SFT 2015 / LECTURE IV — CARLO MACCAFERRI —

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PLAN

1. Summary of TACHYON VACUUM
2. Searching for MARGINAL DEFORMATIONS
3. Exact solutions
4. KOS SOLUTION AND bcc OPERATORS

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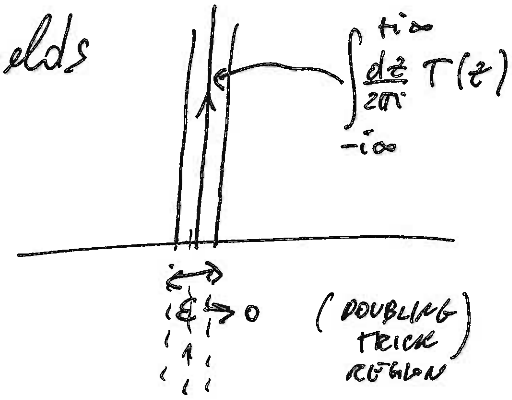
§ 1 The tachyon vacuum solution can be written with just 3 elementary string fields

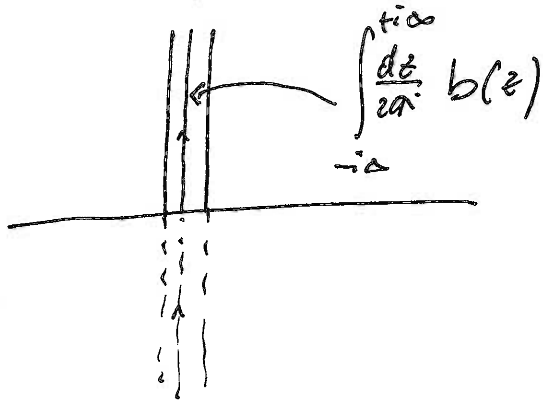
K, B, c

K:



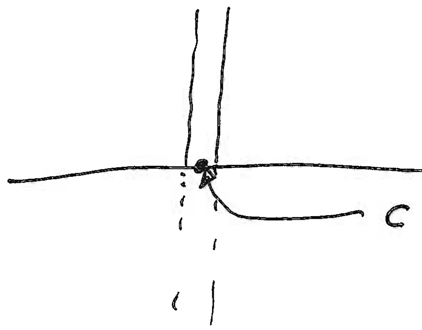
$$Bc + cB = 1$$
$$B^2 = c^2 = 0$$
$$[K, B] = 0$$
$$[K, c] = \partial c$$

B



$$QB = K$$
$$Qc = cKc = c\partial c$$

c



Algebraic solution (Schnabl; Okawa; Erler)

$$\Psi = F(K)\, c \frac{KB}{1-F^2(K)}\, c F(K)$$

$\Rightarrow$ Exercise: Show that $Q\Psi + \Psi^2 = 0$

3rd Sen's conjecture

At the TACHYON VACUUM there are no physical states.

PHYSICAL STATES $\Longleftrightarrow$ COHOMOLOGY OF THE SHIFTED BRST OPERATOR

$$Q_{TV} \equiv Q + [\Psi_{TV}, \cdot\,]$$

3rd Sen's Conjecture: Q_{TV} has TRIVIAL COHOMOLOGY.

Exercise: Show that $Q_{TV}^2 = [Q\Psi_{TV} + \Psi_{TV}^2, \cdot\,] = 0$

Property: If $\exists\, A \,/\, Q_{TV} A = 1$, then Q_{TV} has empty cohomology.

Consider the identity

$$Q_{TV}(A\phi) = (Q_{TV}A)\phi - A(Q_{TV}\phi) = \phi - A(Q_{TV}\phi)$$

$$\Rightarrow \text{ If } Q_{TV}\phi = 0 \Longrightarrow \phi = Q_{TV}(A\phi)$$

$\blacksquare$

For $\Psi_{TV} = F(K)\, c\, \frac{KB}{1-F^2(K)}\, c\, F(K)$ we formally have

$$A = B\,\frac{1-F^2(K)}{K}$$

Exercise: Show that, algebraically,

$$\underline{QA + \Psi_{TV}A + A\Psi_{TV} = 1}$$

However this expression is typically just FORMAL.

