1PI Effective Action for Superstring Field Theory

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PLAN

1. Shortcomings of conventional string perturbation theory

2. 1PI effective string field theory

We shall follow RNS formalism with picture changing operators but it may extend to other approaches also.

<u>References</u>

Zwiebach, hep-th/9206084

Pius, Rudra, A.S., , arXiv:1311.1257, 1401.7014, 1404.6254

A.S., arXiv:1408.0571; A.S., Witten, arXiv:1504.00609

A.S., arXiv: 1411.7478, 1501.00988, work in progress

Conventional approach to computing g-loop S-matrix elements in superstring theory

1. Represent physical states by BRST invariant vertex operators in the world-sheet superconformal field theory of matter and ghost system.

2. Compute correlation functions of vertex operators inserted at ‘punctures’ and additional ghost and ‘picture changing operators’ on a genus g Riemann surface.

3. Integrate the result over the moduli space of the Riemann surface with punctures.

However this approach is insufficient for addressing many issues even within the perturbation theory.

1. Mass renormalization

2. Vacuum shift

LSZ formula for S-matrix elements in QFT

$$\lim_{k_i^2 \to -m_{i,p}^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} \{ Z_i^{-1/2} \times (k_i^2 + m_{i,p}^2) \}$$

$G^{(n)}$: n-point Green's function

$a_1, \cdots a_n$: quantum numbers, $k_1, \ldots k_n$: momenta

$m_{i,p}$: physical mass of the i-th external state

– given by the locations of the poles of two point function in the $-k^2$ plane.

Z_i: wave-function renormalization factors, given by the residues at the poles.

In contrast, string amplitudes compute 'truncated Greens function on classical mass-shell'

$$\lim_{k_i^2 \to -m_i^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} (k_i^2 + m_i^2) \,.$$

m_i: tree level mass of the i-th external state.

$k_i^2 \to -m_i^2$ condition is needed to make the vertex operators BRST invariant.

String amplitudes:

$$\lim_{k_i^2\to -m_i^2} G^{(n)}_{a_1\cdots a_n}(k_1,\cdots k_n)\prod_{i=1}^{n}(k_i^2+m_i^2)\,,$$

The S-matrix elements:

$$\lim_{k_i^2\to -m_{i,p}^2} G^{(n)}_{a_1\cdots a_n}(k_1,\cdots k_n)\prod_{i=1}^{n}\{Z_i^{-1/2}\times(k_i^2+m_{i,p}^2)\}$$

The effect of Z_i can be taken care of. Witten

The effect of mass renormalization is more subtle.

$\Rightarrow$ String amplitudes compute S-matrix elements directly if $m_{i,p}^2 = m_i^2$ but not otherwise.

– Includes BPS states, massless gauge particles and all amplitudes at tree level.

Problem with vacuum shift

Example: In many compactifications of SO(32) heterotic string theory on Calabi-Yau 3-folds, one loop correction generates a Fayet-Ilioupoulos term.

Effect: Generate a potential of a charged scalar ϕ of the form

$$\mathbf{c}(\phi^*\phi - \mathbf{K}\,\mathbf{g_s}^2)^2$$

c, K: positive constants, $\mathbf{g_s}$: string coupling

Dine, Seiberg, Witten; Atick, Dixon, A.S.; Dine, Ichinose, Seiberg
Atick, A.S.; Witten; D'Hoker, Phong; Berkovits, Witten

Correct vacuum: $|\phi| = \mathbf{g_s}\sqrt{\mathbf{K}}$

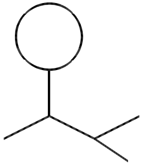
– not described by a world-sheet CFT

– conventional perturbation theory fails.

Even in absence of mass renormalization and vacuum shift we have to deal with infrared divergences in the integration over moduli space at intermediate stages.

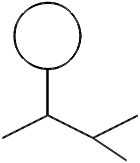
Witten

Consider a tadpole diagram in a QFT:



This diverges if a massless state propagates along the vertical propagator.

In SUSY theoies the result often vanishes after loop integration.



In string theory, this translates to a specific regularization procedure for integration over moduli spaces of Riemann surfaces.

