SM predictions for electroweak precision observables

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1. **Introduction**

2. **Electroweak precision observables**

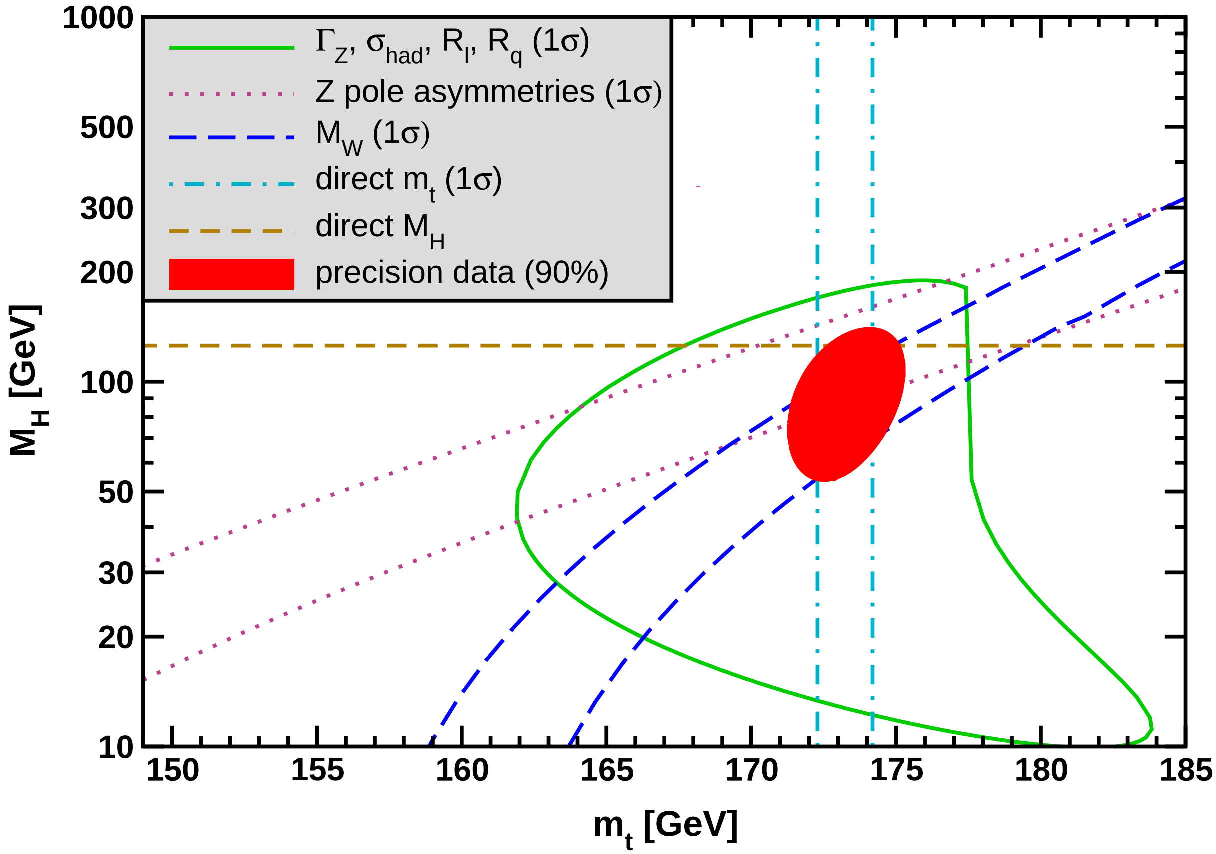
3. **Current status of SM loop results**

4. **Future projections**

Standard Model after Higgs discovery:

- Good agreement between measured mass and indirect prediction
- Very good agreement over large number of observables

Erler '13



Direct measurements:

