Asymptotic symmetries of higher-spin gauge theories: the metric approach

Andrea Campoleoni
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A.C., M. Henneaux, arXiv:1412.6774

& work in progress with M. Henneaux, S. Hörtner, A. Leonard

International Conference on String Field Theory and Related Aspects VII, Chengdu, China, 15/5/2015

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3D

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Asymptotic symmetries in D=2+1

- The asymptotic symmetries of 3D (higher-spin) gravity are remarkably rich
  - pure gravity → two copies of the Virasoro algebra — Brown, Henneaux (1986)
  - $s = 2,3,\ldots,N$ → two copies of the W_N algebra — Henneaux, Rey; A.C., Pfenninger, Fredenhagen, Theisen (2010)
  - $s = 2,3,\ldots,\infty$ → one parameter family of W_∞ algebras — Gaberdiel, Hartman (2011)
- This richness triggered several developments in the last few years
  - AdS_3/CFT_2: minimal model holography — Gaberdiel, Gopakumar (2010)
  - Exact solutions: black holes with HS charges — Gutperle, Kraus (2011)

What's special about D=2+1?

- Gauge sector described by a Chern-Simons action

Achúcarro, Townsend (1986); Witten (1988)

$$\begin{cases} e = \frac{\ell}{2}(A - \tilde{A}) \\ \omega = \frac{1}{2}(A + \tilde{A}) \end{cases}$$

$$I_{EH} = I_{CS}[A] - I_{CS}[\tilde{A}]$$

  - from gravity to higher spins: sl(2) → sl(N) (or hs[λ])

Blencowe (1989)

  - asymptotic symmetries ↔ Hamiltonian (aka DS) reduction

Balog, Fehér, O'Raifeartaigh, Forgács, Wipf (1990)

- Gain: full non-linear theory & action principle

- Loss: ad hoc techniques, non directly applicable neither in higher space-time dimensions nor in 3D extensions

  - Matter couplings (crucial in minimal model holography!)

Prokushkin, Vasiliev (1999)

  - Topological mass

Chen, Long, Wu; Bagchi, Lal, Saha, Sahoo (2011)

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Asymptotic symmetries beyond Chern-Simons

- One can describe higher spins (at least) in two ways
  - Frame-like setup: $e_\mu{}^{a_1\cdots a_{s-1}}\,,\ \omega_\mu{}^{b,\,a_1\cdots a_{s-1}}\,,\ \Theta_\mu{}^{b_1\cdots b_k,\,a_1\cdots a_{s-1}}$
  - Metric-like setup: $\varphi_{\mu_1\cdots\mu_s}$

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  - In D = 2+1: { frame-like → Chern-Simons action; metric-like → two-derivative action }
- Goal of this talk: revisit asymptotic symmetries in D = 2+1 in the metric formulation
  - For the moment only gauge fields of spin s≥2 (like in CS)
  - Similar techniques can be used in higher space dimensions, or with matter couplings or topological masses

More on the strategy

- One could ignore the CS approach and work directly with metric-like variables
  - One can (at least in principle) reconstruct the action perturbatively…

    $$I = I_2[\phi^2] + I_3[\phi^3] + \cdots$$

  - …and set up boundary conditions for the fields

    $$\phi = r^\alpha h_0(x^i) + r^{\alpha-2} h_1(x^i) + \cdots$$

  - We'll see that higher order corrections in the action tend to be subleading at the boundary (one doesn't need the full action!)
- This procedure requires a certain amount of guessing work: better to check each step comparing with CS!

Outline

- **3D gravity coupled to a spin-3 field**
  - Comparison between Chern-Simons and metric-like
  - Asymptotic symmetries in the metric-like formulation

- **Intermezzo: charges from Fronsdal Hamiltonian**

- **Including higher spins**
  - Role of HS interactions: the spin-4 example
  - Conformal Killing tensors revisited

Chern-Simons
vs
metric-like*

*up to spin-3

Rewriting of the Chern-Simons action

- Chern-Simons action (focus on sl(3,R)⊕sl(3,R))

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3\ell^2}\, e\wedge e\wedge e\right)$$

$$\begin{cases} e = \left(e_\mu{}^a J_a + e_\mu{}^{ab}\, T_{ab}\right) dx^\mu \\ \omega = \left(\omega_\mu{}^a J_a + \omega_\mu{}^{ab}\, T_{ab}\right) dx^\mu \end{cases}$$

sl(3,R) algebra:

$$[J_a, J_b] = \epsilon_{abc} J^c$$

$$[J_a, T_{bc}] = \epsilon^m{}_{a(b} T_{c)m}$$

$$[T_{ab}, T_{cd}] = -\left(\eta_{a(c}\epsilon_{d)bm} + \eta_{b(c}\epsilon_{d)am}\right) J^m$$

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- Gauge transformations

$$\delta e = d\xi + [\,\omega\,,\,\xi\,] + [\,e\,,\,\Lambda\,]\,,$$
$$\delta\omega = d\Lambda + [\,\omega\,,\,\Lambda\,] + \frac{1}{\ell^2}\,[\,e\,,\,\xi\,]$$

