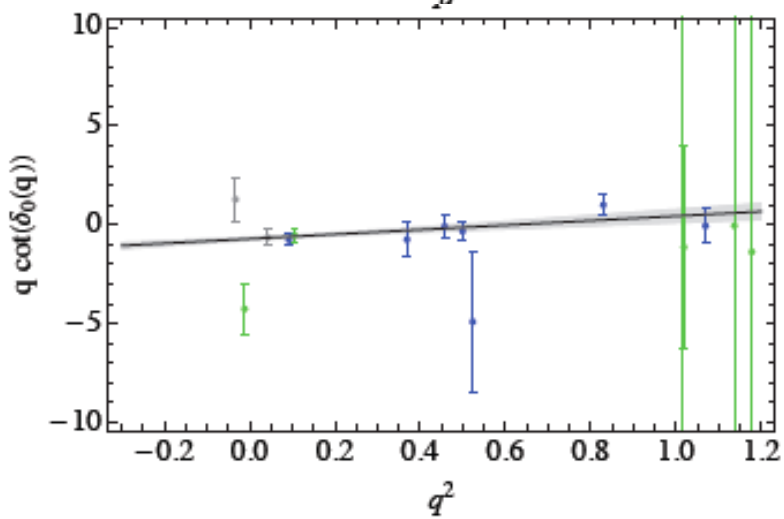




TABLE VI. The values for $a_0$ and $r_0$ in physical units obtained from the numbers for the correlated fit in Table IV.

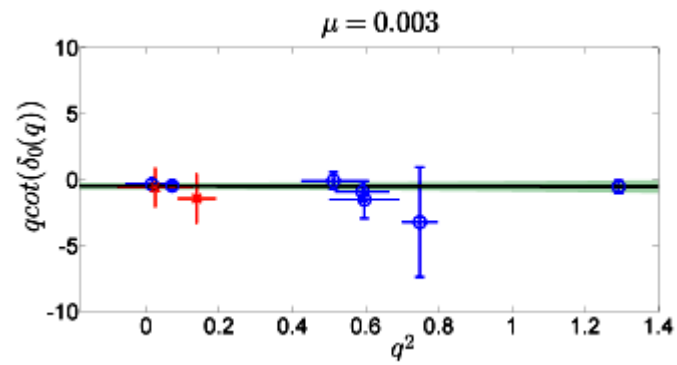
| | $\mu = 0.003$ | $\mu = 0.006$ | $\mu = 0.008$ |
|---|---|---|---|
| $a_0$[fm] | −0.67(1) | −2.1(1) | −0.51(7) |
| $r_0$[fm] | −0.78(3) | −0.27(7) | 0.82(27) |

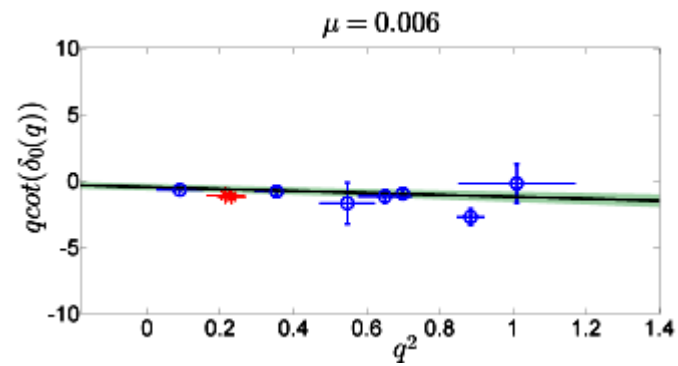
In the J^P=1^+ channel, the scattering lengths are negative, indicating a weak repulsive interaction between D and D*bar. Our result does not support a bound state in this channel. However, since the pion mass is still much higher than the physical pion mass, we cannot rule out the possible appearance of a bound state. A more systematic lattice study is demanding.

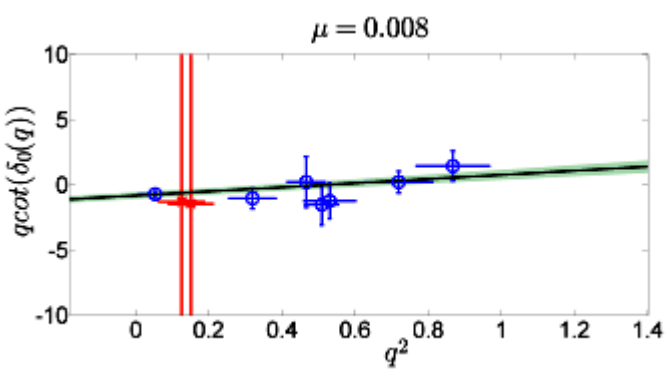
# 2. Zc(4025)

Zc(4025) is observed in D*D*bar and h_c pi system. It has definitely a ccbar component and is close to D*D*bar threshold, it can be a DD*bar molecule. We carry out a similar study on the D*-D*bar scattering in order to learn about the propery of the interaction between D* and D*bar.

Y. Chen et al., (CLQCD Collaboration), arXiv:1503.02371(hep-lat)




| | $\mu = 0.003$ | $\mu = 0.006$ | $\mu = 0.008$ |
|---|---|---|---|
| $a_0$[fm] | $-0.76^{+0.14}_{-0.21}$ | $-0.86^{+0.22}_{-0.22}$ | $-0.59^{+0.19}_{-0.25}$ |
| $r_0$[fm] | $-0.0022^{+0.18}_{-0.19}$ | $-0.14^{+0.15}_{-0.18}$ | $0.64^{+0.50}_{-0.51}$ |

Weak repulsive interaction again.

