3,4D SCFT on Conic Space as hologram of 4,5D Charged AdS topological Black Hole

Yang Zhou (Tel Aviv U.)

Sichuan University

with Xing Huang, Soo Jong Rey, JHEP 1403:127 (arXiv: 1401.5421)
also with Xing Huang, JHEP 1502:068 (arXiv:1408.3393)

A motivation: (but not all)

Q1: In interacting field theory in flat space, how to compute entanglement entropy (or Renyi entropy)?

A1: …

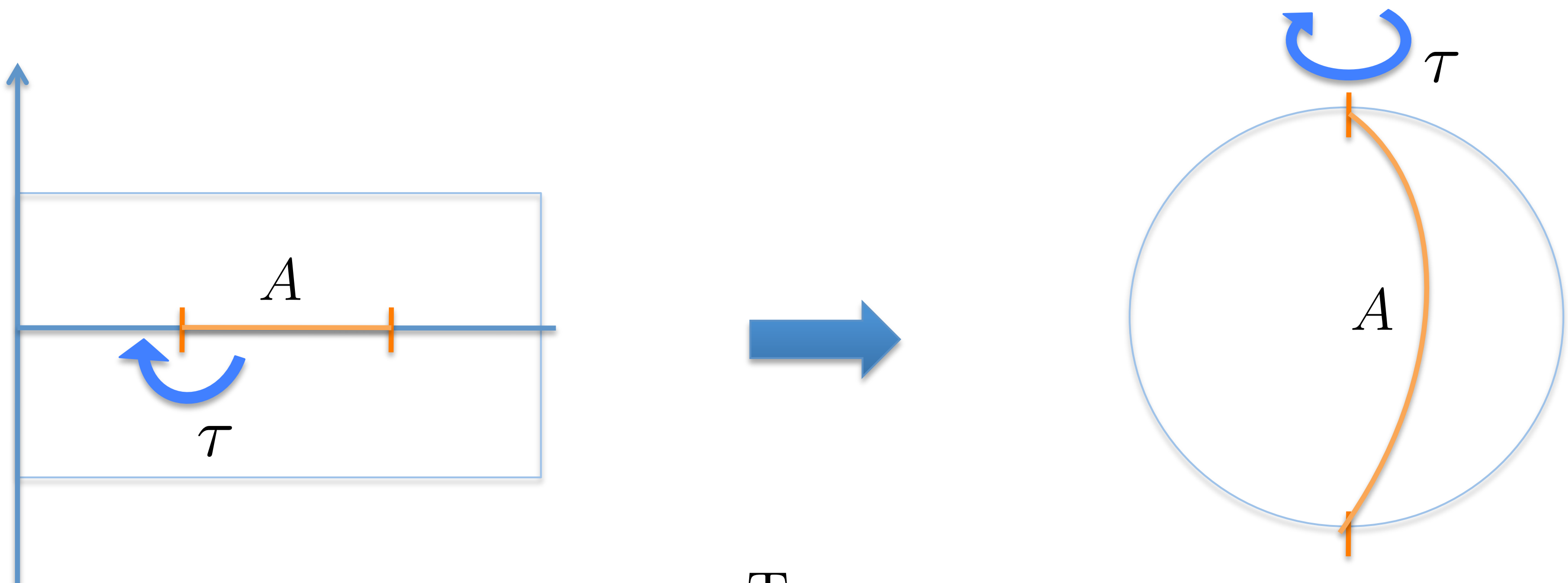
----- perturbative way, with small coupling λ

----- numerical study

A2: For CFT, formulate the problem on sphere and compute the path integral exactly (with supersymmetry)!

Casini-Huerta-Myers '2011

Renyi entropy of CFT in 1+1,

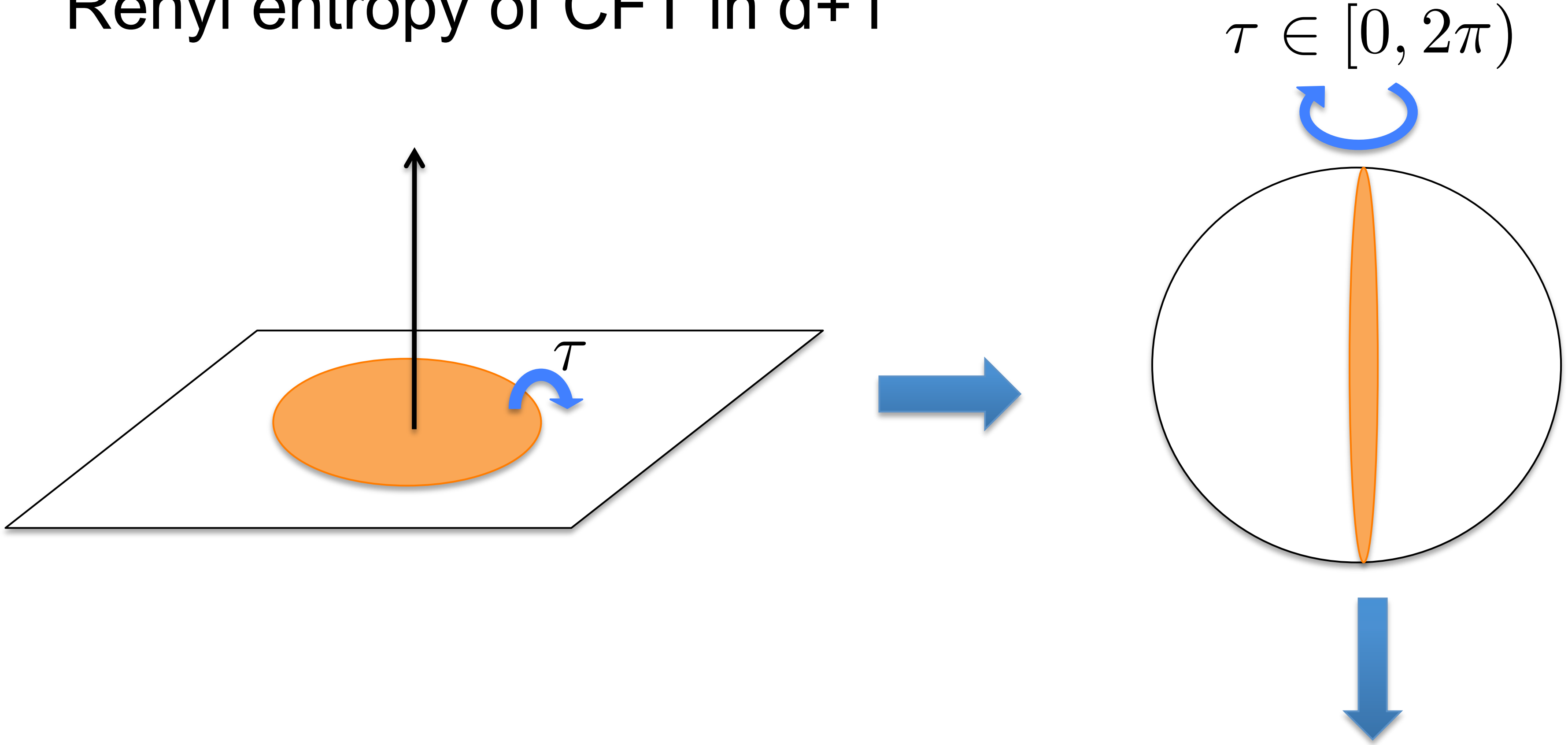


$$\rho_A = \mathrm{Tr}_B \rho$$

$$S_q = \frac{1}{1-q} \log \mathrm{Tr}(\rho_A^q) \qquad \mathrm{Tr}(\rho_A^q) = Z_q/(Z_1)^q$$

