SFT, May 8th 2015

SICHUAN UNIVERSITY

Plan

* 1 REWRITING OF Ψ_λ^{KOS} USING bcc operators
* 2 short detour on BOUNDARY FIELDS in BCFT
* 3 EM SOLUTION

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* 1 We have seen the KOS solution ($\lambda j \to j$) — ABSORB λ in j.

$$\Psi^{(KOS)} = \frac{1}{\sqrt{1+K}} c j \frac{B}{1+K+j}(1+K) c \frac{1}{\sqrt{1+K}}$$

$$\left\{ \begin{array}{l} [B,c]=1, \; c^2=B^2=1, \; [B,j]=[c,j]=0 \\ Qc = c\partial c, \; QB = K, \; Qj = \partial(cj) \end{array} \right\}$$

Let us now INTRODUCE a couple of $h=0$ matter fields $(\sigma, \bar\sigma)$ with the property that $\boxed{\bar\sigma\sigma = \sigma\bar\sigma = 1}$.

They are DEFINED (up to obvious relative normalization) by

$$\boxed{\sigma\partial\bar\sigma = \sigma[K,\bar\sigma] = j}$$

$$\left.\begin{array}{l} Q\sigma = c\partial\sigma \\ Q\bar\sigma = c\partial\bar\sigma \end{array}\right\} \text{BRST VARIATION of } h=0 \text{ primary matter fields}$$

Consider now

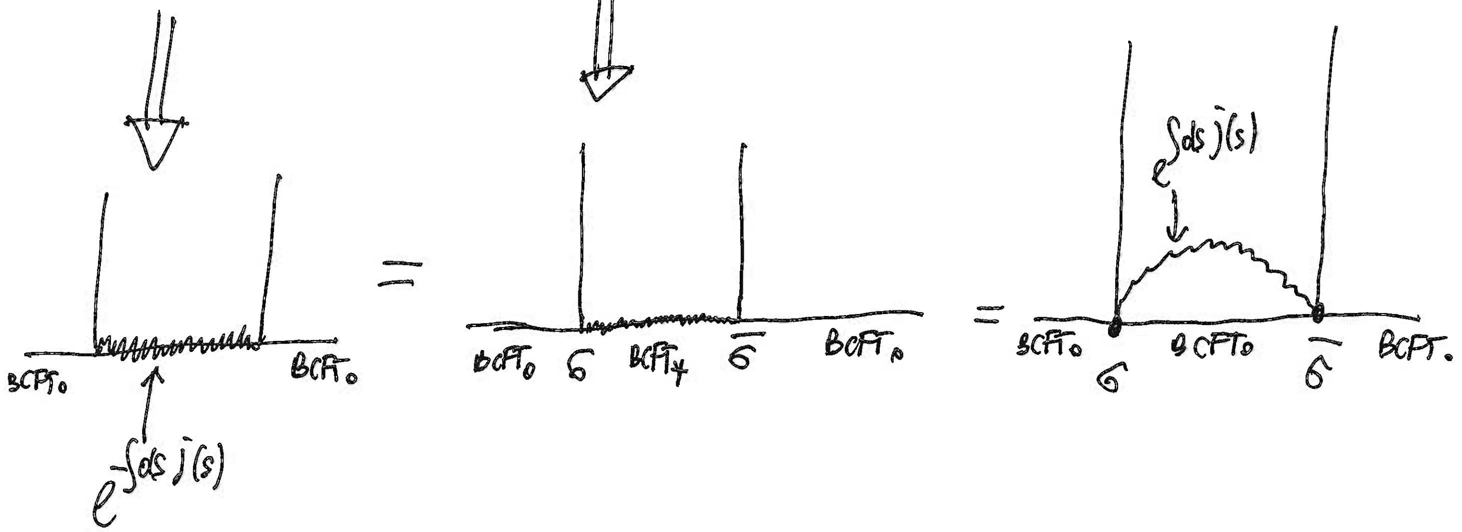
$$K + j = K + \sigma\partial\bar{\sigma} = K + \sigma\left(K\bar{\sigma} - \bar{\sigma}K\right) = \underbrace{(1-\sigma\bar{\sigma})}_{0}K + \sigma K\bar{\sigma}$$