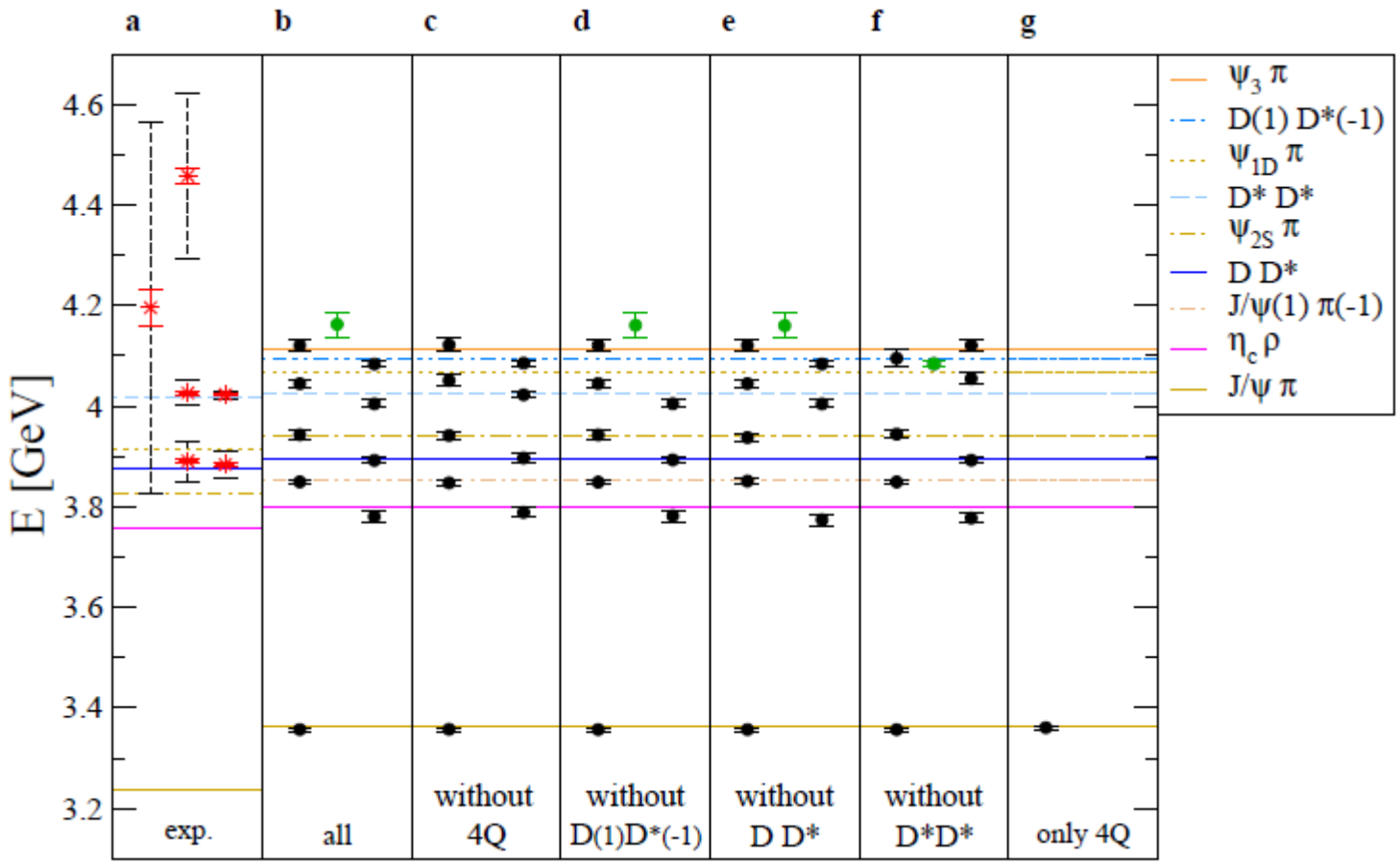
# 3. Lattice QCD studies on Zc particles from other groups

## 1) Study of the Zc+ channel using lattice QCD

S. Prelovsek et al., Phys. Rev. D 91, 014504 (2015) arXiv:1405.7623(hep-lat)

- Spectroscopic study
- Quite a lot of two-particle operators and tetraquark operators are involved

$$
\begin{aligned}
\mathcal{O}_1^{\psi(0)\pi(0)} &= \bar{c}\gamma_i c(0)\ \bar{d}\gamma_5 u(0)\,, \qquad (4)\\
\mathcal{O}^{\psi(1)\pi(-1)} &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k)\ \bar{d}\gamma_5 u(-e_k)\,,\\
\mathcal{O}^{\psi(2)\pi(-2)} &= \sum_{|u_k|^2=2} \bar{c}\gamma_i c(u_k)\ \bar{d}\gamma_5 u(-u_k)\,,\\
\mathcal{O}^{\eta_c(0)\rho(0)} &= \bar{c}\gamma_5 c(0)\ \bar{d}\gamma_i u(0)\,,\\
\mathcal{O}_1^{D(0)D^*(0)} &= \bar{c}\gamma_5 u(0)\ \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,\\
\mathcal{O}^{D^*(0)D^*(0)} &= \epsilon_{ijk}\ \bar{c}\gamma_j u(0)\ \bar{d}\gamma_k c(0)\,,\\
\mathcal{O}_1^{4q} &\propto \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\gamma_5 \bar{d}_c\ c_{b'}\gamma_i C u_{c'} - \bar{c}_b C\gamma_i \bar{d}_c\ c_{b'}\gamma_5 C u_{c'})\,,\\
\mathcal{O}_2^{4q} &\propto \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\bar{d}_c\ c_{b'}\gamma_i\gamma_5 C u_{c'} - \bar{c}_b C\gamma_i\gamma_5 \bar{d}_c\ c_{b'} C u_{c'})\ ,
\end{aligned}
$$




All the scatering states below 4.2 GeV are obtained.

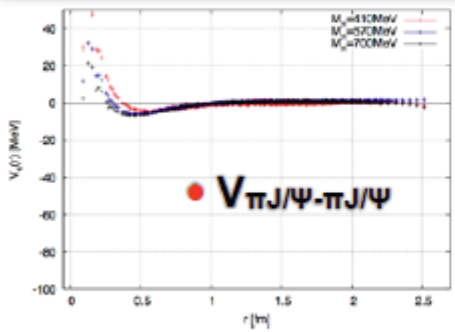
They concluded that No convincing exotic Zc state is observed.

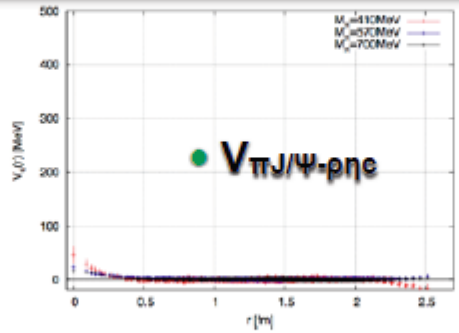
## 2) On the structure of Zc(3900) from lattice QCD

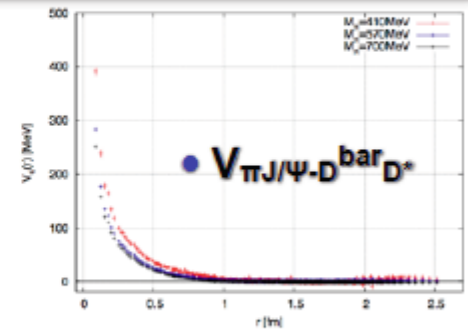
talk given by Yiochi Ikeda (HAL Collab.) on NSTAR2015(Osaka)

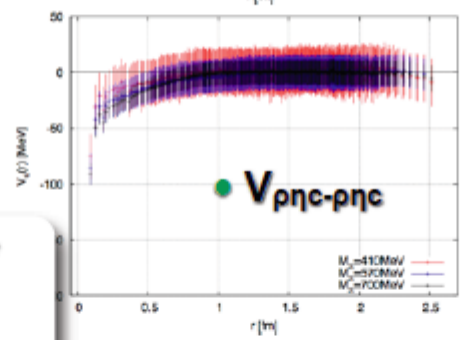
a) BS wave functions of two-meson systems are extracted from lattice QCD.
b) Subsequently, the interaction potentials are obtained.
c) Couple channel effects of J/psi pi and DD*bar is considered
d) They conclude the near DD*bar threshold enhancement is due to the J/psi pi and DD*bar coupling.

