CFT defects in Open String Field Theory

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BCFT's from OSFT: Brief review

- String field theory is a great tool to understand new things about BCFT from a novel perspective.

- Let us consider OSFT for strings 'propagating' in a background given by $\mathrm{BCFT}_c \otimes \mathrm{BCFT}_{26\text{-}c}$ and look for classical solutions which do not excite any primaries in $\mathrm{BCFT}_{26\text{-}c}$. Such solutions will describe new BCFT_c^*.

BCFT's from OSFT: Brief review

- The full boundary state can be constructed rather explicitly as

$$|B_\Psi\rangle = \sum_\alpha B_\Psi^\alpha |V_\alpha\rangle\rangle$$

where $|V_\alpha\rangle\rangle$ are the Ishibashi states and

$$B_\Psi^\alpha = 2\pi i \langle I|\mathcal{V}^\alpha(i,-i)|\Psi - \Psi_{TV}\rangle^{\mathrm{BCFT}_c \otimes \mathrm{BCFT}_{26-c} \otimes \mathrm{BCFT}_{\mathrm{aux}}}$$

$$\mathcal{V}^\alpha = c\bar{c}V^\alpha \, e^{2i\sqrt{1-h_\alpha}\,Y} \, w$$

BCFT's from OSFT: Brief review

- Last year we observed a curious fact following from the linearity of the formula for the boundary state:

  Let us assume that the solution $|\Psi\rangle$ describes the boundary state $||B_x\rangle\rangle$ as seen from $||B_0\rangle\rangle$. Assuming that the Verma modules "turned on" are present also on $||B_y\rangle\rangle$ and the structure of boundary operators is identical, then:

$$\frac{B_x^\beta B_y^\beta}{B_0^\beta} = \sum_z N_{xy}{}^z B_z^\beta$$

- This formula is valid for rational theories, and in some cases also for irrational theories (e.g. chiral marginal deformations)

BCFT's from OSFT: Brief review

- This formula is remarkable for number of reasons:
  - Using explicit Cardy solution for diagonal minimal models $$\|B_i\rangle\rangle = \sum_j \frac{S_i^{\ j}}{\sqrt{S_0^{\ j}}} |j\rangle\rangle$$ one derives the enigmatic Verlinde formula $$\sum_k S_k^{\ p} N_{ij}^{\ \ k} = \frac{S_i^{\ p} S_j^{\ p}}{S_0^{\ p}}$$ for the modular matrix $S_i^{\ j}$
  - It tells us that D-brane energies (g-function) form a ring, i.e. they can be both added and multiplied. For example for the Ising model $\sqrt{2} \times \sqrt{2} = 1 + 1$

Topological defects

- The formula

$$\frac{B_x^\beta B_y^\beta}{B_0^\beta} = \sum_z N_{xy}{}^z B_z^\beta$$

can be very nicely derived in terms of *topological defects* as we shall now explain.

Topological defects: Brief review

- Defects naturally describe disorder lines on the Riemann surface. A particularly nice class are closed topological defects which can be freely deformed on the surface. They give rise to closed string operators obeying

$$[L_n, D] = [\tilde{L}_n, D] = 0, \qquad \forall n$$

- For diagonal minimal models they are labeled by the same index as primary fields. By Schur's lemma they are constant on every Verma module and can be easily constructed as

$$D_a = \sum_i \frac{S_{ai}}{S_{0i}} P^i$$

where P^i are projectors on the *i-th* Verma module

Topological defects: Brief review

- These obey fusion algebra

$$\begin{aligned} D_a D_b &= \left(\sum_i \frac{S_{ai}}{S_{0i}} P^i\right)\left(\sum_j \frac{S_{bj}}{S_{0j}} P^j\right) \\ &= \sum_i \frac{S_{ai}}{S_{0i}} \frac{S_{bi}}{S_{0i}} P^i \\ &= \sum_i \sum_c N_{ab}{}^c \frac{S_{ci}}{S_{0i}} P^i \\ &= \sum_c N_{ab}{}^c D_c \end{aligned}$$

←Verlinde formula used

just like the conformal families $[\phi_a] \times [\phi_b] = \sum_c N_{ab}{}^c [\phi_c]$