<u>Exercise</u>:

- $(K+j)^n = \sigma K^n \bar{\sigma}$
- $f(K+j) = \sigma f(K)\bar{\sigma}$

<u>A new look of (marginally deformed) wedges</u>

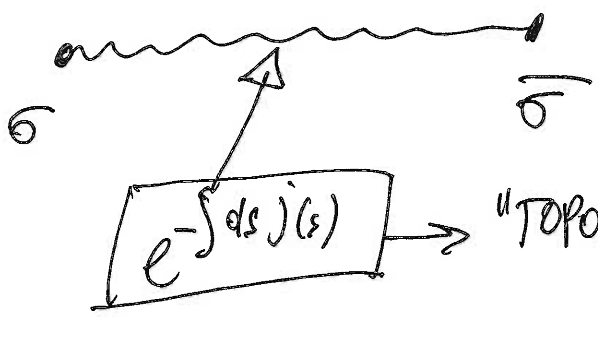
$$e^{-(K+j)} = \sigma e^{-K}\bar{\sigma}$$



$\sigma, \bar{\sigma} \Rightarrow$ <u>BOUNDARY CONDITION CHANGING OPERATORS</u>

$\Rightarrow$ THE ~~????~~ BOUNDARY CONDITION $BCFT_0$ IS CHANGED BY A TOPOLOGICAL DEFECT $e^{-\int ds\, j(s)}$

~~THE TOP. DEFECT END~~



$\rightarrow$ "TOPOLOGICAL DEFECT"

⇒ The BOUNDARY CONDITION CHANGING OPERATORS are THE <u>END-POINTS</u> OF THE TOPOLOGICAL DEFECT WHICH CHANGES THE BOUNDARY CONDITIONS.

—— , ——

The KOS solution can be REWRITTEN in terms of $\sigma, \bar{\sigma}$ to give:

$$\Psi^{(KOS)} = \frac{-1}{\sqrt{1+K}}\, c\partial\sigma \frac{B}{1+K}\, \bar{\sigma}(1+K)c \frac{1}{\sqrt{1+K}}$$

<u>Exercise</u> Show that $Q\Psi^{(KOS)} + \Psi^2 = 0$

by using K, B, c ⊕ $\sigma\bar{\sigma} = \bar{\sigma}\sigma = 1$ ⊕ $Q\sigma = c\partial\sigma$, $Q\bar{\sigma} = c\partial\bar{\sigma}$

⊕ $[B,\sigma] = [c,\sigma] = 0$

#2

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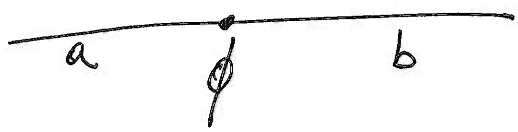
<u>BOUNDARY FIELDS</u>



Labeled by 3 indices

ϕ^{ab}_j

- a → LEFT-BOUNDARY CONDITION
- b → RIGHT-BOUNDARY CONDITION
- j → REPRESENTATION NUMBER + Multiplicity

$a \neq b$: ϕ^{ab} IS A BOUNDARY CONDITION CHANGING OPERATOR/FIELD.

⇒ Most general classification: j ⇒ IRREP OF THE VIRASORO ALGEBRA

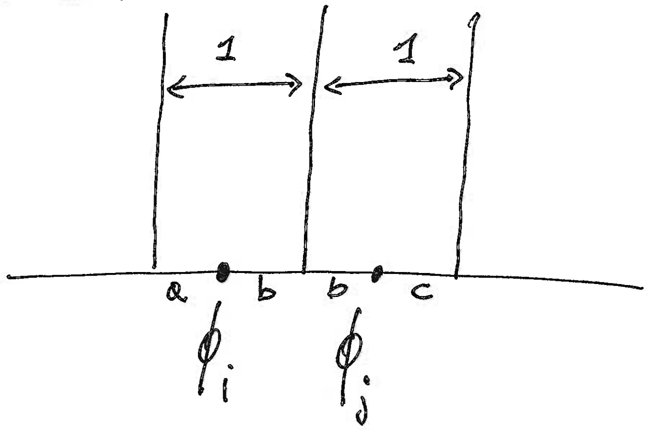
Boundary OPERATOR ALGEBRA

$$\phi_i^{ab}(x)\,\phi_j^{bc}(0) = \sum_k C_{ij}^{(abc)k}\,\phi_k^{ac}(0)\,x^{-h_i-h_j+h_k} + [\text{Virasoro Descendants}]$$