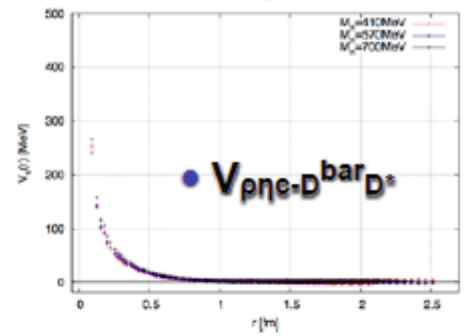
# Potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$)

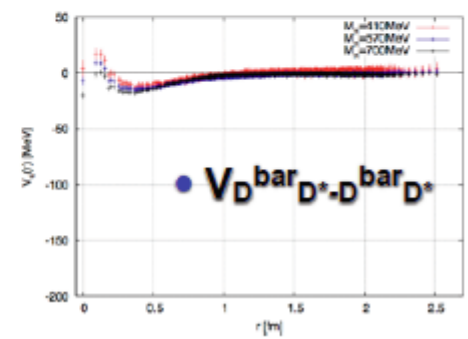


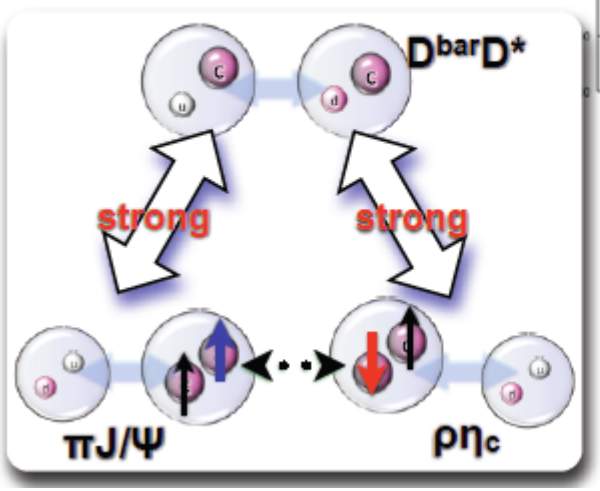



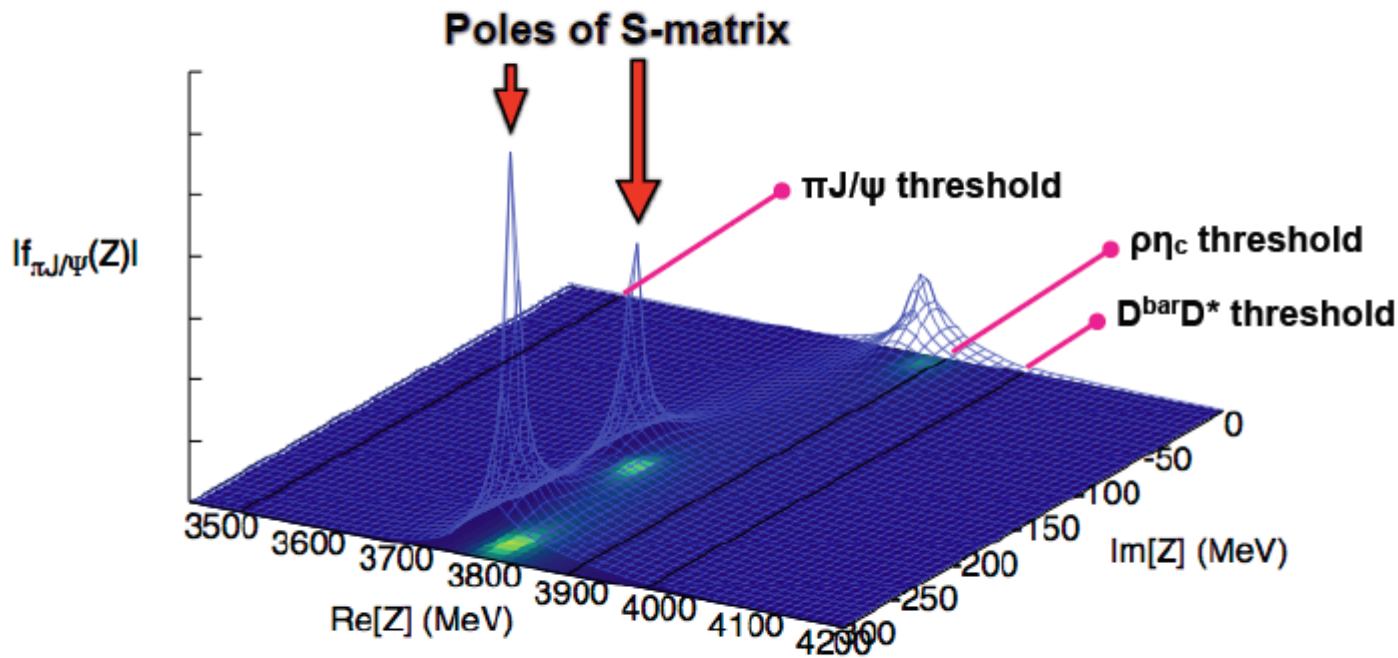



# Pole search ($\pi J/\psi$ :2nd, $\rho\eta_c$ :2nd, $D^{bar}D^*$:2nd)

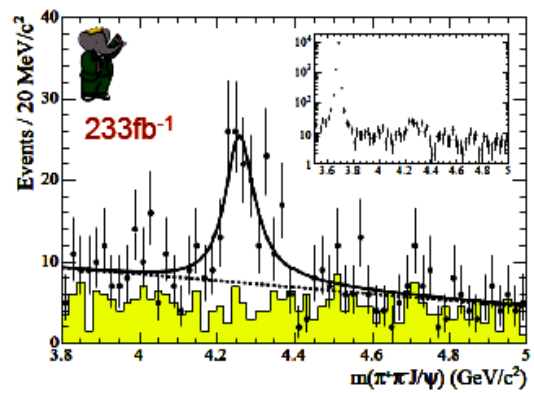
✤ input : LQCD potential matrix @ $m_\pi$=410MeV



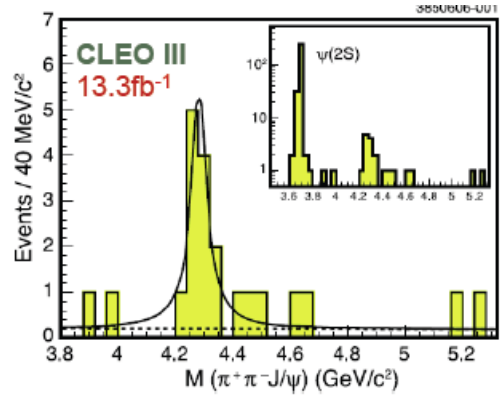

✓ "Virtual (shadow)" poles on the most adjacent complex energy plane for $Z_c(3900)$ energy region are found

✓ These poles contribute to threshold enhancement of amplitude

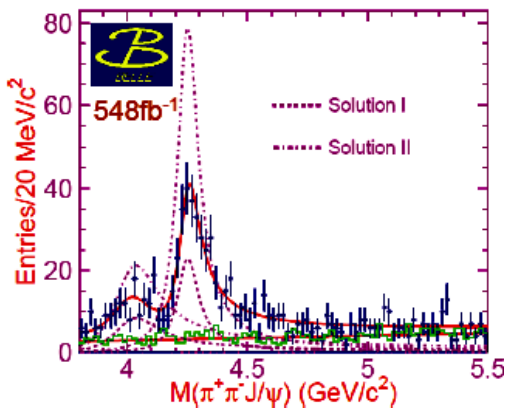
# 4. Y(4260) relevant study from quenched lattice QCD