- A MUST BE AN ALLOWED ELEMENT OF $\mathcal{H}_{BCFT_0}$.
- But $\frac{1}{K}$ IS NOT! *Exercise*: Show that if there exists the $\frac{1}{K}$ string field, then $Q\frac{B}{K} = 1$ and therefore Q should have TRIVIAL COHOMOLOGY!

$\Rightarrow$ FOR MOST CHOICES OF $F(K)$, THERE IS NOT A WELL-DEFINED A.

TV: $F^2(K) = 1 + F^{2\prime}(0)\,K + O(K^2)$ "$K \to 0$"

Then $A = B\,\frac{1 - 1 - (F^2)'(0)\,K + O(K^2)}{K} \approx -B\left(F^{2\prime}(0) + O(K) \ldots\right)$

Exercise: Show that we need $(F^2)'(0) \neq 0$ to avoid "$\frac{1}{K}$" factors when expanding $\frac{K}{1-F^2(K)}$.

$$\Rightarrow \boxed{\text{TV is CHARACTERIZED BY } \begin{cases} F^2(0) = 1 \\ (F^2)'(0) \neq 0 \end{cases}}$$

$\to$ SCHNABL (2005): $F^2(K) = e^{-K} = |0\rangle_{SL(2\ldots}$

$\to$ ERLER SCHNABL: $F^2(K) = \frac{1}{1+K}$ (SIMPLER!)

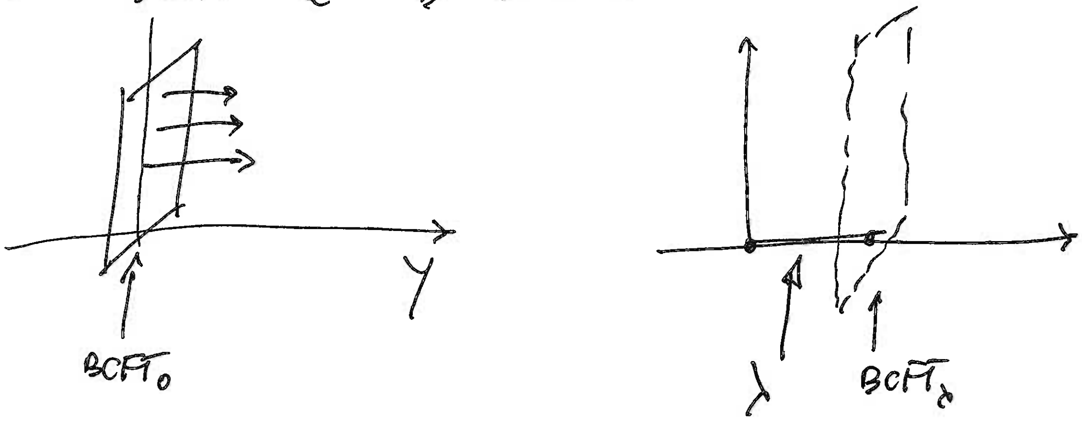
*2) TV is a <u>VERY DRASTIC</u> CHANGE in the OPEN STRING BACKGROUND

The initial D-brane system DISAPPEARS.

$\Rightarrow$ Much milder changes: MARGINAL DEFORMATIONS

<u>Simplest example</u>:

Translate a D-brane in the transverse space



Infinitesimally this translation is generated by

$$|V\rangle = \lambda\, c\,\partial_\perp X(0)\,|0\rangle^{(BCFT_0)}_{SL(2,\mathbb{R})} \qquad Q|V\rangle = 0$$

$$|V\rangle \neq Q|\Lambda\rangle$$

We expect the <u>FINITE</u> translation to be generated by $\Psi_\lambda = \sum_{n=1}^{\infty} \lambda^n \Psi_n$ $\quad \Big|$ N.B. $\Psi_{\lambda\to 0} = 0 \Big|$

$Q\Psi_\lambda + \Psi_\lambda^2 = 0 \quad \Longrightarrow$ QUADRATIC EQUATION (DIFFICULT)

$\Downarrow$

$$Q\Psi_1 = 0$$
$$Q\Psi_2 + \Psi_1^2 = 0$$
$$Q\Psi_3 + \Psi_2\Psi_1 + \Psi_1\Psi_2 = 0$$
$$\vdots \qquad \vdots$$

$\Longrightarrow$ LINEAR EQUATIONS WHICH CAN BE SOLVED 1 by 1!! (SIMPLER!)