$M_{\rm H} = 125.6 \pm 0.4$ GeV

$m_{\rm t} = 173.24 \pm 0.95$ GeV

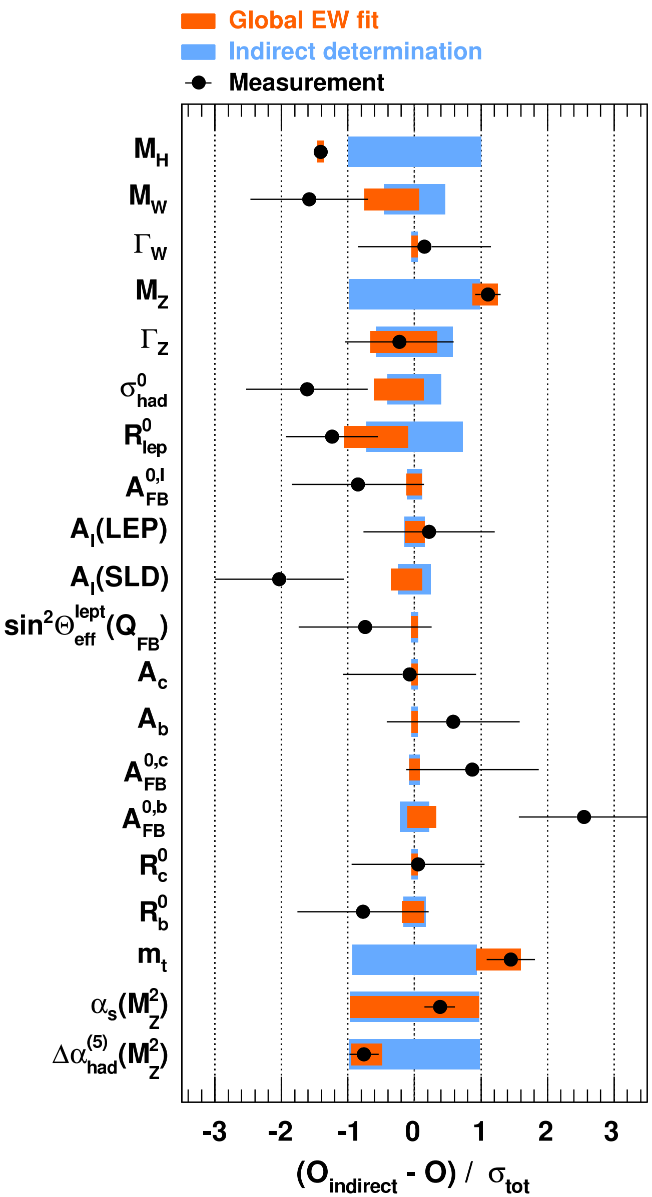
Indirect prediction:

$M_{\rm H} = 123.7 \pm 2.3$ GeV (with LHC BRs)

$M_{\rm H} = 89^{+22}_{-18}$ GeV (w/o LHC data)

$m_{\rm t} = 177.0 \pm 2.1$ GeV

Current status of electroweak precision tests

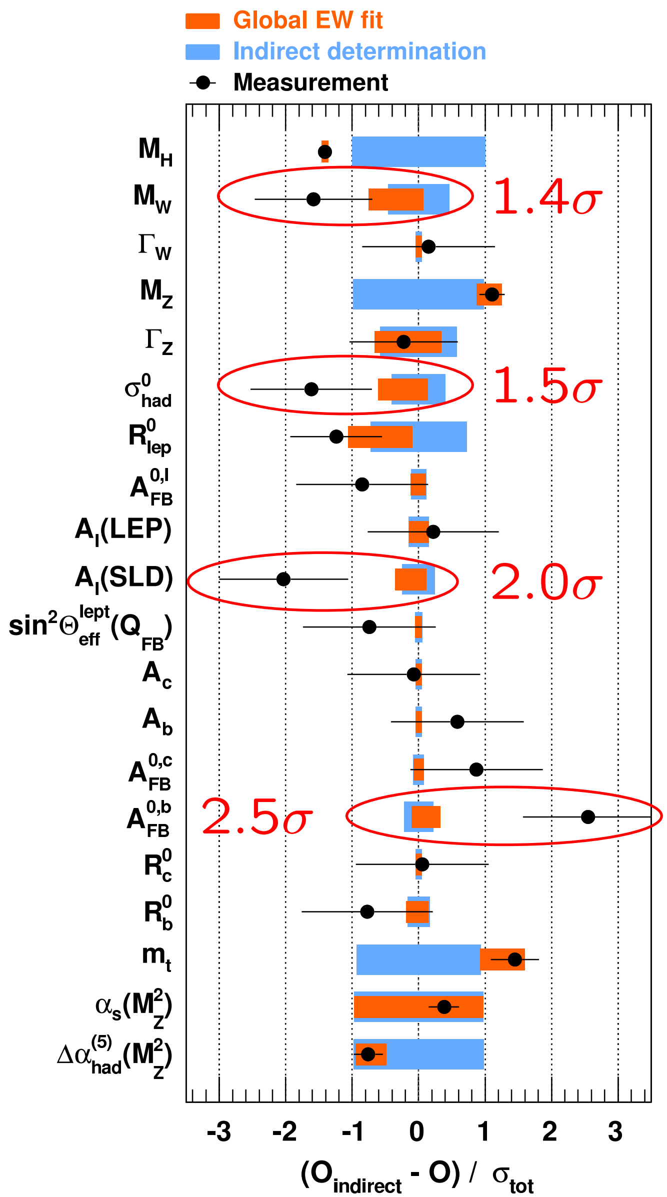


Surprisingly good agreement:
χ^2/d.o.f. $= 18.1/14$ ($p = 20\%$)

