Equivalence of open/closed strings

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Outlines

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Motiviations from relations between open/closed strings and their low energy effective theories:

1. Open/closed relation: relates the open/closed string metrics and couplings. The low energy effective version is Seiberg-Witten map, connecting non-commutative and commutative gauge theories.

2. Open/closed string channel duality: one-loop open string amplitudes = tree-level closed string amplitudes.

3. Gauge/Gravity: AdS/CFT, higher spin.

4. Gauge/Gauge: Electromagnetic duality, Montonen and Olive duality, Seiberg duality.

Motivations from M-theory

1. The five different string theories describe the same object from different limits.
2. Type I string includes both open and closed strings, but the other four have closed strings only.
3. In $D = 5$, M theory has $SO(5,5)$ symmetry.

D	E_D	H_D
3	$SL(3) \times SL(2)$	$SO(3) \times SO(2)$
4	$SL(5)$	$SO(5)$
5	$SO(5,5)$	$SO(5) \times SO(5)$
6	E_6	$USp(8)$
7	E_7	$SU(8)$
8	E_8	$SO(16)$

4. T-duality is $O(d,d;\mathbb{Z})$

Motivation

How about an intermediate theory: Polyakov $+ O(D, D)$?

Some points about the symmetries:

1. The continuous $O\left(D,D\right)$ symmetry is defined as $\Omega\eta\Omega^T=\eta$,

$$\eta_{MN}=\left(\begin{array}{cc}0 & 1\\ 1 & 0\end{array}\right).$$

2. Compactification of $d=D-n$ dimensions breaks the continuous $O\left(D,D\right)$ into an $O\left(n,n\right)\times O\left(d,d;\mathbb{Z}\right)$ group.
3. $O\left(n,n\right)$ relates flat background, and $O\left(d,d;\mathbb{Z}\right)$ represents T-duality in the compactified background.

Motivation

$O(D,D)$ invariant extension of Polyakov action is the Tseytlin's action (Tseytlin 1990PLB; 1991 NPB)

$$S = -\frac{1}{4\pi\alpha'}\int_\Sigma \left(-\partial_1 X^M \mathcal{H}_{MN}\partial_1 X^N + \partial_1 X^M \eta_{MN}\partial_0 X^N\right),$$

where $\partial_0 = \partial_\tau$, $\partial_1 = \partial_\sigma$ and

$$\mathcal{H}_{MN} = \begin{pmatrix} g & -gB^{-1} \\ B^{-1}g & g^{-1} - B^{-1}gB^{-1} \end{pmatrix}, \quad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad X^M = \begin{pmatrix} X^i \\ \tilde{X}_i \end{pmatrix},$$

where $M, N = 1, 2, \ldots, 2D$ are $O\left(D,D\right)$ indices,

g is D dimensional spacetime metric,

B is the anti-symmetric Kalb-Ramond field.

The boundary conditions

The EOM and boundary conditions can be obtained by varying the action,

$$\begin{aligned}\delta S &= -\frac{1}{2\pi\alpha'}\int_{\Sigma}\delta X^M\partial_1\left(\mathcal{H}_{MN}\partial_1 X^N-\eta_{MN}\partial_0 X^N\right)\\ &\quad -\frac{1}{2\pi\alpha'}\int_{\Sigma}\partial_1\left[\delta X^M\left(\mathcal{H}_{MN}\partial_1 X^N-\frac{1}{2}\eta_{MN}\partial_0 X^N\right)\right]\\ &\quad -\frac{1}{4\pi\alpha'}\int_{\Sigma}\partial_0\left[\delta X^N\eta_{MN}\partial_1 X^M\right],\end{aligned}$$

where we kept the spatial boundary for reasons becoming clear soon. For simplicity, we consider vanishing B field at first. The EOM is

$$\partial_1\left(\mathcal{H}_{MN}\partial_1 X^N-\eta_{MN}\partial_0 X^N\right)=0,$$

which leads to

$$\begin{aligned}g_{ij}\partial_1 X^j-\partial_0\tilde{X}_i &= f_1(\tau),\\ g^{ij}\partial_1\tilde{X}_j-\partial_0 X^i &= f_2(\tau).\end{aligned}$$