- Metric-like fields

A.C., Pfenninger, Fredenhagen, Theisen (2010)

$$g = \frac{1}{2}\,\mathrm{tr}\,(\,e_\mu e_\nu\,)\,dx^\mu dx^\nu\,, \qquad \phi = \frac{1}{6}\,\mathrm{tr}\,(\,e_\mu e_\nu e_\rho\,)\,dx^\mu dx^\nu dx^\rho$$

Metric-like action and gauge transformations

- Action (spin-3 field coupled to gravity)

A.C., Pfenninger, Fredenhagen, Theisen (2012)

$$I_{\{3\}} = \int \frac{d^3x\,\sqrt{-g}}{16\pi G}\left\{\left(R + \frac{2}{\ell^2}\right) + \phi^{\mu\nu\rho}\left(\mathcal{F}_{\mu\nu\rho} - \frac{3}{2}\,g_{(\mu\nu}\,\mathcal{F}_{\rho)}\right) + \mathcal{L}_{NM}\right\} + \mathcal{O}(\phi^4)$$

Covariantized Fronsdal operator:

$$\mathcal{F}_{\mu\nu\rho} = \Box\,\phi_{\mu\nu\rho} - \frac{3}{2}\left(\nabla^\lambda\nabla_{(\mu}\,\phi_{\nu\rho)\lambda} + \nabla_{(\mu}\nabla^\lambda\,\phi_{\nu\rho)\lambda}\right) + 3\,\nabla_{(\mu}\nabla_\nu\,\phi_{\rho)}$$

L_{NM} collects all terms of the form Rϕϕ

- Gauge transformations

$$\delta_2\,g_{\mu\nu} = \nabla_{(\mu}v_{\nu)} \qquad \delta_2\,\phi_{\mu\nu\rho} = v^\alpha\partial_\alpha\phi_{\mu\nu\rho} + \phi_{\alpha(\mu\nu}\partial_{\rho)}v^\alpha$$

$$\delta_3\,g_{\mu\nu} = \mathcal{O}(\phi) \qquad \delta_3\,\phi_{\mu\nu\rho} = \nabla_{(\mu}\xi_{\nu\rho)} + \mathcal{O}(\phi^2)$$

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- Drawback of the metric formulation: higher orders in ϕ not under control

- Not a problem for asymptotic symmetries: they are subleading in r

- Gauge transformations

$$\delta_2\, g_{\mu\nu} = \nabla_{(\mu} v_{\nu)} \qquad \delta_2\, \phi_{\mu\nu\rho} = v^\alpha \partial_\alpha \phi_{\mu\nu\rho} + \phi_{\alpha(\mu\nu}\partial_{\rho)} v^\alpha$$

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Asymptotic symmetries: general strategy

- Impose boundary conditions on the fields
- Look for gauge variations that preserve them
- Compute charges

Chern-Simons	Metric-like
fix $A_\mu(r,x^i)$ and look for λ s.t.	fix $g_{\mu\nu}(r,x^i)$, $\phi_{\mu\nu\rho}(r,x^i)$ and look for v^μ, $\xi^{\mu\nu}$
$\delta A = d\lambda + [\,A, \lambda\,] \sim A$	$\delta_2\, \phi_{\mu\nu\rho} = v^\alpha \partial_\alpha \phi_{\mu\nu\rho} + \phi_{\alpha(\mu\nu}\partial_{\rho)} v^\alpha \sim \phi_{\mu\nu\rho}$ $\delta_3\, \phi_{\mu\nu\rho} = \nabla_{(\mu}\xi_{\nu\rho)} + \mathcal{O}(\phi^2) \sim \phi_{\mu\nu\rho}$
Charges are manifestly finite, they don't depend on r thanks to a smart (but standard) gauge fixing	Harder to compute charges, but one can get them by comparison with CS

Asymptotic symmetries: general strategy

- Impose boundary conditions on the fields ←
- Look for gauge variations that preserve them ←
- Compute charges

We'll use info from CS

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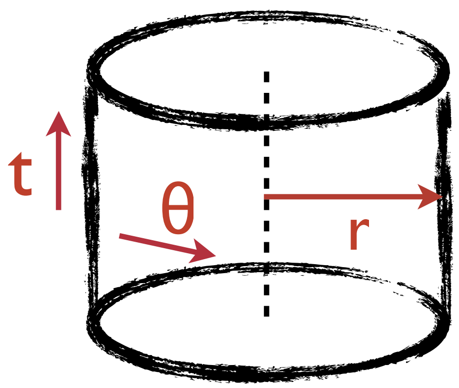
Boundary conditions I

