Symmetries and Feynman Rules for Ramond Sector in WZW-type Superstring Field Theories

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1. Introduction

♠ WZW-type Superstring field theory

♢ A formulation with *no explicit picture changing operator* and working well for NS sector:

- Open [Berkovits (1995)]
- Hetetrotic [Berkovits, Okawa and Zwiebach (2004)]
- Type II [Matsunaga (2014)]

♢ Difficult to construct an action for R sector

- EOM [Berkovits (2001)] [HK (2014)]
- A pseudo-action [Michishita (2005)] [HK (2014)]

  We can *construct* the (self-dual) Feynman rules reproducing the on-shell four point amplitudes.

  However, these rules do not reproduce the five-point amplitudes [Michishita (2007)], or have an ambiguity [HK (2013)].

Q: Can we construct Feynman rules for R sector reproducing correct on-shell physical amplitudes?

Plan of the Talk

1. Introduction

2. New Feynman rules in the WZW-type open Super SFT

   2.1 EOM and the pseudo-action

   2.2 Gauge fixing and the self-dual Feynman rules

   2.3 Gauge symmetries and the new Feynman rules

3. amplitude with the external fermions

4. Summary and discussion

2. New Feynman rules in the WZW-type Open Super SFT

2.1 EOM [Berkovits] *and the pseudo-action* [Michishita]

Large Hilbertspace : $(\eta \equiv \eta_0)$

$$\mathcal{H}_{large} \ni A = \alpha + \xi_0 a, \quad \text{with} \quad \alpha, a \in \mathcal{H}_{small} \quad (i.e.\ \ \eta\alpha = \eta a = 0).$$

String fields :

$$\text{NS} \ : \Phi \ \in \mathcal{H}^{(NS)}_{large}, \quad (G, P) = (0, 0), \quad |\Phi| = 0,$$
$$\text{R} \ : \Psi \ \in \mathcal{H}^{(R)}_{large}, \quad (G, P) = (0, 1/2), \quad |\Psi| = 0,$$

EOM including the R sector :

$$\eta(g^{-1}Qg) + (\eta\Psi)^2 = 0, \qquad Q'\eta\Psi = 0, \tag{1}$$

where $g = e^{\Phi}$ and $Q'A = QA + A(g^{-1}Qg) - (-1)^{|A|}(g^{-1}Qg)A$.

Gauge symmetry :

$$g^{-1}\delta g = Q'\Lambda_0 + \eta\Lambda_1 - (\eta\Psi)\Lambda_{\frac{1}{2}} - \Lambda_{\frac{1}{2}}(\eta\Psi), \tag{2a}$$

$$\delta\Psi = Q'\Lambda_{\frac{1}{2}} + \eta\Lambda_{\frac{3}{2}} + \Psi(\eta\Lambda_1) - (\eta\Lambda_1)\Psi. \tag{2b}$$

NS action :

$$S_{NS} = \frac{1}{2}\langle(g^{-1}Qg)(g^{-1}\eta g) - \int_0^1 dt(\hat{g}^{-1}\partial_t\hat{g})\{(\hat{g}^{-1}Q\hat{g}),(\hat{g}^{-1}\eta\hat{g})\}\rangle,$$

where $\hat{g} = g(t) = e^{t\Phi}$.

Auxiliary field :

$$\Xi \quad \in \mathcal{H}^{(R)}_{large}, \quad (G,P) = (0,-1/2), \quad |\Xi| = 0.$$

The pseudo-action :

$$S_R = -\frac{1}{2}\langle(Q\Xi)g(\eta\Psi)g^{-1}\rangle. \tag{3}$$

The variation of $S = S_{NS} + S_R$ yields

$$\eta(g^{-1}Qg) + \frac{1}{2}\{\eta\Psi, Q'\Xi'\} = 0, \qquad Q'\eta\Psi = 0, \qquad \eta Q'\Xi' = 0. \tag{4}$$

where $\Xi' = g^{-1}\Xi g$. If we eliminate Ξ by imposing a constraint

$$Q'\Xi' = \eta\Psi,$$

EOMs (4) become EOMs (1).

2.2 Gauge fixing and the self-dual Feynman rules

Since the pseudo-action (3) is not the true action we cannot derive the Feynman rules, but we can construct them.

Quadratic part of the action

$$S = \frac{1}{2}\langle (Q\Phi)(\eta\Phi)\rangle - \frac{1}{2}\langle (Q\Xi)(\eta\Psi)\rangle, \tag{5}$$

has gauge symmetries

$$\delta\Phi = Q\Lambda_0 + \eta\Lambda_1, \qquad \delta\Psi = Q\Lambda_{\frac{1}{2}} + \eta\Lambda_{\frac{3}{2}}, \qquad \delta\Xi = Q\Lambda_{-\frac{1}{2}} + \eta\tilde{\Lambda}_{\frac{1}{2}}. \tag{6}$$

Fix them by (the simplest) gauge conditions

$$b_0\Phi = \xi_0\Phi = 0, \qquad b_0\Psi = \xi_0\Psi = 0, \qquad b_0\Xi = \xi_0\Xi = 0.$$

Then (5) can be inverted as

$$\overbracket{\Phi\Phi} = \frac{\xi_0 b_0}{L_0} = \int_0^\infty d\tau(\xi_0 b_0)e^{-\tau L_0}, \qquad \overbracket{\Psi\Xi} = \overbracket{\Xi\Psi} = -2\,\frac{\xi_0 b_0}{L_0} = -2\,\overbracket{\Phi\Phi}.$$

The self-dual Feynman rules : [Michishita]