Petkova, Zuber 2000

Topological defects: Brief review

- Action on Cardy boundary states is straightforward

$$
\begin{aligned}
D_a \| B_b \rangle\!\rangle &= \sum_i \frac{S_{ai}}{S_{0i}} P^i \left(\sum_j \frac{S_{bj}}{\sqrt{S_{0j}}} |j\rangle\!\rangle \right) \\
&= \sum_i \frac{S_{ai}}{S_{0i}} \frac{S_{bi}}{\sqrt{S_{0i}}} |i\rangle\!\rangle \\
&= \sum_i \sum_c N_{ab}{}^c \frac{S_{ci}}{\sqrt{S_{0i}}} |i\rangle\!\rangle \\
&= \sum_c N_{ab}{}^c \| B_c \rangle\!\rangle
\end{aligned}
$$

←Verlinde formula used

Graham, Watts 2003

Topological defects: Brief review

- Back to our curious formula. Notice that

$$\frac{\langle V^{\beta}|D\|R\rangle\rangle}{\langle V^{\beta}\|R\rangle\rangle} = \frac{\langle V^{\beta}|D\|X\rangle\rangle}{\langle V^{\beta}\|X\rangle\rangle}$$

for any $\|R\rangle\rangle$ and $\|X\rangle\rangle$, and hence

$$\frac{B_X^{\beta} B_{DR}^{\beta}}{B_R^{\beta}} = B_{DX}^{\beta} = \sum_Z N_{DX}{}^{Z} B_Z^{\beta}$$

This is same as the formula we derived from OSFT by reinterpreting a solution $\Psi_{R\to X}$ on the *DR* brane.

Topological defects in OSFT

- It would be very nice to understand this derivation from an OSFT perspective. Intuitively we search for an operator $\mathcal{D}$ acting on the open string fields as

$$\mathcal{D}\Psi_{X\to Y} = \Psi_{DX\to DY}$$

It will manifestly map OSFT solutions to solutions provided that

$$\begin{aligned}[Q,\mathcal{D}] &= 0 \\ \mathcal{D}(\phi * \chi) &= (\mathcal{D}\phi) * (\mathcal{D}\chi) \qquad \forall \phi,\chi\end{aligned}$$

Topological defects in OSFT

- One such class of operators comes from considering symmetries of the OSFT action and the underlying BCFT. They are described by automorphisms of the star algebra

$$S(\phi * \chi) = S(\phi) * S(\chi)$$

In the case of continuous symmetries they are given just by exponentials of derivatives $S_\alpha = e^{\alpha P}$ where

$$P(\phi * \chi) = (P\phi) * \chi + \phi * P\chi$$

Assuming $[Q_B, P] = 0$ then S_α indeed maps solutions to solutions.

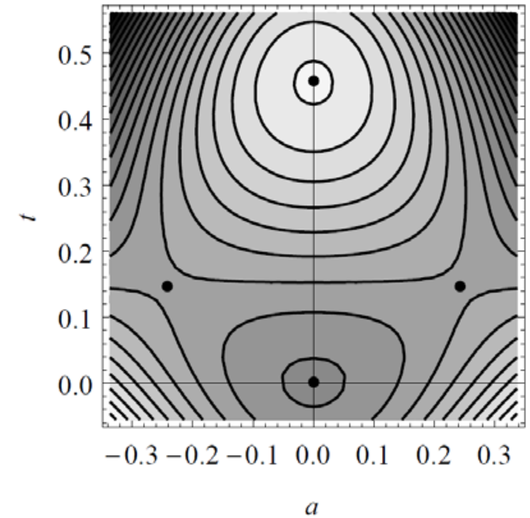
Topological defects in OSFT

- In general for defects the operator $\mathcal{D}$ will not be an automorphism. Recall the solution we found in the Ising model on the σ-brane. At level ½ the string field

$$\Psi = tc_1|0\rangle + ac_1|\varepsilon\rangle$$

leads to a potential

$$V(t,a) = -\frac{1}{2}t^2 - \frac{1}{4}a^2 + \frac{27\sqrt{3}}{64}t^3 + \frac{27}{16}ta^2$$



The various critical points have been interpreted as perturbative and tachyon vacua, and $\mathbb{1}$ or ε branes

Kudrna, Rapčák, M.S. 2014

Topological defects in OSFT

- Decomposing the complete solution as

$$\Psi_{\sigma\to 1} = \psi_{\mathbb{1}} + \psi_{\varepsilon}$$

it is obvious that the two components obey

$$\begin{aligned} Q\psi_{\mathbb{1}} + \psi_{\mathbb{1}} * \psi_{\mathbb{1}} + \psi_{\varepsilon} * \psi_{\varepsilon} &= 0 \\ Q\psi_{\varepsilon} + \psi_{\mathbb{1}} * \psi_{\varepsilon} + \psi_{\varepsilon} * \psi_{\mathbb{1}} &= 0 \end{aligned}$$

and hence $\psi_{\mathbb{1}} - \psi_{\varepsilon}$ and $\begin{pmatrix} \psi_{\mathbb{1}} & \pm\psi_{\varepsilon} \\ \pm\psi_{\varepsilon} & \psi_{\mathbb{1}} \end{pmatrix}$ are also solutions!