Most quantities measured with
1%–0.1% precision

GFitter coll. '14

Current status of electroweak precision tests



Surprisingly good agreement:
χ^2/d.o.f. $= 18.1/14$ ($p = 20\%$)

Most quantities measured with 1%–0.1% precision

A few interesting deviations:

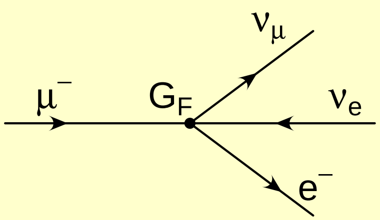
M_W	$(\sim 1.4\sigma)$
σ^0_{had}	$(\sim 1.5\sigma)$
A_ℓ(SLD)	$(\sim 2\sigma)$
A^b_{FB}	$(\sim 2.5\sigma)$
$(g_\mu - 2)$	$(\gtrsim 3\sigma)$

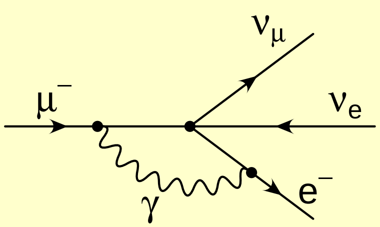
GFitter coll. '14

Electroweak precision observables

W mass

μ decay in Fermi Model



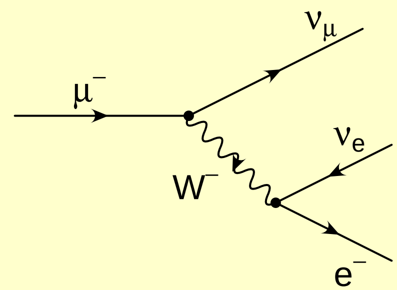


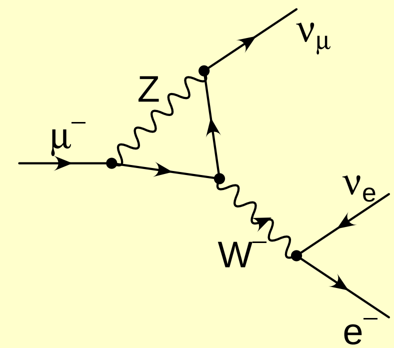
← QED corr. (2-loop)

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)(1 + \Delta q)$$

Ritbergen, Stuart '98
Pak, Czarnecki '08

μ decay in Standard Model





$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1 + \Delta r)$$

electroweak corrections

Z-pole observables

$e^+e^- \to f\bar{f}$ for $\sqrt{s} \sim m_Z$:

$$\sigma = \mathcal{R}_{\text{ini}} \left[12\pi \frac{\Gamma_{ee}\Gamma_{ff}}{(s - m_Z^2)^2 - m_Z^2 \Gamma_Z^2} (1 + \delta X)\,(1 - \mathcal{P}_e \mathcal{A}_e) \;+\; \sigma_{\text{non-res}} \right],$$

$$\Gamma_{ff} = \mathcal{R}_V^f g_{Vf}^2 + \mathcal{R}_A^f g_{Af}^2, \qquad \Gamma_Z = \sum_f \Gamma_{ff},$$

$$\mathcal{A}_f = 2 \frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2} = \frac{1 - 4|Q_f| \sin^2\theta_{\text{eff}}^f}{1 - 4|Q_f| \sin^2\theta_{\text{eff}}^f + 8(|Q_f| \sin^2\theta_{\text{eff}}^f)^2}.$$

$$A_{\text{FB}}^f = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f \qquad A_{\text{LR}} = \mathcal{A}_e$$

Z-pole observables

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$$A_{\text{FB}}^f = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \qquad A_{\text{LR}} = \mathcal{A}_e$$

QED/QCD corrections on ext. fermions

Chetyrkin, Kataev, Tkachov ’79
Dine, Saphirstein ’79
Celmaster, Gonsalves ’80
Gorishnii, Kataev, Larin ’88,91
Chetyrkin, Kühn ’90
Surguladze, Samuel ’91
Kataev ’92
Chetyrkin ’93
etc...