- Chern-Simons: $$A = b^{-1} a_+ b \, dx^+ + b^{-1} db$$

Henneaux, Rey; A.C., Pfenninger, Fredenhagen, Theisen (2010)

$$a_+ = L_1 - \frac{2\pi}{k} \mathcal{L}(x^+) \, L_{-1} - \frac{\pi}{2k} \mathcal{W}(x^+) \, W_{-2}$$



$$x^\pm = \frac{t}{l} \pm \theta$$

sl(3,R) algebra:

$$[L_i, L_j] = (i-j) L_{i+j}$$

$$[L_i, W_m] = (2i - m) W_{i+m}$$

$$[W_m, W_n] = \frac{n-m}{3} (2m^2 + 2n^2 - mn - 8) L_{m+n}$$

-1 ≤ i,j ≤ 1 and -2 ≤ m,n ≤ 2

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- Relations among gauge connections and Fronsdal fields

$$g \, = \, \frac{1}{2} \, \mathrm{tr} \left(e_\mu e_\nu \right) dx^\mu dx^\nu \, , \qquad \phi \, = \, \frac{1}{6} \, \mathrm{tr} \left(e_\mu e_\nu e_\rho \right) dx^\mu dx^\nu dx^\rho$$

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$$b(r) = e^{\log\left(\frac{r}{\ell}\right) L_0} \;\; \Rightarrow \;\; \begin{cases} g \, = \, \frac{dr^2}{r^2} + \left\{ \frac{r^2}{2} \, \eta_{ij} + \frac{2\pi}{k} \mathcal{L}_{ij}(x^k) \right\} dx^i dx^j + \mathcal{O}(r^{-2}) \\ \phi \, = \, \frac{\pi}{4k} \, \mathcal{W}_{ijk}(x^k) dx^i dx^j dx^k + \mathcal{O}(r^{-2}) \end{cases}$$

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Boundary conditions II

- Can we derive/guess these boundary conditions directly in the metric formulation?
- Consider the free eom:

$$\begin{cases} \left[\Box - \dfrac{s^2+(d-6)s-2(d-3)}{\ell^2}\right]\phi_{\mu_1\cdots\mu_s} = 0 \\ \nabla\cdot\phi_{\mu_1\cdots\mu_{s-1}} = 0 \\ \phi_{\mu_1\cdots\mu_{s-2}\alpha}{}^{\alpha} = 0 \end{cases}$$

- <u>They admit two solutions</u>

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Conserved current	Source/shadow
$\varphi_{i_1\cdots i_s} = \mathcal{W}_{i_1\cdots i_s}(x^n) + \mathcal{O}(r^{-2})$	$\varphi_{i_1\cdots i_s} = r^{2(s-1)}\mathcal{S}_{i_1\cdots i_s}(x^n) + \mathcal{O}(r^{-2})$
$\varphi_{r\cdots r\, i_1\cdots i_{s-k}} = \mathcal{O}(r^{-3k})$	$\varphi_{r\cdots r\, i_1\cdots i_{s-k}} = \mathcal{O}(r^{2(s-1)-3k})$
with	with
$\partial \cdot \mathcal{W}_{ij\cdots} = \eta^{kl}\mathcal{W}_{klij\cdots} = 0$	$\delta\mathcal{S}_{i_1\cdots i_s} = s\{\partial_{(i_1}\epsilon_{i_2\cdots i_s)} - \eta_{(i_1 i_2}\partial\cdot\epsilon_{i_3\cdots i_s)}\}$

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Asymptotic symmetries*

*up to spin-3

Boundary conditions III

- Boundary conditions in <u>contravariant</u> form

$$g^{rr} = r^2 + \mathcal{O}(r^{-2}) \qquad \phi^{rrr} = \mathcal{O}(r^{-3})$$

$$g^{ri} = \mathcal{O}(r^{-3}) \qquad \phi^{rri} = \mathcal{O}(r^{-4})$$

$$g^{ij} = \frac{2}{r^2}\,\eta^{ij} - \frac{8\pi}{k\,r^4}\,\mathcal{L}^{ij}(x^k) + \mathcal{O}(r^{-6}) \qquad \phi^{rij} = \mathcal{O}(r^{-5})$$

$$\phi^{ijk} = \frac{2\pi}{k\,r^6}\,\mathcal{W}^{ijk}(x^k) + \mathcal{O}(r^{-8})$$

- Both $\mathcal{L}^{ij}$ and $\mathcal{W}^{ijk}$ are traceless and conserved

$$\mathcal{L}_{++} = \mathcal{L}(x^+)\,, \qquad \mathcal{L}_{++} = \tilde{\mathcal{L}}(x^-)\,, \qquad \mathcal{L}_{+-} = 0$$

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Gauge transformations

- Gauge transformations:

$$\delta_2\, g^{\mu\nu} = [\, g, v\,]^{\mu\nu} \qquad \delta_2\, \phi^{\mu\nu\rho} = [\, \phi, v\,]^{\mu\nu\rho}$$

$$\delta_3\, g^{\mu\nu} = \mathcal{O}(\phi) \qquad \delta_3\, \phi^{\mu\nu\rho} = [\, g, \xi\,]^{\mu\nu\rho} + \mathcal{O}(\phi^2)$$