The boundary terms are:

$$\begin{aligned}\delta X^i\left(g_{ij}\partial_1 X^j-\frac{1}{2}\partial_0\tilde{X}_i\right)+\delta\tilde{X}_i\left(g^{ij}\partial_1\tilde{X}_j-\frac{1}{2}\partial_0 X^i\right)\Big|_{\sigma} &= 0,\\ \delta X^i\partial_1\tilde{X}_i+\delta\tilde{X}_i\partial_1 X^i|_{\tau} &= 0,\end{aligned}$$

The boundary conditions

B.C.

$$\delta X^i \left(g_{ij}\partial_1 X^j - \frac{1}{2}\partial_0 \tilde{X}_i \right) + \delta \tilde{X}_i \left(g^{ij}\partial_1 \tilde{X}_j - \frac{1}{2}\partial_0 X^i \right) \Bigg|_\sigma = 0,$$

$$\delta X^i \partial_1 \tilde{X}_i + \delta \tilde{X}_i \partial_1 X^i |_\tau = 0,$$

The usual selections of boundary condition are closed-closed or open-open:

1. Closed-closed boundary condition

$$\tilde{X}(\sigma,\tau) = \tilde{X}(\sigma + 2\pi, \tau), \qquad \text{and} \qquad X(\sigma,\tau) = X(\sigma+2\pi,\tau).$$

EOM

$$\begin{aligned} g_{ij}\partial_1 X^j - \partial_0 \tilde{X}_i &= 0, \\ g^{ij}\partial_1 \tilde{X}_j - \partial_0 X^i &= 0. \end{aligned}$$

- Unifying commutative/non-commutative closed strings.
- The low energy limit is DFT.

The boundary conditions

B.C.

$$\begin{aligned} \delta X^i \left(g_{ij} \partial_1 X^j - \frac{1}{2} \partial_0 \tilde{X}_i \right) + \delta \tilde{X}_i \left(g^{ij} \partial_1 \tilde{X}_j - \frac{1}{2} \partial_0 X^i \right) \bigg|_\sigma &= 0, \\ \delta X^i \partial_1 \tilde{X}_i + \delta \tilde{X}_i \partial_1 X^i |_\tau &= 0, \end{aligned}$$

2. Open-open boundary condition (Polyakov, Wang, Wu and Yang arXiv:1501.01550)

$$\partial_0 X|_\sigma = \partial_1 \tilde{X}|_\sigma = 0, \quad \text{or} \quad \partial_1 X|_\sigma = \partial_0 \tilde{X}|_\sigma = 0.$$

EOM

$$\begin{aligned} g_{ij} \partial_1 X^j - \partial_0 \tilde{X}_i &= 0, \\ g^{ij} \partial_1 \tilde{X}_j - \partial_0 X^i &= 0. \end{aligned}$$

- Unifying non-commutative/commutative open strings through open/closed relation.
- In low energy limit, they reduce to the non-commutative/commutative gauge theories, related by the Seiberg-Witten map.

Questions

- Is the open-closed configuration allowed?

- Why does the open/closed relation connect open-open or closed-closed but not open-closed, just as the name implies?

Crucial boundary condition

We missed the third $O(D, D)$ covariant boundary condition!

$$\left(g_{ij}\partial_1 X^j - \frac{1}{2}\partial_0 \tilde{X}_i\right)\Big|_\sigma = \left(g^{ij}\partial_1 \tilde{X}_j - \frac{1}{2}\partial_0 X^i\right)\Big|_\sigma = 0.$$

It is neither open nor closed B.C.