Z-pole observables

$e^+e^- \to f\bar{f}$ for $\sqrt{s} \sim m_Z$:

$$\sigma = \mathcal{R}_{\rm ini}\left[12\pi \frac{\Gamma_{ee}\Gamma_{ff}}{(s-m_Z^2)^2 - m_Z^2\Gamma_Z^2}(1+\delta X)\,(1-\mathcal{P}_e\mathcal{A}_e)\ +\ \sigma_{\rm non-res}\right],$$

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$$A_{\rm FB}^f = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \qquad A_{\rm LR} = \mathcal{A}_e$$

additional initial-state QED corrections

Kuraev, Fadin ’85
Berends, Burgers, v. Neerven ’88
Kniehl, Krawczyk, Kühn, Stuart ’88
Beenakker, Berends, v. Neerven ’89
Bardin et al. ’89,91
Montagna, Nicrosini, Piccinini ’97
etc...

Z-pole observables

$e^+e^- \to f\bar{f}$ for $\sqrt{s} \sim m_Z$:

$$\sigma = \mathcal{R}_{\text{ini}} \left[12\pi \frac{\Gamma_{ee}\Gamma_{ff}}{(s-m_Z^2)^2 - m_Z^2\Gamma_Z^2}(1+\delta X)\,(1-\mathcal{P}_e\mathcal{A}_e) \;+\; \sigma_{\text{non-res}} \right],$$

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$$A_{\text{FB}}^f = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f \qquad A_{\text{LR}} = \mathcal{A}_e$$

electroweak corrections

Correction term first at NNLO:

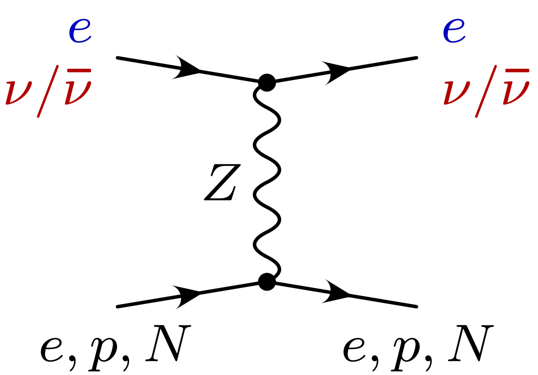
Grassi, Kniehl, Sirlin '01
Freitas '13

$$\delta X_{(2)} = -(\text{Im}\,\Sigma'_{Z(1)})^2 - 2\overline{\Gamma}_Z\overline{M}_Z\,\text{Im}\,\Sigma''_{Z(1)}$$

Test of running $\overline{\text{MS}}$ weak mixing angle $\sin^2\bar{\theta}(\mu)$

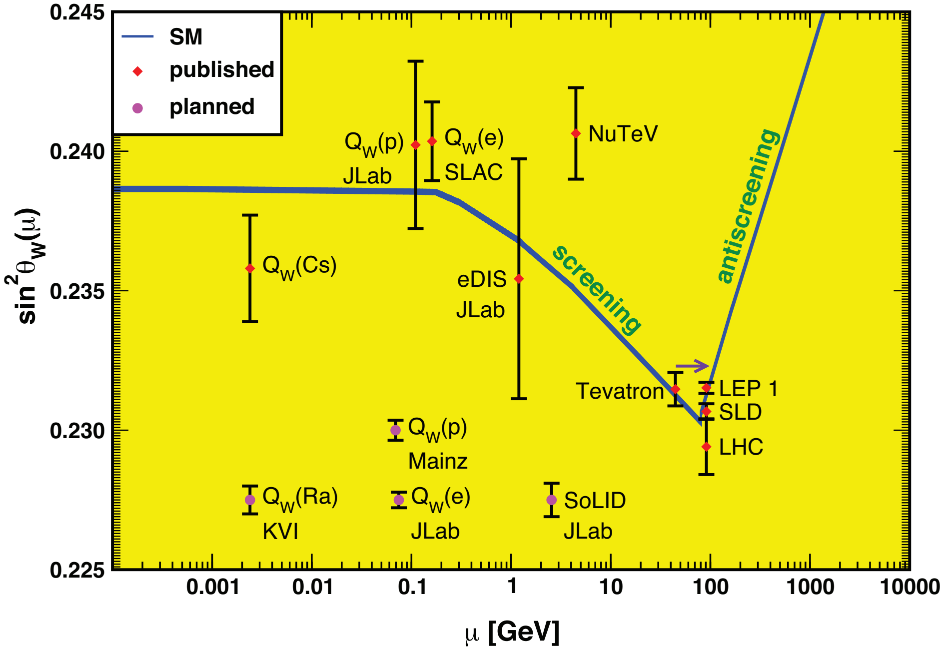
- Polarized ee, ep, ed scattering
  ($Q_W(e)$, $Q_W(p)$, eDIS)
  E158 '05; Qweak '13; JLab Hall A '13