An essential prescription

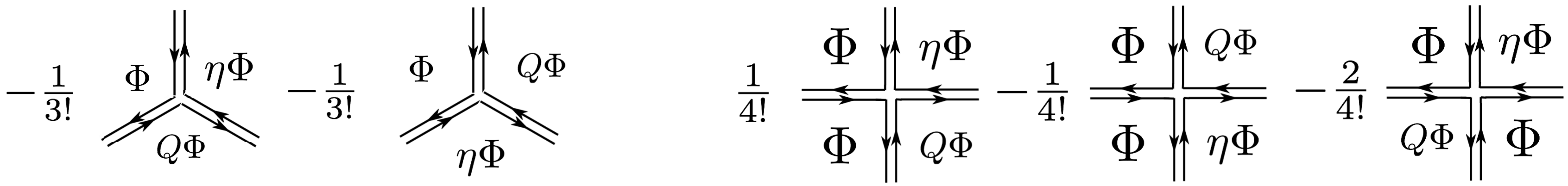
Taking into account the constraint by replacing $\eta\Psi$ and $Q\Xi$ in vertices by their (linearized) "self-dual" part $\omega = (\eta\Psi + Q\Xi)/2$.

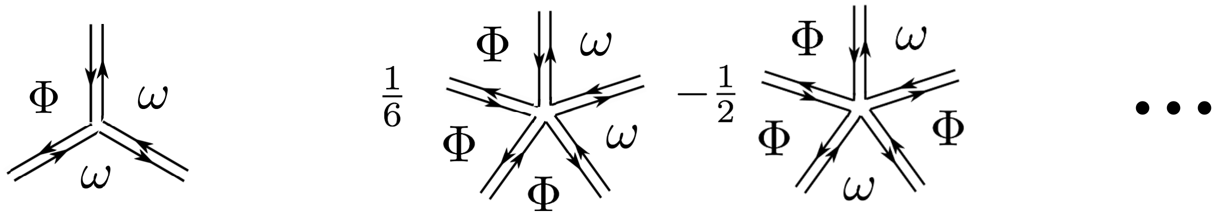
(∼ A part vanishing under the constraint should be decoupled.)

Propagator :

$$\mathrm{NS} \;:\; \overbrace{\Phi\Phi} = \frac{\xi_0 b_0}{L_0}, \qquad \mathrm{R} \;:\; \overbrace{\omega\omega} = \frac{1}{2}\left(Q\left(\frac{\xi_0 b_0}{L_0}\right)\eta + \eta\left(\frac{\xi_0 b_0}{L_0}\right)Q\right) \quad \text{(diagonal)}$$

Vertices :





On-shell external fermions : $\omega = \eta\Psi$

by imposing the linearized constraint $Q\Xi = \eta\Psi$.
(It automatically implies the on-shell condition $Q\eta\Psi = 0$.)

The self-dual Feynman rules give the well-known (on-shell) 4-point amplitudes but do not reproduce the 5-point amplitudes. [Michishita]

2.3 Gauge symmetries and the new Feynman rules

Gauge symmetries :

The (pseudo-) action $S = S_{NS} + S_R$ is invariant under

$$g^{-1}\delta g = Q'\Lambda'_0 + \eta\Lambda_1,$$
$$\delta\Psi = \eta\Lambda_{\frac{3}{2}} + [\Psi, \eta\Lambda_1], \qquad \delta\Xi = Q\Lambda_{-\frac{1}{2}} + [Q, \Lambda_0].$$

These are compatible with the self-dual/anti-self-dual decomposition.

$$\delta(Q'\Xi' \pm \eta\Psi) = [(Q'\Xi' \pm \eta\Psi), \Lambda_1],$$

and so respected by the self-dual rules. However, these symmetries are not enough to gauge away all the un-physical states. They do not contain all the symmetries of the quadratic action (6).

The missing "symmetries" generated by $\Lambda_{\frac{1}{2}}$ and $\tilde{\Lambda}_{\frac{1}{2}}$ can be extended to

$$g^{-1}\delta g = -\frac{1}{2}\{Q'\Xi', \Lambda_{\frac{1}{2}}\} + \frac{1}{2}\{\eta\Psi, \tilde{\Lambda}_{\frac{1}{2}}\}, \tag{7a}$$
$$\delta\Psi = Q'\Lambda_{\frac{1}{2}}, \qquad \delta\Xi = g(\eta\tilde{\Lambda}_{\frac{1}{2}})g^{-1}. \tag{7b}$$

The variation of the action under these transformations becomes

$$\delta S = \frac{1}{4}\langle \Lambda_{\frac{1}{2}}\, [(Q'\Xi')^2, (Q'\Xi' - \eta\Psi)]\rangle + \frac{1}{4}\langle \tilde{\Lambda}_{\frac{1}{2}}\, [(\eta\Psi)^2, (Q'\Xi' - \eta\Psi)]\rangle,$$

which vanishes under the constraint.

These "symmetries" (7) are not respected by the self-dual rules because they are not compatible with the self-dual/anti-self-dual decomposition.

Ansatz

The correct Feynman rules should respect these "symmetries" too.