1. Put an upper cut-off L on certain modulus corresponding to the Schwinger parameter of the vertical propagator.

2. Do integration over the other moduli first.

3. Then let L go to infinity.

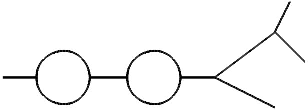
Witten

This works but requires an IR cut-off at the intermediate stages of calculation.

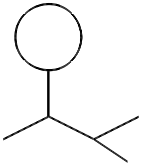
How do we circumvent these difficulties / need for IR cut-off?

All of these problems arise from one particle reducible (1PR) diagrams.

If we use the tree level mass in LSZ procedure, the amplitude diverges from insertion of two point function on external legs.



If we do not work in the correct vacuum, the amplitude diverges from tadpole diagrams.



In a quantum field theory we know how to solve these problems.

1. Solve equations of motion of the effective action to go to the correct vacuum

– no tadpoles.

2. Use $k^2 + m_p{}^2 = 0$ for external legs instead of $k^2 + m^2 = 0$.

m_p: Renormalized mass

Try to follow the same strategy in string theory.

First guess

Use string field theory Witten; Zwiebach; $\cdots$

1. Take external states off-shell.

2. Construct interaction vertices by restricting the integration over moduli space of Riemann surfaces to a subspace of the full moduli space.

3. Ensure that when we construct the full amplitude using Feynman rules we get back integral over the full moduli space.

– achieved for bosonic string theory but seems difficult for heterotic / superstring theory.

However for addressing perturbative issues this is an overkill.

World-sheet approach already tells us how to compute amplitudes in terms of integral over full moduli space of Riemann surfaces.

There is no reason why we should try to reproduce this starting from a string field theory.

At the same time we should try to avoid ‘bad’ regions of the moduli space associated with tadpole and mass renormalization diagrams and use field theory intuition to deal with them.

Our proposal

Use one particle irreducible (1PI) string field theory

– similar in spirit to string field theory, but not the same.

Elementary vertices are again given by integrals of off-shell amplitudes over a subspace of the moduli space of Riemann surfaces.

However now we demand that tree amplitudes computed from the vertices reproduce the full amplitude given by the integral over full moduli spaces of Riemann surfaces.

Specifics

Off-shell amplitudes in superstring theory involve

1. Choice of local coordinates at the punctures.

2. Choice of locations of picture changing operators (PCO).

We shall focus on heterotic string theory but analysis for superstrings is identical.

Some notations:

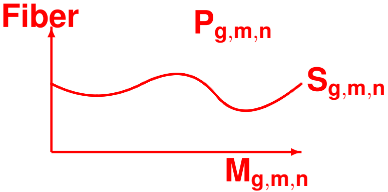
Q_B: BRST charge

$\mathcal{X}(z)$: PCO

$M_{g,m,n}$: (6g-6+2m+2n) dimensional moduli space of genus g Riemann surfaces with (m,n) punctures of (NS,R) type in $(-1,-1/2)$ picture number.

$P_{g,m,n}$: A fiber bundle with $M_{g,m,n}$ as the base and possible choices of local coordinates at punctures and PCO locations as fibers.

Number of PCO's: $2g-2+m+n/2$



A choice of local coordinate system and PCO locations corresponds to a section $S_{g,m,n}$ of this fiber bundle.

Dimension of $S_{g,m,n} = 6g - 6 + 2m + 2n$.

Note: We could also choose formal weighted average of multiple sections.

Plumbing fixture provides a natural map from $P \times P \to P$.

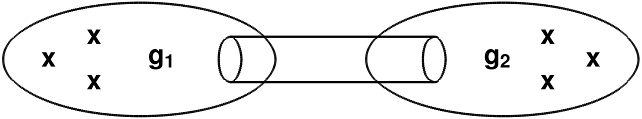
Consider a genus g_1, $(m_1 + n_1)$-punctured Riemann surface and a genus g_2, $(m_2 + n_2)$-punctured Riemann surface.

Take one puncture from each of them, and let w_1, w_2 be the local coordinates around the punctures at $w_1 = 0$ and $w_2 = 0$.