- $\nu N/\bar{\nu}N$ scattering NuTeV '02



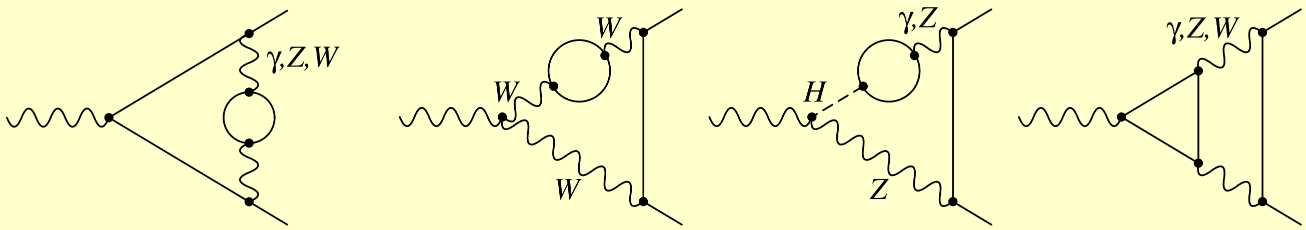
- Atomic parity violation
  ($Q_W(^{133}\text{Cs})$) Wood et al. '97
  Guéna, Lintz, Bouchiat '05

Erler '14



Current status of SM loop results

Known corrections to Δr, $\sin^2\theta^f_{\rm eff}$, g_{Vf}, g_{Af}:



- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\rm eff}$) — Freitas, Hollik, Walter, Weiglein '00; Awramik, Czakon '02; Onishchenko, Veretin '02; Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06; Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- "Fermionic" NNLO corrections (g_{Vf}, g_{Af}) — Czarnecki, Kühn '96; Harlander, Seidensticker, Steinhauser '98; Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_s^3)$ — Chetyrkin, Kühn, Steinhauser '95; Faisst, Kühn, Seidensticker, Veretin '03; Boughezal, Tausk, v. d. Bij '05; Schröder, Steinhauser '05; Chetyrkin et al. '06; Boughezal, Czakon '06

  $\left(\alpha_t \equiv \frac{y_t^2}{4\pi}\right)$

Current uncertainties

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_s$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	α^2_{bos}, α^3, $\alpha^2\alpha_s$, $\alpha\alpha_s^2$
σ^0_{had}	41540 ± 37 pb	6 pb	α^2_{bos}, α^3, $\alpha^2\alpha_s$
$R_b \equiv \Gamma^b_Z/\Gamma^{had}_Z$	0.21629 ± 0.00066	0.00015	α^2_{bos}, α^3, $\alpha^2\alpha_s$
$\sin^2\theta^\ell_{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_s$

Methods for theory error estimates:

- Parametric factors, $i.e.$ factors of α, N_c, N_f, ...
- Geometric progression, $e.g.$ $\frac{\mathcal{O}(\alpha^3)}{\mathcal{O}(\alpha^2)} \sim \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}$
- Renormalization scale dependence (often underestimates error)
- Renormalization scheme dependence (may underestimate error)

Renormalization scheme dependence

Use of $\overline{\text{MS}}$ renormalization for m_t reduces h.o. QCD corrections of $\mathcal{O}(\alpha_t \alpha_s^n)$:

loops $(n+1)$	$\Delta\rho^{\overline{\text{MS}}}_{(n)} \Big/ \left(\frac{3G_F \overline{m}_t^2}{8\sqrt{2}\pi^2}\right)$	$\Delta\rho^{\text{OS}}_{(n)} \Big/ \left(\frac{3G_F m_t^2}{8\sqrt{2}\pi^2}\right)$	
2	$-0.193 \left(\frac{\alpha_s}{\pi}\right)$	$-3.970 \left(\frac{\alpha_s}{\pi}\right)$	Djouadi, Verzegnassi '87 Kniehl '90
3	$-2.860 \left(\frac{\alpha_s}{\pi}\right)^2$	$-14.59 \left(\frac{\alpha_s}{\pi}\right)^2$	Avdeev, Fleischer, et al. '94 Chetyrkin, Kühn, Steinhauser '95
4	$-1.680 \left(\frac{\alpha_s}{\pi}\right)^3$	$-93.15 \left(\frac{\alpha_s}{\pi}\right)^3$	Schröder, Steinhauser '05 Chetyrkin, Faisst, Kühn, et al. '06 Boughezal, Czakon '06