First equation:

$Q\Psi_1 = 0 \quad \Rightarrow \quad \Psi_1$ is in the cohomology

ex $\Psi_1 = c\,j(0)\,|0\rangle$

with j: MATTER PRIMARY FIELD OF $h=1$.

To solve the equations we can FIX A GAUGE:

Ex Siegel Gauge $b_0 \Psi_\lambda = 0 \qquad b_0 = \oint \frac{dz}{2\pi i}\, z\, b(z)$

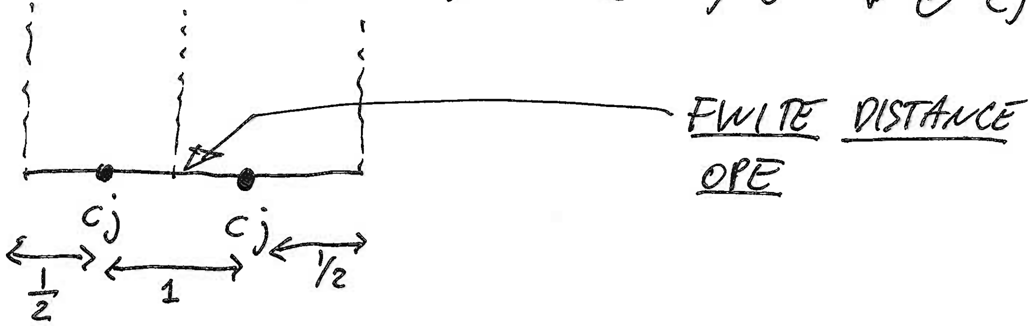
$$\Rightarrow \Psi_2 = -\frac{b_0}{L_0}\left(\Psi_1^2\right)$$

$$\Psi_3 = -\frac{b_0}{L_0}\left(\Psi_1\Psi_2 + \Psi_2\Psi_1\right)$$

$$\vdots \qquad \vdots$$

However IF Ψ_1^2 contains a weight-zero component, then $\frac{1}{L_0}$ is not defined on it and the solution is OBSTRUCTED.

$$\Psi_1^2 = c\,j(0)|0\rangle * c\,j(0)|0\rangle = e^{-\frac{K}{2}}\, c j\, e^{-\frac{K}{2}} * e^{-\frac{K}{2}}\, c j\, e^{-\frac{K}{2}} =$$

$=$ 

Suppose that $\exists\, W$ (matter) s.t. $\langle j\, j\, W\rangle \neq 0 \;\oplus\; \langle j\, j\rangle \neq 0$
weight 1

$$\Rightarrow c\,j(z)\, c\,j(0) \sim z^{-1} \underbrace{c\partial c(0)}_{h=-1} + \underbrace{c\partial c\, W(0)}_{h=0} + \dots$$

The presence of $c\partial c W$ in the $cj\, cj$ OPE is an OBSTRUCTION in computing $\frac{b_0}{L_0}(cj * cj)$.

$\Rightarrow$ But this is a WELL KNOWN necessary condition for exactly marginal boundary deformations!!

j exactly marginal $\Longrightarrow \langle j\, j\, W \rangle = 0, \ \forall W$ primary of $h=1$.

$\Rightarrow$ other more complicated conditions arise from solving the higher order equations ...

$\Rightarrow$ CONDITIONS ON EXACT MARGINALITY CAN BE IN PRINCIPLE OBTAINED BY ENFORCING THE EXISTENCE OF OSFT SOLUTIONS!

e.g. Ψ_1 = BRST CLOSED

Ψ_1^2 = BRST EXACT

$\Psi_1 \Psi_2 + \Psi_2 \Psi_1$ = BRST EXACT ... etc..