$C_{ij}^{(abc)k}$ ⇒ BOUNDARY STRUCTURE CONSTANTS

STAR PRODUCT

$$|\phi_i^{ab}\rangle * |\phi_j^{bc}\rangle =$$



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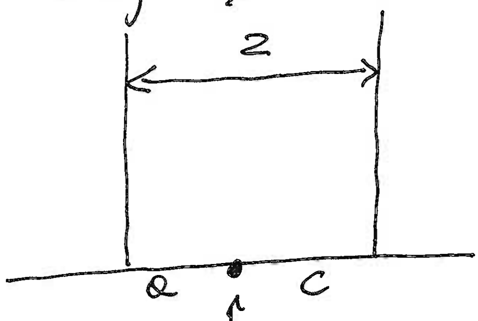
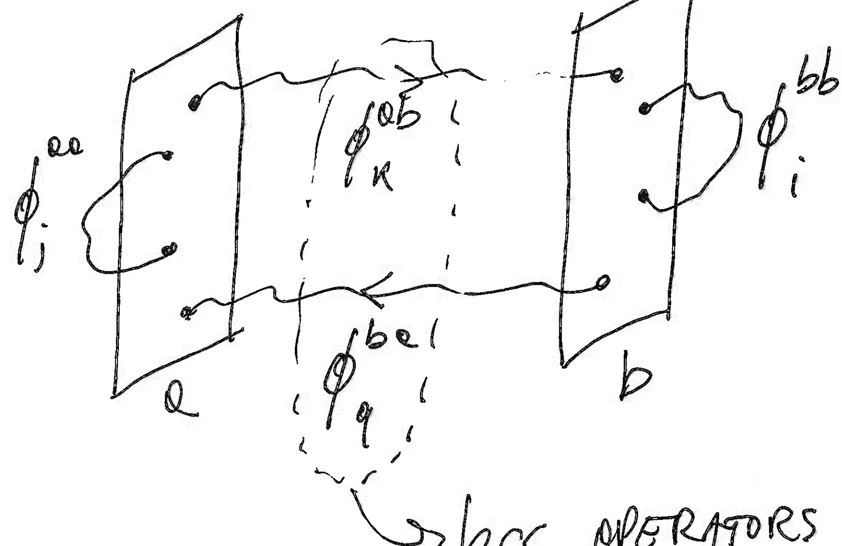


$\sum C_{ij}^{(abc)k}\,\phi_k^{ac} + \ldots$

⇒ IN THE ARCTAN-FRAME, THE STAR PRODUCT IS "JUST" THE BOUNDARY OPE!

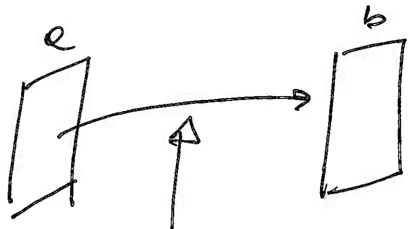
D-branes

→ Boundary conditions ↔ D-brane's systems



→ bcc OPERATORS: they are the STATES/OPERATORS for strings stretched between DIFFERENT D-branes.

(#3) Can we use generic $(\phi_i^{ab}, \phi_i^{ba})$ to get a OSFT solution



$\Psi_{a\to b} \,/\, Q\Psi+\Psi^2=0$ (??)