No clear pattern of this kind known for $\mathcal{O}(\alpha^n)$

$\longrightarrow$ Only few results available that allow direct comparison

e.g. Faisst, Kühn, Seidensticker, Veretin '03

Future projections

ILC: $\sqrt{s} \approx M_Z$ with 30 fb^{-1}, $\sqrt{s} \approx 2M_W$ with 100 fb^{-1}

FCC-ee: $\sqrt{s} \approx M_Z$ with 4×3000 fb^{-1}, $\sqrt{s} \approx 2M_W$ with 4×3000 fb^{-1}

CEPC: $\sqrt{s} \approx M_Z$ with 100–1000 fb^{-1}, $\sqrt{s} \approx 2M_W$ with 100 fb^{-1}

	Current exp.	ILC	FCC-ee	CEPC	Current perturb.
M_W [MeV]	15	3–5	~ 1	3–5	4
Γ_Z [MeV]	2.3	~ 1	~ 0.1	~ 0.5	0.5
R_b [10^{-5}]	66	15	$\lesssim 5$	17	15
$\sin^2 \theta^\ell_{eff}$ [10^{-5}]	16	1.3	0.3	~ 3	4.5

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC, but not future e^+e^- machines!

Theory and parametric uncertainties

	ILC	CEPC	perturb. error with 3-loop†	Param. error ILC*	Param. error CEPC**
$M_{\rm W}$ [MeV]	3–5	3–5	1	2.6	2.1
Γ_Z [MeV]	~ 1	~ 0.5	$\lesssim 0.2$	0.5	0.09
R_b [10^{-5}]	15	17	5–10	< 1	< 1
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	1.3	3	1.5	2	2

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_{\rm s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_{\rm s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_{\rm s})$

(N_f^n = at least n closed fermion loops)

Parametric inputs:

*** ILC:** $\delta m_t = 100$ MeV, $\delta\alpha_{\rm s} = 0.001$, $\delta M_{\rm Z} = 2.1$ MeV

****CEPC:** $\delta m_t = 600$ MeV, $\delta\alpha_{\rm s} = 0.0001$, $\delta M_{\rm Z} = 0.5$ MeV

also: $\delta(\Delta\alpha) = 5\times 10^{-5}$

- Subtraction of QED radiation contributions

  $\longrightarrow$ Known to $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha^3 L^3)$ for **ISR**, $\mathcal{O}(\alpha^2)$ for **FSR** and $\mathcal{O}(\alpha^2 L^2)$ for **A_{FB}**

  $(L = \log \frac{s}{m_e^2})$

  Berends, Burgers, v.Neerven '88
  Kniehl, Krawczyk, Kühn, Stuart '88
  Beenakker, Berends, v.Neerven '89
  Skrzypek '92; Montagna, Nicrosini, Piccinini '97

  $\longrightarrow$ $\mathcal{O}(0.1\%)$ uncertainty on σ_Z, A_{FB}

  $\longrightarrow$ Improvement needed for ILC/FCC-ee

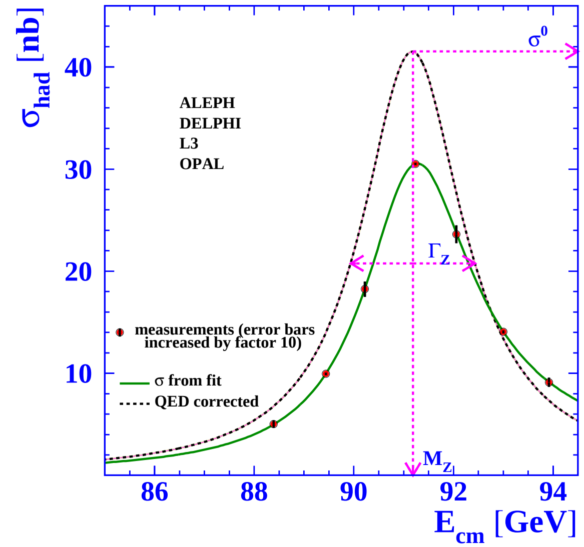
- Subtaction of non-resonant γ-exchange, γ–Z interf., box contributions, Bhabha scattering

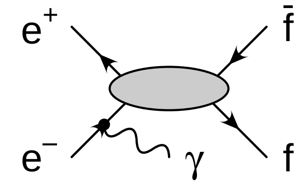
  see, e.g., Bardin, Grünewald, Passarino '99

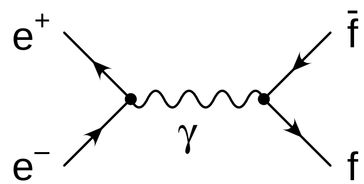
  $\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed)