BaBar,PRL95,142001(05)




CLEO-III,PRD74, 091104(R)(2006)




Belle,PRL99,182004(07)

$$e^+e^- \to \gamma_{ISR} X$$
$$X \to J/\psi\pi^+\pi^-$$
$$J/\psi \to l^+l^-$$

1. Observed in the initial state radiation process

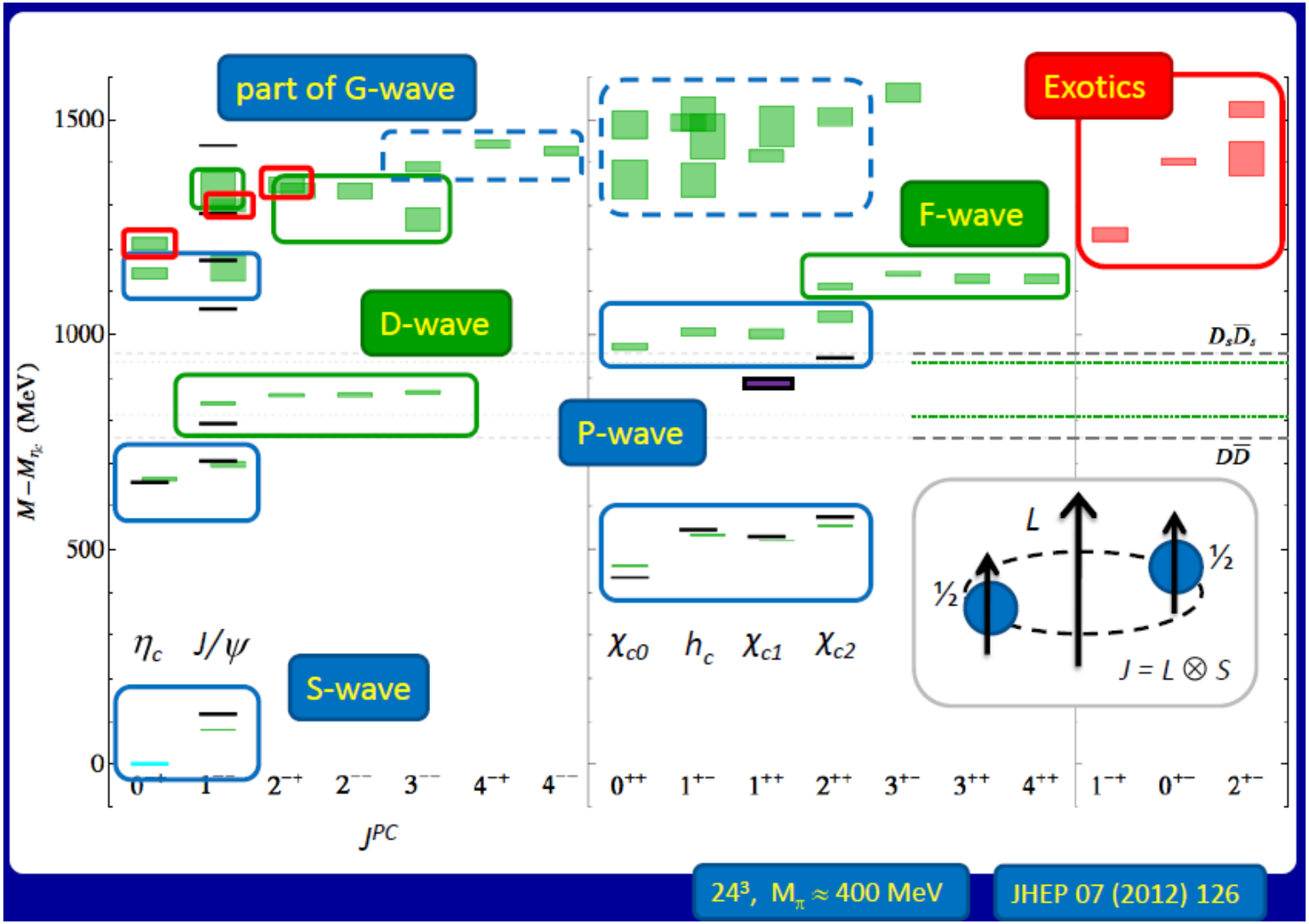
2. The resonance parameter (PDG2012)

$$M_X = 4263(8) MeV$$
$$\Gamma_X = 95(14) MeV$$

3. The leptonic decay width

$$\Gamma(Y(4260) \to e^+e^-)\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)/\Gamma_{tot} = 5.8 eV$$

# II) Latest charmonium spectrum from lattice QCD



## A. 1– hybrid-like interpolation field operator

$$\overline{\psi}^{a}\gamma_5\psi^{b}B_i^{ab}$$

### Nonrelativistic decomposition

$$
\begin{aligned}
q = \quad & e^{\frac{\gamma\cdot D}{2m}}\begin{pmatrix}\psi\\ \chi\end{pmatrix} = \left[1+\frac{\gamma\cdot D}{2m}+\frac{\gamma\cdot\vec{D}\,\gamma\cdot D}{8m^2}O(1/m^3)\right]\begin{pmatrix}\psi\\ \chi\end{pmatrix}\\
& = \begin{pmatrix}\psi\\ \chi\end{pmatrix}+\frac{i}{2m}\begin{pmatrix}-\sigma\cdot\vec{D}\chi\\ \sigma\cdot\vec{D}\psi\end{pmatrix}+\frac{(\vec{D}^2+\sigma\cdot B)}{8m^2}\begin{pmatrix}\psi\\ \chi\end{pmatrix}+O(1/m^3),\\
\bar{q} = \quad & \begin{pmatrix}\psi^\dagger & -\chi^\dagger\end{pmatrix}e^{-\frac{\gamma\cdot\overleftarrow{D}}{2m}} = \begin{pmatrix}\psi^\dagger & -\chi^\dagger\end{pmatrix}+\frac{i}{2m}\begin{pmatrix}\chi^\dagger\sigma\cdot\overleftarrow{D}^\dagger & \psi^\dagger\sigma\cdot\overleftarrow{D}^\dagger\end{pmatrix}\\
& +\frac{(\overleftarrow{D}^2+\sigma\cdot B)}{8m^2}\begin{pmatrix}\psi^\dagger & -\chi^\dagger\end{pmatrix}+O(1/m^3),
\end{aligned}
$$