The new Feynman rules :

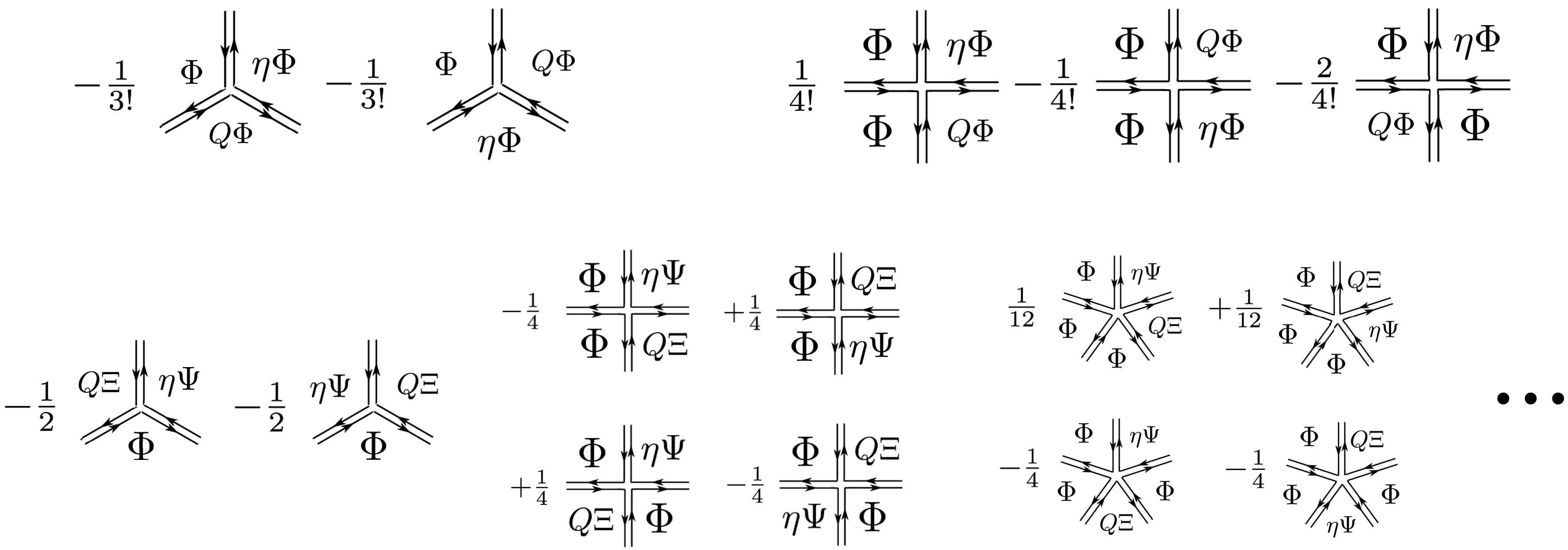
An essential prescription

Do not impose any restriction on the off-shell (propagating) states.

Propagator :

$$\text{NS} \;:\; \overline{\Phi\Phi} = \frac{\xi_0 b_0}{L_0}, \qquad \text{R} \;:\; \overline{\Psi\Xi} = \overline{\Xi\Psi} = -2\,\frac{\xi_0 b_0}{L_0} \equiv \Pi_R, \quad \text{(off-diagonal)}$$

Vertices :



Notes :

∘ Fermion number, $F(\Psi) = 1,\ F(\Xi) = -1$, is conserved.

∘ Then R-propagator is directed.

On-shell external fermions : $\eta\Psi$

by imposing the linearized constraint $Q\Xi = \eta\Psi$, after adding (averaging) two possibilities of the external fermions, $\eta\Psi$ and $Q\Xi$.

We can show that the new Feynman rules correctly reproduce all the on-shell physical 4- and 5-point amplitudes.

3. Amplitudes with the external fermions

Four-point amplitudes are correctly reproduced as in the self-dual rules.

Difference between two rules comes from the diagram with either

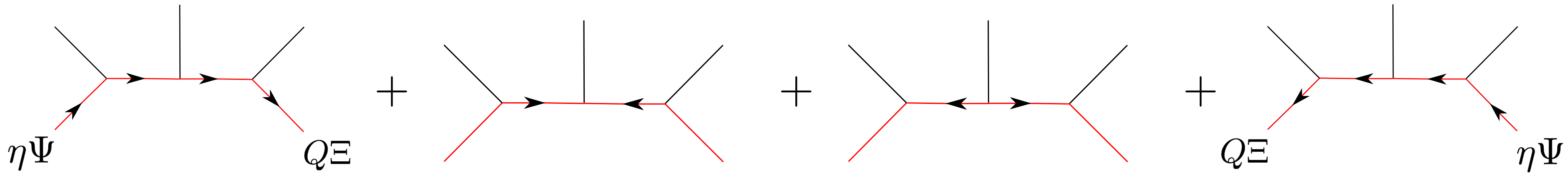
(i) at least two fermion propagators. The amplitude is schematically

$$\mathcal{A} \sim \langle \cdots (Q\Pi_R\eta + \eta\Pi_R Q) \cdots (Q\Pi_R\eta + \eta\Pi_R Q) \cdots \rangle_W,$$

in the self-dual rules and

$$\mathcal{A} \sim \langle \cdots Q\Pi_R\eta \cdots Q\Pi_R\eta \cdots \rangle_W + \langle \cdots \eta\Pi_R Q \cdots \eta\Pi_R Q \cdots \rangle_W,$$

in the new rules, where we can assign a direction to fermion lines,



or,

(ii) two-fermion-even-boson interactions,

$$e.g. \qquad S_R^{(4)} = -\frac{1}{4}\Big(\langle\Phi^2(Q\Xi)(\eta\Psi)\rangle - \langle\Phi^2(\eta\Psi)(Q\Xi)\rangle\Big) + \frac{1}{4}\Big(\langle\Phi(Q\Xi)\Phi(\eta\Psi)\rangle - \langle\Phi(\eta\Psi)\Phi(Q\Xi)\rangle\Big),$$