Glue them via the identification (plumbing fixture)

$$w_1 w_2 = e^{-s+i\theta}, \quad 0 \le s < \infty, \quad 0 \le \theta < 2\pi$$

– gives a family of new Riemann surfaces of genus $g_1 + g_2$ with 2 less punctures together with choice of local coordinates and PCO locations



$\to$ a new point in $P_{g_1+g_2, m_1+m_2-2, n_1+n_2}$ or $P_{g_1+g_2, m_1+m_2, n_1+n_2-2}$ (almost)

A counting problem

For gluing at NS puncture the no of PCO's on the final surface is the sum of the number of PCO's on the individual surfaces.

$$(2g_1 - 2 + m_1 + n_1/2) + (2g_2 - 2 + m_2 + n_2/2)$$

$$= 2(g_1 + g_2) - 2 + (m_1 + m_2 - 2) + (n_1 + n_2)/2$$

$\rightarrow$ no further action necessary

For gluing at R-puncture the sum of the number of PCO's on the individual surfaces is <u>one less</u> than the required number.

$$(2g_1 - 2 + m_1 + n_1/2) + (2g_2 - 2 + m_2 + n_2/2)$$

$$= 2(g_1 + g_2) - 2 + (m_1 + m_2) + (n_1 + n_2 - 2)/2 - 1$$

A consistent prescription: Insert

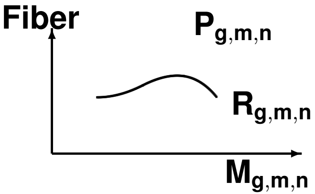
$$\mathcal{X}_0 \equiv \oint \frac{dw_1}{w_1} \mathcal{X}(w_1) = \oint \frac{dw_2}{w_2} \mathcal{X}(w_2)$$

around either puncture.

$\mathcal{X}_0$ has been used earlier for other purposes.

Berkovits, Zwiebach; Erler, Konopka, Sachs

1PI vertices are represented by specific subspaces $R_{g,m,n}$ of $S_{g,m,n}$ satisfying appropriate gluing relations.



All the Riemann surfaces corresponding to the full section $S_{g,m,n}$ are given by the Riemann surfaces in $R_{g,m,n}$ and their plumbing fixture in all possible ways.

We do not allow plumbing fixture of punctures on the same Riemann surface.

Systematic construction of 1PI regions

1. Begin with 3-punctured sphere and one punctured torus.

The first one has 0-dimensional moduli space and the second one has two dimensional moduli space.

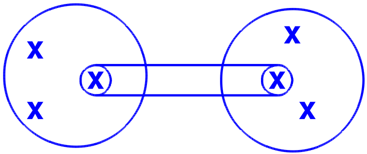
Declare them to be 1PI.

2. Choose local coordiates at the punctures and PCO locations arbitrarily consistent with symmetries

– exchange of punctures on the 3-punctured sphere

– modular transformation for the 1-punctured torus.

3. Now take two 3-punctured spheres and glue them using plumbing fixture.



$$\mathbf{w_1 w_2 = q}, \quad \mathbf{q} \equiv \mathbf{e}^{-\mathbf{s}+\mathbf{i}\theta}, \quad \mathbf{0} \leq \mathbf{s} < \infty, \quad \mathbf{0} \leq \theta < \mathbf{2}\pi$$

Declare these to be 1PR 4-punctured spheres and choose the local coordinates and PCO locations to be those induced from 3-punctured spheres.

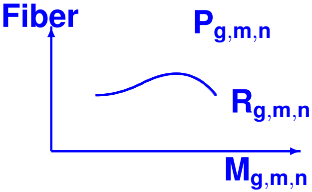
Repeat this for inequivalent permutations of the four punctures i.e. ‘sum over s, t and u-channel diagrams’.

The 1PR 4-punctured spheres will typically cover part of the moduli space of 4-punctured spheres.

Declare the rest of the 4-punctured spheres to be 1PI 4-punctured spheres.

On them choose local coordinates and PCO locations arbitrarily consistent with symmetries and continuity.

Proceeding this way, for all $P_{g,m,n}$ we can choose 1PI subspace $R_{g,m,n}$ of $P_{g,m,n}$



Via plumbing fixture of Riemann surfaces in $R_{g,m,n}$ we generate the full section $S_{g,m,n}$.