Schouten bracket:

$$[v, w]^{\mu_1 \cdots \mu_{p+q-1}} = \frac{(p+q-1)!}{p!\, q!} \left(p\, v^{\alpha(\mu_1 \cdots} \partial_\alpha w^{\cdots \mu_{p+q-1})} - q\, w^{\alpha(\mu_1 \cdots} \partial_\alpha v^{\cdots \mu_{p+q-1})} \right)$$

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- We assume that <u>asymptotic Killing tensors</u> (i.e. the gauge parameters that preserve the boundary conditions) behave at r→∞ as the exact Killing tensors of AdS_3

$$v^r = r\,\zeta(x^k) + \frac{\zeta_1(x^k)}{r} + \cdots \qquad \xi^{rr} = r^2\,\lambda(x^k) + \lambda_1(x^k) + \cdots$$

$$v^i = \epsilon^i(x^k) + \frac{{\epsilon_1}^i(x^k)}{r^2} + \cdots \qquad \xi^{ri} = r\,w^i(x^k) + \frac{{w_1}^i(x^k)}{r} + \cdots$$

$$\xi^{ij} = \chi^{ij}(x^k) + \frac{{\chi_1}^{ij}(x^k)}{r^2} + \cdots$$

Warming up with pure gravity

- Variations of the components of the metric:

$$\delta g^{rr} = -4\,\zeta_1 + \mathcal{O}(r^{-2})\,,$$

$$\delta g^{ri} = \frac{2}{r}\,\{-\,\epsilon_1{}^i + \partial^i\zeta\} + \mathcal{O}(r^{-3})\,,$$

$$\delta g^{ij} = \frac{4}{r^2}\{\partial^{(i}\epsilon^{j)} + \eta^{ij}\zeta\} + \frac{4}{r^4}\left\{\partial^{(i}\epsilon_1{}^{j)} + \eta^{ij}\zeta_1 - \frac{2\pi}{k}[\,\mathcal{L},\epsilon\,]^{ij} - \frac{8\pi}{k}\mathcal{L}^{ij}\zeta\right\} + \mathcal{O}(r^{-6})\,.$$

- Resulting conditions:

$$2\,\partial^{(i}\epsilon^{j)} - \eta^{ij}\partial\cdot\epsilon = 0\,,\qquad \zeta = -\frac{1}{2}\,\partial\cdot\epsilon\,.$$

$$\epsilon^+ = \epsilon(x^+)\,,\quad \epsilon^- = \tilde{\epsilon}(x^-)$$

i.e. ϵ^i is a conformal Killing vector for the flat boundary metric

- Action on the space of solutions:

$$\delta\mathcal{L}^{ij} = [\,\mathcal{L},\epsilon\,]^{ij} - 2\,\mathcal{L}^{ij}\partial\cdot\epsilon + \frac{k}{4\pi}\,\partial^i\partial^j\partial\cdot\epsilon$$

δΦ under spin-3 transformations

- What remains similar
  - $\delta\phi^{ijk} = \frac{6}{r^2}\left\{\partial^{(i}\chi^{jk)} + 2\,\eta^{(ij}w^{k)}\right\} + \mathcal{O}(r^{-4})\,.$
  - χ^{ij} is a conformal Killing tensor

$$\partial^{(i}\chi^{jk)} - \frac{1}{2}\,\eta^{(ij}\partial\cdot\chi^{k)} = 0 \;\rightarrow\; \chi^{++} = \chi(x^+)\,,\quad \chi^{--} = \tilde{\chi}(x^-)\,,\quad \chi^{+-} = 0$$

- What changes

$$h^{ij} = -\,\partial^{(i}h^{j)r} - \frac{1}{2}\,\eta^{ij}h^{rr} + \frac{24\pi^2}{k^2}\,\mathcal{L}^i{}_k\mathcal{L}^{jk}$$

$$g^{rr} = r^2 + r^{-2}\,h^{rr} + \mathcal{O}\left(r^{-4}\right)\,,$$

$$g^{ri} = r^{-3}\,h^{ri} + \mathcal{O}\left(r^{-5}\right)\,,$$

$$g^{ij} = \frac{2}{r^2}\,\eta^{ij} - \frac{8\pi}{k\,r^4}\,\mathcal{L}^{ij} + r^{-6}\,h^{ij} + \mathcal{O}\left(r^{-8}\right)$$

δΦ under spin-3 transformations

- What remains similar
  - $\delta\phi^{ijk} = \frac{6}{r^2}\{\partial^{(i}\chi^{jk)} + 2\,\eta^{(ij}w^{k)}\} + \mathcal{O}(r^{-4})\,.$
  - χ^{ij} is a conformal Killing tensor

$$\partial^{(i}\chi^{jk)} - \frac{1}{2}\eta^{(ij}\partial\cdot\chi^{k)} = 0 \;\rightarrow\; \chi^{++} = \chi(x^+)\,, \quad \chi^{--} = \tilde{\chi}(x^-)\,, \quad \chi^{+-} = 0$$

- What changes
  - one has to "dig deeper into the variation" → non-linearities

    e.g. $\lambda = \frac{1}{12}\partial\cdot\partial\cdot\chi - \frac{2\pi}{3k}\mathcal{L}_{ij}\chi^{ij}$
  - the subleading components of the metric enter the variation → the tracelessness of $\mathcal{W}^{ijk}$ is preserved only on shell