Casini-Huerta-Myers '2011

Renyi entropy of CFT in d+1



$$ds^2/\ell^2 = d\theta^2 + q^2 \sin^2\theta d\tau^2 + \cos^2\theta(d\phi^2 + \sin\phi^2 d\chi^2)$$

$$S_q = \frac{1}{1-q} \log \mathrm{Tr}(\rho_A^q) \qquad \mathrm{Tr}(\rho_A^q) = Z_q/(Z_1)^q$$

Refine the Renyi entropy to be supersymmetric and compute it exactly.

Part 1. qSCFT3

Claim:

3D N=2 SCFT on q-branched 3-sphere $\mathbb{S}^3_q$

$$ds^2 = \ell^2 \left(d\theta^2 + \cos^2\theta d\phi^2 + q^2 \sin^2\theta d\tau^2\right)$$

dual to BPS topological black hole in AdS4 (Euclidean)

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Sigma(\mathbb{H}^2)$$

$$f(r) = \frac{r^2}{L^2} + \kappa - \frac{2m}{r} + \frac{Q^2}{r^2}$$

Plan:

- ✧ 3D N=2 Killing spinor equation
- ✧ localization of partition function
- ✧ 4D charged topological black hole
- ✧ qSCFT3/TBH4 correspondence

1.1. 3D N=2 Killing spinor equation

Killing Spinors on $\mathbb{S}^3_q$

Nishioka-Yaakov '2013

-- characterize rigid SUSY on curved spacetime

3D N=2 Killing spinors with ± U(1) R charge on 3-sphere (or deformed) satisfy generally

$$\begin{aligned}(\nabla_\mu - iA_\mu)\,\zeta &= -\frac{1}{2}H\gamma_\mu\zeta\ ,\\ (\nabla_\mu + iA_\mu)\,\widetilde{\zeta} &= -\frac{1}{2}H\gamma_\mu\widetilde{\zeta}\ .\end{aligned}$$

solutions:

$\mathbb{S}^3$

2 constant Killing spinors with vanishing background vector field, and $H = -i$

$\mathbb{S}^3_q$

$$\zeta = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \widetilde{\zeta} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$H(\mathbb{S}^3_q) = -i\ , \qquad A(\mathbb{S}^3_q) = \frac{1}{2}\,(q-1)\,\mathrm{d}\tau$$

Resolved sphere $\widehat{\mathbb{S}}_q^3(\epsilon)$

$$\mathrm{d}s^2 = f_\epsilon(\theta)^2\,\mathrm{d}\theta^2 + q^2\ell^2\sin^2\theta\mathrm{d}\tau^2 + \ell^2\cos^2\theta\mathrm{d}\phi^2$$

$$f_\epsilon(\theta) = \begin{cases} q\ell\,, & \theta \to 0 \\ \ell\,, & \epsilon < \theta \leq \frac{\pi}{2} \end{cases}$$

Killing spinors:

$$\zeta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\,, \quad \widetilde{\zeta} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$H = -\frac{i}{f_\epsilon(\theta)}\,, \quad A = \frac{1}{2}\left(\frac{q\ell}{f_\epsilon(\theta)} - 1\right)\mathrm{d}\tau + \frac{1}{2}\left(\frac{\ell}{f_\epsilon(\theta)} - 1\right)\mathrm{d}\phi$$

$\mathbb{S}_q^3$ as the $\epsilon \to 0$ limit of $\widehat{\mathbb{S}}_q^3(\epsilon)$, as we will see the partition function on resolved space $Z[\widehat{\mathbb{S}}_q^3(\epsilon)]$ by localization will not depend on resolving function $f_\epsilon(\theta)$, therefore not sensitive to the singular limit $\epsilon \to 0$.

General branched spaces $\widetilde{\mathbb{S}}^3_{p,q}$ with U(1)×U(1)

branched ellipsoid
$$\frac{x_1^2+x_2^2}{\ell^2}+\frac{x_3^2+x_4^2}{\tilde{\ell}^2} = 1$$

$$\mathrm{d}s^2 = f(\theta)^2\mathrm{d}\theta^2 + p^2\ell^2\cos^2\theta\mathrm{d}\phi^2 + q^2\tilde{\ell}^2\sin^2\theta\mathrm{d}\tau^2 \ , \quad f(\theta) = \sqrt{\ell^2\sin^2\theta + \tilde{\ell}^2\cos^2\theta}$$

branched squashed sphere
$$\mathrm{d}s^2 = \ell^2\left(\frac{1}{v^2}\mu^1\mu^1 + \mu^2\mu^2 + \mu^3\mu^3\right)$$

$$\mathrm{d}s^2 = \mathrm{d}\theta^2 + \frac{1}{v^2}\left(\cos^4\theta q^2\mathrm{d}\tau^2 + \sin^4\theta p^2\mathrm{d}\phi^2\right) + \cos^2\theta\sin^2\theta(q^2\mathrm{d}\tau^2 + p^2\mathrm{d}\phi^2) - \frac{pq\sin^2 2\theta}{2}\left(-\frac{1}{v^2}+1\right)\mathrm{d}\phi\mathrm{d}\tau$$

$f(\theta)^2$

A general space:
with various choices of f function, this metric can cover all the known spheres with U(1)×U(1) isometry, including different resolved spheres. The same Killing spinors exist, provided the following backgrounds

$$H = -\frac{i}{vf(\theta)} \ ,$$
$$A = \left(-\frac{q}{2v^2f(\theta)}\left((v^2-1)\cos 2\theta - 1\right) - \frac{1}{2}\right)\mathrm{d}\tau + \left(\frac{p}{2v^2f(\theta)}\left((v^2-1)\cos 2\theta + 1\right) - \frac{1}{2}\right)\mathrm{d}\phi$$

1.2 SUSY localization of partition function

Localization principle

Witten '1988, Pestun '07, Kapustin-Willett-Yaakov '09

δ represents a fermionic symmetry (off shell) $\delta S = 0$.

Preserved operator $\delta \mathcal{O} = 0$.