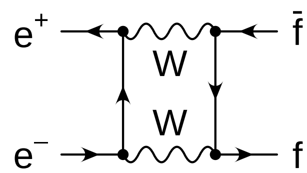
  $\longrightarrow$ Sensitivity to some NP beyond EWPO

LEP EWWG '05





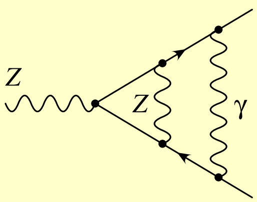


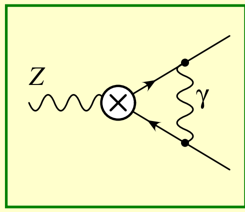


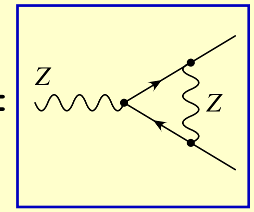
Non-factorizable contributions

Factorization of massive and QED/QCD FSR:

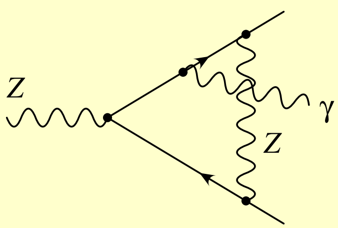
$$\Gamma_{ff} \propto \mathcal{R}_V^f g_{Vf}^2 + \mathcal{R}_A^f g_{Af}^2$$



$=$


$+$ finite, with $\otimes =$


Additional non-factorizable contributions, e.g.



$\longrightarrow$ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98

$\longrightarrow$ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty on Γ_Z, σ_Z, maybe larger for A_b
(improvements may be needed)

Theory challenges

Full SM corrections at $>$2-loop:

- Large number of diagrams and tensor integrals, $\mathcal{O}(1000) - \mathcal{O}(10000)$
  $\longrightarrow$ Use of computer algebra tools
- Many different scales (masses and ext. momenta)
  $\longrightarrow$ In general not possible analytically
  $\longrightarrow$ Numerical methods must be automizable, stable, fastly converging
  $\longrightarrow$ Need procedure for isolating divergent pieces

Analytic calculations

- Useful for diagrams with up to two scales
  ($e.g.$ M_W & m_t or M_W & M_Z)
- Reduce to master integrals with integration-by-parts and Lorentz-invariance identities — Chetyrkin, Tkachov '81; Gehrmann, Remiddi '00; Laporta '00; ...
- Evaluate master integrals with differential equations or Mellin-Barnes representations — Kotikov '91; Remiddi '97; Smirnov '00,01; ...

Current status:

Single-scale problems: $Zf\bar{f}$ QED/QCD vertex corrections up to 4-loop

Gorishnii, Kataev, Larin '88,91; Chetyrkin, Kühn, Kwiatkowski '96
Baikov, Chetyrkin, Kühn, Rittinger '12

Two-scale problems: $Zf\bar{f}$ electroweak 2-loop vertex diagrams with $m_f = 0$

Awramik, Czakon, Freitas, Weiglein '04

Extendability: Possible, but much work needed

Asymptotic expansions

- Exploit large mass ratios, *e. g.* $M_Z^2/m_t^2 \approx 1/4$
- Fast numerical evaluation

Current status:

Two-scale problems: $\mathcal{O}(\alpha\alpha_s^n)$ corr. to $\Delta\rho$, Δr, ...

$\longrightarrow$ Several expansion terms up to 3-loop, leading term up to 4-loop

Djouadi, Verzegnassi '87; Bardin, Chizhov '88

Chetyrkin, Kühn, Steinhauser '95

Faisst, Kühn, Seidensticker, Veretin '03

...

Up to three-scale problems: $Zf\bar{f}$ ew. 2-loop vertex corrections

Barbieri et al. '92,93; Fleischer, Tarasov, Jegerlehner '93,95

Degrassi, Gambino, Sirlin '97; Awramik, Czakon, Freitas, Weiglein '04

Extendability: Promising, mostly limited by computing/algorithmic power

Direct numerical integration

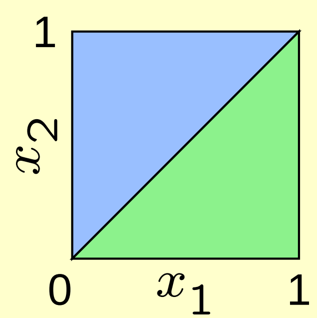
General form of Feynman integral:

$$I = \int_0^1 dx_1 \ldots dx_n \, \delta(1 - \sum_i x_i) \, \frac{N(x_i)}{D(x_i)^{r+\varepsilon}}$$

$\longrightarrow$ Can be integrated numerically (if finite)

Alternatives: Integration in momentum space, Mellin-Barnes space

Treatment of divergencies:



- **Sector decomposition:** Sub-divide integration space such that divergent terms factorize
  Binoth, Heinrich '00,03

- **Subtraction terms:** Remove divergencies with simple terms that can be integrated analytically

Nagy, Soper '03
Becker, Reuschle, Weinzierl '10; Freitas '12

Direct numerical integration

- Automizable, but computing intensive
- Internal thresholds reduce numerical convergence

Current status:

Several 2-loop applications with many scales

Anastasiou et al., Petriello et al., Borowka et al., ...