FURTHER REWRITING OF KOS

$$\Psi = -\frac{1}{\sqrt{1+K}}\, c\partial\sigma \frac{B}{1+K}\bar\sigma(1+K)c\frac{1}{\sqrt{1+K}} = *$$

$$\partial\sigma = [K,\sigma] = [1+K,\sigma] = (1+K)\sigma - \sigma(1+K)$$

$[\sigma\bar\sigma=1]$ ↓

$$* = \frac{1}{\sqrt{1+K}}c(1+K)Bc\frac{1}{\sqrt{1+K}} - \frac{1}{\sqrt{1+K}}c(1+K)\sigma\frac{B}{1+K}\bar\sigma(1+K)c\frac{1}{\sqrt{1+K}} \equiv \Psi_*$$

This last expression allows for a nicer representation and [MOST IMPORTANTLY] only NEEDS $\bar\sigma\sigma=1$ to solve the EOM ($\sigma\bar\sigma$ = NOT FIXED)

- Consider the tachyon vacuum solution (Erler-Schnabl)

$$\Psi_{TV} = \frac{1}{\sqrt{1+K}}c(1+K)Bc\frac{1}{\sqrt{1+K}}$$

- Consider

$$\Sigma \equiv Q_{TV}\left(\frac{B}{\sqrt{1+K}}\sigma\frac{1}{\sqrt{1+K}}\right) \qquad Q_{TV} = Q + [\Psi_{TV}, \cdot\,]$$

$$\bar\Sigma = Q_{TV}\left(\frac{B}{\sqrt{1+K}}\bar\sigma\frac{1}{\sqrt{1+K}}\right)$$

Exercise

Show that $\bar{\Sigma}\Sigma = \bar{\sigma}\sigma$ and $\Sigma\bar{\Sigma} = \sigma\bar{\sigma}$.

Use $[B, \sigma] = 0$ ⊕ $Q_{TV}\frac{B}{1+K} = 1$

Exercise: show that
→ Page 5

$\Psi_* = \Psi_{TV} - \Sigma \Psi_{TV} \bar{\Sigma}$ (ERLER, CM 2014)

EOM:

To show that $Q\Psi_* + \Psi_*^2 = 0$ it is enough to show that $Q_{TV}(-\Sigma\Psi_{TV}\bar{\Sigma}) + (-\Sigma\Psi_{TV}\bar{\Sigma})^2 = 0$ (Exercise: prove the above claim)

$$\Downarrow$$

$$-\Sigma(Q_{TV}\Psi_{TV})\bar{\Sigma} + \Sigma\Psi_{TV}(\bar{\Sigma}\Sigma)\Psi_{TV}\bar{\Sigma}$$

$$\downarrow$$

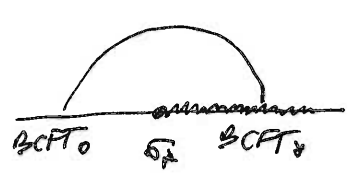
$$\Sigma\Big(\underbrace{-Q_{TV}\Psi_{TV} + \Psi_{TV}^2}_{=0}\Big)\bar{\Sigma} = 0$$

⇒ Ψ_* is a solution only using $\bar{\Sigma}\Sigma = \bar{\sigma}\sigma = 1$

⇒ Ψ_* is MORE GENERAL than $\Psi^{(KOS)}$.

$\Psi_\star^{(cm)}$ can be used to describe GENERIC OPEN STRING BACKGROUNDS [BUT TIME INDEPENDENT! (... as of today!...)]

$\sigma_\star, \bar\sigma_\star \Rightarrow$ BOUNDARY CONDITION CHANGING OPERATORS



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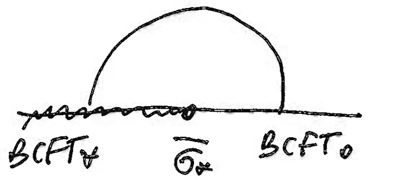
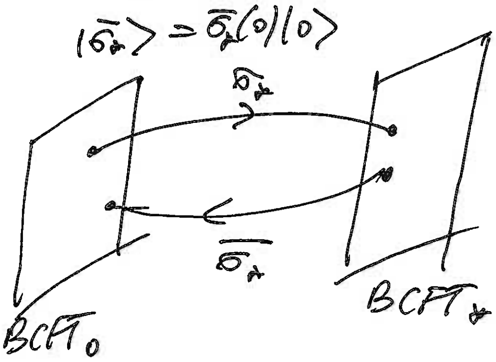
$|\sigma_\star\rangle = \sigma_\star(0)|0\rangle$