The path integral value of the operator $< \mathcal{O} > = \int \mathcal{D}[\phi] e^{iS} \mathcal{O}$

does not change under the deformation $< \mathcal{O} >_t \equiv \int \mathcal{D}[\phi] e^{iS + t\delta V} \mathcal{O}$,

This can be examined by

$$\frac{d}{dt} < \mathcal{O} >_t \equiv \int \mathcal{D}[\phi](\delta V) e^{iS + t\delta V} \mathcal{O} = \delta\left(\int \mathcal{D}[\phi] V e^{iS + t\delta V} O\right) = 0$$

Take $t \to \infty$ the actual integral we need to compute becomes

$$< \mathcal{O} >_{Euc} = \lim_{t \to \infty} \int \mathcal{D}[\phi] e^{-S - t\delta V} \mathcal{O} = \int \mathcal{D}[\phi_\alpha] J[\phi_\alpha] \frac{1}{\sqrt{\mathrm{sdet}_{\phi_\beta} V[\phi_\alpha]}} e^{-S[\phi_\alpha]} \mathcal{O}(\phi_\alpha)$$

saddle point (locus): $\delta V[\phi_\alpha] = 0$

quadratic fluctuations around locus.

Lagrangians of 3D N=2 CS-matter

Hama-Hosomichi-Lee '11,
Closset-Dumitrescu-Festuccia-Komargodski '12

vector multiplet $(a_\mu, \lambda, \bar{\lambda}, \sigma, D)$

$$\mathcal{L}_{\rm YM} = {\rm Tr}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\sigma D^\mu\sigma - i\bar{\lambda}\gamma^\mu D_\mu\lambda - \frac{1}{2}(D+\sigma H)^2 - i\bar{\lambda}[\sigma,\lambda] + \frac{i}{2}H\bar{\lambda}\lambda\right]$$

YM: δ exact, used to localize the vector part. Solving it = 0 gives

locus: $a_\mu = 0\ , \qquad \sigma = \sigma_0\ , \qquad D = -H\sigma_0$

nontrivial classical contribution: $\mathcal{L}_{\rm CS} = \frac{k}{4\pi}{\rm Tr}\left[i\epsilon^{\mu\nu\rho}(a_\mu\partial_\nu a_\rho + \frac{2i}{3}a_\mu a_\nu a_\rho) - 2D\sigma + 2i\bar{\lambda}\lambda\right]$

chiral multiplet (ϕ, ψ, F)

matter Lagrangian $\zeta\widetilde{\zeta}\ \mathcal{L}_{\rm matter} = \delta_\zeta\delta_{\widetilde{\zeta}}\left(\bar{\psi}\psi + 2i\bar{\phi}\sigma\phi\right)$

: δ exact, used to localize matter part. It gives

$$\phi = 0, \qquad F = 0$$

Partition function

general form after localization: CS+FI are the only classic contribution on the locus. And one loop determinants only come from δ exact deformation.

$$Z[k,N;\mathfrak{g},\Delta;\mathcal{M}_3(b_1,b_2)] = \int [\mathrm{d}\sigma_0]\, e^{ikf(b_1,b_2)\mathrm{Tr}\,\sigma_0^2}\mathrm{Det}_{\mathrm{v}}(\sigma_0,b_1,b_2;\alpha)\mathrm{Det}_{\mathrm{ch}}(\sigma_0,b_1,b_2,\Delta;\rho)\ ,$$

Round sphere: Nishioka-Yaakov '2013

Ellipsoid:

$$Z_{\mathrm{saddle}} = e^{\frac{i\pi k}{b_1 b_2}\mathrm{Tr}\,\sigma_0^2}\ , \quad b_1^{-1} = q\tilde{\ell},\ b_2^{-1} = p\ell$$

$$\mathrm{Det}_{\mathrm{ch}}(\sigma_0,b_1,b_2,\Delta;\rho) = s_b\left[\frac{iQ(1-\Delta)}{2} + \frac{\rho(\sigma_0)}{\sqrt{b_1 b_2}}\right]$$

$$\mathrm{Det}_{\mathrm{v}}(\sigma_0,b_1,b_2;\alpha) \sim \prod_{\alpha>0}\left[\frac{1}{\alpha(\sigma_0)^2}\times 4\sinh\frac{\pi\alpha(\sigma_0)}{b_1}\sinh\frac{\pi\alpha(\sigma_0)}{b_2}\right]$$

Squashed sphere:

$$Z_{\mathrm{saddle}} = e^{\frac{i\pi k}{b_1 b_2}\mathrm{Tr}\,\sigma_0^2}\ , \quad b_1 = \frac{v}{q}\ ,\ b_2 = \frac{v}{p}$$

$$\mathrm{Det}_{\mathrm{ch}}(\sigma_0,b_1,b_2,\Delta;\rho) = s_b\left[\frac{iQ(1-\Delta)}{2} + \frac{\rho(\sigma_0)}{\sqrt{b_1 b_2}}\right]$$

$$\mathrm{Det}_{\mathrm{v}}(\sigma_0,b_1,b_2;\alpha) \sim \prod_{\alpha>0}\left[\frac{1}{\alpha(\sigma_0)^2}\times 4\sinh\frac{\pi\alpha(\sigma_0)}{b_1}\sinh\frac{\pi\alpha(\sigma_0)}{b_2}\right]$$

One-loop determinant, chiral matter

matter Lagrangian on branched ellipsoid

$$\zeta\widetilde{\zeta}\,\mathcal{L}_{\rm matter} = \delta_\zeta \delta_{\widetilde{\zeta}}\left(\bar{\psi}\psi + 2i\bar{\phi}\sigma\phi\right)$$

kinetic operators:

$$\Delta_\phi = -D_\mu D^\mu - \frac{2i(\Delta-1)}{f(\theta)} v^\mu D_\mu + \sigma_0^2 + \frac{2\Delta^2 - 3\Delta}{2f(\theta)^2} + \frac{\Delta R}{4}$$

$$\Delta_\psi = -i\gamma^\mu D_\mu - i\sigma_0 - \frac{1}{2f(\theta)} + \frac{\Delta - 1}{f(\theta)}\gamma^\mu v_\mu$$

$$\begin{aligned} D_\mu \phi &= (\nabla_\mu - i\Delta A_\mu)\phi\ , \\ D_\mu \psi &= (\nabla_\mu - i(\Delta-1)A_\mu)\psi \end{aligned}$$

$$v_\mu = \zeta\gamma_\mu\widetilde{\zeta}$$

EOMs

$$\Delta_\psi \psi = \lambda_f \psi$$

$$\Delta_\phi \phi = \lambda_s \phi$$

matching condition: $\lambda_s = \lambda_f(\lambda_f + 2i\sigma_0)$

un-cancelled modes give the one-loop determinant

$$\mathrm{Det}_{\rm ch} = \frac{\det \Delta_\psi}{\det \Delta_\phi} = \prod_{m,n\geq 0} \frac{\frac{n}{p\ell} + \frac{m}{q\tilde{\ell}} - \frac{\Delta-2}{2}\left(\frac{1}{p\ell} + \frac{1}{q\tilde{\ell}}\right) - i\sigma_0}{\frac{n}{p\ell} + \frac{m}{q\tilde{\ell}} + \frac{\Delta}{2}\left(\frac{1}{p\ell} + \frac{1}{q\tilde{\ell}}\right) + i\sigma_0} = s_b\left[\frac{iQ(1-\Delta)}{2} + \frac{\rho(\sigma_0)}{\sqrt{b_1 b_2}}\right]$$

$$b = \sqrt{\frac{b_2}{b_1}} := b_0\sqrt{\frac{q}{p}}\ , \quad b_0 = \sqrt{\frac{\tilde{\ell}}{\ell}}\ , \quad Q = b + 1/b$$

