HS and Matter Interacting in D=3

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Intro and Goals

Vasiliev's equations in 3d can be considered as a non-trivial toy model for their higher-dimensional relatives while still displaying all non-trivial features as in d>3

In this talk:

- Extract the associated matter couplings from 3d Vasiliev's equations
- Link the above couplings to an ordinary Lagrangian formulation

In the near future:

- Test Gaberdiel-Gopakumar conjecture beyond symmetry considerations
- Improve our understanding of non-localities (functional class) in Vasiliev's theory

$$J_{\underline{m}(s)} = \sum_l g_l \,\Box^l \left(\ldots + \nabla_{\underline{m}(s-k)} \Phi^* \nabla_{\underline{m}(k)} \Phi + \ldots \right)$$

*Any implicit reference is to Vasiliev's works

Ingredients

$$\omega_{\underline{m}}^{a(s-1)} = \epsilon^{a}{}_{bc}\, \omega_{\underline{m}}^{a(s-2)b,c}$$

- The frame-like formalism in 3d deals with (Vasiliev's 1980):

$$e_{\underline{m}}^{a(s-1)} \qquad\qquad \omega_{\underline{m}}^{a(s-1),b}$$

- Linear equations read as:

$$T^{a(s-1)} = \nabla e^{a(s-1)} - \epsilon^{a}{}_{bc} h^{b} \wedge \omega^{a(s-2)c} = 0$$

$$R^{a(s-1)} = \nabla \omega^{a(s-1)} + \epsilon^{a}{}_{bc} h^{b} \wedge e^{a(s-2)c} = 0$$

- The standard metric-like Fronsdal field is recovered as:

$$\phi_{\underline{m}_1 \ldots \underline{m}_s} = e_{\underline{m}_1}^{a(s-1)} h_{a\underline{m}_2} \ldots h_{a\underline{m}_s} \qquad\qquad \delta\phi_{\underline{m}(s)} = \nabla_{\underline{m}} \epsilon_{\underline{m}(s-1)}$$

- 3d isomorphism:

$$so(2,2) \sim sp(2) \oplus sp(2) \quad \Rightarrow \quad V^{a(k)} \longleftrightarrow V^{\alpha(2k)}$$

Ingredients

$$so(2,2) \sim sp(2) \oplus sp(2)$$
$$U(sp_2)/(C_2 - (\lambda^2 - 1))$$
$$hs(\lambda) \oplus hs(\lambda)$$

The HS algebra is conveniently formulated in the sp(2) language (Vasiliev 1992)

$$[L_{\alpha\alpha}, L_{\beta\beta}] = \epsilon_{\alpha\beta} L_{\alpha\beta}\,, \qquad [L_{\alpha\alpha}, P_{\beta\beta}] = \epsilon_{\alpha\beta} P_{\alpha\beta}\,, \qquad [P_{\alpha\alpha}, P_{\beta\beta}] = \epsilon_{\alpha\beta} L_{\alpha\beta}\,.$$

For $\lambda=1/2$ we have a "standard" oscillator realization:

$$[\hat{y}_\alpha, \hat{y}_\beta] = 2i\epsilon_{\alpha\beta} \qquad \phi^2 = 1 \qquad \psi^2 = 1 \qquad \{\phi, \psi\} = 0$$

$$L_{\alpha\beta} = -\frac{i}{4}\{\hat{y}_\alpha, \hat{y}_\beta\} \qquad P_{\alpha\beta} = \phi L_{\alpha\beta}$$

$$f \in hs: \qquad f(\hat{y}, \phi) = f_0(\hat{y}, \phi) + \psi f_1(\hat{y}, \phi)$$

One can also use a Moyal realization of the operator product

$$(f \star g)(y) = f(y) \exp i \left(\frac{\overleftarrow{\partial}}{\partial y^\alpha} \epsilon^{\alpha\beta} \frac{\overrightarrow{\partial}}{\partial y^\beta} \right) g(y)$$