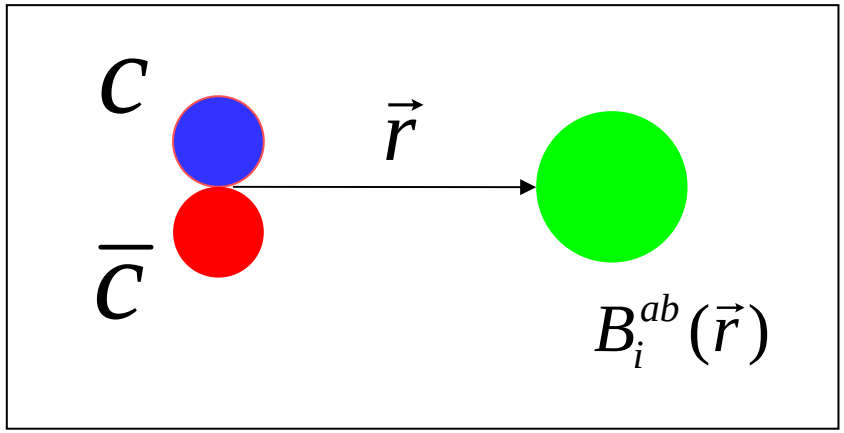
| | | |
|---|---|---|
| $0^{-+}$ | $\overline{\psi}\gamma_5\psi$ | $\chi^{+}\varphi$ |
| $1^{--}$ | $\overline{\psi}\gamma_i\psi$ | $\chi^{+}\sigma_i\varphi$ |
| $1^{--}{}_H$ | $\overline{\psi}^{a}\gamma_5\psi^{b}B_i^{ab}$ | $\chi^{a+}\varphi^{b}B_i^{ab}$ |

$$O_i^{(H)} \equiv \bar{c}^a\gamma_5 c^b B_i^{ab} \to \chi^{a\dagger}\phi^b B_i^{ab} + O(\frac{1}{m_c}),$$ → c-cbar spin singlet

$$O_i^{(M)} \equiv \bar{c}^a\gamma_i c^a \to \chi^{a\dagger}\sigma_i\phi^a + O(\frac{1}{m_c}).$$ → c-cbar spin triplet

## B. Spatially extended interpolation field operator for the vector charmonium-like state

In the Coulomb gauge, $$O(\vec{r}) = (\bar{c}^a \gamma_5 c^b)(0) B_i^{ab}(\vec{r})$$



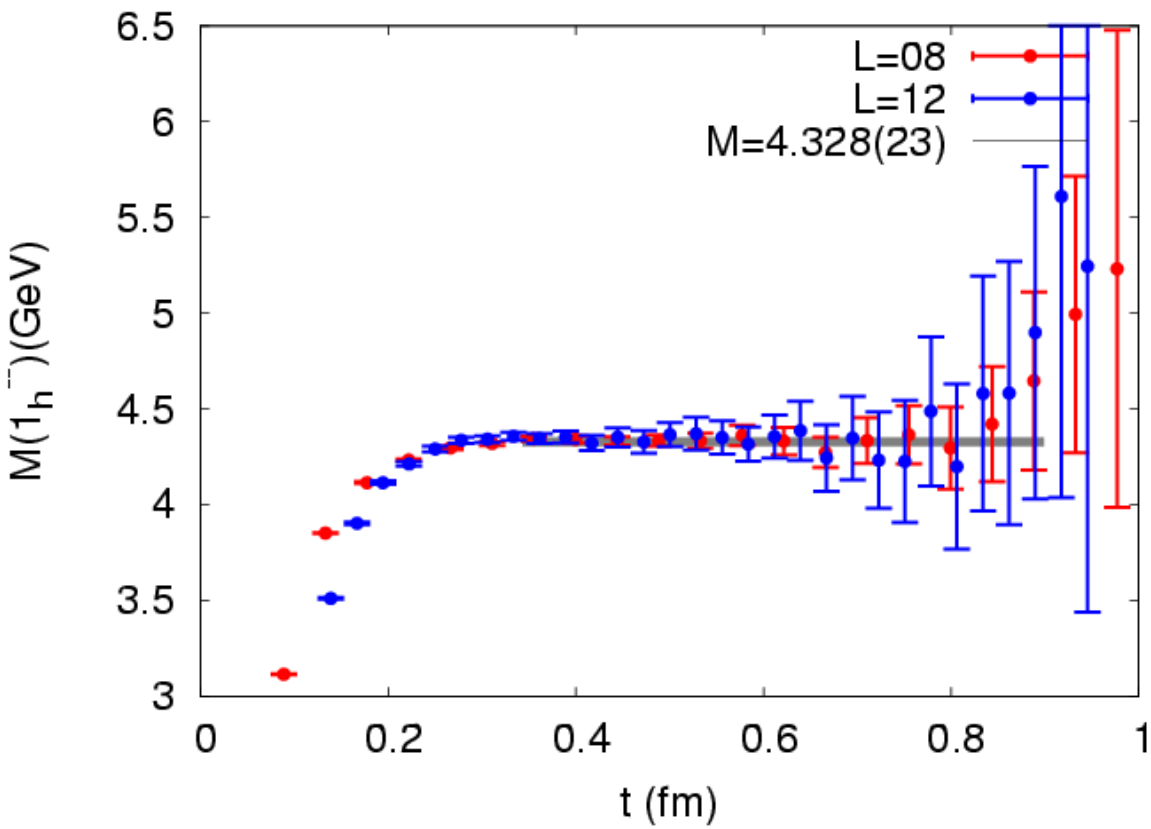

This is equivalent to giave a c-cbar center of mass motion, which describes the recoil of the c-cbar against additional degrees of freedom.

Intuitively, the coupling of this kind of operators to conventional vector charmonia can be suppressed from two aspects:

a) spin states of the c-cbar (spin flipping is suppressed by the heavy quark mass.

b) center-of-mass motion ( to the leading order of NR, there is no cneter-of-mass motions for conventional charmonia.)

This kind of operator does couple strongly to a state with mass near 4.3 GeV, as seen in the effective mass plateau



## IV) The leptonic decay width of the exotic vector charnomium

1. The leptonic decay width of this exotic vector chamonium is an important quantity, which can shed light on the nature of Y(4260).

$$\boxed{\Gamma(Y(4260) \to e^+e^-)\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)/\Gamma_{tot} = 5.8eV}$$

2. The leptonic decay constant of the exotic state can be calculated directly in lattice QCD.

The decay constant of a vector meson is defined as

$$\boxed{\langle 0|\bar{q}\gamma_\mu q|V(\vec{p},r)\rangle = m_V f_V \varepsilon_\mu(\vec{p},r)}$$

where the matrix element on the left can be derived by calculate the two point function

$$\sum_{\vec{x}} \langle 0|\bar{q}\gamma_\mu q(\vec{x},t)O^{(w)}(0)|0\rangle = \sum_{i,r} \frac{1}{2M_i} \langle 0|\bar{q}\gamma_\mu q|V_i,r\rangle\langle V_i,r|O^{(w)}|0\rangle e^{-M_i t}$$

| $\beta$ | $M(J/\psi)$(GeV) | $f_{J/\psi}$ (MeV) | $M(Y)$(GeV) | $f_Y$ (MeV) |
|---|---|---|---|---|
| 2.4 | 3.076(4) | 428(7) | 4.43(7) | 32(20) |
| 2.8 | 3.082(1) | 378(6) | 4.40(7) | 31(11) |
| Exp. | 3.097 | 407(5) | ... | ... |