However proceeding in Siegel gauge is VERY COMPLICATED

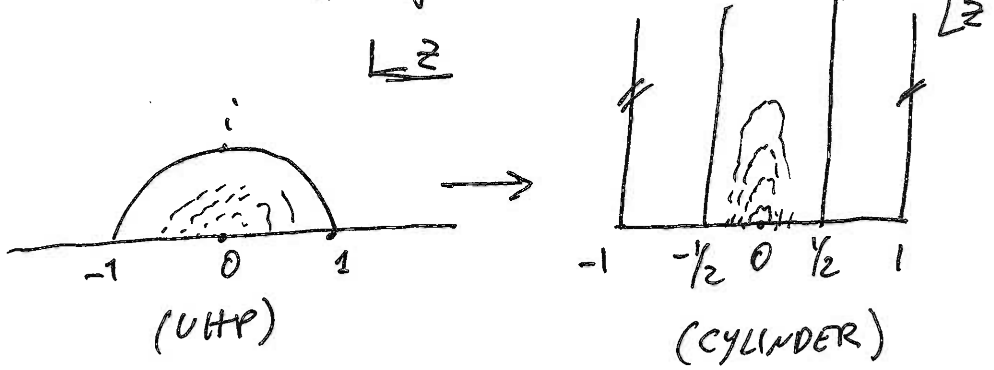
ex. $\Psi_2 = -\frac{b_0}{L_0}\left(cj(0)|0\rangle * cj(0)|0\rangle\right)$ is NOT KNOWN in closed form.

Ψ_3 even more complicated ...

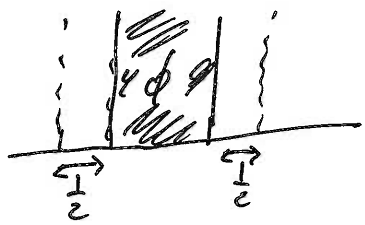
Strategy : Fix the $\mathcal{B}_0$ gauge instead !

$$\mathcal{B}_0 = \oint \frac{d\tilde{z}}{2\pi i}\, \tilde{z}\, \tilde{b}(\tilde{z}) \qquad \tilde{z} = \frac{2}{\pi} \operatorname{tg}^{-1} z$$

$\boxed{\mathcal{B}}$



Advantage : $\dfrac{\mathcal{B}_0}{\mathcal{L}_0}$ is EXACTLY COMPUTABLE on STATES of the form $\Phi = e^{-\frac{K}{2}} \phi\, e^{-\frac{K}{2}} =$



Remember from lecture III (M. SCHNABL)

$$\mathcal{B}_0 \left(e^{-\frac{K}{2}} \phi\, e^{-\frac{K}{2}} \right) = e^{-\frac{K}{2}} (\mathcal{B}^- \phi)\, e^{-\frac{K}{2}} \qquad \mathcal{B}^- = \frac{1}{2}(\mathcal{B}_0 - \mathcal{B}_0^*)$$

$$\mathcal{L}_0 \left(e^{-\frac{K}{2}} \phi\, e^{-\frac{K}{2}} \right) = e^{-\frac{K}{2}} (\mathcal{L}^- \phi)\, e^{-\frac{K}{2}} \qquad \mathcal{L}^- = \frac{1}{2}(\mathcal{L}_0 - \mathcal{L}_0^*)$$

$\Downarrow$

$\boxed{\text{BOTH } \mathcal{B}^- \text{ and } \mathcal{L}^- \text{ are } *\text{-algebra DERIVATIONS!}}$

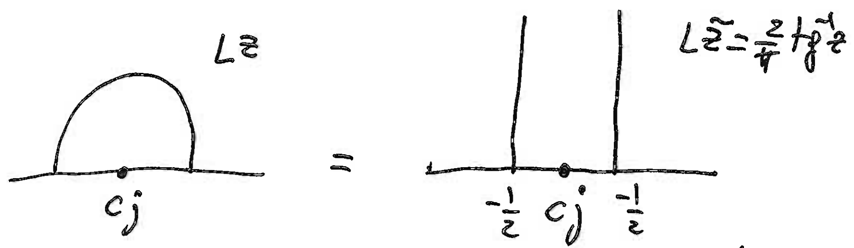
Moreover we have

$$\begin{aligned} \mathcal{B}^- c &= 0 & \mathcal{L}^- c &= -c \\ \mathcal{B}^- B &= 0 & \mathcal{L}^- B &= B \\ \mathcal{B}^- K &= B & \mathcal{L}^- K &= K \\ \mathcal{B}^- j &= 0 & \mathcal{L}^- j &= j \end{aligned}$$

$\mathcal{L}^-$ counts the "effective scale dimension" of the corresponding insertion.