HS algebra and Free Fields

Vacuum solution = flat so(2,2) Connection of HS algebra

$$d\Omega = \Omega \star \Omega \quad \longrightarrow \quad \Omega = \frac{1}{2}\varpi^{\alpha\alpha}L_{\alpha\alpha} + \frac{1}{2}h^{\alpha\alpha}P_{\alpha\alpha}$$

Vacuum AdS_3 solution

Free equations have a plain algebraic meaning in terms of HS-algebra:

$$d\omega = \{\Omega, \omega\} \qquad\qquad d\mathbf{C} = [\Omega, \mathbf{C}]$$

The fields are 1-form and 0 form modules of the HS-algebra satisfying covariant constancy conditions

Physical and Twisted sectors

The covariant constancy conditions written before are reducible due to the absence of ψ in Ω:

$$[\phi f, g\psi] = \phi\{f, g\}\psi$$

One can then split fields according to whether or not they involve ψ

$$\boldsymbol{\omega} = \omega + \tilde{\omega}\psi \qquad \mathbf{C} = \tilde{C} + C\psi$$

$$\widetilde{D}\tilde{\omega} = 0 \qquad D\tilde{C} = 0$$

$$D\omega = 0 \qquad \widetilde{D}C = 0 \Rightarrow (\Box + \tfrac{3}{4})\Phi(x) = 0$$

- Gauge fields $\omega(y, \phi) \sim \phi e + \omega$
- A scalar (and spin ½ fermion) $C(\phi) + y^\alpha C_\alpha(\phi)$
- A constant + killing tensor fields $\tilde{C}(y, \phi)$
- twisted one forms leaving in infinite dimensional modules $\tilde{\omega}(y, \phi)$

$$D = \nabla - \frac{1}{2}h^{\alpha\alpha}[P_{\alpha\alpha}, \bullet]$$

$$\widetilde{D} = \nabla - \frac{1}{2}h^{\alpha\alpha}\{P_{\alpha\alpha}, \bullet\}$$

Unfolding and Interactions

From free theories to interacting theories Vasiliev's prescription is:

$$\begin{cases} d\boldsymbol{\omega} = F^{\omega}(\boldsymbol{\omega}, \mathbf{C}) \\ d\mathbf{C} = F^{C}(\boldsymbol{\omega}, \mathbf{C}) \end{cases}$$

$$\boxed{d^2 = 0}$$

$$F^{\omega}(\boldsymbol{\omega}, \mathbf{C}) = \mathcal{V}(\boldsymbol{\omega}, \boldsymbol{\omega}) + \mathcal{V}(\boldsymbol{\omega}, \boldsymbol{\omega}, \mathbf{C}) + \mathcal{V}(\boldsymbol{\omega}, \boldsymbol{\omega}, \mathbf{C}, \mathbf{C}) + \ldots$$

$$F^{C}(\boldsymbol{\omega}, \mathbf{C}) = \mathcal{V}(\boldsymbol{\omega}, \mathbf{C}) + \mathcal{V}(\boldsymbol{\omega}, \mathbf{C}, \mathbf{C}) + \mathcal{V}(\boldsymbol{\omega}, \mathbf{C}, \mathbf{C}, \mathbf{C}) + \ldots$$

C-expansion

First cocycle governed by HS algebra (hs covariantization of free eqs.):

$$\mathcal{V}(\boldsymbol{\omega}, \boldsymbol{\omega}) = \boldsymbol{\omega} \star \boldsymbol{\omega} \qquad \mathcal{V}(\boldsymbol{\omega}, \mathbf{C}) = \boldsymbol{\omega} \star \mathbf{C} - \mathbf{C} \star \boldsymbol{\omega}$$