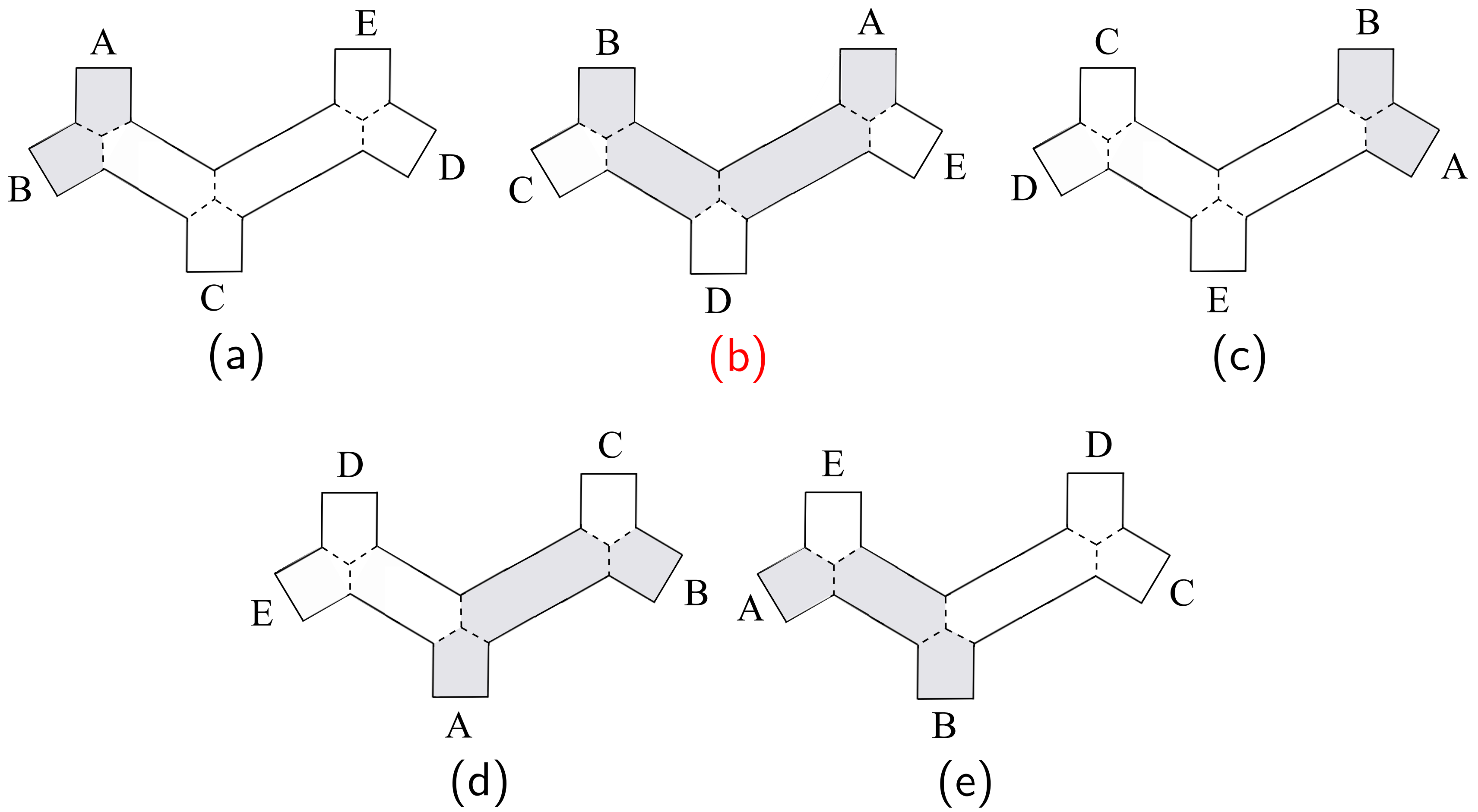
with at least one off-shell fermion, which gives non-trivial contribution to the amplitude.

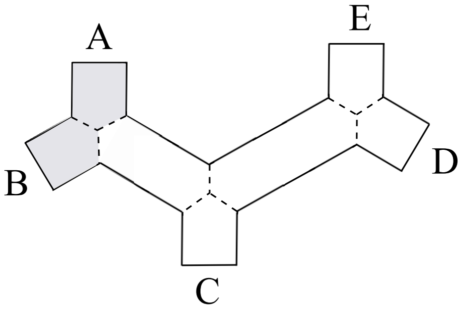
These differences improve the discrepancy in the five-point amplitudes!

3.2 Five-point amplitudes

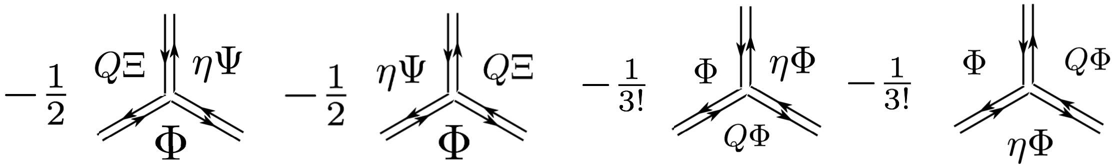
<u>Example</u> : $(ABCDE) = (FFBBB)$

Five two-propagator (2P) diagrams

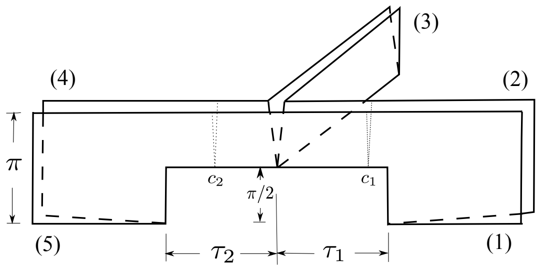




(a)



$$
\begin{aligned}
\mathcal{A}^{(2\mathrm{A})(a)}_{FFBBB} = \frac{1}{8}\int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \\
\times \Big(\langle (Q\Xi_A \eta\Psi_B + \eta\Psi_A Q\Xi_B)(\xi_{c_1} b_{c_1} Q)\Phi_C (\eta \xi_{c_2} b_{c_2})(Q\Phi_D \eta\Phi_E + \eta\Phi_D Q\Phi_E)\rangle_W \\
+ \langle (Q\Xi_A \eta\Psi_B + \eta\Psi_A Q\Xi_B)(\xi_{c_1} b_{c_1} \eta)\Phi_C (Q \xi_{c_2} b_{c_2})(Q\Phi_D \eta\Phi_E + \eta\Phi_D Q\Phi_E)\rangle_W \Big).
\end{aligned}
$$