[exercise: $\mathcal{B}^- \partial c = 1$]

$$\Psi_1 = cj(0)|0\rangle = e^{-\frac{K}{2}}\, cj\, e^{-\frac{K}{2}}$$



This obeys $\mathcal{B}_0 \Psi_1 = 0 = e^{-\frac{K}{2}} B^-(cj)\, e^{-\frac{K}{2}} = 0$ ok

$$\Psi_2 = -\frac{\mathcal{B}_0}{\mathcal{L}_0}\left(cj(0)|0\rangle * cj(0)|0\rangle \right) = -\frac{\mathcal{B}_0}{\mathcal{L}_0}\left(e^{-\frac{K}{2}} cj\, e^{-K} cj\, e^{-\frac{K}{2}} \right) =$$

$$= -e^{-\frac{K}{2}} \frac{B^-}{L^-}\left(cj\, e^{-K} cj \right) e^{-\frac{K}{2}} = (*)$$

EXERCISE: PROVE THIS

Now represent $\frac{1}{L^-} = \int_0^1 \frac{dt}{t} t^{L^-}$ ← (EXERCISE: PROVE THIS FORMAL EXPRESSIONS)

$$t^{L^-} cj = cj \quad ; \quad t^{L^-} e^{-K} = e^{-tK} \qquad t^{L^-}(AB) = (t^{L^-}A)(t^{L^-}B)$$

$$(*) = -e^{-\frac{K}{2}} \left[B^- \int_0^1 \frac{dt}{t}\, cj\, e^{-tK} cj \right] e^{-\frac{K}{2}} = \quad \text{→ ASSUMPTION: } j(z)j(0) = \text{REGULAR!}$$

$$= -e^{-\frac{K}{2}} \int_0^1 \frac{dt}{t} \left[-cj \underbrace{\left(B^- e^{-tK} \right)}_{\frac{d}{dK}\left(e^{-tK}\right) B} cj \right] e^{-\frac{K}{2}} =$$

(CHAIN RULE OF DERIVATIONS)

$$= \boxed{-e^{-\frac{K}{2}} cj\, B \left[\int_0^1 dt\, e^{-tK} \right] cj\, e^{-\frac{K}{2}} = \Psi_2}$$

Calling $A \equiv B\int_0^1 dt\, e^{-tK} = B\,\frac{1-e^{-K}}{K}$

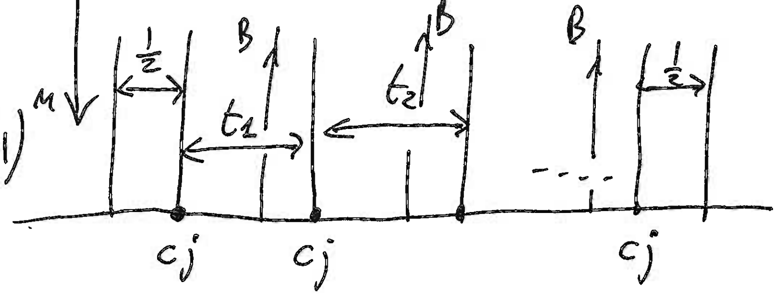
(YES, IT IS THE HOMOTOPY FIELD FOR THE TACHYON VACUUM with $F'(K)=e^{-K}$)

Show that:

$$\Psi_m = (-1)^{m-1} e^{-\frac{K}{2}} \underbrace{cj\,A\,cj\,A \cdots cj}_{m-cj\text{'s}}\, e^{-\frac{K}{2}} = (-1)^{m-1} e^{-\frac{K}{2}} cj\,(A\,cj)^{m-1}\, e^{-\frac{K}{2}}$$

$$\int_0^1 dt_1 \cdots dt_m$$

$$= (-1)^m$$



EXPLICIT SURFACE WITH INSERTIONS.