The system is endowed with fully non-linear gauge symmetries

$$\delta\boldsymbol{\omega} = d\boldsymbol{\xi} + \boldsymbol{\xi}\frac{\partial}{\partial\boldsymbol{\omega}}F^{\omega}(\boldsymbol{\omega}, \mathbf{C}) = d\boldsymbol{\xi} - [\boldsymbol{\omega}, \boldsymbol{\xi}]_{\star} + O(\mathbf{C}) \qquad \delta\mathbf{C} = \boldsymbol{\xi}\frac{\partial}{\partial\boldsymbol{\omega}}F^{C}(\boldsymbol{\omega}, \mathbf{C}) = \boldsymbol{\xi} \star \mathbf{C} - \mathbf{C} \star \boldsymbol{\xi} + O(\mathbf{C}^2)$$

Goal: Expand Vasiliev's equations and study the above cocycles to second order

Vasiliev's Equations

The cocycles are resummed in Vasiliev's equations with the help of an additional Z oscillator:

$$\mathcal{W} = \mathcal{W}_{\underline{m}}(y,z,\phi,\psi|x)\,dx^{\underline{m}} \qquad \mathcal{B} = \mathcal{B}(y,z,\phi,\psi|x) \qquad \mathcal{S}_\alpha = \mathcal{S}_\alpha(y,z,\phi,\psi|x)$$

$$f(y,z)\star g(y,z) = \frac{1}{(2\pi)^2}\int d^2u\,d^2v\;f(y+u,z+u)\,g(y+v,z-v)\exp\left(iv^\alpha u_\alpha\right)$$

$$\begin{aligned} d\mathcal{W} &= \mathcal{W}\star\mathcal{W} \\ d\mathcal{B} &= [\mathcal{W},\mathcal{B}] \\ d\mathcal{S}_\alpha &= [\mathcal{W},\mathcal{S}_\alpha] \\ 0 &= [\mathcal{B},\mathcal{S}_\alpha] \\ [\mathcal{S}_\alpha,\mathcal{S}_\beta] &= -2i\epsilon_{\alpha\beta}(1+\mathcal{B}) \end{aligned}$$

Vasiliev 1992

(well known) subtleties:

- Naive Lorentz generators fail beyond linear approximation
- The right Lorentz generators acquire non-linear corrections

$$L_{\alpha\alpha} = -\frac{i}{2}(y_\alpha y_\alpha - z_\alpha z_\alpha) - \frac{i}{4}\{\mathcal{S}_\alpha,\mathcal{S}_\alpha\}$$

Vasiliev

- Pseudo-local redefinition restores Lorentz

Puzzle for the AdS/CFT

In 3d Gaberdiel and Gopakumar conjectured it has been argued that a scalar coupled to HS gauge sector is required:

$$\begin{cases} \Box C = -\frac{3}{4} C \\ \omega(y,\phi) = \sum_s \frac{1}{(2s-2)!} y_{\alpha(2s-2)} \left(\omega^{\alpha(2s-2)} + \phi e^{\alpha(2s-2)}\right) \end{cases}$$

The role of Killing tensors has not been investigated so far from this perspective [In PV the constant is related to λ in hs(λ)]:

Can one embed twisted one forms and killing tensors in AdS/CFT?

$$\begin{cases} \nabla \tilde{C}_{\pm}^{\alpha(n)} \mp h^{\alpha}{}_{\gamma}\, \tilde{C}_{\pm}^{\gamma\alpha(n-1)} = 0 \\ (\widetilde{D}\tilde{\omega}_{\pm})^{\alpha(2s)} = 0 \end{cases}$$

Since we do not understand this sector we will try to truncate the theory to only include gauge fields and scalars

Integration Flow

About the possibility of truncating away the twisted sector we already know a yes go result to all orders → Prokushkin-Vasiliev 1998