Using the formula

$$\Gamma(V_{c\bar{c}} \to e^+e^-) = \frac{16\pi}{27}\alpha_{\rm QED}^2 \frac{f_V^2}{M_V}$$

One can predict the leptonic decay width of the exotic vector charmonium

$$\boxed{\Gamma(Y \to e^+e^-) \approx 25(20)\,\text{eV}}$$

(*very reliminary*)

$$\boxed{\Gamma(Y(4260) \to e^+e^-)\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)/\Gamma_{tot} = 5.8eV}$$

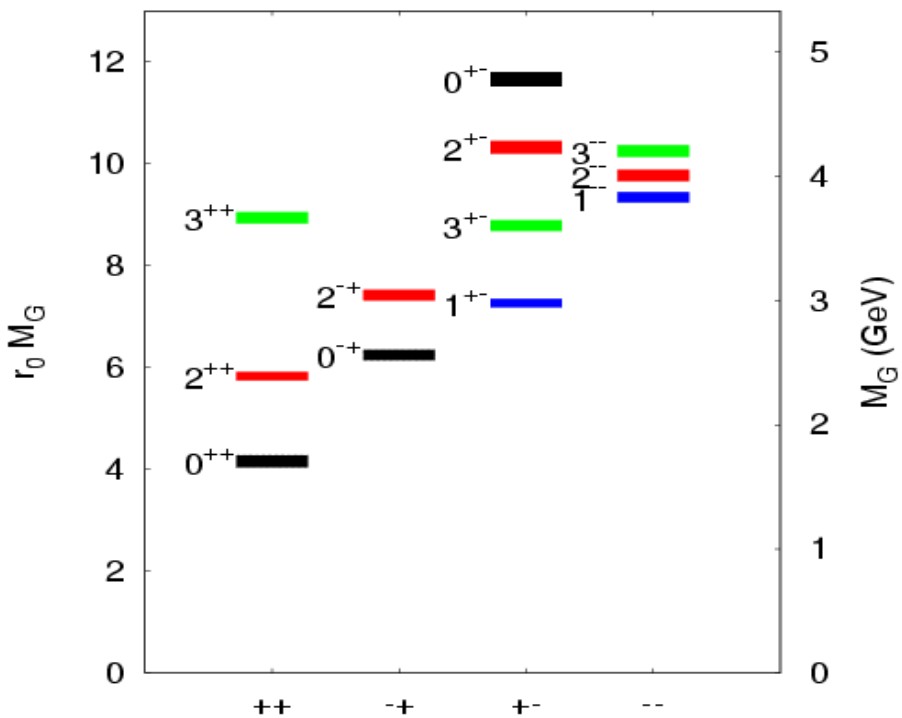
$\sim 20 - 30\%$

Note: Please do not take too seriously the precise value above. We just want to say that it can be very small.

# III. Glueballs on the lattice

## I). Glueball mass spectrum

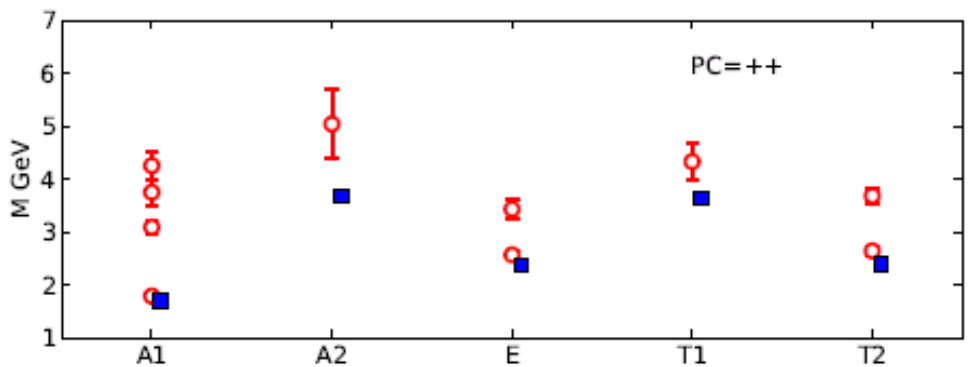
- Quenched LQCD predicts glueball spectrum
  Lowest-lying glueballs have masses in the range 1~3GeV




| $J^{PC}$ | $rM_G$ | $M_G$ (MeV) |
|---|---|---|
| $0^{++}$ | 4.16(11)(4) | 1710(50)(80) |
| $2^{++}$ | 5.83(5)(6) | 2390(30)(120) |
| $0^{-+}$ | 6.25(6)(6) | 2560(35)(120) |
| $1^{+-}$ | 7.27(4)(7) | 2980(30)(140) |
| $2^{-+}$ | 7.42(7)(7) | 3040(40)(150) |
| $3^{+-}$ | 8.79(3)(9) | 3600(40)(170) |
| $3^{++}$ | 8.94(6)(9) | 3670(50)(180) |
| $1^{--}$ | 9.34(4)(9) | 3830(40)(190) |
| $2^{--}$ | 9.77(4)(10) | 4010(45)(200) |
| $3^{--}$ | 10.25(4))(10) | 4200(45)(200) |
| $2^{+-}$ | 10.32(7)(10) | 4230(50)(200) |
| $0^{+-}$ | 11.66(7)(12) | 4780(60)(230) |

**Y. Chen et al, Phys. Rev. D 73, 014516 (2006)**

- Latest results of glueball masses from 2+1 flavor dynamical lattice QCD study, which confirm the prediction of the quenched lattice QCD.
  [E.Gregory et al, JHEP 10 (2012) 170, arXiv:1208.1858(hep-lat)]




Open circles are full-QCD results, and the filled squares are from quenched lattice QCD studies

## II). The production rates of glueballs in the J/psi radiative decays

- Radiative decay width:

$$\Gamma(i \to \gamma f) = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{M_i^2} \frac{1}{2J_i+1} \times \sum_{r_i, r_j, r_\gamma} \left| M_{r_i, r_j, r_\gamma} \right|^2,$$

- Transition amplitudes:

$$M_{r_i, r_f, r_\gamma} = \epsilon_\mu^*(\vec{q}, r_\gamma) \langle f(\vec{p}_f, r_f) | j_{\rm em}^\mu(0) | i(\vec{p}_i, r_i) \rangle$$

- Multipole decomposition:

$$\langle f(\vec{p}_f, r_f) | j_{\rm em}^\mu(0) | i(\vec{p}_i, r_i) \rangle = \sum_k \alpha_k^\mu(p_i, p_f) F_k(Q^2).$$

- Decay width expressed in terms of the form factors

$$\Gamma(i \to \gamma f) \propto \sum_k F_k^2(0).$$

- So the major task is to calculate the matrix elements, which can be derived from the three-point functions

$$\Gamma^{(3)\mu i}(\vec{p}_f, \vec{q}; t_f, t) = \frac{1}{T} \sum_{\tau=0}^{T-1} \sum_{\vec{y}} e^{+i\vec{q}\cdot\vec{y}} \left\langle O_G(\vec{p}_f, t_f + \tau) j^\mu(\vec{y}, t+\tau) O_{J/\psi}^{i,+}(\tau) \right\rangle$$