We can formally resum:

$$\Psi_\lambda = \sum_{m=1}^{\infty} \lambda^m \Psi_m = e^{-\frac{K}{2}} (\lambda cj) \sum_{m=1}^{\infty} (-A\lambda cj)^m\, \cancel{e}\, e^{-\frac{K}{2}}$$

$$= e^{-\frac{K}{2}}\, \lambda cj\, \frac{1}{1+\lambda A cj}\, e^{-\frac{K}{2}}$$

EXERCISE $\longrightarrow$

$$= e^{-\frac{K}{2}}\, \frac{1}{1+\lambda cj A}\, \lambda cj\, e^{-\frac{K}{2}}$$

EXERCISE $\longrightarrow$

$$= e^{-\frac{K}{2}}\, \lambda cj\, \frac{B}{1+\frac{1-e^{-K}}{K}\lambda j}\, c\, e^{-\frac{K}{2}}$$

M. Schnabl 2007

KIERMAIER OKAWA RASTELLI ZWIEBACH 2007

Remember, the solution is <u>OK</u> as for $j(z)j(0)$ OPE <u>is REGULAR</u>.

<u>examples</u>: $j_1(z) \sim i\partial X^+(z)$ (light-cone current)

$j_2(z) \sim e^{\pm X^0}(z)$ (Rolling-tachyon)

<u>Exercise</u>: Show that $j_{\#}j$ is regular.

For most marginal deformations this is not true. The corresponding solutions has been addressed in

1) KIERMAIER, OKAWA, RASTELLI, ZWIEBACH: hep-th/0701249
2) FUCHS, KROYTER, POTTING: 0704.2222
3) KIERMAIER, OKAWA: 0707.4472

} PERTURBATIVE IN THE MARGINAL PARAMETER $\Psi_\lambda = \sum_M \lambda^M \Psi_M$

4) C.M. 1402.3546 } NON-PERTURBATIVE $\Psi_\lambda = \Psi(\lambda)$

All solutions 1) — 4) deal with the divergence in the j-j OPE with <u>APPROPRIATE REGULARIZATIONS</u>

[... a bit too advanced for these basic lectures ...]

The solution can be generalized for general (Tachyon vacuum) $F(K)$
[Erler 2007]

$$\Psi_\lambda = F(K)\,\lambda c j \, \frac{B}{1+\frac{1-F^2(K)}{K}\lambda j}\, c\, F(K)$$

Exercise: Show that $Q\Psi_\lambda + \Psi_\lambda^2 = 0$ [Use $Qj = \partial(cj) = [K, cj]$]

A striking simplification arises for $F^2(K) = \frac{1}{1+K}$

[KIERMAIER-OKAWA-SOLER 2010]

EXERCISE $F^2(K) = \frac{1}{1+K}$

$$\Psi_\lambda^{(KOS)} = \frac{1}{\sqrt{1+K}}\lambda c j \, \frac{B}{1+K+\lambda j}\,(1+K)\, c\, \frac{1}{\sqrt{1+K}} =$$

$$= \frac{1}{\sqrt{1+K}}\, c\,(1+K)\, \frac{B}{1+K+\lambda j}\,\lambda c j\, \frac{1}{\sqrt{1+K}}$$

EXERCISE

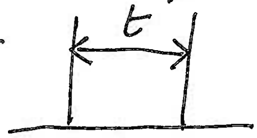
$$j\frac{1}{1+K+j}(1+K) = (1+K)\frac{1}{1+K+j}j = j - j\frac{1}{1+K+j}j$$

EXERCISE: Show that $Q\Psi_\lambda^{KOS} + \left(\Psi_\lambda^{KOS}\right)^2 = 0$

What are $\frac{1}{1+K}$ and $\frac{1}{1+K+j\lambda}$??

Schwinger parametrization

WEDGE-STATE OF WIDTH "t"

$$\frac{1}{1+K} = \int_0^\infty dt\, e^{-t}\, e^{-tK} = \int_0^\infty dt\, e^{-t}$$ 

it suppresses the (singular) $t\to\infty$ REGION.