Consider a vacuum solution where: $\tilde{C}(y=0)=\nu$

$$\mathcal{B}=\nu+\mu\mathcal{B}'(\mu,\nu) \qquad \mathcal{W}=\mathcal{W}(\mu,\nu) \qquad S_\alpha=S_\alpha(\mu,\nu)$$

$$\left\{\begin{array}{l}
\dfrac{\partial\mathcal{W}}{\partial\mu}=\dfrac{1}{2}\mathcal{B}'\star\dfrac{\partial\mathcal{W}}{\partial\nu}+\dfrac{1}{2}\dfrac{\partial\mathcal{W}}{\partial\nu}\star\mathcal{B}' \\
\dfrac{\partial\mathcal{B}'}{\partial\mu}=\dfrac{1}{2}\mathcal{B}'\star\dfrac{\partial\mathcal{B}'}{\partial\nu}+\dfrac{1}{2}\dfrac{\partial\mathcal{B}'}{\partial\nu}\star\mathcal{B}' \\
\dfrac{\partial\mathcal{S}_\alpha}{\partial\mu}=\dfrac{1}{2}\mathcal{B}'\star\dfrac{\partial\mathcal{S}_\alpha}{\partial\nu}+\dfrac{1}{2}\dfrac{\partial\mathcal{S}_\alpha}{\partial\nu}\star\mathcal{B}'
\end{array}\right.$$

μ is the perturbative expansion parameter

Solution at μ=1 obtained from solution at μ=0

Compatible with the equations: It can be thought of as field redefinition mapping the non-linear system to the free one (also Fronsdal and Scalar)

Yes-go: twisted fields can be truncated away -- can we say something more?

1^{st} Order

$$\begin{cases} D\omega = 0\,, \\ \widetilde{D}\tilde{\omega} = \frac{1}{8}H^{\alpha\alpha}(y_\alpha + i\partial^w_\alpha)(y_\alpha + i\partial^w_\alpha)C(w,\phi)\Big|_{w=0} \\ D\tilde{C} = 0\,, \\ \widetilde{D}C = 0 \end{cases}$$

We search for a consistent truncation of the theory in which the twisted sector is not present (Vasiliev, 92)

$$\Delta\tilde{\omega} = \frac{1}{4}\phi h^{\alpha\alpha}\int_0^1 dt\ (t^2-1)(y_\alpha + it^{-1}\partial^y_\alpha)(y_\alpha + it^{-1}\partial^y_\alpha)C(yt,\phi)$$

- On top of the above we have an ambiguity given by $\mathbb{H}^1(\widetilde{D}, C) \neq \emptyset$

$$\Delta\tilde{\omega} = \frac{1}{4}\phi h^{\alpha\alpha}\int_0^1 dt\ (t^2-1)\Big[(g_0 y_\alpha y_\alpha + 2it^{-1}y_\alpha t^{-1}\partial^y_\alpha - g_0 t^{-2}\partial^y_\alpha\partial^y_\alpha)C_{\text{bose}}(ty)$$
$$+ (y_\alpha y_\alpha + 2id_0 y_\alpha t^{-1}\partial^y_\alpha - t^{-2}\partial^y_\alpha\partial^y_\alpha)C_{\text{fermi}}(ty)\Big]$$

2^{nd} Order

The ambiguity in truncating the twisted sector induces an ambiguity at the level of second order interactions while we can set to zero all first order twisted fields

$$
\begin{cases}
\widetilde{D}C^{(2)} = \omega^{(1)} \star C^{(1)} - C^{(1)} \star \omega^{(1)}(-\phi) \\
D\widetilde{C}^{(2)} = \widetilde{\mathcal{V}}(\Omega, C^{(1)}, C^{(1)}) \\
\widetilde{D}\tilde{\omega}^{(2)} = \widetilde{\mathcal{V}}(\Omega, \omega^{(1)}, C^{(1)}) + \widetilde{\mathcal{V}}(\Omega, \Omega, C^{(2)}) \\
D\omega^{(2)} = \omega^{(1)} \star \omega^{(1)} + \mathcal{V}(\Omega, \Omega, C^{(1)}, C^{(1)})
\end{cases}
$$

Cocycles depend on

$g_0 \, , d_0$

The twisted fields are sourced again at second order by physical fields!

So far:

Truncating away the twisted sector at linear order seems to introduce ambiguities... Furthermore: is it possible to redefine away the sources to the twisted sector again?