Off-shell and 1PI amplitudes

Once $R_{g,m,n}$ have been constructed, we can define 1PI off shell amplitude of external states $|A_1\rangle, \cdots |A_{m+n}\rangle$ as

$$\int_{R_{g,m,n}} \omega_{6g-6+2m+2n}(|A_1\rangle, \cdots |A_{m+n}\rangle)$$

ω_p is a p-form in $P_{g,m,n}$ constructed from appropriate correlation functions of off-shell vertex operators and ghost and PCO insertions on the Riemann surface.

Zwiebach; A.S.; A.S., Witten

Full off-shell amplitude at genus g:

$$\int_{S_{g,m,n}} \omega_{6g-6+2m+2n}(|A_1\rangle, \cdots |A_{m+n}\rangle)$$

All separating type degenerations come from 1PR amplitudes

$\Rightarrow$ the 1PI amplitudes are free from all IR divergences associated with tadpoles, and mass or wave-function renormalization.

We shall now use these to construct gauge invariant 1PI action.

Definitions and identities

H: Hilbert space of GSO even states of the matter-ghost CFT annihilated by

$$b_0 - \bar{b}_0, \qquad L_0 - \bar{L}_0$$

$\langle A|B\rangle$: BPZ inner product between CFT states

NS-sector vertex operators are grassman even for even ghost number and grassmann odd for odd ghost number

For R-sector it is opposite.

$$G|s\rangle \equiv \begin{cases} |s\rangle & \text{if } |s\rangle \in H_{NS} \\ \mathcal{X}_0\,|s\rangle & \text{if } |s\rangle \in H_R \end{cases},$$

More definitions (following bosonic closed SFT)

Given N states $|A_1\rangle, \cdots |A_N\rangle \in H$, of which m are NS states and $n = N - m$ are R-states we define a multi-linear function

$$\{A_1 \cdots A_N\} = \sum_{g=0}^{\infty} {g_s}^{2g} \int_{R_{g,m,n}} \omega_{6g-6+2(m+n)}(|A_1\rangle, \cdots |A_N\rangle)$$

We also define $[A_1 \cdots A_N] \in H$ via

$$\langle A_0|c_0^-|[A_1 \cdots A_N]\rangle = \{A_0 A_1 \cdots A_N\}, \quad c_0^- \equiv (c_0 - \bar{c}_0)/2$$

$$b_0^\pm = b_0 \pm \bar{b}_0, \quad L_0^\pm \equiv L_0 \pm \bar{L}_0, \quad c_0^\pm \equiv (c_0 \pm \bar{c}_0)/2$$

NS sector string field: An arbitrary state $|\psi_{\text{NS}}\rangle \in$ H carrying ghost number 2 and picture number -1 with grassmann even expansion coefficients.

R sector string field: An arbitrary state $|\psi_{\text{R}}\rangle \in$ H carrying ghost number 2 and picture number $-1/2$ with grassmann odd expansion coefficients.

$$|\psi\rangle \equiv |\psi_{\text{NS}}\rangle + |\psi_{\text{R}}\rangle$$

Both ψ_{NS} and ψ_{R} are grassmann even.

1PI effective action for NS sector fields.

$$S(|\psi_{NS}\rangle) = g_s{}^{-2}\left[\frac{1}{2}\langle\psi_{NS}|c_0^- Q_B|\psi_{NS}\rangle + \sum_{n=1}^{\infty}\frac{1}{n!}\{\psi_{NS}{}^n\}\right]$$

$\{\psi_{NS}{}^n\}$: $\{\psi_{NS}\psi_{NS}\cdots\psi_{NS}\}$ with n copies of ψ_{NS} inside $\{\ \}$.

Invariant under infinitesimal gauge transformation

$$\delta|\psi_{NS}\rangle = Q_B|\lambda_{NS}\rangle + \sum_{n=0}^{\infty}\frac{1}{n!}[\psi_{NS}{}^n\lambda_{NS}]$$

$|\lambda_{NS}\rangle$: is an element of H with ghost number 1, picture number -1.

Gauge invariance of $S(|\psi_{NS}\rangle)$ can be proved using the identities involving $\{\cdots\}$ and $[\cdots]$.

For Ramond sector states writing down an action requires using fields with <u>additional constraints</u>.

Alternatively we can write down the equations of motion.

– related to the fact that Ramond sector states carry picture number $-1/2$ and the inner product between two such states vanish by picture number conservation.