δg under spin-3 transformations

- Unfortunately not everything has a nice geometric interpretation (at least at present…)

$$
\begin{aligned}
\delta g^{\mu\nu} = -\,3\Big\{ & 2k_2\,\phi_\rho{}^{\mu\nu}\,\nabla\cdot\xi^\rho + \alpha\,\phi_{\rho\sigma}{}^{(\mu}\nabla^{\nu)}\xi^{\rho\sigma} + 2(2k_1-3)\,\phi^{\rho\sigma(\mu}\nabla_\rho\,\xi^{\nu)}{}_\sigma \\
& + (4k_3+3)\,\phi^{(\mu}\nabla\cdot\xi^{\nu)} + \beta\,\phi_\rho\nabla^{(\mu}\xi^{\nu)\rho} + k_2\,\phi_\rho\,\nabla^\rho\xi^{\mu\nu} \\
& + 4\,\xi^{\rho\sigma}\,\nabla_\rho\,\phi^{\mu\nu}{}_\sigma - (2k_1+5-\alpha)\,\xi^{\rho\sigma}\nabla^{(\mu}\phi^{\nu)}{}_{\rho\sigma} + 8\,\xi^{\rho(\mu}\nabla\cdot\phi^{\nu)}{}_\rho \\
& - 8\,\xi^{\rho\,(\mu}\nabla_\rho\,\phi^{\nu)} - 2(k_2+4-\beta)\,\xi^{\rho(\mu}\nabla^{\nu)}\phi_\rho + 2\,\xi^{\mu\nu}\,\nabla\cdot\phi \\
& - g^{\mu\nu}\Big[3(2k_1+4k_4-1)\,\phi_{\rho\sigma\lambda}\nabla^\rho\xi^{\sigma\lambda} + 4\,\xi^{\rho\sigma}\,\nabla\cdot\phi_{\rho\sigma} \\
& + (4k_2+4k_3+8k_5+3)\,\phi_\rho\,\nabla\cdot\xi^\rho - 8\,\xi^{\rho\sigma}\,\nabla_\rho\,\phi_\sigma\Big]\Big\} + \mathcal{O}(\phi^2)\,.
\end{aligned}
$$

A.C., Pfenninger, Fredenhagen, Theisen (2012)

δg under spin-3 transformations

- Unfortunately not everything has a nice geometric interpretation (at least at present…)
- Tuning the free parameters in the action one obtains however a simple variation at the boundary

$$\delta g^{rr} = \mathcal{O}(r^{-2})\,,$$

$$\delta g^{ri} = \mathcal{O}(r^{-3})\,,$$

$$\delta g^{ij} = \frac{36}{r^4}\, h^r{}_k{}^k \chi^{ij} - \frac{6\pi}{k\, r^4} \left\{ \underbrace{\chi^{kl}\left(\partial_k \mathcal{W}_l{}^{ij} + \partial^{(i}\mathcal{W}^{j)}{}_{kl}\right) + 3\, \mathcal{W}_{kl}{}^{(i}\partial^{j)}\chi^{kl}}_{\delta_\xi \mathcal{L}^{ij}} \right\} + \mathcal{O}(r^{-6})\,.$$

A.C., Henneaux (2014)

Summary

- Covariant rewriting of W3⊗W3 — A.C., Henneaux (2014)

$$\delta_v \mathcal{L}^{ij} = [\,\mathcal{L}, \epsilon\,]^{ij} - 2\,\mathcal{L}^{ij}\partial\cdot\epsilon + \frac{k}{4\pi}\,\partial^i\partial^j\partial\cdot\epsilon\,,$$

$$\delta_v \mathcal{W}^{ijk} = [\,\mathcal{W}, \epsilon\,]^{ijk} - 3\,\mathcal{W}^{ijk}\partial\cdot\epsilon\,,$$

$$\delta_\xi \mathcal{L}^{ij} = \frac{4}{3}\left\{\chi^{kl}\left(\partial_k \mathcal{W}_l{}^{ij} + \partial^{(i}\mathcal{W}^{j)}{}_{kl}\right) + 3\,\mathcal{W}_{kl}{}^{(i}\partial^{j)}\chi^{kl}\right\},$$