Cohomology Analysis

Non-trivial cohomologies cannot be removed by a (pseudo-)local redefinition

$$\mathcal{V} \neq D(\ldots)$$

$$\mathbb{H}^1(D, CC) \neq \emptyset$$

(Prokushkin-Vasiliev 1999)

$$D\widetilde{C}^{(2)} = \widetilde{\mathcal{V}}(\Omega, C^{(1)}, C^{(1)})$$

2s+1 cohomologies for each irreducible AdS irrep

$$\mathbb{H}^2(\widetilde{D}, \omega C) \neq \emptyset$$

$$\widetilde{D}\tilde{\omega}^{(2)} = \widetilde{\mathcal{V}}(\Omega, \omega^{(1)}, C^{(1)})$$

The backreaction of the scalar on the twisted sector might be irremovable

$$\mathbb{H}^2(\widetilde{D}, C) = \emptyset$$

but

$$\mathbb{H}^1(\widetilde{D}, C) \neq \emptyset$$

$$\widetilde{D}\tilde{\omega}^{(2)} = \widetilde{\mathcal{V}}(\Omega, \Omega, C^{(2)})$$

Further ambiguity is introduced to remove

$$\widetilde{\mathcal{V}}(\Omega, \Omega, C^{(2)}) \longrightarrow g_1\,,\, d_1$$

and shows up already in $\widetilde{\mathcal{V}}(\Omega, \omega^{(1)}, C^{(1)})$

A uniqueness result

The cohomology analysis allowed us to explicitly determine weather or not the backreaction on the twisted sector could or could not be removed

Result:

$$\left.\begin{array}{l}\widetilde{\mathcal{V}}(\Omega,\omega^{(1)},C^{(1)}) \\ \\ \widetilde{\mathcal{V}}(\Omega,C^{(1)},C^{(1)})\end{array}\right\} \text{ exact if: }$$

$$g_0 = d_0 = g_1 = d_1 = \beta$$

$$\beta = 0$$

- The ambiguity introduced at first order is fixed uniquely by requiring the consistency of the truncation at the next order
- Few parameters kill infinitely many D-cohomologies. Not surprising -- after all those should arise from a single HS cohomolgy branched with respect to the AdS subalgebra
- The above point must coincide with Integration flow restricted to the twisted sector

Generalized stress tensors

Having fixed the ambiguity in the second order theory and having truncated the twisted sector we can now move to the physical backreaction (stress tensor)

$$D\omega^{(2)} = \ldots + \mathcal{V}(\Omega, \Omega, C^{(1)}, C^{(1)}) \quad \Rightarrow \quad \Box \phi_{\underline{m}(s)} = J_{\underline{m}(s)}$$

The result takes the form:

$$\begin{cases} \mathcal{V}_{\alpha(2s)} = H_{\alpha\alpha} J'_{\alpha(2s-2)} + H_{\alpha}{}^{\beta} J^{\text{hook}}_{\beta\alpha(2s-1)} + H^{\beta\beta} J_{\beta\beta\alpha(2s)} \\ J_{\alpha(n+m)} = \sum_{l} a^{n,m,l} C_{\alpha(n)\nu(l)}(\phi) C^{\nu(l)}{}_{\alpha(m)}(-\phi) \end{cases}$$

$$H^{\alpha\alpha} = h^{\alpha}{}_{\gamma} \wedge h^{\gamma\alpha}$$

$$C_{\alpha(2s)} \sim (\sigma^{m}_{\alpha\alpha} \nabla_{\underline{m}})^s \Phi(x)$$

The expression decomposes in various independently conserved pieces in correspondence with improvement tensor structures or canonical currents, but dressed with an infinite derivative tail

$$\sim \sum_{l} \Box^{l} (\Phi^* \nabla^s \Phi)$$

A convenient basis

The decomposition of the backreaction in the various currents and improvements is most easily obtained up to a choice of basis:

