Towards UV complete SFT based theory of gravity

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Instead of outline

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- **GR is not a complete theory in several respects**

  However:

- **IR tests of GR are extremely solid**

Problems of GR

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- **GR is not geodesically complete**
- **Either of the above has been overcome sacrificing the unitarity**

  Ghosts appeared in the theory

Why Einstein's GR is not enough?

- **Raychaudhuri equation:**
  Consider a congruence of null geodesics characterized by a null vector k_α, such that $k_\alpha k^\alpha = 0$. Then
  $$R_{\mu\nu}k^\mu k^\nu < 0$$
  for a non-singular space-time
- **GR equations of motion are:**
  $$M_P^2 G_{\mu\nu} = T_{\mu\nu}$$
  And usually matter is taken in the form of a perfect fluid
  $$T^\mu_\nu = \mathrm{diag}(-\rho, p, p, p)$$
- $$\textbf{Then} \ \Rightarrow 0 > M_P^2 R_{\mu\nu}k^\mu k^\nu = \rho + p \Rightarrow \ \textbf{NEC is violated}$$
  This is a reflection of the Hawking-Penrose theorem.

Who are ghosts?

Terminology in $(-,+,+,+)$ signature:

$$\textbf{good: } L = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \dots$$
$$\textbf{ghost: } L = +\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \dots$$

Ostrogradski statement says that higher (>2) derivatives in the Lagrangian are equivalent to the presence of ghosts.

This statement is not absolutely rigorous. There are systems with higher derivatives which have no ghosts.

Ghosts lead to a very rapid vacuum decay.

Exorcising ghosts

- **In some cases ghosts do not appear, like in $f(R)$ gravity for special parameters.**

  This is because the system is constrained.

- **There are special field theories which have higher derivatives in the Lagrangian but no more than 2 derivatives act on a field in the equations of motion. For example KGB models or galileons.**

  The fine-tuning is required.

- **Propagators can be modified and be non-local without changing the physical excitations**

$$\Box - m^2 \Rightarrow (\Box - m^2)e^{\gamma(\Box)}$$

  $\gamma(\Box)$ **must be an entire function. This guarantees that no extra degress of freedom appear.**

Why strings? Why SFT?

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Perhaps, I can skip the detailed answer in this room

History

- **Classical gravity and GR; also Ostrogradski 1850**
- **Stelle, 1977,1978, renormalizable R^2 type gravity (containing ghosts)**
- **Starobinsky, 1980-s, R^2 inflation**
- **Witten, 1986, String Field Theory which by construction contains non-local vertexes**
- **Aref'eva, AK, 2004, models of non-local stringy inspired scalar fields coupled to gravity**
- **Biswas, Mazumdar, Siegel, 2005, first explicit non-local gravity modification**

$\mathcal{G}(\Box)$ physics, SFT motivation

Low level example action from SFT:

$$L \sim \frac{1}{2}\phi(\Box - m^2)\phi + \frac{\lambda}{4}\left(e^{-\beta\Box}\phi\right)^4 \Rightarrow \frac{1}{2}\varphi(\Box - m^2)e^{2\beta\Box}\varphi + \frac{\lambda}{4}\varphi^4$$

The Lagrangian to understand is

$$S = \int d^Dx \left(\frac{1}{2}\varphi\mathcal{G}(\Box)\varphi - \lambda v(\varphi) + \dots\right)$$

$\mathcal{G}(\Box) = \sum_{n\geq 0} g_n\Box^n$, **i.e. it is an analytic function.**

Canonical physics has $\mathcal{G}(\Box) = \Box - m^2$, **i.e.** $L = \frac{1}{2}\varphi\Box\varphi - \frac{m^2}{2}\varphi^2$

Ghosty example $\mathcal{G}(\Box) = \Box - m^2 + g_2\Box^2$

SFT motivated non-local gravity

We start with GR, which must be the IR limit anyway

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2}\right), \; M_P^2 = \frac{1}{8\pi G_N}$$

We proceed by modifying it in a covariant way, containing higher derivatives in a form of $\Box$ operator and (as a zero try) focus on terms contributing to the propagator on the Minkowski background.