- While the first one is clearly result of $\mathcal{D}_{\varepsilon}$ action, the second one should be the result of the $\mathcal{D}_{\sigma}$.

Topological defects in OSFT

- So the essential difference between D and $\mathcal{D}$ is

$$D^d : \mathcal{H}_{closed} \rightarrow \mathcal{H}_{closed}$$

$$\mathcal{D}^d : \mathcal{H}^{(ab)} \rightarrow \bigoplus_{\substack{a' \in d \times a \\ b' \in d \times b}} \mathcal{H}^{(a'b')}$$

- Since the defect is topological and we demand $[Q, \mathcal{D}] = 0$ we expect that $[L_n, \mathcal{D}] = 0$ and hence by Schur's lemma

$$\mathcal{D}^d \, \psi_i^{(ab)} = \sum_{\substack{a' \in d \times a \\ b' \in d \times b}} X^{dab}_{ia'b'} \, \psi_i^{(a'b')}$$

Topological defects in OSFT

- To satisfy constraint $\mathcal{D}(\phi * \chi) = (\mathcal{D}\phi) * (\mathcal{D}\chi) \qquad \forall \phi, \chi$
  it is enough to require for primary fields

$$\mathcal{D}^d \left(\phi_i^{(ab)}(x) \phi_j^{(bc)}(y) \right) = \left(\mathcal{D}^d \phi_i^{(ab)}(x) \right) \left(\mathcal{D}^d \phi_j^{(bc)}(y) \right)$$

from which it follows

$$X^{dac}_{ka'c'} C^{(abc)k}_{ij} = \sum_{b' \in d \times b} C^{(a'b'c')k}_{ij} X^{dab}_{ka'b'} X^{dbc}_{kb'c'}$$

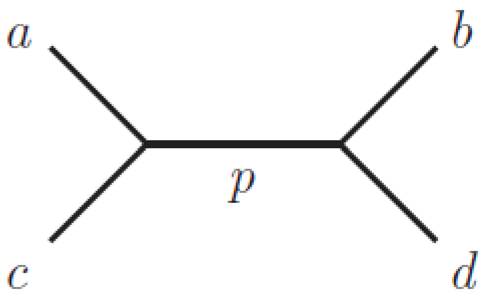
For minimal models Runkel found the boundary structure constants

$$C^{(abc)k}_{ij} = F_{bk} \begin{bmatrix} a & c \\ i & j \end{bmatrix}$$

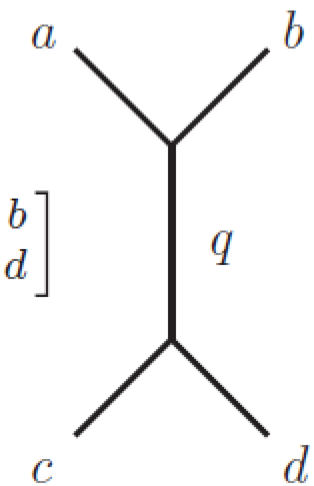
(for the A-series)

Topological defects in OSFT

- Where the fusion matrices *F* are defined by the transformation properties of 4pt-conformal blocks



$$= \sum_q F_{pq}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



Topological defects in OSFT

- Inserting this explicit solution into the constraint we found a general solution

$$X^{dab}_{ia'b'} = \frac{N(d,a,a')}{N(d,b,b')} F_{b'a}\begin{bmatrix} i & b \\ a' & d \end{bmatrix}$$

Generalizes result by Graham and Watts (2003)

thanks to the *pentagon identity* of rational CFT.