$$\begin{aligned}\delta_\xi \mathcal{W}^{ijk} = -\frac{1}{2}\Big\{ & 2\,\partial^i\partial^j\partial^k(\mathcal{L}_{mn}\chi^{mn}) + 3\,\partial_m\partial^{(i}\mathcal{L}^{jk)}\partial\cdot\chi^m + 9\,\partial_m\mathcal{L}^{(ij}\partial^{k)}\partial\cdot\chi^m \\ & + 6\left(\partial^m\mathcal{L}^{(ij}\partial_m\partial\cdot\chi^{k)} - \partial^{(i}\mathcal{L}^{j|m}\partial_m\partial\cdot\chi^{|k)}\right) + \left(8\,\mathcal{L}^{(ij}\partial^{k)} + 5\,\eta^{(ij}\mathcal{L}^{k)m}\partial_m\right)\partial\cdot\partial\cdot\chi \\ & - 18\,\mathcal{L}^{m(i}\partial_m\partial^j\partial\cdot\chi^{k)} - \frac{k}{4\pi}\,\partial^i\partial^j\partial^k\partial\cdot\partial\cdot\chi - \frac{8\pi}{k}\Big[\left(8\,\mathcal{L}^{(ij}\partial^{k)} + 5\,\eta^{(ij}\mathcal{L}^{k)p}\partial_p\right)(\mathcal{L}_{mn}\chi^{mn}) \\ & - 9\,\chi^{m(i|}\partial_m\left(\mathcal{L}_n{}^{|j}\mathcal{L}^{k)n}\right) - \frac{27}{2}\,\mathcal{L}^{m(i}\mathcal{L}_m{}^j\partial\cdot\chi^{k)} + 9\,\mathcal{L}^m{}_n\mathcal{L}^{n(i}\partial_m\chi^{jk)}\Big]\Big\}.\end{aligned}$$

Summary

- Covariant rewriting of $W_3 \otimes W_3$

A.C., Henneaux (2014)

$$\delta_v \mathcal{L}^{ij} = [\,\mathcal{L}, \epsilon\,]^{ij} - 2\,\mathcal{L}^{ij}\partial\cdot\epsilon + \frac{k}{4\pi}\,\partial^i\partial^j\partial\cdot\epsilon\,,$$

$$\delta_v \mathcal{W}^{ijk} = [\,\mathcal{W}, \epsilon\,]^{ijk} - 3\,\mathcal{W}^{ijk}\partial\cdot\epsilon\,,$$

$$\delta_\xi \mathcal{L}^{ij} = \frac{4}{3}\left\{\chi^{kl}\left(\partial_k \mathcal{W}_l{}^{ij} + \partial^{(i}\mathcal{W}^{j)}{}_{kl}\right) + 3\,\mathcal{W}_{kl}{}^{(i}\partial^{j)}\chi^{kl}\right\},$$

$$\begin{aligned}\delta_\xi \mathcal{W}^{ijk} = -\frac{1}{2}\Big\{ & 2\,\partial^i\partial^j\partial^k(\mathcal{L}_{mn}\chi^{mn}) + 3\,\partial_m\partial^{(i}\mathcal{L}^{jk)}\partial\cdot\chi^m + 9\,\partial_m\mathcal{L}^{(ij}\partial^{k)}\partial\cdot\chi^m \\ & + 6\left(\partial^m\mathcal{L}^{(ij}\partial_m\partial\cdot\chi^{k)} - \partial^{(i}\mathcal{L}^{j|m}\partial_m\partial\cdot\chi^{|k)}\right) + \left(8\,\mathcal{L}^{(ij}\partial^{k)} + 5\,\eta^{(ij}\mathcal{L}^{k)m}\partial_m\right)\partial\cdot\partial\cdot\chi \\ & - 18\,\mathcal{L}^{m(i}\partial_m\partial^j\partial\cdot\chi^{k)} - \frac{k}{4\pi}\,\partial^i\partial^j\partial^k\partial\cdot\partial\cdot\chi - \frac{8\pi}{k}\Big[\left(8\,\mathcal{L}^{(ij}\partial^{k)} + 5\,\eta^{(ij}\mathcal{L}^{k)p}\partial_p\right)(\mathcal{L}_{mn}\chi^{mn}) \\ & - 9\,\chi^{m(i|}\partial_m\left(\mathcal{L}_n{}^{|j}\mathcal{L}^{k)n}\right) - \frac{27}{2}\,\mathcal{L}^{m(i}\mathcal{L}_m{}^j\partial\cdot\chi^{k)} + 9\,\mathcal{L}^m{}_n\mathcal{L}^{n(i}\partial_m\chi^{jk)}\Big]\Big\}\,.\end{aligned}$$

Summary

- Covariant rewriting of W3⊗W3

A.C., Henneaux (2014)

$$\delta_v \mathcal{L}^{ij} = [\,\mathcal{L},\epsilon\,]^{ij} - 2\,\mathcal{L}^{ij}\partial\cdot\epsilon + \frac{k}{4\pi}\,\partial^i\partial^j\partial\cdot\epsilon\,,$$

$$\delta_v \mathcal{W}^{ijk} = [\,\mathcal{W},\epsilon\,]^{ijk} - 3\,\mathcal{W}^{ijk}\partial\cdot\epsilon\,,$$

$$\delta_\xi \mathcal{L}^{ij} = \frac{4}{3}\left\{\,\chi^{kl}\left(\partial_k \mathcal{W}_l{}^{ij} + \partial^{(i}\mathcal{W}^{j)}{}_{kl}\right) + 3\,\mathcal{W}_{kl}{}^{(i}\partial^{j)}\chi^{kl}\,\right\},$$