↳ CONTINUOUS SUPERPOSITION OF WEDGE STATES.

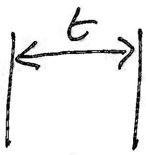
In the same way we can define

$$\frac{1}{1+K+j} = \int_0^\infty dt\, e^{-t}\, e^{-t(K+j\lambda)}$$

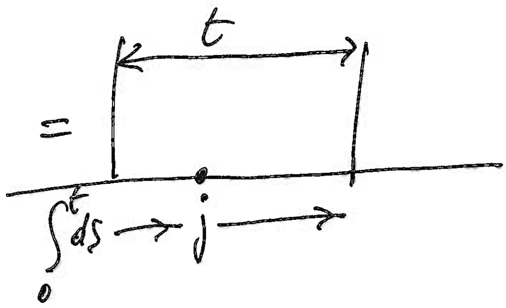
but what is $e^{-t(K+j\lambda)}$?

expand perturbatively:

$$e^{-t(K+\lambda j)} = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n}{d\lambda^n}\left(e^{-t(K+\lambda j)}\right)\Big|_{\lambda=0} \lambda^n$$

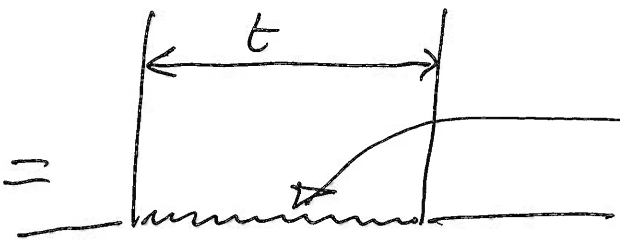
$n=0 \rightarrow e^{-tK} =$ 

$$n=1 \quad = \frac{d}{d\lambda}\left(e^{-t(K+\lambda j)}\right)\Big|_{\lambda=0} = \int_0^t ds\, e^{-sK}\, j\, e^{-(t-s)K} =$$

$=$ 

⋮

$\sum_{n=0}^{\infty} \longrightarrow$ INSERTION OF $e^{-\lambda\int_0^t ds\, j(s)}$!

$e^{-t(K+\lambda j)} =$ 

$$\underbrace{e^{-\lambda\int_0^t ds\, j(s)}}_{\text{BOUNDARY INTERACTION!!}}$$

This rings a bell!

Suppose I want to use the primary matter operator j to DEFORM the original world-sheet theory $BCFT_0$

$$S_{WS}^{(\lambda)} = \underbrace{\underbrace{\int_{DISK} dz\,d\bar{z}\ \partial X \bar{\partial} X}_{BCFT_0} + \lambda \int_{\partial DISK} ds\ j(s)}_{BCFT^{(\lambda)}}$$

$BCFT^{(\lambda)}$ → CONTINUOUS DEFORMATION OF THE INITIAL BOUNDARY CONDITIONS.

When I compute (for example) the DISK partition function of the deformed theory

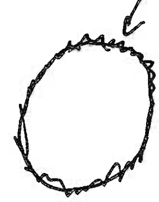
$$\langle 1 \rangle^{(\lambda)} = \int [dX \cdots]\, e^{-S_{WS}^{(\lambda)}} = \int [dX]\, e^{-S_{WS}^{(0)}}\, e^{-\lambda \int_{\partial D} ds\, j(s)} =$$

$$= \langle e^{-\lambda \int_{\partial D} ds\, j(s)} \rangle^{(0)}$$

... But this is precisely what I get by computing

$$\mathrm{Tr}\left[e^{-(K+\lambda j)}\right] = \langle e^{-\lambda \int_0^{2\pi} d\theta\, j(\theta)} \rangle^{(BCFT_0)}_{DISK} ! =$$

$e^{-\lambda \int ds\, j(s)}$



⇒ The solution $\Psi_\lambda^{(KOS)}$ DEFORMS the WS boundary conditions!

⇒ VERY DIRECT WAY TO SEE THAT OSFT solutions correspond to DIFFERENT BOUNDARY CONDITIONS!