Universal result on conic spheres with U(1)×U(1)

Killing vector (Reeb) $K = \zeta\gamma^{\mu}\widetilde{\zeta}\partial_{\mu} = b_1\partial_{\tau} + b_2\partial_{\phi}$

partition function solely depends on b1 and b2:

$$Z[k, N; \mathfrak{g}, \Delta; b_1, b_2] = \int \prod_{i=1}^{r} \mathrm{d}(\sigma_0)_i \, e^{\frac{i\pi k}{b_1 b_2}\mathrm{Tr}\,\sigma_0^2} \prod_{\alpha>0} 4\sinh\frac{\pi\alpha(\sigma_0)}{b_1}\sinh\frac{\pi\alpha(\sigma_0)}{b_2} \prod_{\rho} s_b\left(\frac{iQ}{2}(1-\Delta) + \frac{\rho(\sigma_0)}{\sqrt{b_1 b_2}}\right)$$

List of K vectors:

round	$K\ell = \partial_{\tau} + \partial_{\phi}$
ellipsoid	$K\ell = \frac{\ell}{\tilde{\ell}}\partial_{\tau} + \partial_{\phi}$
squashed	$K\ell = v\partial_{\tau} + v\partial_{\phi}$
branched round	$K\ell = \frac{1}{q}\partial_{\tau} + \frac{1}{p}\partial_{\phi}$
branched ellipsoid	$K\ell = \frac{\ell}{q\tilde{\ell}}\partial_{\tau} + \frac{1}{p}\partial_{\phi}$
branched squashed	$K\ell = \frac{v}{q}\partial_{\tau} + \frac{v}{p}\partial_{\phi}$

Large N (for fixed k)

define an effective parameter $b=\sqrt{\frac{b_2}{b_1}}$, the partition functions only depend on b, up to an overall constant.

For a certain class of CS matter theories (non-chiral, Σ k_i=0,..), which have M theory dual, the following scaling law is satisfied in the large N limit   Imamura-Yokoyama '11, Martelli-Passias-Sparks '11

$$\log Z[\widetilde{\mathbb{S}}_b^3] = \frac{1}{4}\left(b+\frac{1}{b}\right)^2 \log Z_{b=1}$$

Particularly, for q-branched sphere, the scaling law becomes ($b=\sqrt{q}$):

$\mathbb{S}_q^3$

$$\log Z_q = \frac{(q+1)^2}{4q} \log Z_1$$

Renyi entropy $\quad S_q = \frac{q\log Z_1 - \log Z_q}{q-1} = \frac{3q+1}{4q} S_1$

q=1 gives entanglement entropy (= free energy on round sphere)

$$S_1 = \log Z_1 = -F_1 \xrightarrow{\text{ABJM}} \frac{\pi\sqrt{2}}{3} k^{1/2} N^{3/2}$$

From CFT on $\mathbb{S}_q^3$ to CFT on $\mathbb{S}_q^1 \times \mathbb{H}^2$

$$\mathrm{d}s^2 = \ell^2 \left(\mathrm{d}\theta^2 + \cos^2\theta \mathrm{d}\phi^2 + q^2 \sin^2\theta \mathrm{d}\tau^2\right)$$

$\mathbb{S}_q^3$

$$\sinh\eta = -\cot\theta$$
$$\tau_E = q\tau\ell, \quad \tau_E \in [0, 2\pi q\ell)$$

$$\mathrm{d}s^2 = \sin^2\theta \left(\mathrm{d}\tau_E^2 + \ell^2(\mathrm{d}\eta^2 + \sinh^2\eta \mathrm{d}\phi^2)\right)$$

dropping a Weyl factor

$$\mathrm{d}s^2 = \mathrm{d}\tau_E^2 + \ell^2(\mathrm{d}\eta^2 + \sinh^2\eta \mathrm{d}\phi^2)$$

$\mathbb{S}_q^1 \times \mathbb{H}^2$

no Weyl anomaly in odd dimension:

$$Z[\mathbb{S}_q^3] = Z[\mathbb{S}_q^1 \times \mathbb{H}^2]$$

which motivates us to search for AdS dual with boundary $\mathbb{S}_q^1 \times \mathbb{H}^2$!

Note: the AdS with original q-deformed sphere boundary is difficult to find!

1.3. charged topological black hole in AdS4

Euclidean black hole with boundary $\mathbb{S}^1 \times \mathbb{H}^2$

proceed in Lorentzian first, solution in AdS4

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2 \mathrm{d}\Sigma(\mathbb{H}^2)$$

$$\mathrm{d}\Sigma(\mathbb{H}^2) = \mathrm{d}\eta^2 + \sinh^2\eta\,\mathrm{d}\phi^2$$

such solution exists in 4D N=2 gauged supergravity, with $L = \frac{1}{g}$

$$f(r) = \frac{r^2}{L^2} + \kappa - \frac{2m}{r} + \frac{Q^2}{r^2} \qquad \kappa = -1 \text{ for } \mathbb{H}^2$$

$$A_{\mathrm{TBH}} = \left(\frac{Q}{r} - \mu\right)\mathrm{d}t$$

properties of black hole

$$\mu = \frac{Q}{r_{\mathrm{h}}} \qquad f(r_{\mathrm{h}}) = 0 \qquad T = \frac{f'(r_{\mathrm{h}})}{4\pi}$$

Killing spinor equation

$$\hat{\nabla}_\mu \epsilon = 0 \qquad \hat{\nabla}_\mu = \nabla_\mu - igA_\mu + \frac{1}{2}g\gamma_\mu + \frac{i}{4}F_{\nu\rho}\gamma^{\nu\rho}\gamma_\mu$$

BPS condition

$$\Omega_{\mu\nu}\epsilon = 0 \qquad \det\Theta = \frac{\left(m^2 - \kappa Q^2\right)^2}{Q^4} = 0$$

more about topological black hole

massless uncharged:

$$f(r) = \frac{r^2}{L^2} - 1 \qquad r_{\rm h} = L \qquad T_0 = \frac{1}{2\pi L} \qquad L = \ell$$

Hyperbolic horizon can be mapped to Ryu-Takayanagi surface in pure AdS.

charged BPS:

$$Q^2 = \kappa m^2 \qquad f(r) = \frac{r^2}{L^2} + \kappa \left(1 - \frac{m}{\kappa r}\right)^2$$

to find explicit Killing spinors, it is helpful to use the integrability condition, through a projection operator

$$P := \frac{\Theta}{2\sqrt{f(r)}}$$

$$\Theta\epsilon = 0 \ , \quad \Theta = \sqrt{f(r)} + gr\gamma_1 + \left(\frac{1}{r} - \frac{1}{\kappa m}\right) i\gamma_0 Q$$