$$\delta_\xi \mathcal{W}^{ijk} = -\frac{1}{2}\Big\{\,2\,\partial^i\partial^j\partial^k(\mathcal{L}_{mn}\chi^{mn}) + 3\,\partial_m\partial^{(i}\mathcal{L}^{jk)}\partial\cdot\chi^m + 9\,\partial_m\mathcal{L}^{(ij}\partial^{k)}\partial\cdot\chi^m$$
$$+\,6\left(\partial^m\mathcal{L}^{(ij}\partial_m\partial\cdot\chi^{k)} - \partial^{(i}\mathcal{L}^{j|m}\partial_m\partial\cdot\chi^{|k)}\right) + \left(8\,\mathcal{L}^{(ij}\partial^{k)} + 5\,\eta^{(ij}\mathcal{L}^{k)m}\partial_m\right)\partial\cdot\partial\cdot\chi$$
$$-\,18\,\mathcal{L}^{m(i}\partial_m\partial^j\partial\cdot\chi^{k)} - \frac{k}{4\pi}\,\partial^i\partial^j\partial^k\partial\cdot\partial\cdot\chi - \frac{8\pi}{k}\Big[\left(8\,\mathcal{L}^{(ij}\partial^{k)} + 5\,\eta^{(ij}\mathcal{L}^{k)p}\partial_p\right)(\mathcal{L}_{mn}\chi^{mn})$$
$$-\,9\,\chi^{m(i|}\partial_m\left(\mathcal{L}_n{}^{|j}\mathcal{L}^{k)n}\right) - \frac{27}{2}\,\mathcal{L}^{m(i}\mathcal{L}_m{}^j\partial\cdot\chi^{k)} + 9\,\mathcal{L}^m{}_n\mathcal{L}^{n(i}\partial_m\chi^{jk)}\Big]\Big\}\,.$$

Charges

- We saw how symmetries act on the space of solutions
- How to compute charges? What is their algebra?
- In CS: $Q(\lambda) = \int d\theta \epsilon(\theta)\mathcal{L}(\theta) + \int \delta\theta \chi(\theta)\mathcal{W}(\theta)$
- Once one knows the variations $\delta\mathcal{L}$, $\delta\mathcal{W}$ one can compute the algebra of charges thanks to

$$\delta_\lambda F = \{Q(\lambda), F\}$$

- The "covariant" variations $\delta\mathcal{L}_{ij}$, $\delta\mathcal{W}_{ijk}$ reproduce those of the independent chiral functions as computed in CS

Intermezzo

Charges from Hamiltonian formulation

Charges à la Regge-Teitelboim

- Example: linearised gravity action in Hamiltonian form
  - Canonical variables: h_{ij} & conj. momenta Π^{ij} ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$)
  - Lagrange multipliers: $N = h_{00}\,, \quad N_i = h_{0i}$

$$\mathcal{L} = \Pi^{ij}\dot{h}_{ij} - \mathcal{H} - N\,\mathcal{C} - N_i\,\mathcal{C}^i$$

- Charges as boundary terms which make the functional derivative of the generators of gauge transformations (aka smeared constraints) well defined

$$\mathcal{G}[v, v^i] = \int d^3x \left(v\,\mathcal{C} + v^i\,\mathcal{C}_i\right) + Q$$

Regge, Teitelboim (1974)

Charges à la Regge-Teitelboim

- Example: linearised gravity action in Hamiltonian form
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$$\mathcal{L} = \Pi^{ij}\dot{h}_{ij} - \mathcal{H} - N\,\mathcal{C} - N_i\,\mathcal{C}^i$$

- Charges as boundary terms which make the functional derivative of the generators of gauge transformations (aka smeared constraints) well defined

$$\mathcal{G}[v, v^i] = \int d^3x \left(v\,\mathcal{C} + v^i\,\mathcal{C}_i\right) + Q$$

Regge, Teitelboim (1974)

$$\delta\mathcal{G} = \int d^3x \left(A^{ij}\delta h_{ij} + B_{ij}\delta\Pi^{ij}\right) + \int ds_k(\cdots) + \delta Q$$

Fronsdal Hamiltonian (spin-3)

- Almost same structure as in the spin-2 example
  - Canonical variables: $\phi_{ijk}\,,\ \alpha = \phi_{000} - 3\,\phi_{0i}{}^{i}$ & momenta $\Pi^{ijk}\,,\ \tilde{\Pi}$
  - Lagrange multipliers: $N_i = \phi_{00i}\,, \quad N_{ij} = \phi_{0ij}$
- Action in Hamiltonian form

Metsaev (2011)

$$\mathcal{L} = \Pi^{ijk}\dot{\varphi}_{ijk} + \tilde{\Pi}\,\dot{\alpha} - \mathcal{H} - N_i\,\mathcal{C}^i - N_{ij}\,\mathcal{C}^{ij}$$

- Generator of gauge transformations

$$\mathcal{G}[\xi^i,\xi^{ij}] = \int d^3x \left(\xi^i\,\mathcal{C}_i + \xi^{ij}\,\mathcal{C}_{ij}\right) + Q$$

- Well defined setup to compute charges!