Witten diagram

Using

$$\int_0^\infty d\tau \{Q, b_0\} e^{-\tau L_0} = -\int_0^\infty d\tau \frac{\partial}{\partial\tau} e^{-\tau L_0},$$

we can align three external bosons as $(Q\Phi_C, Q\Phi_D, \eta\Phi_E)$:

$$\mathcal{A}^{(2\mathrm{A})(a)}_{FFBBB} = \frac{1}{2}\int_0^\infty d^2\tau \, \langle \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big) \, (\xi_{c_1} b_{c_1}) \, Q\Phi_C \, b_{c_2} \, Q\Phi_D \, \eta\Phi_E \rangle_W$$

$$+ \frac{1}{8}\int_0^\infty d\tau \, \Bigg(\langle \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big)$$

$$\times (\xi_c b_c) \, \Phi_C \, \Big(Q\Phi_D \, \eta\Phi_E + \eta\Phi_D \, Q\Phi_E\Big) \rangle_W$$

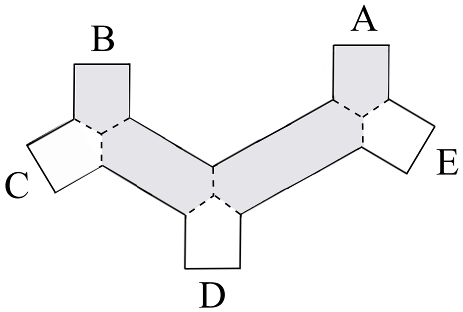
$$- 2\langle \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big) \, (\xi_c b_c) \, Q\Phi_C \, \eta(\Phi_D \, \Phi_E) \rangle_W$$

$$- \langle \Big(Q\Phi_D \, \eta\Phi_E + \eta\Phi_D \, Q\Phi_E\Big)$$

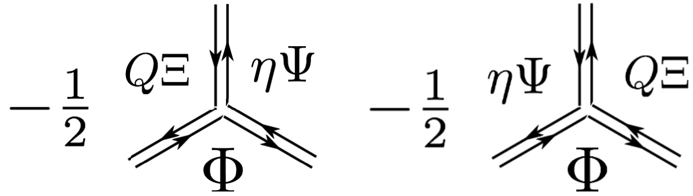
$$\times (\xi_c b_c) \, \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big) \, \Phi_C \rangle_W$$

$$- 2\langle \eta(\Phi_D \, \Phi_E) \, (\xi_c b_c) \, \Big(Q\Xi_A \, \eta\Psi_B + \eta\Psi_A \, Q\Xi_B\Big) \, Q\Phi_C \rangle_W \Bigg).$$

Similarly, the contribution of the diagram (b) is given by



(b)



$$\mathcal{A}^{(2\mathrm{P})(b)}_{FFBBB} = -\frac{1}{2}\int_0^\infty d\tau_1 \int_0^\infty d\tau_2$$

$$\times \bigg(\langle Q\Xi_B\ \Phi_C\ (\eta\xi_{c_1} b_{c_1} Q)\ \Phi_D\ (\eta\xi_{c_2} b_{c_2} Q)\ \Phi_E\ \eta\Psi_A\rangle_W$$

$$+ \langle \eta\Psi_B\ \Phi_C\ (Q\xi_{c_1} b_{c_1} \eta)\ \Phi_D\ (Q\xi_{c_2} b_{c_2} \eta)\ \Phi_E\ Q\Xi_A\rangle_W \bigg)$$

$$
\begin{aligned}
&= \frac{1}{2}\int d^2\tau \Bigg(\langle Q\Xi_B \; Q\Phi_C \; (\xi_{c_1} b_{c_1}) \; Q\Phi_D \; b_{c_2} \; \eta\Phi_E \; \eta\Psi_A \rangle_W \\
&\qquad\qquad + \langle \eta\Psi_B \; Q\Phi_C \; (\xi_{c_1} b_{c_1}) \; Q\Phi_D \; b_{c_2} \; \eta\Phi_E \; Q\Xi_A \rangle_W \Bigg) \\
&\quad - \frac{1}{2}\int d\tau \Bigg(\langle Q\Xi_B \; \Phi_C \; (\xi_c b_c) \; \eta\Phi_D \; Q\Phi_E \; \eta\Psi_A \rangle_W \\
&\qquad - \langle Q\Xi_B \; Q\Phi_C \; (\xi_c b_c) \; \eta(\Phi_D \; \Phi_E) \; \eta\Psi_A \rangle_W \\
&\qquad - \langle Q\Phi_E \; \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \; \eta(\Phi_C \; \Phi_D) \rangle_W \\
&\qquad + \langle \eta\Phi_E \; Q\Xi_A \; (\xi_c b_c) \; \eta\Psi_B \; Q\Phi_C \; \Phi_D \rangle_W \\
&\qquad + \langle \eta\Phi_E \; \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \; Q\Phi_C \; \Phi_D \rangle_W \\
&\qquad - \langle \Phi_E \; \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \; Q\Phi_C \; \eta\Phi_D \rangle_W \Bigg),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}^{(2\mathrm{P})(c)}_{FFBBB} =& \frac{1}{2}\int_0^\infty d^2\tau\ \langle Q\Phi_C\ Q\Phi_D\ (\xi_{c_1}b_{c_1})\ \eta\Phi_E\ b_{c_2}\ \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B\Big)\rangle_W \\
&+ \frac{1}{8}\int_0^\infty d\tau \Bigg(2\langle \Phi_C\ Q\Phi_D\ (\xi_c b_c)\ \eta\Phi_E\ \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B\Big)\rangle_W \\
&\quad + \langle \Big(Q\Phi_C\ \eta\Phi_D - \eta\Phi_C\ Q\Phi_D\Big) \\
&\qquad\qquad \times (\xi_c b_c)\ \Phi_E\ \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B\Big)\rangle_W \\
&\quad - 2\langle Q\Phi_C\ \eta\Phi_D\ (\xi_c b_c)\ \Phi_E\ \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B\Big)\rangle_W \\
&\quad - 2\langle Q\Phi_C\ \Phi_D\ (\xi_c b_c)\ \eta\Phi_E\ \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B\Big)\rangle_W \\
&\quad - 2\langle \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B\Big)\ (\xi_c b_c)\ \Phi_C\ Q\Phi_D\ \eta\Phi_E\rangle_W \\
&\quad - \langle \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B\Big) \\
&\qquad\qquad \times (\xi_c b_c)\ \Big(Q\Phi_C\ \eta\Phi_D - \eta\Phi_C\ Q\Phi_D\Big)\ \Phi_E\rangle_W \\
&\quad + 2\langle \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B\Big)\ (\xi_c b_c)\ Q\Phi_C\ \eta(\Phi_D\ \Phi_E)\rangle_W \Bigg),
\end{aligned}
$$