For a string field theory this would be problematic since we would not know how to quantize the theory.

However for 1PI theory this is not a problem since we only need to work at the tree level.

General structure (including NS and R-sector):

A general string field configuration corresponds to a state $|\psi\rangle \in$ H of ghost number 2 and picture number $(-1, -1/2)$ in (NS,R) sector.

1PI equation of motion:

$$\mathbf{Q_B}|\psi\rangle + \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \mathbf{G}[\psi^{n-1}] = 0$$

$\mathbf{Q_B}$: BRST operator

G: identity in NS sector

$\mathcal{X}_0 \equiv \oint z^{-1} dz\, \mathcal{X}(z)$ in R sector

Gauge transformations

The infinitesimal gauge transformation parameters correspond to states $|\lambda\rangle$ of ghost number 1 and picture number $(-1,-1/2)$ in (NS,R) sector.

Gauge transformation law

$$\delta|\psi\rangle = \mathbf{Q_B}|\lambda\rangle + \sum_{\mathbf{n=0}}^{\infty} \frac{\mathbf{1}}{\mathbf{n}!}\mathbf{G}[\psi^{\mathbf{n}}\,\lambda]$$

– includes general coordinate transformation and local supersymmetry.

Action including Ramond sector

Introduce an extra set of fields $|\widetilde{\psi}\rangle$ carrying ghost number 2 and picture number $(-1, -1/2)$.

$$\mathbf{S} = \mathbf{g_s}^{-2} \left[-\frac{1}{2} \langle\widetilde{\psi}|\mathbf{c_0^-} \mathbf{Q_B} \mathbf{G}|\widetilde{\psi}\rangle + \langle\widetilde{\psi}|\mathbf{c_0^-} \mathbf{Q_B}|\psi\rangle + \sum_{n=1}^{\infty} \frac{1}{n!} \{\psi^n\} \right]$$

– reproduces the equations of motion but has more states than needed.

– can be used to derive Feynman rules.

Constraint on external states:

$$|\psi\rangle - \mathbf{G}|\widetilde{\psi}\rangle = 0$$

In Siegel gauge, <u>tree amplitudes</u> computed from this 1PI effective theory give the full quantum corrected amplitudes of string theory if we <u>ignore</u> tadpoles and mass renormalization.

Equations of motion in NS sector:

$$\mathbf{Q_B}|\psi_{\mathbf{NS}}\rangle + \sum_{\mathbf{n=1}}^{\infty} \frac{\mathbf{1}}{(\mathbf{n-1})!}[\psi_{\mathbf{NS}}{}^{\mathbf{n-1}}] = \mathbf{0}$$

Note: $\{\psi_{\mathbf{NS}}\}$ and [ ] are non-zero from genus 1 onwards

$|\psi_{\mathbf{NS}}\rangle = \mathbf{0}$ is not a solution to equations of motions.

We have to first solve the equations of motion and then expand the 1PI action around the solution.

Special importance: Vacuum solution carrying zero momentum

Iterative construction of the vacuum solution:

Suppose $|\psi_k\rangle$ is the solution to order ${g_s}^k$. $(|\psi_0\rangle = 0)$

P: projection operator to $L_0^+ \equiv L_0 + \bar{L}_0 = 0$ states.

Then

$$|\psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+}\sum_{n=1}^{\infty}\frac{1}{(n-1)!}(1-P)[\psi_k^{n-1}] + |\phi_{k+1}\rangle\,,$$

$|\phi_{k+1}\rangle$ is an $L_0^+ = 0$ state satisfying

$$Q_B|\phi_{k+1}\rangle = -\sum_{n=1}^{\infty}\frac{1}{(n-1)!}P[\psi_k^{n-1}] + \mathcal{O}({g_s}^{k+2})\,.$$

$$|\psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+}\sum_{n=1}^{\infty}\frac{1}{(n-1)!}(1-P)[\psi_k^{n-1}] + |\phi_{k+1}\rangle\,,$$

$$Q_B|\phi_{k+1}\rangle = -\sum_{n=1}^{\infty}\frac{1}{(n-1)!}P[\psi_k^{n-1}] + \mathcal{O}({g_s}^{k+2})\,.$$

Possible obstruction / ambiguity to solving these arise from the last equation.

rhs could contain a component along a non-trivial element of BRST cohomology.