Prokushkin-Vasiliev 1999

$$C(y) = \int d^2\xi e^{iy\xi} \hat{C}(\xi)$$

$$J(y) \sim \int d^2\xi d^2\eta K(\xi, \eta, y) C(\xi) C(\eta)$$

Three possible tensor contraction can be defined:

$$\xi^\alpha \eta_\alpha \quad y^\alpha \eta_\alpha \quad y^\alpha \xi_\alpha$$

D is diagonal if we consider monomials of the type:

$$[y^\alpha(\xi+\eta)_\alpha]^n [y^\alpha(\xi-\eta)_\alpha]^m f(\xi^\alpha \eta_\alpha)$$

$$D\left([y^\alpha(\xi+\eta)_\alpha]^n [y^\alpha(\xi-\eta)_\alpha]^m f(\xi^\alpha \eta_\alpha)\right) = [y^\alpha(\xi+\eta)_\alpha]^n [y^\alpha(\xi-\eta)_\alpha]^m (\mathcal{D} f)(\xi^\alpha \eta_\alpha)$$

Spin-1 Backreaction

In the spin-1 case we find a pseudo-local source to the Maxwell tensor:

$$C_{\alpha(2s)} \sim (\sigma^{m}_{\alpha\alpha} \nabla_{\underline{m}})^s \Phi(x)$$

$$d\omega^{(2)} = H^{\beta\beta} \Big[\sum_{l \in 2\mathbb{N}} a_l \Big(C_{\beta\beta\nu(l)}(\phi) C^{\nu(l)}(-\phi) + C_{\nu(l)}(\phi) C^{\nu(l)}{}_{\beta\beta}(-\phi) \Big) - \sum_{l \in 2\mathbb{N}+1} a_l\, C_{\beta\nu(l)}(\phi) C^{\nu(l)}_{\beta}(-\phi) \Big]$$

$$H^{\alpha\alpha} = h^{\alpha}{}_{\gamma} \wedge h^{\gamma\alpha}$$

The coefficients are: ⟶ $$a_l = \frac{i(-i)^l}{l!} \frac{1}{(l+2)^2 (l+4)}$$

Puzzle:

At linear order we do not need to add a spin-1 field but at second order a source appears. Can we redefine it away or redefine away the higher-derivative tail?

Spin-2 Backreaction

To make contact with standard symmetric canonical current we need to solve torsion

$$\phi \leftrightarrow -\phi$$

We then obtain the manifestly symmetric backreaction:

$$R^{(2)}_{\alpha\alpha} = J^{\text{canonical}}_{\alpha\alpha} + J^{\text{Improvement}}_{\alpha\alpha} \qquad H^{\alpha\alpha} = h^{\alpha}{}_{\gamma} \wedge h^{\gamma\alpha}$$

$$C_{\alpha(2s)} \sim (\sigma^{m}_{\alpha\alpha}\nabla_{\underline{m}})^{s}\Phi(x)$$

$$\begin{cases} J^{\text{canonical}}_{\alpha\alpha} = H^{\beta\beta} J_{\beta\beta\alpha\alpha} \\ J_{\beta\beta\alpha\alpha} = \sum_{l\in 2\mathbb{N}} a_l \left(C_{\alpha(4)\nu(l)}(\phi) C^{\nu(l)}(-\phi) + 3\, C_{\alpha(2)\nu(l)}(\phi) C^{\nu(l)}{}_{\alpha(2)}(-\phi) \right) \end{cases}$$

$$a_l = \frac{i^{l-1}}{4\, l!} \left(\frac{1}{l+1} - \frac{6}{l+2} + \frac{9}{(l+3)^2} + \frac{19}{4(l+3)} - \frac{6}{l+4} + \frac{7}{l+5} - \frac{3}{4(l+7)} \right)$$

Can we redefine away the higher-derivative tail?