Individual 3-loop integrals

Extendability: Likely, but more work needed

Conclusions

- **Current SM predictions** for electroweak precision observables under good control (compared to experimental uncertainties)

- **LHC** will provide independent results for $\sin^2\theta_{\rm eff}$ and $M_{\rm W}$,
  but overall precision not substantially improved

- **ILC/CEPC/FCC-ee** with $\sqrt{s} \sim M_{\rm Z}$ will reduce exp. error of some EWPO by $\mathcal{O}(10)$
  $\longrightarrow$ 3-loop (and maybe some 4-loop) corrections needed!

- **Asymptotic expansion and numerical integration techniques** are promising but more work needed

- **Open questions** in evaluation of theory errors, resummation and optimal choice of inputs

Backup slides

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z \alpha^2 \sim 0.1 \text{ MeV}$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi} \alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.5$ MeV

Example: Error estimation for M_W

- Renormalization scheme dependence:

a) Uncertainty of $\mathcal{O}(\alpha^2)$ corrections beyond leading $\alpha^2 m_\mathrm{t}^4$ and $\alpha^2 m_\mathrm{t}^2$ from comparison of $\overline{\mathrm{MS}}$ and OS schemes: Degrassi, Gambino, Sirlin '96

$\delta M_\mathrm{W} \sim 2$ MeV (for $M_\mathrm{H} \sim 100$ GeV)

Actual remaining $\mathcal{O}(\alpha^2)$ corrections: Freitas, Hollik, Walter, Weiglein '00

$\delta M_\mathrm{W} \sim 3$ MeV (for $M_\mathrm{H} \sim 100$ GeV)

b) Estimate of missing $\mathcal{O}(\alpha^3)$ corrections from comparison of $\overline{\mathrm{MS}}$ and OS results: Awramik, Czakon, Freitas, Weiglein '03; Degrassi, Gambino, Giardino '14

$\delta M_\mathrm{W} \sim 4...5$ MeV (after accounting for $\mathcal{O}(\alpha_\mathrm{t}\alpha_\mathrm{s}^3)$ corrections)

$\longrightarrow$ Saturates previous δM_W estimate!

Note: Differences in (implicitly) resummed higher-order contributions

Parametrization: Resummation

Parametrization of perturbation series: $\boldsymbol{\alpha}$ **vs.** $\boldsymbol{G_F}$**?**

G_F can resum some leading one-loop terms

$$\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 \qquad \Delta\rho = \frac{3\alpha}{16\pi s^2 c^2}\frac{m_t^2}{M_Z^2}$$

But: Strong cancellations between $\Delta\alpha$ and $\Delta\rho$ terms beyond one-loop:

$$\begin{aligned}\Delta r^{(3)}_{\text{res}} &= (\Delta\alpha)^3 \quad -3(\Delta\alpha)^2\left(\tfrac{c^2}{s^2}\Delta\rho\right) \quad +6(\Delta\alpha)\left(\tfrac{c^2}{s^2}\Delta\rho\right)^2 \quad -5\left(\tfrac{c^2}{s^2}\Delta\rho\right)^3 \\ &\approx (2.05 \quad -3.40 \quad +3.74 \quad -1.72) \times 10^{-4} \\ &= 0.68 \times 10^{-4}\end{aligned}$$

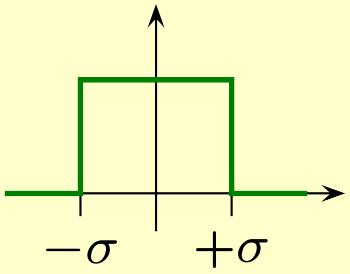
$\longrightarrow$ Not *the* numerically leading contribution anymore

Combination of theory errors

- Add theory errors from each source **linearly**:

  Idea: each value within error range in equally likely

  $\longrightarrow$ Use flat prior in global fits



- Add theory errors from each source **quadratically**:

  Idea: different error sources are uncorrelated

  $\longrightarrow$ Use Gaussian prior in global fits (central limit theorm)