1.4. qSCFT3/TBH4 correspondence

black hole computation

first fix the temperature and chemical potential by matching the boundary conditions

$$T(q) = T_0/q$$

$$\mu(q) = -\left(\frac{q-1}{2q}\right) i \qquad g A_{\mathrm{TBH}}(r \to \infty) = A(\mathbb{S}^3_q)$$

state variables: (I is Euclidean on-shell action $I := \log Z(\mu, T)$, $\beta = 1/T$)

$$\begin{aligned} E &= \left(\frac{\partial I}{\partial \beta}\right)_\mu - \frac{\mu}{\beta}\left(\frac{\partial I}{\partial \mu}\right)_\beta , \\ S &= \beta\left(\frac{\partial I}{\partial \beta}\right)_\mu - I , \\ \hat{Q} &= -\frac{1}{\beta}\left(\frac{\partial I}{\partial \mu}\right)_\beta . \end{aligned}$$

holographic super-Renyi entropy:

$$S_q = \frac{q}{q-1}\left(\frac{\log Z_1}{1} - \frac{\log Z_q}{q}\right) = \frac{q}{q-1}\int_q^1 \partial_n \left(\frac{\log Z(T,\mu)}{n}\right) \mathrm{d}n$$

black hole results

total derivative inside the integral

$$\partial_q \left(\frac{\log Z(T,\mu)}{q} \right) = \frac{S}{q^2} - \frac{\widehat{Q}\mu'(q)}{T_0}$$

given charge and Bekestein-Hawking entropy

$$\widehat{Q} = \frac{2V_\Sigma}{\ell_p^2} Q = \left(\frac{2V_\Sigma}{\ell_p^2} \right) \mu(q) r_{\rm h} \qquad\qquad S_{\rm BH} = 2\pi \frac{V_\Sigma}{\ell_p^2} r_{\rm h}^2$$

where horizon is also evaluated as a function of q

$$x(q) := \frac{r_{\rm h}}{L} = \frac{1}{2}\left(1 + \frac{1}{q}\right)$$

Putting all together, holographic Renyi entropy is obtained

$$S_q = 2\pi \left(\frac{L}{\ell_p} \right)^2 V_\Sigma \frac{q}{q-1} \int_q^1 \left(\frac{x(n)^2}{n^2} - 2x(n)\mu(n)\mu'(n) \right) {\rm d}n = \frac{3q+1}{4q} S_1$$

It immediately gives the holographic free energy

$$I_q = \frac{(q+1)^2}{4q} I_1$$

precisely agree with field theory results!

further details

mass-charge relation

$$Q(q) = -\frac{i}{4}L\left(1-\frac{1}{q^2}\right)$$
$$m(q) = -\frac{1}{4}L\left(1-\frac{1}{q^2}\right)$$

BPS !

4d Killing spinor eq.

$$\hat{\nabla}_t = \partial_t - i\frac{1}{L}\left(\frac{Q}{r}-\frac{Q}{r_{\rm h}}\right) + \frac{1}{2L}\sqrt{f(r)}\gamma_0 - i\frac{Q}{2r^2}\sqrt{f(r)}\gamma_1 + \frac{1}{4}f'(r)\gamma_{01},$$
$$\hat{\nabla}_r = \partial_r + \frac{1}{2L}\sqrt{f(r)}^{-1}\gamma_1 - i\frac{Q}{2r^2}\sqrt{f(r)}^{-1}\gamma_0,$$
$$\hat{\nabla}_\eta = \partial_\eta - \frac{1}{2}\sqrt{f(r)}\gamma_{12} + \frac{r}{2L}\gamma_2 - i\frac{Q}{2r}\gamma_{01}\gamma_2,$$
$$\hat{\nabla}_\phi = \partial_\phi - \frac{1}{2}\sqrt{f(r)}\gamma_{13}\sinh\eta - \frac{1}{2}\gamma_{23}\cosh\eta + \frac{1}{2L}r\gamma_3\sinh\eta - i\frac{Q}{2r}\sinh\eta\gamma_{01}\gamma_3 \ .$$

simplified as (with the help of projection operator P)

$$\begin{aligned}\left(\partial_t - \frac{1}{2L}\left(1+2m/r_{\mathrm{h}}\right)\right)\epsilon &= 0\\ \left(\partial_r + \frac{m}{2r(r+m)} + \frac{1}{2L\sqrt{f(r)}}(1+\frac{m}{r+m})\gamma_1\right)\epsilon &= 0\\ \left(\partial_\eta - \frac{1}{2}\gamma_0\gamma_1\gamma_2\right)\epsilon &= 0\\ \left(\partial_\phi - \frac{1}{2}\cosh\eta\gamma_{23} - \frac{1}{2}\sinh\eta(\gamma_0\gamma_1\gamma_3)\right)\epsilon &= 0\end{aligned}$$

solution:

$$\epsilon(t,r,\eta,\phi) = e^{\frac{1}{2qL}t} e^{\frac{\eta}{2}\gamma_0\gamma_1\gamma_2} e^{\frac{\phi}{2}\gamma_{23}}\epsilon(r)$$

$$\epsilon(r) = \left(\sqrt{\frac{r}{L}+\sqrt{f(r)}} - \gamma_0\sqrt{\frac{r}{L}-\sqrt{f(r)}}\right)\left(\frac{1-\gamma_1}{2}\right)\epsilon_0'$$

KSE reduce to 3D, which is identifiable with KSE on S1×H2

$$\left(\nabla_\mu - iA_\mu + \frac{i}{2\ell}e^{\bar{\nu}}_\mu\gamma_{\bar{\nu}}\gamma_{\bar{\tau}_E}\right)\epsilon = 0$$

Part 2. qSCFT4

Claim:

4D N=4 SYM on q-branched 4-sphere $\mathbb{S}_q^4$

$$\mathrm{d}s^2/\ell^2 = \mathrm{d}\theta^2 + q^2 \sin^2\theta \mathrm{d}\tau^2 + \cos^2\theta(\mathrm{d}\phi^2 + \sin\phi^2 \mathrm{d}\chi^2)$$

dual to STU topological black hole in AdS5

$$\mathrm{d}s^2 = -\mathcal{H}^{-4/3} f(r) \mathrm{d}t^2 + \mathcal{H}^{2/3}\left(\frac{1}{f(r)}\mathrm{d}r^2 + r^2 \mathrm{d}\Sigma_{3,k}\right)$$

$$f(r) = k - \frac{m}{r^2} + \frac{r^2}{L^2}\mathcal{H}^2 \ , \quad \mathcal{H}^2 = H_1 H_2 H_3 \ , \quad H_i = 1 + \frac{Q_i}{r^2}$$

Plan:

✧ Killing spinor equation on $\mathbb{S}_q^4$

✧ partition function (heat kernel + localization)

✧ 5D STU topological black hole

✧ TBH5/qSCFT4 correspondence

2.1. 4D Killing spinor equation

Killing Spinors on $\mathbb{S}^4_q$

-- characterize rigid SUSY on curved spacetime

4-sphere (deformed) conformal Killing spinors generally satisfy

$$D_\mu \zeta = +\frac{1}{2\ell}\gamma_\mu \zeta'$$
$$D_\mu \zeta' = -\frac{1}{2\ell}\gamma_\mu \zeta$$