Cohomology and backreaction

We can ask the same question we asked for the twisted sector backreaction

It was already noticed by Prokushkin-Vasiliev that canonical s-derivative currents are exact in cohomology

$$(\Phi^* \nabla^s \Phi) \sim D \left(\sum_l^\infty \Box^l \ldots \right)$$

Projushkin-Vasiliev '99

This result is puzzling since we know canonical currents to be physically meaningful but the crux is that the redefinition is pseudo-local

Extending the analysis of Prokushkin-Vasiliev we find:

$$\mathbb{H}^2(D, CC) = \emptyset$$

Any quadratic backreaction on the physical sector can be expressed as *D* of some pseudo-local expression and hence can formally be removed by a pseudo-local field redefinition

Puzzle: physically we expect only to be able to relate pseudo-local backreactions to canonical currents

Ordinary Lagrangian Formulation

We would like to link the result obtained from Vasiliev's equations to a (perturbative) action principle.

$$S_{CS} = \frac{k}{4\pi} \int tr \left(\omega \wedge d\omega - \frac{2}{3} \omega \wedge \omega \wedge \omega \right)$$

$$S_{sc} = \int \det |h| \left(|\nabla \Phi|^2 + m^2 |\Phi|^2 \right)$$

$$S_{int} = \int tr \left[\omega(y, \phi) \star \wedge \mathbf{J}(y, \phi) \right] \sim \sum_s \frac{2g_s}{s} \int \phi_{\underline{m}(s)} j^{\underline{m}(s)}$$

$$S = S_{CS} + S_{sc} + S_{int}$$

The scalar field acquires some gauge transformations at this order

$$\delta^{(s)} \Phi = 2 i g_s \, \xi_{\underline{m}(s-1)} (2 i \nabla^{\underline{m}})^{s-1} \Phi$$

Ordinary Lagrangian Formulation

We can now compare the induced gauge transformations at the action level with those appearing in Vasiliev's equations

$$\delta^{(s)}\Phi = 2ig_s\,\xi_{\underline{m}(s-1)}(2i\nabla^{\underline{m}})^{s-1}\Phi \quad \Longleftrightarrow \quad \delta C = [\xi_\omega, C] + \{\xi_e, C\}$$

The comparison allows fixing the constant g_s otherwise arbitrary at cubic order

$$g_s = \frac{1}{(2s-2)!}$$

A redefinition bringing from Vasiliev's backreaction to the above currents and coefficient is still pseudo-local but should be physically acceptable

Local Cohomology

To have a better understanding of redefinitions it is also useful to study local cohomologies

Idea:

$$\int \omega^{(s)}(\Box^l J) \sim \int (\Box^l \omega^{(s)}) J \sim C_l^{(s)} \int \omega^{(s)} J$$

In more detail in terms of frame-like formalism and covariant derivatives we want find coefficients C_l such that:

$$C_l^{(s)} J^{can}_{\alpha(2s)} - \frac{1}{l!} \Box^l J^{can}_{\alpha(2s)} = (DK)_{\alpha(2s)}$$

(DK) → Local

Solution:

$$C_l^{(s)} = (-1)^l \frac{s\,(2s+l)!}{(2s)!(l+1)!} \Big[2\,(l+s)\,{}_2F_1(1, l+2s+1; l+2; -1) - l - 1 \Big] + 4^{-s}(l+s)$$

$$C_l^{(s)} \sim l^{2s-1} \quad (l \to \infty)$$

Summary & Outlook

- Truncating the twisted sector in PV is a non-trivial task and requires a knowledge of HS cohomology order by order (see also Integration Flow) – puzzle for holography
- Twisted sector can be added in any d-dim Vasiliev's theory but the way this is done is different than in PV
- In particular the d-dim theory at d=3 describes a HS theory w. hs(λ=1) without the twisted sector from the start (while twisted field sources on physical fields are never quadratic). Generically the latter theory is expected to be different from PV (?)
- Leaving open the question whether a consistent non-trivial truncation of the PV theory where the twisted sector is not present exists, it will certainly be useful to define from the start a theory without twisted sector generalizing the d-dim theory at d=3 presumably also for other values of λ
- Understanding functional class is still an important problem and would be a key element for a better understanding of the pseudo-local tails we see in the backreaction (see e.g. Vasiliev 2015)
- Physical way to check if a redefinition is allowed: compute observables (CFT correlators via Witten diagrams). Acceptable pseudo-local redefinitions should not have effect on those by definition. (in progress...)