The boundary conditions

To consider the last boundary condition, we can again absorb $f_i(\tau)$ by shifting X and $\tilde{X}$

$$\begin{aligned} \tilde{X} &\rightarrow \tilde{X} - \int d\tau f_1(\tau), \\ X &\rightarrow X - \int d\tau f_2(\tau). \end{aligned}$$

Then the decoupled second order EOM is

$$\begin{aligned} ({\partial_1}^2 - {\partial_0}^2)X &= 0, \\ ({\partial_1}^2 - {\partial_0}^2)\tilde{X} &= 0, \end{aligned}$$

with the first order constraint,

$$\begin{aligned} g\partial_1 X - \partial_0 \tilde{X} &= 0, \\ g^{-1}\partial_1 \tilde{X} - \partial_0 X &= 0, \end{aligned}$$

and the boundary conditions (good news and bad news: B.C. is the same as EOM),

$$\begin{aligned} \delta X^i \left(g_{ij}\partial_1 X^j - \partial_0 \tilde{X}_i\right) + \delta\tilde{X}_i \left(g^{ij}\partial_1 \tilde{X}_j - \partial_0 X^i\right)|_\sigma &= 0, \\ g_{ij}\delta X^i \partial_0 X^j + g^{ij}\delta\tilde{X}_i \partial_0 \tilde{X}_j|_\tau &= 0. \end{aligned}$$

To see the picture clearer, we consider the string propagating between two Dp branes. We use the notations:

$$\begin{aligned} \mu, \nu, \ldots &= 0, \ldots, p, \\ a, b, \ldots &= p+1, \ldots, D-1. \end{aligned}$$

The boundary conditions become

$$\begin{aligned} \delta X^a\left(g_{ab}\partial_1 X^b - \partial_0\tilde{X}_a\right) + \delta\tilde{X}_a\left(g^{ab}\partial_1\tilde{X}_b - \partial_0 X^a\right) & \\ +\delta X^\mu\left(g_{\mu\nu}\partial_1 X^\nu - \partial_0\tilde{X}_\mu\right) + \delta\tilde{X}_\mu\left(g^{\mu\nu}\partial_1\tilde{X}_\nu - \partial_0 X^\mu\right)|_\sigma &= 0, \\ g_{ab}\delta X^a\partial_0 X^b + g^{ab}\delta\tilde{X}_a\partial_0\tilde{X}_b & \\ +g_{\mu\nu}\delta X^\mu\partial_0 X^\nu + g^{\mu\nu}\delta\tilde{X}_\mu\partial_0\tilde{X}_\nu|_\tau &= 0. \end{aligned}$$

How to get Open-Closed? Decoupling of X and $\tilde{X}$ near the boundary only!!

$$g_{ab}|_{\partial\Sigma} \gg 1 \qquad \text{or} \qquad g_{ab}|_{\partial\Sigma} \ll 1$$

General guidances:

- Near the boundaries, $g_{ab} \gg 1$ or $g_{ab} \ll 1$.
- Generalize Tseytlins action to nonlinear double sigma model.
- For D-branes, $g_{\mu\nu}$ is reciprocal of g_{ab} and $g_{ab} = 0$.
- Metric on D-branes is conformally flat.
- $D = 5$ from the symmetry group of M theory.
- Consistent with Einstein equation.

The only choice is AdS_5:

$$ds^2 = \frac{r^2}{c^2}\eta_{ab}dx^a dx^b + \frac{c^2}{r^2}dr^2$$

It is crucial to remember:

- X and $\tilde{X}$ are always $O(D,D)$ related.
- EOM

$$
\begin{aligned}
g\partial_1 X - \partial_0 \tilde{X} &= 0, \\
g^{-1}\partial_1 \tilde{X} - \partial_0 X &= 0,
\end{aligned}
$$

After realizing the decoupling of X and $\tilde{X}$ near the boundaries, it is easy to understand that open/closed strings are $O(D,D)$ equivalent!

Open-Closed configuration

Both compact and non-compact open string worldsheets respect the B.C. Let us focus on the compact case. To maintain the same dimensionalities of the D-branes under different limits, we can choose

- Near $r \gg c$, X open and $\tilde{X}$ closed.
- Near $r \ll c$, X closed and $\tilde{X}$ open.

or vice versa

Open-Closed configuration

We then have four B.C. :
For $r \gg c$:

$$\text{①}: \quad \begin{aligned} \eta_{ab}\partial_1 X^b|_\sigma &= 0, \\ X^\mu|_\sigma &= Y^\mu, \\ X(\sigma,\tau) &= X(\sigma,\tau+2\pi t). \end{aligned}$$

$$\text{②}: \quad \begin{aligned} \eta^{ab}\partial_0 \tilde{X}_b|_\tau &= 0, \\ \tilde{X}_\mu|_\tau &= \tilde{Y}_\mu, \\ \tilde{X}(\sigma,\tau) &= \tilde{X}(\sigma+2\pi,\tau). \end{aligned}$$