To have solution on q-deformed sphere, we need to add background U(1) field through the covariant derivative

$$D_\mu = \nabla_\mu \pm i A_\mu$$

We take the following gamma matrices (in terms of Pauli matrices)

$$\gamma_1 = \begin{pmatrix} 0 & i\tau_1 \\ -i\tau_1 & 0 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 0 & i\tau_2 \\ -i\tau_2 & 0 \end{pmatrix}$$
$$\gamma_3 = \begin{pmatrix} 0 & i\tau_3 \\ -i\tau_3 & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & 1_{2\times 2} \\ 1_{2\times 2} & 0 \end{pmatrix}$$

Round 4-sphere can be considered as 3-sphere fibered on a segment

$$\mathrm{d}s^2/\ell^2 = \mathrm{d}\rho^2 + \sin\rho^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\tau^2 + \cos^2\theta\mathrm{d}\phi^2)$$

We take the vielbein

$$\begin{aligned}
e^1/\ell &= \sin\rho\sin(\tau+\phi)\mathrm{d}\theta + \sin\rho\cos(\tau+\phi)\sin\theta\cos\theta(\mathrm{d}\tau - \mathrm{d}\phi) \ , \\
e^2/\ell &= -\sin\rho\cos(\tau+\phi)\mathrm{d}\theta + \sin\rho\sin(\tau+\phi)\sin\theta\cos\theta(\mathrm{d}\tau - \mathrm{d}\phi) \ , \\
e^3/\ell &= \sin\rho\,(\sin\theta^2\mathrm{d}\tau + \cos\theta^2\mathrm{d}\phi) \ , \qquad e^4/\ell = \mathrm{d}\rho \ .
\end{aligned}$$

and find the identity

$$\frac{1}{4}\omega_\mu = \gamma_\mu T(\rho) \ , \qquad \mu = \theta, \tau, \phi$$

where T matrix is defined as

$$T(\rho) = \begin{pmatrix} 0 & 0 & -\frac{1}{2}\tan\frac{\rho}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2}\tan\frac{\rho}{2} \\ \frac{1}{2}\cot\frac{\rho}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\cot\frac{\rho}{2} & 0 & 0 \end{pmatrix}$$

It implies constant 4-spinor satisfy $\mu = \theta, \tau, \phi$ components of KSE, provided

$$\frac{1}{2\ell}\zeta' := T(\rho)\zeta$$

After taking care of the ρ-component KSE, the solution is

$$\zeta = S(\rho)\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} \qquad S(\rho) = \begin{pmatrix} \sin\frac{\rho}{2} & 0 & 0 & 0 \\ 0 & \sin\frac{\rho}{2} & 0 & 0 \\ 0 & 0 & \cos\frac{\rho}{2} & 0 \\ 0 & 0 & 0 & \cos\frac{\rho}{2} \end{pmatrix}$$

When the 4-sphere is deformed by q, the above solution still valid provided

$$D_\mu = \nabla_\mu + iA_\mu \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad A_{\mathbb{S}^4_q} = \frac{q-1}{2}\mathrm{d}\tau$$

U(1)^3 R-symmetry of N=4 SYM

There could be 3 different U(1) background field A^i, (i=1,2,3), which couple to dynamical fields as

Table 1: charges under three U(1)'s

	ψ^1	ψ^2	ψ^3	ψ^4	$\mathcal{A}_\mu$	ϕ^1	ϕ^2	ϕ^3
k_1	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	$+1$	0	0
k_2	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	$+1$	0
k_3	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	$+1$

Note: Killing spinors charged under all 3 U(1)s, ±1/2. We are particularly interested in those sharing the same sign for different U(1).

2.2. partition function (heat kernel + localization)

4D Renyi entropy with SUSY

The external gauge field we found before by solving the KSE, provides a chemical potential, which makes the partition function SUSY invariant.

Therefore SUSY Renyi entropy is defined as

$$S_q = \frac{1}{1-q} \log \left(\frac{Z_q(\mu)}{Z_1(0)^q} \right)$$

For N=4 SYM, there are 3 external gauge fields, the effective chemical potential for each field is given by the weighted sum of individuals

$$\mu = k_i \mu_i$$

$\mu_i = A^i$, by definition.

For Killing spinors, the effective chemical potential has to be

$$\boxed{\frac{q-1}{2} = k_i \mu_i}$$

From CFT on $\mathbb{S}^4_q$ to CFT on $\mathbb{S}^1 \times \mathbb{H}^3$

$$ds^2/\ell^2 = d\theta^2 + q^2 \sin^2\theta d\tau^2 + \cos^2\theta d\Sigma_{d-2,+1}$$

$$\downarrow \quad \sinh\eta = -\cot\theta$$

$$ds^2 = \sin^2\theta \left(d\tau^2 + \ell^2(d\eta^2 + \sinh^2\eta d\Sigma_{d-2,+1})\right)$$

$$\downarrow$$

$$ds^2 = d\tau^2 + \ell^2(d\eta^2 + \sinh^2\eta d\Sigma_{d-2,+1})$$

conformal invariance+
unitary transformation:

$$Z[\mathbb{S}^d_q] = Z[\mathbb{S}^1_q \times \mathbb{H}^{d-1}]\,, \quad d \text{ odd}$$

$$a[\mathbb{S}^d_q] = a[\mathbb{S}^1_q \times \mathbb{H}^{d-1}]\,, \quad d \text{ even}$$

Super-Renyi entropy in free limit

The partition function Z(β) on $\mathbb{S}^1_\beta \times \mathbb{H}^d$ can be computed by heat kernel of the Laplacian operator, $\beta = 2\pi q$.

$$\log Z(\beta) = \frac{1}{2}\int_0^\infty \frac{dt}{t} K_{\mathbb{S}^1\times\mathbb{H}^d}(t)$$

$$K_{\mathbb{S}^1\times\mathbb{H}^d}(t) = K_{\mathbb{S}^1}(t)\, K_{\mathbb{H}^d}(t) e^{(d-1)^2\pi^2 t}$$

$$K_{\mathbb{S}^1}(t) = \frac{\beta}{\sqrt{4\pi t}} \sum_{n\neq 0, \in\mathbb{Z}} e^{\frac{-\beta^2 n^2}{4t}}$$

$$K_{\mathbb{H}^3}(t) = \int d^3x\sqrt{g}\; K_{\mathbb{H}^3}(x,x,t) := V K_{\mathbb{H}^3}(0,t)$$

for a complex scalar: $$K^b_{\mathbb{H}^3}(0,t) = \frac{2}{(4\pi t)^{d/2}} e^{-(d-1)^2\pi^2 t}\,, \qquad d = 3$$

for a Weyl fermion: $$K^f_{\mathbb{H}^3}(0,t) = \frac{2(1+\frac{t}{2})}{(4\pi t)^{d/2}} e^{-(d-1)^2\pi^2 t}\,, \qquad d = 3$$