- Further demanding twist symmetry fixes the form

$$X^{dab}_{ia'b'} = F_{di}\begin{bmatrix} a & b \\ a' & b' \end{bmatrix} \frac{\sqrt{F_{1a'}\begin{bmatrix} a & d \\ a & d \end{bmatrix} F_{1b'}\begin{bmatrix} b & d \\ b & d \end{bmatrix}}}{F_{1i}\begin{bmatrix} a & b \\ a & b \end{bmatrix}}$$

Topological defects in OSFT

- Interestingly it turns out that

$$\mathcal{D}^d \mathcal{D}^c \neq \bigoplus_e N_{dc}{}^e \mathcal{D}_e$$

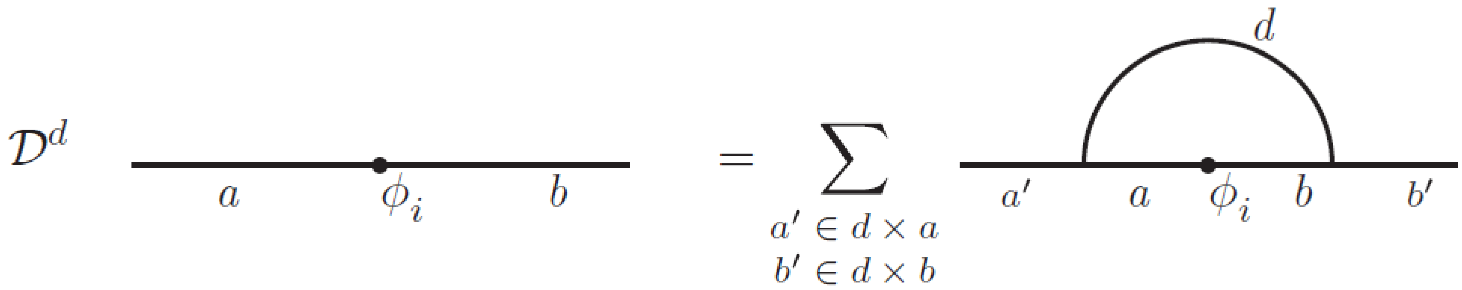
but fortunately at least

$$\mathcal{D}^d \mathcal{D}^c = U \left(\bigoplus_e N_{dc}{}^e \mathcal{D}_e \right) U^{-1}$$

is true! The matrices *U* are simply given by the fusion matrix, they square to 1, but most importantly they do not contribute to bulk observables.

Topological defects in OSFT

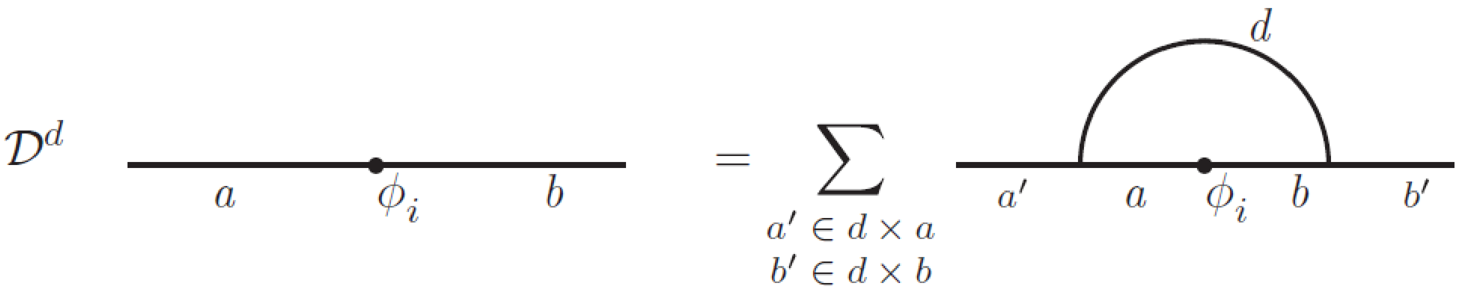
- To understand these extra factors it is convenient to develop a geometric formalism. Moving topological defect towards boundary, we get new boundary conditions. When we want to understand the action on boundary operators we need to fuse it only partway:



- We have to understand the CFT with defects ending on boundaries and possible operator insertions at junctions

Topological defects in OSFT

- Let us assume that the original boundary conditions *a* and *b* arise from the action of defect on the identity boundary condition (if it exists). Then one can reinterpret the same diagram

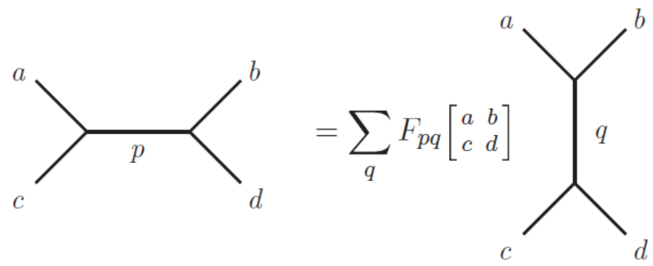


as one for *defect action* on *defect changing operators*

Topological defects in OSFT

- For that one can use the powerful TFT approach to rational CFT developed by Felder, Fröhlich, Fuchs, Runkel and Schweigert.