$$\mathcal{A}^{(2\mathrm{P})(d)}_{FFBBB} = \frac{1}{2}\int_0^\infty d^2\tau\ \langle Q\Phi_D\ \eta\Phi_E\ (\xi_{c_1} b_{c_1})\ \Big(Q\Xi_A\ b_{c_2}\ \eta\Psi_B + \eta\Psi_A\ b_{c_2}\ Q\Xi_B\Big)\ Q\Phi_C\rangle_W$$

$$+ \frac{1}{4}\int_0^\infty d\tau \Bigg(\langle \Big(Q\Phi_D\ \eta\Phi_E + \eta\Phi_D\ Q\Phi_E\Big)\ (\xi_c b_c)\ \eta\Psi_A\ Q\Xi_B\ \Phi_C\rangle_W$$

$$+ \langle \eta(\Phi_D\ \Phi_E)\ (\xi_c b_c)\ \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B\Big)\ Q\Phi_C\rangle_W$$

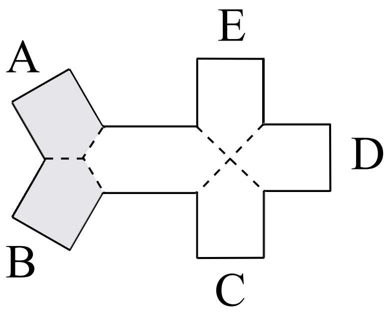
$$+ \langle Q\Xi_B\ \Phi_C\ (\xi_c b_c)\ \Big(Q\Phi_D\ \eta\Phi_E + \eta\Phi_D\ Q\Phi_E\Big)\ \eta\Psi_A\rangle_W$$

$$- \langle Q\Xi_B\ Q\Phi_C\ (\xi_c b_c)\ \eta(\Phi_D\ \Phi_E)\ \eta\Psi_A\rangle_W$$

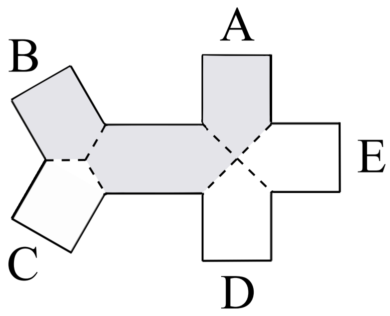
$$- \langle \eta\Psi_B\ Q\Phi_C\ (\xi_c b_c)\ \eta(\Phi_D\ \Phi_E)\ Q\Xi_A\rangle_W \Bigg),$$

$$
\begin{aligned}
\mathcal{A}^{(2\mathrm{P})(e)}_{FFBBB} =& \frac{1}{2}\int_0^\infty d^2\tau \, \Bigg(\langle \eta\Phi_E \; Q\Xi_A \; (\xi_{c_1} b_{c_1}) \; \eta\Psi_B \; b_{c_2} \; Q\Phi_C \; Q\Phi_D \rangle_W \\
&\qquad + \langle \eta\Phi_E \; \eta\Psi_A \; (\xi_{c_1} b_{c_1}) \; Q\Xi_B \; b_{c_2} \; Q\Phi_C \; Q\Phi_D \rangle_W \Bigg) \\
& - \frac{1}{4}\int_0^\infty d\tau \Bigg(\langle \Phi_E \; \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \; \Big(Q\Phi_C \; \eta\Phi_D + \eta\Phi_C \; Q\Phi_D \Big) \rangle_W \\
&\qquad - \langle \eta\Phi_E \; Q\Xi_A \; (\xi_c b_c) \; \eta\Psi_B \; \Big(Q\Phi_C \; \Phi_D - \Phi_C \; Q\Phi_D \Big) \rangle_W \\
&\qquad - \langle \eta\Phi_E \; \eta\Psi_A \; (\xi_c b_c) \; Q\Xi_B \; \Big(Q\Phi_C \; \Phi_D - \Phi_C \; Q\Phi_D \Big) \rangle_W \\
&\qquad - \langle \Big(Q\Phi_C \; \eta\Phi_D + \eta\Phi_C \; Q\Phi_D \Big) \; (\xi_c b_c) \; \Phi_E \; \eta\Psi_A \; Q\Xi_B \rangle_W \\
&\qquad - \langle \Big(Q\Phi_C \; \Phi_D - \Phi_C \; Q\Phi_D \Big) \\
&\qquad\qquad \times (\xi_c b_c) \; \eta\Phi_E \; (Q\Xi_A \; \eta\Psi_B + \eta\Psi_A \; Q\Xi_B) \rangle_W \Bigg) .
\end{aligned}
$$