Turning on a background field along S^1 = a phase shift for the S^1 kernel. Taking use all the above formulae,

$$F^b(\beta,\mu) := -\log Z^b(\beta,\mu) = -V \sum_{n\neq 0,\in\mathbb{Z}} \frac{1}{2}\int_0^\infty \left[\frac{\mathrm{d}t}{t}\frac{\beta}{\sqrt{4\pi t}}e^{\frac{-n^2\beta^2}{4t}}\frac{2}{(4\pi t)^{3/2}}\right]e^{i2n\pi\mu}$$

$$F^f(\beta,\mu) = V \sum_{n\neq 0,\in\mathbb{Z}} \frac{1}{2}\int_0^\infty \left[\frac{\mathrm{d}t}{t}\frac{\beta}{\sqrt{4\pi t}}e^{\frac{-n^2\beta^2}{4t}}\frac{2(1+\frac{t}{2})}{(4\pi t)^{3/2}}\right]e^{i(2\pi\mu-\pi)n}$$

Evaluating this, one obtains

$$F_q^b(\mu) := F^b(2\pi q,\mu) = \frac{V\left(\mu^4+2\mu^3+\mu^2-\frac{1}{30}\right)}{12\pi q^3}$$

$$F_q^f(\mu) = -V\left[\frac{240\mu^4-120\mu^2+\left(30-360\mu^2\right)q^2+7}{2880\pi q^3}\right]$$

Then super-Renyi entropy is obtained by

$$S_q^{\text{super}} = \frac{qF_1(0)-F_q(\mu(q))}{1-q}$$

Reproduce the known result of non-SUSY Renyi entropy of N=4 SYM

$$S_q^{\text{non-SUSY}} = 6 \times \frac{S^b}{2} + 4 \times S^f + S^v = \frac{\left(1+q+7q^2+15q^3\right)V}{48\pi q^3}$$

where we inserted Renyi entropy for a maxwell field

$$S^v = \frac{\left(91q^3+31q^2+q+1\right)V}{360\pi q^3}$$

which is valid even in the later super-Renyi entropy case, since vector field is uncharged under R-symmetry.

For convenience, we extract extra contribution due to the chemical potential

$$\Delta S := S_q - S_q^{\text{non-SUSY}}$$

$$\Delta S^b(\mu) = \frac{(\mu+1)^2\mu^2 V}{12\pi(q-1)q^3}$$

$$\Delta S^f(\mu) = \frac{\mu^2\left(-2\mu^2+3q^2+1\right)V}{24\pi(q-1)q^3}$$

We are now ready to compute the super-Renyi entropy of N=4 SYM.

A single U(1): $|k_3| = \frac{1}{2}, \qquad \mu_3 = q - 1$

$$S_q = S_q^{\text{non-SUSY}} + 4\Delta S_f(\mu = \frac{q-1}{2}) + \Delta S_b(\mu = q-1)$$

$$\boxed{\frac{S_q}{S_1} = 1}$$

Two U(1)s (with equal values): $|k_1 + k_2| = 1 \qquad \mu_1 = \mu_2 = \frac{q-1}{2}$

$$S_q = S_q^{\text{non-SUSY}} + 2\Delta S_f(\mu = \frac{q-1}{2}) + 2\Delta S_b(\mu = \frac{q-1}{2})$$

$$\boxed{\frac{S_q}{S_1} = \frac{3q+1}{4q}}$$

Three U(1)s (different values): $|k_1 + k_2 + k_3| = \frac{3}{2}$

$$\mu_1 = (q-1)\frac{a}{3}\,, \quad \mu_2 = (q-1)\frac{b}{3}\,, \quad \mu_3 = (q-1)\left(1 - \frac{a+b}{3}\right)$$

$$\frac{S_q}{S_1} = \frac{1}{27q^2}\left(q^2 C_2 + q C_1 + C_0\right)$$

$$C_2 = -a^2(-3+b) - a(-3+b)^2 + 3(9 - 3b + b^2) \; ,$$

$$C_1 = a^2(2b-3) + a\left(2b^2 - 9b + 9\right) - 3(b-3)b \; ,$$

$$C_0 = -ab(a+b-3) \; .$$

$$\mu_1 = (q-1)\frac{a}{3} \; , \quad \mu_2 = (q-1)\frac{b}{3} \; , \quad \mu_3 = (q-1)\left(1 - \frac{a+b}{3}\right)$$

$$\frac{S_q}{S_1} = 1$$

$$\frac{S_q}{S_1} = \frac{3q+1}{4q}$$

$$\frac{S_q}{S_1} = \frac{19q^2 + 7q + 1}{27q^2}$$

Exact partition function on resolved 4-sphere

$$\mathrm{d}s^2 = f_\epsilon(\theta)^2\,\mathrm{d}\theta^2 + \ell^2\big(q^2\sin^2\theta\,\mathrm{d}\tau^2 + \cos^2\theta(\mathrm{d}\phi^2 + \sin^2\phi\,\mathrm{d}\chi^2)\big)$$

$$f_\epsilon(\theta) = \begin{cases} q\ell\,, & \theta \to 0 \\ \ell\,, & \epsilon < \theta \leq \frac{\pi}{2} \end{cases}$$

Following the set up in arXiv:1206.6359 by Hama&Hosomichi, one can construct 4D N=2 gauge theory on the resolved sphere.
The particular N=4 SYM case in $\epsilon \to 0$ limit reduce to two U(1)s with equal values we have just discussed,

$$A^{U(1)_J} = \frac{1}{2}(A^1 + A^2) \qquad A^1 = A^2 = \frac{q-1}{2}$$

One can further show that the N=2 partition function is independent of $f_\epsilon(\theta)$ and it is given by

$$Z = \int \prod_i \mathrm{d}(\hat{a}_0)_i\, e^{-\frac{8\pi^2}{g_{\mathrm{YM}}^2}\mathrm{Tr}(\hat{a}_0^2)} \frac{\prod_{\alpha\in\Delta_+}\Upsilon_q(i\hat{a}_0\cdot\alpha)\Upsilon_q(-i\hat{a}_0\cdot\alpha)}{\prod_{\mathcal{I}}\prod_{\rho\in R_{\mathcal{I}}}\Upsilon_q(i\hat{a}_0\cdot\rho + \frac{Q}{2})}|Z_{\mathrm{inst}}|^2$$

Large N (planar limit)

The partition function of N=4 SYM

$$Z = \int \prod_i \mathrm{d}(\hat{a}_0)_i e^{-\frac{8\pi^2 N}{\lambda}\mathrm{Tr}(\hat{a}_0^2)} \prod_{\alpha\in\Delta_+} \frac{\Upsilon_q(i\hat{a}_0\cdot\alpha)\Upsilon_q(-i\hat{a}_0\cdot\alpha)}{\Upsilon_q(i\hat{a}_0\cdot\alpha+\frac{Q}{2})\Upsilon_q(-i\hat{a}_0\cdot\alpha+\frac{Q}{2})}|Z_{\mathrm{inst}}|^2$$

Recall that Υ function is defined as the regularized product

$$\Upsilon_q(x) = \prod_{m,n\geq 0}\left(mq^{1/2}+nq^{-1/2}+Q-x\right)\left(mq^{1/2}+nq^{-1/2}+x\right)\,, \qquad Q\equiv\sqrt{q}+\frac{1}{\sqrt{q}}$$