$|\bar\sigma_\star\rangle = \bar\sigma_\star(0)|0\rangle$



In general

• $\sigma_\star(s)\,\bar\sigma_\star(0) = C^{(0\star0)}_{\sigma\bar\sigma 1}\, s^{-2h_\sigma}\, 1^{(00)} + \sum_{k\neq 1} C^{(0\star0)}_{\sigma\bar\sigma k}\, \phi_k^{(00)}(0)\, s^{-2h_\sigma+h_k} + [\text{desc}\ldots]$

identity representation of Virasoro

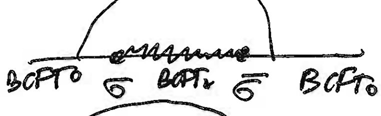
BOUNDARY FIELDS of BCFT0 [NOT CHANGING BC]

• $\bar\sigma_\star(s)\,\sigma_\star(0) = C^{(\star0\star)}_{\bar\sigma\sigma 1}\, s^{-2h_\sigma}\, 1^{(\star\star)} + \sum_{k\neq 1} C^{(\star0\star)}_{\bar\sigma\sigma k}\, \phi_k^{(\star\star)}(0)\, s^{-2h_\sigma+h_k} + [\text{desc}]$

BOUNDARY FIELDS OF BCFT* [NO BCC]

• $C^{(0\star0)}_{\sigma\bar\sigma k}$ $C^{(\star0\star)}_{\bar\sigma\sigma k}$ ARE BOUNDARY STRUCTURE CONSTANTS!

• Notice that $|\sigma\rangle \,\&\, |\bar\sigma\rangle \in \mathcal{H}_{BCFT_0}$



$|\bar\sigma\rangle \,\&\, |\sigma\rangle \in \mathcal{H}_{BCFT_\star}$

Problem: σ_* and $\bar{\sigma}_*$ in general carry non trivial weight (POSITIVE if the CFT is unitary)

$$\Rightarrow \quad \sigma_* \bar{\sigma}_* = \text{DIVERGENT!}$$
$$\bar{\sigma}_* \sigma_* = \text{DIVERGENT!}$$

[CANNOT BE USED in Ψ_*^{EM}]

BUT: Suppose that

$$BCFT_0^{(matter)} = BCFT_{Neumann}^{(X^0)} \otimes BCFT_0^{(c=25)}$$

$$BCFT_*^{(matter)} = BCFT_{Neumann}^{(X^0)} \otimes BCFT_*^{(c=25)}$$

i.e. $BCFT_0$ and $BCFT_*$ are TWO DIFFERENT TIME INDEPENDENT OPEN STRING BACKGROUNDS,

Then I can define

$$\sigma \equiv e^{i\sqrt{h_\sigma}X^0}\sigma_* \qquad (h=0 \quad c=26 \text{ primary})$$

($h = -h_\sigma + h_\sigma = 0$)

$$\tilde{\sigma} \equiv e^{-i\sqrt{h_\sigma}X^0}\tilde{\sigma}_* \qquad (h=0 \quad c=26 \text{ primary})$$

Now the $c=26$ OPE takes the form [Exercise: PROVE IT!]

$$\sigma(s)\,\tilde{\sigma}(0) = C_{\sigma\tilde{\sigma}1}^{(0*0)}\, \mathbb{1}^{(0,0)} + [\text{less-singular} \ldots]$$

$$\tilde{\sigma}(s)\,\sigma(0) = C_{\tilde{\sigma}\sigma 1}^{(*0*)}\, \mathbb{1}^{(**)} + [\text{less-singular}]$$