A.C., Henneaux, Hörtner, Leonard (2015)

Including higher spins

Adding spin 4: peculiarities

- Algebra of charges (spin 3)

$$\left\{\mathcal{W}(\theta)\,,\,\mathcal{W}(\theta')\right\} =$$
$$-\frac{1}{12N_2}\left[2\,\delta(\theta-\theta')\,\mathcal{L}''' - 9\,\delta'(\theta-\theta')\,\mathcal{L}'' + 15\,\delta''(\theta-\theta')\,\mathcal{L}' - 10\,\delta'''(\theta-\theta')\,\mathcal{L}\right]$$
$$-\frac{16\pi}{3kN_2}\left[\delta(\theta-\theta')\,\mathcal{L}\mathcal{L}' - \delta'(\theta-\theta')\,\mathcal{L}^2\right] - \frac{k}{48\pi N_2}\,\delta^{(5)}(\theta-\theta')\,,$$

- It is nonlinear, but the rhs only contains $\mathcal{L}$
  - this is why $\mathcal{O}(\phi^2)$ corrections in $\delta_3\,\phi^{\mu\nu\rho}$ do not play any role

Adding spin 4: peculiarities

- Algebra of charges

$$\left\{\mathcal{W}(\theta)\,,\,\mathcal{W}(\theta')\right\} = -\frac{2N_3}{(N_2)^2}\left[\delta(\theta-\theta')\,\mathcal{U}' - 2\,\delta'(\theta-\theta')\,\mathcal{U}\right]$$
$$-\frac{1}{12N_2}\left[2\,\delta(\theta-\theta')\,\mathcal{L}''' - 9\,\delta'(\theta-\theta')\,\mathcal{L}'' + 15\,\delta''(\theta-\theta')\,\mathcal{L}' - 10\,\delta'''(\theta-\theta')\,\mathcal{L}\right]$$
$$-\frac{16\pi}{3kN_2}\left[\delta(\theta-\theta')\,\mathcal{L}\mathcal{L}' - \delta'(\theta-\theta')\,\mathcal{L}^2\right] - \frac{k}{48\pi N_2}\,\delta^{(5)}(\theta-\theta')\,,$$

- It is nonlinear, but the rhs only contains $\mathcal{L}$
  - this is why $\mathcal{O}(\phi^2)$ corrections in $\delta_3\,\phi^{\mu\nu\rho}$ do not play any role

- Adding a spin-4 field a new contribution appears

- It can and indeed it does come from the deformation of gauge transformations associated to the 3-3-4 vertex

Adding spin 4: overview

- Control over cubic vertices is needed
  - Action: $$I_{\{3,4\}} = \int \frac{d^3x\sqrt{-g}}{16\pi G}\left(\mathcal{L}_{EH} + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_{3-3-4} + \mathcal{L}_{4-4-4}\right) + \cdots$$
  - Gauge transf.: $$\delta g = \nabla v + \phi\nabla\xi + \varphi\nabla\kappa + \phi\varphi\nabla\xi + \phi^2\nabla\kappa + \varphi^2\nabla\kappa + \cdots\,,$$ $$\delta\phi = \nabla\xi + \varphi\nabla\xi + \phi\nabla\kappa + \phi\nabla v + \cdots\,,$$ $$\delta\varphi = \nabla\kappa + \varphi\nabla\kappa + \phi\nabla\xi + \varphi\nabla v + \cdots\,,$$
- Gauge invariance requires higher-order corrections, but these are subleading at r→∞
  - Good news: for a given spin the number of terms to be included in the action is always finite
  - Bad news: the order of the vertices to be considered grows with the spin (like the order of nonlinearities in W-algebras)

Arbitrary spin

- The order of non-linearities grows with the spin
  - What is the metric-like analogue of the quadratic basis for W-algebras?
  - Is there any regularity?
- Strong indications that asymptotic symmetries are generated by conformal Killing tensors on the boundary
  - $\delta\varphi^{i_1\cdots i_s} = \frac{2}{r^2}\left\{ \partial^{(i_1}\lambda_0{}^{i_2\cdots i_s)} + 2\,\eta^{(i_1 i_2}\lambda_1{}^{i_3\cdots i_s)} \right\} + \mathcal{O}(r^{-4})$
  - Not a complete surprise: this is true in arbitrary space-time dimension for the free theory. But one should check that interactions do not spoil this result!

Bekaert, Boulanger (2005); Barnich, Bouatta, Grigoriev (2005); Bekaert (2008);

Conclusions & outlook

- Asymptotic symmetries in the metric-like formulation
  - What do we gain? “Flexibility” and “conformal geometry”
  - What do we lose? Computational power (from spin 4)
  - What is missing? “Fair” identification of charges (in progress)
- Interesting possible developments
  - In D=2+1 and within the gauge sector: recover the quadratic basis (by using field redefinitions in a smart way?), go to flat space, …
  - More broadly in D=2+1: add matter, include topological masses, …
  - Higher space-time dimensions

Thank you

谢谢