– reflects the existence of zero momentum massless tadpoles in perturbation theory.

Unless this equation can be solved we have to declare the vacuum inconsistent.

$$|\psi_{k+1}\rangle = -\frac{b_0^+}{L_0^+}\sum_{n=1}^{\infty}\frac{1}{(n-1)!}(1-P)[\psi_k^{n-1}] + |\phi_{k+1}\rangle\,,$$

$$Q_B|\phi_{k+1}\rangle = -\sum_{n=1}^{\infty}\frac{1}{(n-1)!}P[\psi_k^{n-1}] + \mathcal{O}({g_s}^{k+2})\,.$$

Once these equations have been solved, we do not encounter any further tadpole divergence in perturbation theory.

Note: The full solution $|\psi_v\rangle$ is $|\psi_\infty\rangle$, but in practice we shall stop at some fixed order in g_s.

This also allows us to deal with the cases involving vacuum shift, e.g. when a scalar field χ in low energy theory has potential

$$c(\chi^2 - K\, g_s{}^2)^2 \,.$$

At order g_s we have three solutions $\chi = 0, \pm g_s\sqrt{K}$.

In 1PI effective feld theory this will be reflected in the existence of multiple solutions for $|\phi_1\rangle$.

The solution corresponding to $\chi = 0$ will have non-zero dilaton one point function at higher order

$\Rightarrow$ an obstruction to extending the corresponding 1PI effective field theory solution to higher order.

The solutions corresponding to $\chi = \pm g_s\sqrt{K}$ will not encounter such obstructions.

Once we have a vacuum solution $|\psi_{\mathbf{v}}\rangle$ we can expand the equations of motion around $|\psi_{\mathbf{v}}\rangle$.

Define: $|\chi\rangle \equiv |\psi\rangle - |\psi_{\mathbf{v}}\rangle$

$$\widehat{\mathbf{Q}}_{\mathbf{B}}|\mathbf{A}\rangle \equiv \mathbf{Q}_{\mathbf{B}}|\mathbf{A}\rangle + \sum_{\mathbf{k}=0}^{\infty} \frac{1}{\mathbf{k}!} \mathbf{G}[\psi_{\mathbf{v}}{}^{\mathbf{k}}\mathbf{A}]\,.$$

$\widehat{\mathbf{Q}}_{\mathbf{B}}^2 = 0$ as a consequence of $|\psi_v\rangle$ satisfying equations of motion.

New 'shifted' linearized equations of motion

$$\widehat{\mathbf{Q}}_{\mathbf{B}}|\chi\rangle = 0$$

New shifted linearized gauge transformations

$$\delta|\chi\rangle = \widehat{\mathbf{Q}}_{\mathbf{B}}|\lambda\rangle$$

Linearized equations of motion around $|\psi_{\mathbf{v}}\rangle$:

$$\widehat{\mathbf{Q}}_{\mathbf{B}}|\chi\rangle = \mathbf{0}$$

– has two kinds of solution:

1. Solutions which exist for all momentum k of $|\chi\rangle$

– have the form $\widehat{\mathbf{Q}}_{\mathbf{B}}|\lambda\rangle$ for some $|\lambda\rangle$ and are pure gauge.

2. Solutions which exist for special values of $\mathbf{k}^2$

– represent physical states with the corresponding values of $-\mathbf{k}^2$ giving <u>renormalized mass^2</u>.

This abstract definition can be developed into a fully systematic perturbative scheme.

A similar procedure can be given for the S-matrix elements starting from the LSZ formalism.

Pius, Rudra, A.S.

Gauge transformation laws around the shifted vacuum

$$\delta|\chi\rangle = \widehat{\mathbf{Q}}_{\mathbf{B}}|\lambda\rangle + \mathcal{O}(\chi)$$

Global symmetries are generated by those $|\lambda\rangle$ for which

$$\widehat{\mathbf{Q}}_{\mathbf{B}}|\lambda\rangle = \mathbf{0}$$

For such global symmetries we can derive Ward identities.

e.g. unbroken global SUSY $\Rightarrow$ equality of the renormalized masses of bosons and fermions at all mass levels.

also $\Rightarrow$ absence of obstruction to finding vacuum solution.