The exact form is rather complicated

$$\mathbb{H}^0(D,CC)\neq\emptyset$$

$$\mathcal{V}\sim D(J+\mathbb{H}^0(D,CC))$$

$$J^{(0)}_{m,n}=\frac{\omega^2}{8\,(1-\omega^2)}\int_0^1 dt\Big[2\omega n\left(\frac{(1-t)^{m+2}(1+t)^n}{(m+1)(\omega t-1)}+\frac{(t+1)^{m+2}(1-t)^n}{(m+1)(\omega t+1)}\right)$$
$$-\frac{4\omega\left(1-t^2\right)(m+n+1)(1-t)^{m+n}}{(m+1)(\omega t+1)}$$
$$+\frac{t(2mn+m+3n+2)(1-t)^m(1+t)^n}{(m+1)(n+1)}+\frac{t(2mn+m+3n+2)(1+t)^m(1-t)^n}{(m+1)(n+1)}$$
$$+\frac{(1-t)^m(1+t)^n(2(m+1)(n+1)-\omega(m+n+2))}{\omega(m+1)(n+1)}$$
$$+\frac{(1+t)^m(1-t)^n(\omega(m+n+2)-2(m+1)(n+1))}{\omega(m+1)(n+1)}+\frac{2(m-n)(1-t)^{m+n+1}}{(m+1)(n+1)}\Big]$$

$$J\sim\sum_l g_l\Box^l(\Phi^*\nabla^{n+m}\Phi+\ldots)$$

$$J\sim g_1\omega+g_2\omega^2+\ldots$$

Spin-2 Backreaction

We are also able to identify the precise form of the improvement tensor structure:

$$\begin{cases} J^{\text{improvement}}_{\alpha\alpha} = H^{\beta\beta} J_{\beta\beta\alpha\alpha} + H_{\alpha\alpha} J' \\ J_{\beta\beta\alpha\alpha} = \sum_{l\in 2\mathbb{N}} b_l \left(C_{\alpha(4)\nu(l)}(\phi) C^{\nu(l)}(-\phi) - C_{\alpha(2)\nu(l)}(\phi) C^{\nu(l)}{}_{\alpha(2)}(-\phi) \right) \\ J' = \sum_{l\in 2\mathbb{N}} b'_l C_{\nu(l)}(\phi) C^{\nu(l)}(-\phi) \end{cases}$$

$$C_{\alpha(2s)} \sim (\sigma^{m}_{\alpha\alpha} \nabla_{\underline{m}})^s \Phi(x)$$

$$b_l = -\frac{i^{l-1}}{4\,l!}\left(\frac{1}{l+2} - \frac{1}{(l+3)^2} + \frac{13}{4(l+3)} + \frac{4}{l+4} - \frac{1}{l+5} - \frac{1}{l+6} + \frac{1}{4(l+7)}\right)$$

$$b'_l = \frac{i^{l-1}}{l!}\left(\frac{1}{3(l+2)^2} + \frac{7}{12(l+1)} - \frac{3}{l+2} + \frac{1}{3(l+4)} - \frac{1}{4(l+5)} - \frac{1}{6}\delta_{l,0}\right)$$

Towards a criterion for Functional Class

As a very naive attempt one might combine the asymptotic behavior of the C-coefficients to formulate a convergence criterion after having independently integrated by parts and redefined each box in the pseudo-local tail (Witten-diagram computation)

$$C_l^{(s)} \sim l^{2s-1} \quad (l \to \infty) \qquad\qquad \sum a_l^{(s)} C_l^{(s)} = g_s$$

$$\Downarrow$$

$$a_l^{(s)} \prec \frac{1}{l^{2s}}$$

The above condition ensures that integral over AdS commutes with the sum over *l*