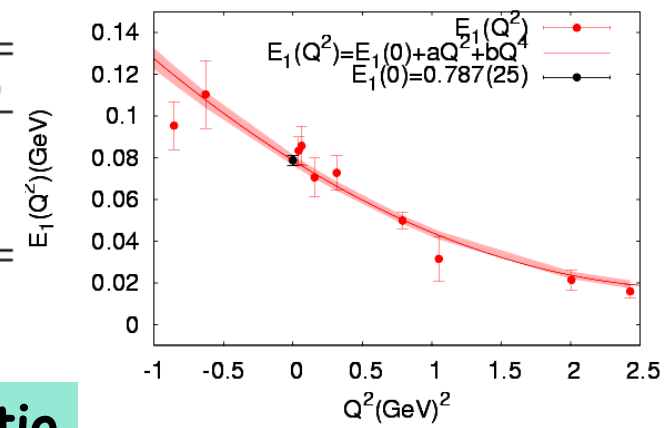
# A). J/psi radiatively decaying to the scalar glueball

(L.Gui, et al. (CLQCD Collaboration), Phys. Rev. Lett. 110, 021601 (2013))

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2$$

**Interpolated on-shell form factor E1(0) and its continuum limit**

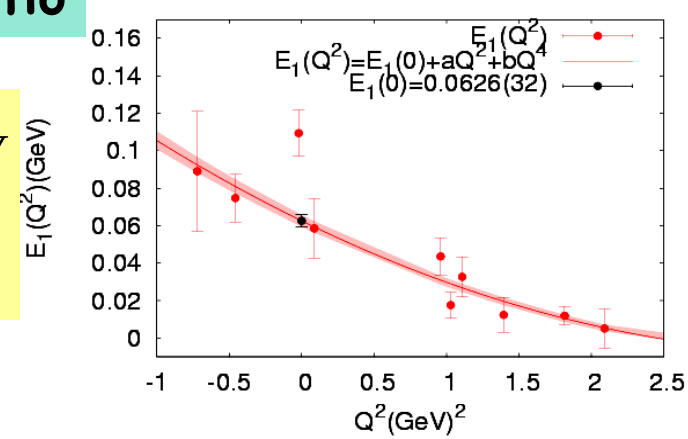
| $\beta$ | $M_G$(GeV) | $Z_V^{(s)}(a)$ | $E_1(0,a)$ (GeV) | $\Gamma$(keV) |
|---|---|---|---|---|
| 2.4 | 1.360(9) | 1.39(2) | 0.0787(25) | - |
| 2.8 | 1.537(7) | 1.11(1) | 0.0626(32) | - |
| $\infty$ | 1.710(90) [3] | - | 0.0536(57) | 0.35(8) |

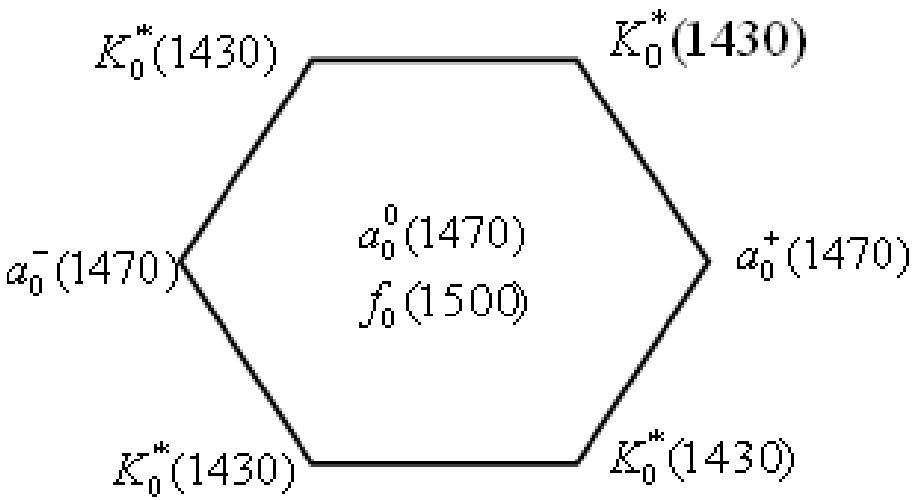



**The predicted width and the branch ratio**

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2 = 0.35(8)keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$






$q\bar{q}$ ??

$+ \quad f_0(1370)$

$f_0(1710)$

These ten scalar mesons can be assigned as a q-barq meson nonet plus a possible scalar glueball which can be either one of the three isoscalars or an admixture of them. There are Many mixing models, the details are out of the scope of this talk.

Lattice prediction:

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2 = 0.35(8)keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$

**Experimental results for J/psi radiatively decaying to scalars**

C. Amsler et al. (Particle Data Group), Phy. Rev. D 86, 010001 (2012)

$$J/\psi \to \gamma f_0(1500) \to \gamma\pi\pi \qquad (1.01 \pm 0.32)\times 10^{-4}$$

$$Br(f_0(1500) \to \pi\pi) = (34.9 \pm 2.3)\% \quad \Rightarrow Br(J/\psi \to \gamma f_0(1500)) = 2.9\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta \qquad (2.35^{+1.27}_{-0.77})\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1500) \to \gamma\eta\eta \qquad (1.65^{+0.57}_{-1.50})\times 10^{-4}$$

BESIII results (PRD87, 092009)

$$J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K} \qquad (8.5^{+1.2}_{-0.9})\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi \qquad (4.0 \pm 1.0)\times 10^{-4}$$

$$J/\psi \to \gamma f_0(1710) \to \gamma\omega\omega \qquad (3.1 \pm 1.0)\times 10^{-4}$$

---

$$J/\psi \to \gamma f_0(1710) \qquad > (1.5 \pm 0.3)\times 10^{-3}$$

**Using $Br(f_0(1710) \to K\underline{K}) = 0.36 \Rightarrow Br(J/\psi \to \gamma f_0(1710)) = 2.4\times 10^{-3}$**

**$Br(f_0(1710) \to \pi\pi) = 0.15 \Rightarrow Br(J/\psi \to \gamma f_0(1710)) = 2.7\times 10^{-3}$**

Our result support f0(1710) as the candidate for the scalar glueball

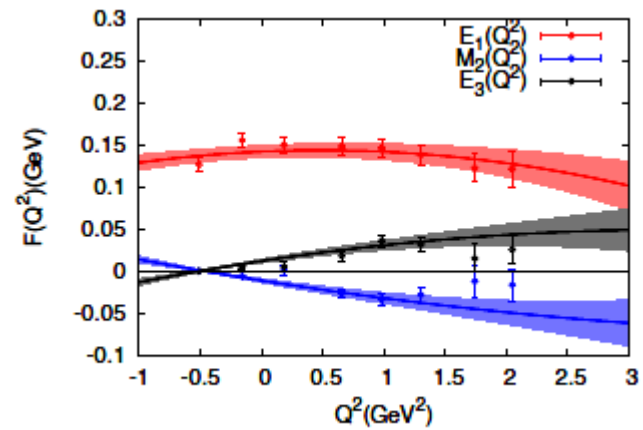
## B). J/psi radiatively decaying to the tensor glueball