Then we arrive at

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2}+\frac{\lambda}{2}R\mathcal{F}_1(\Box)R\right)$$

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and after some deeper thinking

$$S=\int d^4x\sqrt{-g}\left(\frac{M_P^2R}{2}+\frac{\lambda}{2}\left(R\mathcal{F}_1(\Box)R+R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu}+R_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)R^{\mu\nu\lambda\sigma}\right)\right)$$

What is special about such a gravity

- **Ghost-free:**

$\mathcal{F}_4(\Box) = 0$ **(it is your freedom thanks to the Weyl tensor properties)**

$$\mathcal{F}_1(\Box) = -\frac{1}{2}\mathcal{F}_2(\Box)$$

$$\mathcal{F}_1(\Box) = \frac{e^{\gamma(\Box)} - 1}{\Box},\ \gamma(\Box) \text{ is an entire function and } \gamma(0) = 0$$

- **Asymptotically-free:**

$$\gamma(\Box) = -\frac{\Box}{M},\ \Phi \sim -\frac{1}{r}\text{erf}\left(\frac{Mr}{2}\right) \to \begin{cases} \text{const as } r \to 0 \\ \frac{1}{r} \text{ as } r \to \infty \end{cases}$$

- **Singularity-free following the Raychaudhuri equation analysis**
  Conroy, AK, Mazumdar, PRD, 2014 and a joint work in progress

Solutions

Claim: any solution of the local R^2 gravity is a solution here upon 3 algebraic conditions on the action parameters

Accounting $\Box R = r_1 R + r_2$
(which is an EOM rather than a constraint in a local R^2 gravity)

$$\mathcal{F}^{(1)}(r_1) = 0, \;\; \frac{r_2}{r_1} = -\frac{M_P^2 - 6\lambda\mathcal{F}(r_1)r_1}{2\lambda[\mathcal{F}(r_1) - \mathcal{F}(0)]}, \;\; \Lambda = -\frac{r_2 M_P^2}{4r_1},$$

Explicit non-singular bouncing solutions

$$a = a_0 \cosh(\sigma t)$$

Biswas, Muzumdar, Siegel, JCAP, 2006; AK, CQG, 2013

$$a = a_0 \sqrt{\cosh(\sigma t)} \textbf{ and also } a = a_0 e^{-\frac{\sigma}{2} t^2}$$

AK, CQG, 2013

Starobinsky solution

$$a = a_0 \sqrt{t_* - t} e^{\sigma(t_* - t)^2}$$

Craps, De Jonckheere, AK, JCAP, 2014

Quantization of perturbations was studied and shown to reproduce the values close to observable in the cosmological experiments

(a – is the scale factor of the FRW metric)

A wishful solution

Apart from the local R^2 gravity one can arrange such a parameter range that both bounce type and inflation type solutions exist

We are however lack of an explicit construction of such a solution yet

AK, work in progress

Scalar reformulation of the non-local gravity ($\mathcal{F}_1$ piece)

The previous action is equivalent to the following one

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2}(1+\psi) - \frac{M_P^4}{8\lambda}\psi\frac{1}{\mathcal{F}(\Box)}\psi + \dots\right)$$

The conformal transform $(1+\psi)^2 g_{\mu\nu} = \bar{g}_{\mu\nu}$ **allows us to decouple the gravity and the scalar field**

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} - \frac{M_P^2}{4}\frac{3}{(1+\psi)^2}\bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{M_P^4}{8\lambda(1+\psi)^2}\psi\mathcal{G}(\mathcal{P})\psi\right)$$

Here

$$\mathcal{G}(\mathcal{P}) = \frac{1}{\mathcal{F}(\mathcal{P})} \text{ and } \mathcal{P} = (1+\psi)\bar{\Box} - \bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu$$

The ghost-free condition on ψ **implies** $\mathcal{G}(\mathcal{P}) = \sum_{n\geq 0} g_n\mathcal{P}^n$**, i.e. it is an analytic function.**

Limits

The weak field limit

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} + \left(\frac{3}{2} - \frac{g_1 M_P^2}{4\lambda}\right)\psi\bar{\Box}\psi - \frac{g_0 M_P^2}{4\lambda}\psi^2 - \sum_{n>1}\frac{g_n M_P^2}{4\lambda}\left(\bar{\Box}\psi + (\partial\psi)^2\right)\left(\bar{\Box} - \partial^\rho\psi\partial_\rho\right)^{n-1}\psi\right)$$

Here we recognize the KGB models and structures similar to Galileon field theories.

The limit of large field

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} - \frac{3}{2}\frac{(\partial\psi)^2}{\psi^2} - \frac{M_P^2}{4\lambda}\frac{1}{\psi}\mathcal{G}\left(\psi\bar{\Box} - \partial^\rho\psi\partial_\rho\right)\psi\right)$$

As a special limit we restore the p-adic string theory

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} + \frac{\kappa}{2}\psi e^{-\beta\bar{\Box}}\psi\right)$$

Proposal: the non-local theories can be considered as *generating functionals* for other models on the market.

Conclusions and open questioins

- Non-local generalization of Einstein's gravity motivated by SFT is presented
- There are exact analytic solutions including bounce and the Starobinsky inflation in this framework
- Scalar reformulation with a possible connection to other interesting models is discussed
- There are preliminary results for the graviton action in the bosonic closed SFT
  AK, work in progress
- The next major step is to derive in some lowest approximation an action for the massless states in heterotic SFT

Thank you for listening!