For $r \ll c$

$$\text{③}: \quad \begin{aligned} \eta_{ab}\partial_0 X^b|_\tau &= 0, \\ X^\mu|_\tau &= Y^\mu, \\ X^\mu(\sigma,\tau) &= X^\mu(\sigma+2\pi,\tau), \end{aligned}$$

$$\text{④}: \quad \begin{aligned} \eta^{ab}\partial_1 \tilde{X}_b|_\sigma &= 0, \\ \tilde{X}_\mu|_\sigma &= \tilde{Y}_\mu, \\ \tilde{X}(\sigma,\tau) &= \tilde{X}\left(\sigma,\tau+2\pi\frac{1}{t}\right). \end{aligned}$$

Open-Closed configuration

The $O(D, D)$ equivalent configurations are

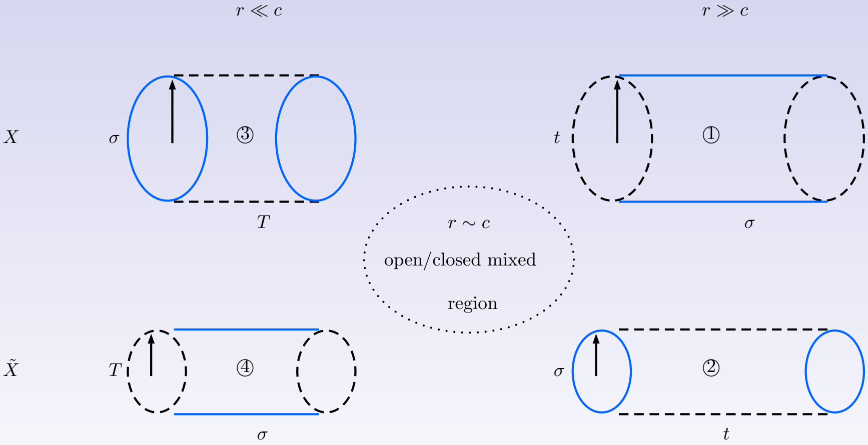

Figure : The four configurations in different limits. The solid line denotes the string and dashed line represents the propagation.

The $B \neq 0$ situation is also valid once fixing gB^{-1} when approaching the asymptotical regions. The open/closed relation is defined as

$$
\begin{aligned}
\hat{g} &\equiv \frac{1}{g^{-1} - B^{-1}} g^{-1} \frac{1}{g^{-1} + B^{-1}}, \\
\hat{B} &\equiv -\frac{1}{g^{-1} - B^{-1}} B^{-1} \frac{1}{g^{-1} + B^{-1}}.
\end{aligned}
$$

Open-Closed configuration

The $O(D, D)$ relations of the four configurations are

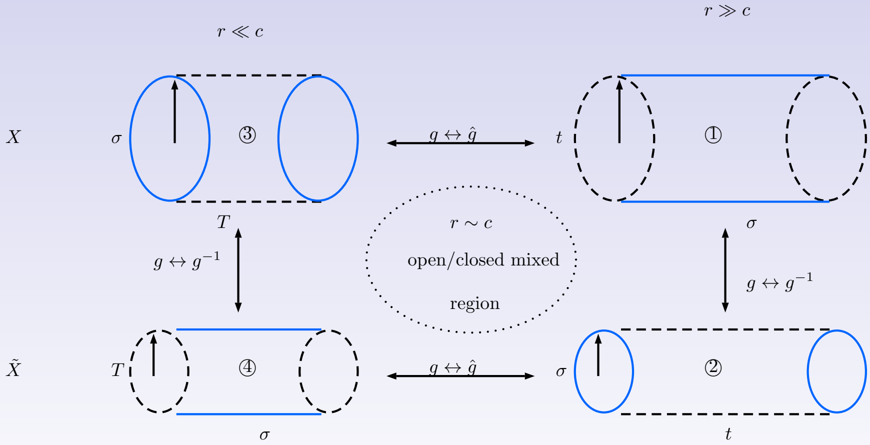

Figure : The $O(D, D)$ transformations between the four configurations. The solid line denotes the string and dashed line represents the propagation.