Then just define:

$$\bar{\sigma} = \frac{1}{C^{(\alpha 0 \alpha)}_{\bar{\sigma} \sigma 1}} \tilde{\sigma} \qquad \text{[Trivial normalization]}$$

to find

$$\bar{\sigma}(s)\, \sigma(0) = \mathbb{1}^{(\alpha\alpha)} + [\text{less-singular}]$$

$$\sigma(s)\, \bar{\sigma}(0) = \frac{C^{(0 \alpha 0)}_{\sigma \bar{\sigma} 1}}{C^{(\alpha 0 \alpha)}_{\bar{\sigma} \sigma 1}}\, \mathbb{1}^{(00)} + [\text{less singular}]$$

The ratio $\dfrac{C^{(0 \alpha 0)}_{\sigma \bar{\sigma} 1}}{C^{(\alpha 0 \alpha)}_{\bar{\sigma} \sigma 1}}$ can be proven [BCFT] to be $\dfrac{g_\alpha}{g_0}$

$$g_a \to \underline{g\text{-function}} \equiv \langle 1 \rangle^{BCFT_a}_{DISK} \equiv g_a$$

In particular the BOUNDARY STATE of $BCFT_a$ is of the form:

$$\| a \rangle\rangle = g_a | 1 \rangle\rangle + \ldots$$

↑ Ishibashi state of the identity representation.

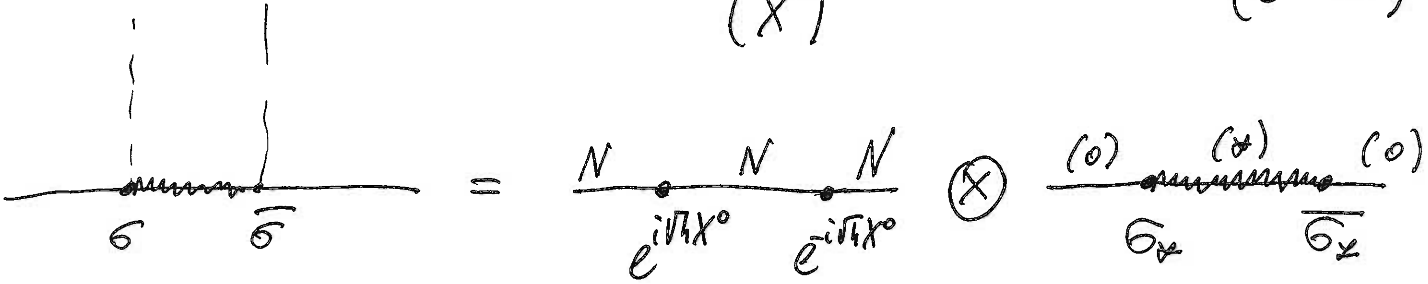
Therefore I can use $\sigma, \bar\sigma$ obeying

$$\begin{cases} \bar\sigma \sigma = 1 \\ \sigma \bar\sigma = \frac{g_*}{g_0} \end{cases} \quad \text{in} \quad \boxed{\Psi_*^{EM} = \Psi_{TV} - \Sigma \Psi_{TV} \bar\Sigma}$$

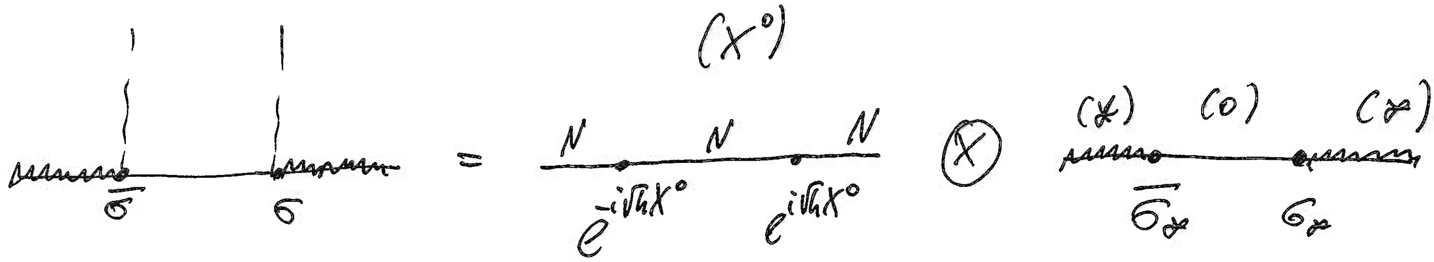
$$\Sigma = Q_{TV}(A\sigma) \qquad A^2 = 0$$
$$\bar\Sigma = Q_{TV}(A\bar\sigma) \qquad Q_{TV} A = 1$$