Five one-propagator diagrams



(a)



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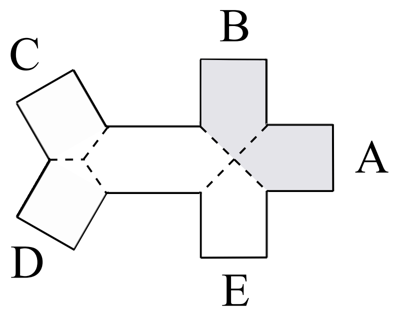

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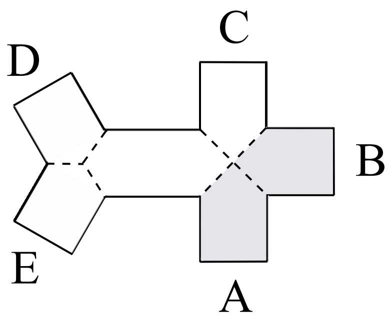

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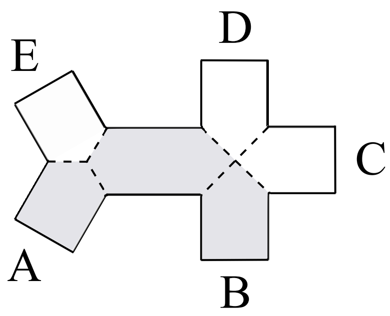


(e)

From the straightforward calculation, we obtain:

$$\mathcal{A}^{(1\mathrm{P})(a)}_{FFBBB} = \frac{1}{24}\int_0^\infty d\tau \Bigg(\langle \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B \Big) \times (\xi_c b_c)\ \Phi_C\ \Big(Q\Phi_D\ \eta\Phi_E - \eta\Phi_D\ Q\Phi_E \Big)\rangle_W - 2\langle \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B \Big) \times (\xi_c b_c)\ \Big(Q\Phi_C\ \Phi_D\ \eta\Phi_E - \eta\Phi_C\ \Phi_D\ Q\Phi_E \Big)\rangle_W + \langle \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B \Big) \times (\xi_c b_c)\ \Big(Q\Phi_C\ \eta\Phi_D - \eta\Phi_C\ Q\Phi_D \Big)\ \Phi_E\rangle_W \Bigg),$$

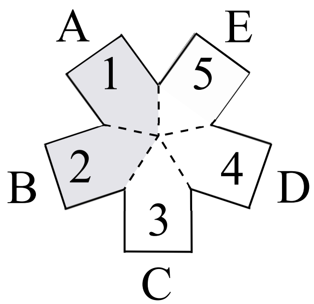
$$
\begin{aligned}
\mathcal{A}^{(1\mathrm{P})(b)}_{FFBBB} =& \frac{1}{4}\int_0^\infty d\tau\ \Big(\langle \eta\Psi_B\ \eta\Phi_C\ (\xi_c b_c)\ Q(\Phi_D\ \Phi_E)\ Q\Xi_A\rangle_W \\
&\qquad - \langle Q\Xi_B\ \eta\Phi_C\ (\xi_c b_c)\ Q(\Phi_D\ \Phi_E)\ \eta\Psi_A\rangle_W \\
&\qquad - \langle \eta\Psi_B\ \Phi_C\ (\xi_c b_c)\ \Big(Q\Phi_D\ \eta\Phi_E - \eta\Phi_D\ Q\Phi_E\Big)\ Q\Xi_A\rangle_W\Big) \\
&- \frac{1}{4}\langle Q\Xi_A\ \eta\Psi_B\ \Phi_C\ \Phi_D\ \Phi_E\rangle_W, \\
\mathcal{A}^{(1\mathrm{P})(c)}_{FFBBB} =& \frac{1}{8}\int_0^\infty d\tau\ \langle\Big(Q\Phi_C\ \eta\Phi_D + \eta\Phi_C\ Q\Phi_D\Big) \\
&\qquad \times (\xi_c b_c)\ \Phi_E\ \Big(Q\Xi_A\ \eta\Psi_B - \eta\Psi_A\ Q\Xi_B\Big)\rangle_W, \\
\mathcal{A}^{(1\mathrm{P})(d)}_{FFBBB} =& \frac{1}{8}\int_0^\infty d\tau\ \langle\Big(Q\Phi_D\ \eta\Phi_E + \eta\Phi_D\ Q\Phi_E\Big) \\
&\qquad \times (\xi_c b_c)\ \Big(Q\Xi_A\ \eta\Psi_B - \eta\Psi_A\ Q\Xi_B\Big)\ \Phi_C\rangle_W,
\end{aligned}
$$

$$\mathcal{A}^{(1\mathrm{P})(e)}_{FFBBB} = \frac{1}{4}\int_0^\infty d\tau\ \Bigg(\langle \Phi_E\ \eta\Psi_A\ (\xi_c b_c)\ Q\Xi_B\ \Big(Q\Phi_C\ \eta\Phi_D - \eta\Phi_C\ Q\Phi_D \Big) \rangle_W$$

$$+ \langle \eta\Phi_E\ \Big(Q\Xi_A\ (\xi_c b_c)\ \eta\Psi_B - \eta\Psi_A\ (\xi_c b_c)\ Q\Xi_B \Big)\ Q(\Phi_C\ \Phi_D) \rangle_W \Bigg)$$

$$- \frac{1}{4} \langle \eta\Psi_A\ Q\Xi_B\ \Phi_C\ \Phi_D\ \Phi_E \rangle_W .$$

One no-propagator diagram,



gives

$$\mathcal{A}^{(\mathrm{NP})}_{FFBBB} = \frac{1}{12} \langle \Big(Q\Xi_A\ \eta\Psi_B + \eta\Psi_A\ Q\Xi_B \Big)\ \Phi_C\ \Phi_D\ \Phi_E \rangle_W .$$