In the planar limit, it is governed the saddle point

$$\fint_{-\mu}^{\mu} dy\,\rho(y)K(x-y) = \frac{8\pi^2}{\lambda}x$$

$$\rho(x) = \frac{1}{N}\sum_i \delta(x-(\hat{a}_0)_i) \qquad K(x) = \frac{1}{2}\partial_x \log\left(\frac{\Upsilon_q(ix)\Upsilon_q(-ix)}{\Upsilon_q(ix+\frac{Q}{2})\Upsilon_q(-ix+\frac{Q}{2})}\right)$$

To solve it, we take the large x expansion, which is essentially the large coupling limit

$$K(x) = \frac{(1+q)^2}{4q}\frac{1}{x}+\frac{(q^2-1)^2}{96q^2}\frac{1}{x^3}+\mathcal{O}(x^{-4})$$

To leading order,

$$K(x) \approx \frac{Q^2}{4}\frac{1}{x}\,, \qquad Q = \sqrt{q} + \frac{1}{\sqrt{q}}$$

The saddle point eq. becomes that of N=4 SYM on round S^4 with a redefined coupling

$$⨍_{-\mu}^{\mu} dy\,\rho(y)\frac{1}{x-y} = \frac{8\pi^2}{\widetilde{\lambda}}x\,, \qquad \widetilde{\lambda} = \frac{Q^2}{4}\lambda$$

It is solved by

$$\rho(x) = \frac{8\pi}{\widetilde{\lambda}}\sqrt{\mu^2 - x^2} \qquad\qquad \mu = \frac{\sqrt{\widetilde{\lambda}}}{2\pi} = \frac{\sqrt{\lambda}}{4\pi}Q$$

Evaluating the partition function

$$\begin{aligned} F_q &= -\log Z_q \\ &= \frac{8\pi^2 N^2}{\lambda}\int_{-\mu}^{\mu}\rho(x)x^2 \mathrm{d}x - \frac{N^2}{2}\frac{Q^2}{4}\int_{-\mu}^{\mu}\rho(x)⨍_{-\mu}^{\mu}\rho(y)\log(x-y)^2 \mathrm{d}x\mathrm{d}y \end{aligned}$$

The relevant log term

$$\boxed{F_q = -\frac{1}{2}N^2\frac{\widetilde{\lambda}}{\lambda}\log\widetilde{\lambda} = -\frac{1}{2}N^2\frac{Q^2}{4}\log\widetilde{\lambda}}$$

Log divergence can be recovered: Russo&Zarembo '12

$$\log\lambda \to \log\lambda - \log\left(\tfrac{\ell}{\Lambda}\right)^2$$

The universal log term

$$F_q = \frac{Q^2}{4}F_1 = \frac{1}{4}\left(\sqrt{q} + \frac{1}{\sqrt{q}}\right)^2 F_1$$

Super-Renyi entropy

$$\boxed{\frac{S_q}{S_1} = \frac{3q+1}{4q}}$$

which agrees with heat kernel result in the free limit.

2.3. STU topological black hole in AdS5

Euclidean black hole with boundary $\mathbb{S}^1_q \times \mathbb{H}^3$

In Lorentzian first, solution in 5D N=2 gauged SUGRA Behrnd,Cvetic,Sabran '98

$$ds^2 = -\mathcal{H}^{-4/3} f(r) dt^2 + \mathcal{H}^{2/3} \left(\frac{1}{f(r)} dr^2 + r^2 d\Sigma_{3,k} \right)$$

$$f(r) = k - \frac{m}{r^2} + \frac{r^2}{L^2} \mathcal{H}^2 \ , \quad \mathcal{H}^2 = H_1 H_2 H_3 \ , \quad H_i = 1 + \frac{Q_i}{r^2}$$

$$X^i = \frac{\mathcal{H}^{2/3}}{H_i} \ , \quad A^i = \left[\sqrt{k + \frac{m}{Q_i}} \left(\frac{1}{H_i} - 1 \right) - \hat{\mu}_i \right] dt$$

We focus on m=0, k=-1. And it can be checked that, it is a BPS solution.
To study this BH, we define rescaled charges

$$\kappa_i := \frac{Q_i}{r_h^2}$$

It turns out that, all other physical quantities can be expressed as function of κ only, including horizon, temperature, total charge, entropy and so on.

Physical quantities

$$T = \frac{1 - \kappa_1\kappa_2 - \kappa_1\kappa_3 - \kappa_2\kappa_3 - 2\kappa_1\kappa_2\kappa_3}{(1+\kappa_1)(1+\kappa_2)(1+\kappa_3)} T_0 \ , \qquad T_0 = \frac{1}{2\pi L}$$

$$S_{\rm BH} = \frac{A}{4G_5} = \frac{V_3 L^3}{4G_5} \frac{1}{(1+\kappa_1)(1+\kappa_2)(1+\kappa_3)}$$

$$\widehat{Q}_i = \frac{V_3 L^2}{8\pi G_5} \frac{i\kappa_i}{(1+\kappa_1)(1+\kappa_2)(1+\kappa_3)}$$

$$\hat{\mu}_i = A^i_t \big|_{r\to\infty} = \frac{i}{\kappa_i^{-1} + 1}$$

2.4. TBH5/qSCFT4 correspondence

Holographic super-Renyi entropy

First express κ in term of q, therefore everything is in terms of q.

$$T = T_0/q$$

Then use the formula YZ-Huang-Rey '14

$$S_q = \frac{-q}{q-1}\int_q^1 \left(\frac{S_{\mathrm{BH}}(n)}{n^2} - \frac{\widehat{Q}(n)\hat{\mu}'(n)}{T_0}\right)\mathrm{d}n$$

which can be derived from

$$I_q := -\log Z(T, \mu_i)$$

$$\begin{aligned}
E &= \left(\frac{\partial I}{\partial \beta}\right)_{\mu} - \frac{\mu}{\beta}\left(\frac{\partial I}{\partial \mu}\right)_{\beta} , \\
S &= \beta\left(\frac{\partial I}{\partial \beta}\right)_{\mu} - I , \\
\hat{Q} &= -\frac{1}{\beta}\left(\frac{\partial I}{\partial \mu}\right)_{\beta} .
\end{aligned}$$

black hole results

For generic 3 charges, holographic super-Renyi entropy is

$$\frac{S_q}{S_1} = \frac{(a^2+ab-3a)(q-1)[3q-b(q-1)]+3q[b(b-3)(q-1)+9q]}{27q^2}$$

$$\frac{S_q}{S_1} = 1$$

$$I_q = I_1$$

$$\frac{S_q}{S_1} = \frac{3q+1}{4q}$$

$$I_q = \frac{(q+1)^2}{4q} I_1$$

$$\frac{S_q}{S_1} = \frac{19q^2+7q+1}{27q^2}$$

$$I_q = \frac{(2q+1)^3}{27q^2} I_1$$

:precisely agree with field theory results!

Conclusion and remarks

We proposed a class of TBH/qSCFT correspondence and show the precise agreements between field theory exact computations and gravity results.

Other BPS observables, such as Wilson loop and correlation functions (involving q) can be tested in TBH/qSCFT.

Thanks for your attention.