(Y.B. Yang ,et al .(CLQCD Collaboration), Phys. Rev. Lett. 111, 091601 (2013))

$$\Gamma(J/\psi \to \gamma G_{2^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left[|E_1(0)|^2 + |M_2(0)|^2 + |E_3(0)|^2\right]$$

- The form factors we obtained from the lattice QCD

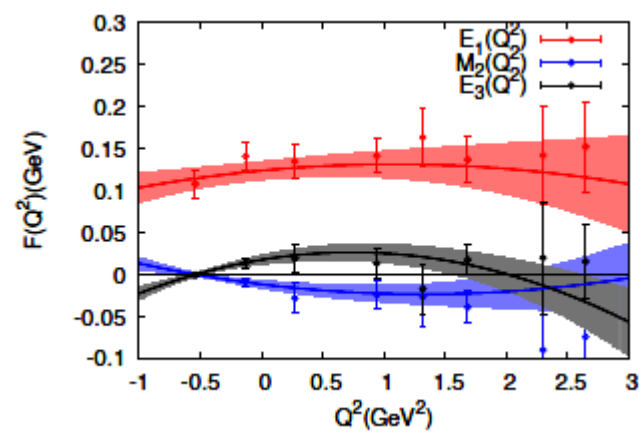
| $\beta$ | $M_T$ (GeV) | $E_1$ (GeV) | $M_2$ (GeV) | $E_3$ (GeV) |
|---|---|---|---|---|
| 2.4 | 2.360(20) | 0.142(07) | −0.012(2) | 0.012(2) |
| 2.8 | 2.367(25) | 0.125(10) | −0.011(4) | 0.019(6) |
| ∞ | 2.372(28) | 0.114(12) | −0.011(5) | 0.023(8) |




- We also carry out a similar lattice study on the tensor glueball production rate in J/psi radiative decay.

$$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22) keV$$

$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1(2)\times 10^{-2}$$



- **Comparing to the experimental observations,** **(PDG2012)**

$$f_J(2220): M = 2231(4) MeV, \Gamma = 23(8) MeV$$

| | | | |
|---|---|---|---|
| $\Gamma_{172}$ | $\gamma f_J(2220)$ | $> 2.50 \times 10^{-3}$ | CL=99.9% |
| $\Gamma_{173}$ | $\gamma f_J(2220) \to \gamma\pi\pi$ | $(8 \pm 4) \times 10^{-5}$ | |
| $\Gamma_{174}$ | $\gamma f_J(2220) \to \gamma K\overline{K}$ | $(8.1 \pm 3.0) \times 10^{-5}$ | |
| $\Gamma_{175}$ | $\gamma f_J(2220) \to \gamma p\overline{p}$ | $(1.5 \pm 0.8) \times 10^{-5}$ | |

$$B(f_J \to p\overline{p}) B(f_J \to \pi\pi) < 1.8 \times 10^{-4}$$

- However, f_J(2220) has not been confirmed by new experiments,
- On the other hand, the tensor glueball can be a broad object.
- With the largest J/psi events sample, our result can provide useful information for BESIII to identify the tensor glueball.

- BESIII new results for $J/\psi \to \gamma\eta\eta$

(M. Ablikim et al. (BES Collaboration),
Phys. Rev. D 87, 092009 (2013) (arXiv:1301.0053)

| Resonance | Mass(MeV/$c^2$) | Width(MeV/$c^2$) | $\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$ | Significance |
|---|---|---|---|---|
| $f_0(1500)$ | $1468^{+14+23}_{-15-74}$ | $136^{+41+28}_{-26-100}$ | $(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$ | 8.2 $\sigma$ |
| $f_0(1710)$ | $1759 \pm 6^{+14}_{-25}$ | $172 \pm 10^{+32}_{-16}$ | $(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$ | 25.0 $\sigma$ |
| $f_0(2100)$ | $2081 \pm 13^{+24}_{-36}$ | $273^{+27+70}_{-24-23}$ | $(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$ | 13.9 $\sigma$ |
| $f_2'(1525)$ | $1513 \pm 5^{+4}_{-10}$ | $75^{+12+16}_{-10-8}$ | $(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$ | 11.0 $\sigma$ |
| $f_2(1810)$ | $1822^{+29+66}_{-24-57}$ | $229^{+52+88}_{-42-155}$ | $(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$ | 6.4 $\sigma$ |
| $f_2(2340)$ | $2362^{+31+140}_{-30-63}$ | $334^{+62+165}_{-54-100}$ | $(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$ | 7.6 $\sigma$ |

In this analysis, the best fit favors the presence of f2(2340) with a mass of 2362(30)MeV and a width of 334(60) MeV. No evident narrow peak around 2.2GeV over the broad bump is observed in the eta-eta mass specturm.

- Flavor-blindness of glueball decays

$$\boxed{\frac{1}{P.S.}\Gamma(G \to \pi\pi : K\overline{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3:4:1:0:1}$$

As such, one can estimate,

$$\Gamma(G \to \eta\eta)/\Gamma(G \to PP) \sim O(10\%)$$

which can be compared with that of f0(1710).

- PP final states in the tensor glueball decays should be in D-wave, considering the centrifugal barrier effects,

$$\Gamma(G \to M\overline{M}) = \eta\alpha \frac{k^{2L+1}}{m_G{}^{2L}} = \frac{\eta\alpha}{m_G}\left(\frac{k}{m_G}\right)^{2L+1}$$

$$\frac{k}{m_G} = \frac{1}{2}\sqrt{1-\left(\frac{2m_M}{m_G}\right)^2} \sim 0.5-0.3$$

So the partial width of G to PP can be suppressed by an order of magnitude, so intutitively one has,

$$Br(G_{2^+} \to PP) \sim O(10\%)$$

- With the BESIII result,

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta\eta) = 5.6(2.3)\times 10^{-5}$$

the production rate of f_2(2340) in the J/psi radiative decay can be 100 times larger, and consistent with our prediction

$$Br(J/\psi \to \gamma f_2(2340) \sim 10^{-2}$$

with the new result of BES

- We look forward to the new results of the analysis the processes

$$J/\psi \to \gamma VV$$

# IV. Summary

- Present Lattice QCD studies do not provide strong support to the existence of Zc particles.

- Lattice QCD studies see the possible existence of an exotic vector charmoium with a mass about 4.3-4.4 GeV.

- We have carried out a first lattice QCD study (in quenched approximation) on the partial decay widths of J/psi radiatively decaying into the scalar and the tensor glueballs, which serve as new criteria for identifying glueball state from experiments.

Thanks!

## The disadvantage and advantage of the Quenched approximation

Disadvantage:

- not a unitary (physical) theory.
- the systematical uncertainties due to the neglect of sea quarks are not under control

Advantages:

- no sea quarks, so no mixing with q-barq mesons
- hybrids and pure gauge glueballs are well-defined
- numerically, very large statistics can be obtained

Anyway,

preliminary full-QCD lattice study shows that the systematical uncertainty got by QA may not be that important.