The boundary contributions are cancelled in the total amplitude:

$$
\begin{aligned}
\mathcal{A}_{FFBBB} &= \sum_{i=a}^{e} \mathcal{A}^{(2\mathrm{P})(i)}_{FFBBB} + \sum_{i=a}^{e} \mathcal{A}^{(1\mathrm{P})(i)}_{FFBBB} + \mathcal{A}^{(\mathrm{NP})}_{FFBBB} \\
&= \int d^2\tau \, \Bigg(\langle \Big(Q\Xi_A \; \eta\Psi_B + \eta\Psi_A \; Q\Xi_B \Big) \; (\xi_{c_1} b_{c_1}) \; Q\Phi_C \; b_{c_2} \; Q\Phi_D \; \eta\Phi_E \rangle_W \\
&\qquad + \langle \eta\Psi_B \; Q\Phi_C \; (\xi_{c_1} b_{c_1}) \; Q\Phi_D \; b_{c_2} \; \eta\Phi_E \; Q\Xi_A \rangle_W \\
&\qquad + \langle Q\Xi_B \; Q\Phi_C \; (\xi_{c_1} b_{c_1}) \; Q\Phi_D \; b_{c_2} \; \eta\Phi_E \; \eta\Psi_A \rangle_W \Bigg) \\
&\qquad + \langle Q\Phi_C \; Q\Phi_D \; (\xi_{c_1} b_{c_1}) \; \eta\Phi_E \; b_{c_2} \; \Big(Q\Xi_A \; \eta\Psi_B + \eta\Psi_A \; Q\Xi_B \Big) \rangle_W \\
&\qquad + \langle Q\Phi_D \; \eta\Phi_E \; (\xi_{c_1} b_{c_1}) \; \Big(Q\Xi_A \; b_{c_2} \; \eta\Psi_B + \eta\Psi_A \; b_{c_2} \; Q\Xi_B \Big) \; Q\Phi_C \rangle_W \\
&\qquad + \langle \eta\Phi_E \; Q\Xi_A \; (\xi_{c_1} b_{c_1}) \; \eta\Psi_B \; b_{c_2} \; Q\Phi_C \; Q\Phi_D \rangle_W \\
&\qquad + \langle \eta\Phi_E \; \eta\Psi_A \; (\xi_{c_1} b_{c_1}) \; Q\Xi_B \; b_{c_2} \; Q\Phi_C \; Q\Phi_D \rangle_W \Bigg)
\end{aligned}
$$

After eliminating Ξ from the external states by imposing $Q\Xi = \eta\Psi$,

$$\begin{aligned}\mathcal{A}_{FFBBB} = \int d^2\tau \Bigg(& \langle \xi(\eta\Psi_A\, \eta\Psi_B\, b_{c_1}\, Q\Phi_C\, b_{c_2}\, Q\Phi_D\, \eta\Phi_E) \rangle_W \\ & + \langle \xi(\eta\Psi_B\, Q\Phi_C\, b_{c_1}\, Q\Phi_D\, b_{c_2}\, \eta\Phi_E\, \eta\Psi_A) \rangle_W \\ & + \langle \xi(Q\Phi_C\, Q\Phi_D\, b_{c_1}\, \eta\Phi_E\, b_{c_2}\, \eta\Psi_A\, \eta\Psi_B) \rangle_W \\ & + \langle \xi(Q\Phi_D\, \eta\Phi_E\, b_{c_1}\, \eta\Psi_A\, b_{c_2}\, \eta\Psi_B\, Q\Phi_C) \rangle_W \\ & + \langle \xi(\eta\Phi_E\, \eta\Psi_A\, b_{c_1}\, \eta\Psi_B\, b_{c_2}\, Q\Phi_C\, Q\Phi_D) \rangle_W \Bigg).\end{aligned}$$

Since this final expression has the same form (up to ξ) as the *bosonic* SFT (with $\eta\Psi, Q\Phi, \eta\Phi \;\leftrightarrow A$), we can conclude that this is nothing but the well-known amplitude.

Similarly, we can calculate the amplitudes with FFFFB and FBFBB, and show that the results agree with the well-known amplitues.

4. Summary and discussion

♠ We have found that the missing gauge-symmetries are realized as those under which the variation of the pseudo-action is proportional to the constraint.

♠ We have proposed the new Feynman rules for the open super SFT respecting all the gauge- "symmetries".

♠ We have shown that the correct on-shell 4- and 5-point amplitudes are reproduced by the new rules.

♣ We can apply the similar argument to the heterotic SFT, and obtain the new Feynman rules which reproduce the correct 4- and 5-point on-shell physical amplitudes.

★ We have to clarify whether the new Feynman rules reproduce all the on-shell physical amplitudes at the tree level.

- general theory of the pseudo-action and its “symmetries”

★ We have to extend the Feynman rules to those applicable beyond the tree level.

- gauge fixing by means of the BV method
- extra factor $1/2$ for each fermion loop?

★ It is interesting to calculate the off-shell amplitudes and to study their properties.