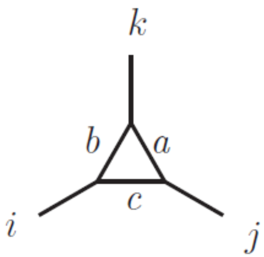
- The upshot of their construction is that for minimal models with selfconjugate representations one can compute correlators of any defect network simply from the rule



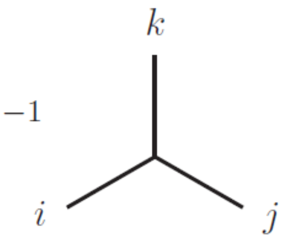
now viewed as the defect network itself. Many relations follow just by imposing mutual consistency.

Topological defects in OSFT

- For us the key relation is



$$= F_{ck}\begin{bmatrix} b & a \\ i & j \end{bmatrix} \times \left(F_{1k}\begin{bmatrix} a & b \\ a & b \end{bmatrix} \right)^{-1}$$



which is actually S_3 symmetric, thanks to nontrivial relations for the fusion matrices.

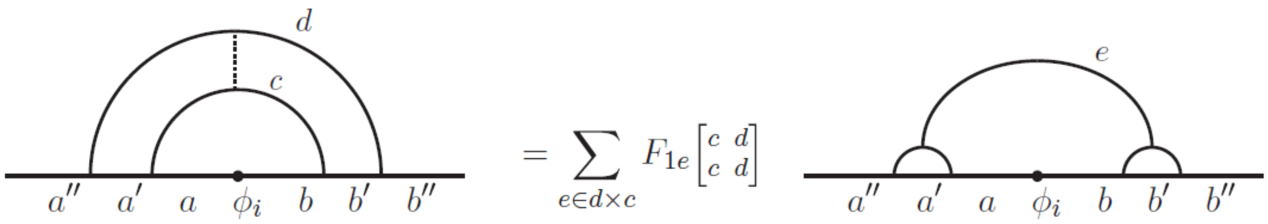
- This prefactor is 1 when one of the internal line is the identity defect. When one of the external defects is 1, we get simple, but nontrivial normalization for the bubble.

Topological defects in OSFT

- So finally the extra factors in

$$\mathcal{D}^d\mathcal{D}^c = U\left(\bigoplus_e N_{dc}{}^e\mathcal{D}_e\right)U^{-1}$$

can be deduced for example from



or better yet by refusing on *a'* and *b'* defect

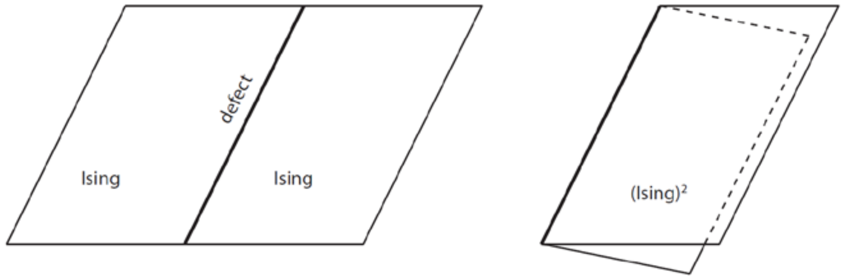
Summary

- CFT defects can be used to relate different solutions in various theories, but one has to be careful when extending their action on open string fields

- The problem of characterizing conformal boundary conditions (or conformal defects) in general CFTs is very interesting, but still unsolved. Open String Field Theory offers a novel approach to the problem, both analytically and numerically.

Comment on Conformal Defects

- Defects can be viewed as boundaries via folding trick



- Two types of defects are particularly useful:
  - factorizing (the two sides are independent)
  - topological or fully transmissive (trivial defect, or the spin flip in the Ising model)