Low energy limit

The couplings of low energy effective theories are determined by the separation of D-branes.

	$r \ll c$	$r \gg c$
X	③ closed string (near-horizon geometry)	① open string (on the boundary)
$\tilde{X}$	④ open string (on the boundary)	② closed string (near-horizon geometry)

Table : The properties of the four configurations

	Short r	Long r
Open string	④ weak gauge theory	① strong gauge theory
Closed string	③ strong gravitational interaction	② weak gravitational interaction

Table : The corresponding low energy effective theories

Open-Closed configuration

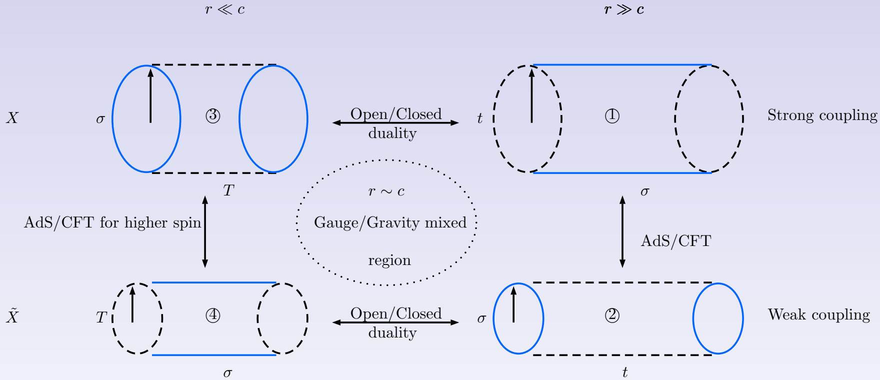

Figure : Open/closed string connections

- (① ↔ ④) is the Seiberg-Witten map.
- (② ↔ ③) is the scale-factor duality in string cosmology.

Comments and discussions

1. We proved that open/closed strings are $O(D, D)$ equivalent. But the whole physical picture is still blurry. More explorations are needed.

2. We are free to have X open on both limits and $\tilde{X}$ closed accordingly. The dimensionalities of the D-branes is then changed from 3 to 2 under different limits. Does this mean a duality between different dimensionalities?

3. For the non-compact open string worldsheet case, it is perfectly ok to have a single D-brane present but not two. Does this correspond to higher spin theory or sth. we are still unaware of?

4. Is the Tseytlins action a must to derive our results? Maybe not. The most important ingredient is the $O(D, D)$ symmetry. It may be possible to circumvent double fields to realize the $O(D, D)$ symmetry. This philosophy has been extensively employed in the studies of string cosmology.
   However, for the third B.C. we applied, the Tseytlin's action can not reduce to the Polyakov action. So before answering this question definitely, we need more works.

Comments and discussions

7. The $r \sim c$ region, where the string is in mixed states of open and closed, is of great importance and interest. It is prompt to study it carefully and one can expect some non-trivial information can be extracted.

8. Constructing the low energy effective theories of the Tseytlin's action is of course very important. It may give us some instructions to verify the various dualities.

9. The dS/CFT correspondence is not compatible with our derivations, but not excluded in other dimensions.

10. It would be of interest to incorporate SUSY into the theory. One may get more information about the required geometry, say, like $AdS_5 \times S^5$? It will provide more evidence for the theory.

Comments and discussions

9. Can we apply this open/closed equivalence to attack the CSFT problems by transferring them to the corresponding OSFT problems?

10. Since we do not know how to define M theory yet, it is a promising way to generalize the Polyakov theory to E_D covariant theories. It is reasonable to expect some non-perturbative features may be captured in these extensions. Furthermore, they may be also of help to the construction of M theory itself.

11. It is very important to consider the B flux to verify the equivalence in the next step.

Thank you!

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