Physical effect of $\sigma, \bar\sigma$

(X^0) $\qquad (c=25)$



(X^0)



- If we take X^0 to be a NON-COMPACT FREE BOSON OF NEGATIVE SIGNATURE [TIME IS NON-COMPACT]

Then Neumann boundary conditions are UNIQUE [NO MODULI (Wilson lines) ASSOCIATED TO THEM]

$$||\text{Neumann}^{(\lambda)}\rangle\rangle^{(R)} = \sum_{w \in \mathbb{Z}} e^{i\lambda w} |w\rangle\rangle^{(R)}$$

$\uparrow$ windings $\qquad$ $\uparrow$ winding modes $U(1)$-Ishibashi states

But for non-compact $(R \to \infty)$ only $w=0$ is in the spectrum

$$||\text{Neumann}^{(\lambda)}\rangle\rangle^{(R=\infty)} = e^{i\lambda \cdot 0}|0\rangle\rangle = |0\rangle\rangle$$

The effect of $e^{i\sqrt{4}X^0}\, e^{-i\sqrt{4}X^0}$ is to switch-on a TIME-LIKE WILSON LINE which is physically INVISIBLE [IT COULD ONLY BE DETECTED BY WINDING MODES, which are DECOUPLED at $R\to\infty$].

$\Rightarrow$ The only effect of $\sigma, \bar\sigma$ is in the $c=25$ sector, where the Boundary Conditions are changed from $BCFT_0^{(c=0)} \longrightarrow BCFT_*^{(c=25)}$.

$\Rightarrow$ ALL TIME INDEPENDENT (FUNDAMENTAL) D-BRANES CONFIGURATIONS CAN BE CONNECTED BY $\Psi_*^{(EM)}$.

PEDAGOGICAL EXAMPLE

Ising Model D-branes

Ising Model CFT : Virasoro minimal model with $c=\frac{1}{2}$

REPRESENTATIONS : $1 \to h=0$, $\quad \sigma \to h=\frac{1}{16}$, $\quad \varepsilon \to h=\frac{1}{2}$

FUSION RULES
$1\times 1 = 1$
$\varepsilon\times\varepsilon = 1$
$\varepsilon\times\sigma = \sigma$
$\sigma\times\sigma = 1+\varepsilon$

Bulk fields :

$$1(z,\bar z) = 1(z)\otimes 1(\bar z) \qquad h=(0,0)$$

$$\sigma(z,\bar z) = \sigma(z)\otimes \sigma(\bar z) \qquad h=\left(\tfrac{1}{16},\tfrac{1}{16}\right)$$

$$\varepsilon(z,\bar z) = \varepsilon(z)\otimes \varepsilon(\bar z) \qquad h=\left(\tfrac{1}{2},\tfrac{1}{2}\right)$$

FUNDAMENTAL BOUNDARY CONDITIONS

They are labelled by the Virasoro irreps [This is common to Virasoro Minimal Models with DIAGONAL PARTITION FUNCTION

$$Z_{(q,\bar{q})} = \sum_i \chi_i^{(q)} \bar{\chi}_i^{(\bar{q})} \to \text{A-series}]$$

⇒ 3 boundary states

$$\|\mathbb{1}\rangle\rangle = \frac{1}{\sqrt{2}}|\mathbb{1}\rangle\rangle + \frac{1}{\sqrt{2}}|\varepsilon\rangle\rangle + \frac{1}{2^{1/4}}|\sigma\rangle\rangle$$

$$\|\varepsilon\rangle\rangle = \frac{1}{\sqrt{2}}|\mathbb{1}\rangle\rangle + \frac{1}{\sqrt{2}}|\varepsilon\rangle\rangle - \frac{1}{2^{1/4}}|\sigma\rangle\rangle$$