Section dependence

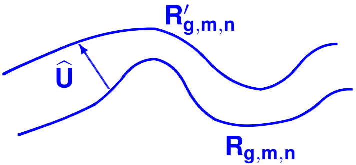
The definition of $\{A_1 \cdots A_N\}$ and all subsequent analysis depends on the choice of 1PI subspace $R_{g,m,n}$.

A different choice of '1PI sections' $R_{g,m,n}$

$\Rightarrow$ a different set of equations of motion.

Do the renormalized masses and S-matrix elements depend on this choice?

We shall consider the case of infinitesimal deformations from $\mathbf{R_{g,m,n}}$ to $\mathbf{R'_{g,m,n}}$ labelled by some tangent vector $\widehat{\mathbf{U}}$ of $\mathbf{P_{g,m,n}}$ at every point of $\mathbf{R_{g,m,n}}$.



Result: The change in the equation of motion can be compensated by a <u>field redefinition</u> $|\psi\rangle \to |\psi\rangle + \delta|\psi\rangle$ where

$$\langle\phi|\mathbf{c_0^-}|\delta\psi\rangle = -\sum_{\mathbf{g=0}}^{\infty} \mathbf{g_s}^{\mathbf{2g}} \sum_{\mathbf{m,n=0}}^{\infty} \frac{\mathbf{1}}{\mathbf{m!n!}}$$

$$\Bigg[\int_{\mathbf{R_{g,m+1,n}}} \omega_{\mathbf{6g-5+2m+2n+2}}[\widehat{\mathbf{U}}](\mathbf{G}|\phi_{\mathbf{NS}}\rangle\rangle, |\psi_{\mathbf{NS}}\rangle^{\otimes\mathbf{m}}, |\psi_{\mathbf{R}}\rangle^{\otimes\mathbf{n}})$$

$$+\int_{\mathbf{R_{g,m,n+1}}} \omega_{\mathbf{6g-5+2m+2n+2}}[\widehat{\mathbf{U}}](|\psi_{\mathbf{NS}}\rangle^{\otimes\mathbf{m}}, \mathbf{G}|\phi_{\mathbf{R}}\rangle, |\psi_{\mathbf{R}}\rangle^{\otimes\mathbf{n}})\Bigg]$$

Thus renormalized masses and S-matrix elements remain unchanged (Generalization of Hata-Zwiebach result)

Future prospects

1. Demand of infinite dimensional gauge invariance more or less fixes the perturbative scattering amplitude

– integral over the full integration cycle $S_{g,m,n}$.

Could it constrain the structure of non-perturbative corrections to the 1PI effective action?

2. Can we use the off-shell action to study string theory in weak RR background field perturbatively?

1. Under exchange of A_i and A_j, $\{A_1 \cdots A_N\}$ pick up a sign

$$(-1)^{\gamma_i \gamma_j}$$

γ_i: grassmannality of A_i i.e. 0 if A_i is grassmann even and 1 if A_i is grassmann odd.

2.
$$\sum_{i=1}^{N} (-1)^{\gamma_1 + \cdots \gamma_{i-1}} \{A_1 \cdots A_{i-1} (Q_B A_i) A_{i+1} \cdots A_N\}$$
$$= -\frac{1}{2} \sum_{\substack{\ell, k \geq 0 \\ \ell + k = N}} \sum_{\substack{\{i_a; a=1, \cdots \ell\}, \{j_b; b=1, \cdots k\} \\ \{i_a\} \cup \{j_b\} = \{1, \cdots N\}}} \sigma(\{i_a\}, \{j_b\})$$
$$\{A_{i_1} \cdots A_{i_\ell} (G[A_{j_1} \cdots A_{j_k}])\}$$

$\sigma(\{i_a\}, \{j_b\})$: the sign that one picks up while rearranging $b_0^-, A_1, \cdots A_N$ to $A_{i_1}, \cdots A_{i_\ell}, b_0^-, A_{j_1}, \cdots A_{j_k}$

$$G|s\rangle \equiv \begin{cases} |s\rangle & \text{if } |s\rangle \in H_{NS} \\ \mathcal{X}_0 |s\rangle & \text{if } |s\rangle \in H_R \end{cases},$$