Z_2 symmetry

$$\|\sigma\rangle\rangle = |\mathbb{1}\rangle\rangle - |\varepsilon\rangle\rangle$$

Boundary Fields : They fall in REPRESENTATIONS dictated by the FUSION RULES (Cardy)

On $\|\mathbb{1}\rangle\rangle$: $\mathbb{1} \times \mathbb{1} = \mathbb{1}^{11}$

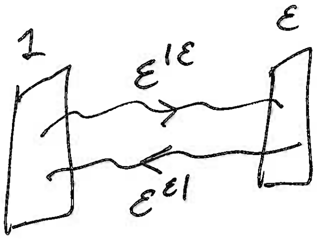
On $\|\varepsilon\rangle\rangle$: $\varepsilon \times \varepsilon = \mathbb{1}^{\varepsilon\varepsilon}$

On $\|\sigma\rangle\rangle$: $\sigma \times \sigma = \mathbb{1}^{\sigma\sigma} + \varepsilon^{\sigma\sigma}$

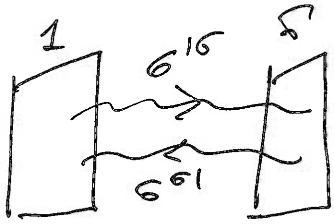
These boundary fields DON'T CHANGE THE BOUNDARY CONDITIONS.

Ising bcc-operators

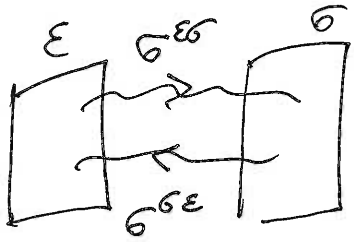
→ Stretched strings



$$1 \times \varepsilon = \varepsilon \nearrow \varepsilon^{1\varepsilon}, \quad \searrow \varepsilon^{\varepsilon 1}$$



$$1 \times \sigma = \sigma \nearrow \sigma^{1\sigma}, \quad \searrow \sigma^{\sigma 1}$$



$$\varepsilon \times \sigma = \sigma \nearrow \sigma^{\varepsilon\sigma}, \quad \searrow \sigma^{\sigma\varepsilon}$$

Define OSFT on the $\|1\rangle\rangle$-brane:

$$BCFT_0 = BCFT_{gh} \otimes BCFT^{(X^0)} \otimes BCFT_{1\!\!1}^{(Ising)} \otimes BCFT_0^{(c=25-\frac{1}{2})}$$

$$c=-26 \qquad c=1 \qquad c=\tfrac{1}{2} \qquad c=25-\tfrac{1}{2}$$

$$\Psi_{1\to\varepsilon} = \Psi_{TV} - \Sigma^{(1\varepsilon)}\,\Psi_{TV}\,\bar{\Sigma}^{(1\varepsilon)}$$

(ε-Brane described with 1-BRANE D.O.F.)

$$\Psi_{1\to\sigma} = \Psi_{TV} - \Sigma^{(1\sigma)}\,\Psi_{TV}\,\bar{\Sigma}^{(1\sigma)}$$

(σ-Brane described with 1-BRANE D.O.F)

$$\Sigma^{ab} = Q_{TV}\left(A\,\sigma^{ab}\right) \;;\; \sigma^{ab} = e^{i\sqrt{h_{ab}}X^0}\,\sigma_{*}^{ab}$$

$\Uparrow$ $BCFT^{Ising}$

More "realistic" changes in b.c. are described in EM (1406.3021)

→ NEUMANN - DIRICHLET TRANSITIONS ("TACHYON LUMPS")

→ MULTI-BRANES SOLUTIONS.

⇒ Challenges

(1) Eliminate the fake dependence on X^0

(2) Generalize to SUPER STRING FIELD THEORY

More ambitiously:

(3) We have shown that given $BCFT_0$, $BCFT_*$, there exists a solution $\Psi_{0\to *}$.

Can we show that given a solution on $BCFT_0$, this DEFINES a NEW $BCFT_*$?? (POSSIBLY UNKNOWN!)

THE END