# Tachyon Vacuum on a separated $D$-$\bar{D}$ system

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I discuss an analytic tachyon vacuum solution in the Berkovits WZW OSFT formulated on a separated $D$-$\bar{D}$ branes pair. This is part of an ongoing collaboration with Ted Erler.

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# 1 Introduction

- Tachyon condensation to the tachyon vacuum is a universal phenomenon for bosonic D-branes. The corresponding solution in OSFT can be expressed in the universal sector by making only use of ghosts and Virasoro operators.
- In the superstring the situation is richer. An unstable D-brane system has tachyons in the $GSO(-)$ sector. However the spectrum and operator algebra of this sector typically depends on the details of the system and it could not contain a universal sector.
- In 2008, Bagchi and Sen found a numerical, level truncated *non-universal* solution in the WZW-like OSFT on a separated $D$-$\bar{D}$ system, describing the tachyon vacuum.
- A universal tachyon vacuum analytic solution in the WZW-like OSFT has been found by Erler (2013) in the universal sector for a non-BPS D-brane and, with straightforward generalization, on a coincident $D$-$\bar{D}$ pair.
- I am going to present an exact non-universal solution for the separated $D$-$\bar{D}$ system.

# 2 TV on a separated $D$-$\bar{D}$ system

## 2.1 Separated $D$-$\bar{D}$ system

A brane and an anti-brane are separated along a flat non-compact direction $Y$, by a distance equal to $2\pi d$. The string field on this system is a 2x2 matrix of the form

$$\Psi = \begin{pmatrix} GSO(+) & GSO(-) \\ GSO(-) & GSO(+) \end{pmatrix} \tag{2.1}$$

The $GSO(+)$ sector has internal Chan-Paton's factor $\{1, \sigma_3\}$ for Grassmann even/odd string fields and the $GSO(-)$ carries $\{\sigma_1, \sigma_2\}$. The derivations $Q$ and $\eta = \eta_0$ carries internal multiplicative Chan-Paton $\sigma_3$. These CP factors will be understood in the following. The 11 entry contains the boundary fields of the D-brane boundary super conformal field theory with Dirichlet boundary condition $Y = 0$, and the 22 entry the boundary fields of the anti-D brane, with Dirichlet boundary condition $Y = 2\pi d$. The off diagonal terms 12 and 21 are twisted sector, describing the strings stretching from $D$ to $\bar{D}$ and from $\bar{D}$ to $D$, respectively. The ground state of the GSO(+) sector is given by the SL(2,R) vacuum while, in the GSO(-) sector, the lowest lying states are given by the bcc operators that change the Dirichlet boundary condition $Y = 0$ to $Y = 2\pi d$, and viceversa. These are given respectively by exponentials of the dual $Y$ coordinate $\tilde{Y}(z, \bar{z}) \equiv Y(z) - \bar{Y}(\bar{z})$.

$$\Delta = e^{id\tilde{Y}} \tag{2.2}$$

$$\bar{\Delta} = e^{id\tilde{Y}}, \tag{2.3}$$

they are superconformal primaries of weight $h = d^2$, and they are tachyonic for $d < \frac{1}{\sqrt{2}}$. In the small Hilbert space and at picture $-1$, the zero momentum tachyon fields are given by

$$t_\theta = e^{-\phi} c \begin{pmatrix} 0 & e^{i\theta}\Delta \\ e^{-i\theta}\bar{\Delta} & 0 \end{pmatrix}, \tag{2.4}$$

where $\theta$ is a phase describing an hermitian linear combination of the two real tachyons, connecting $D$ with $\bar{D}$ and viceversa.

In the large Hilbert space we have

$$\zeta'_\theta = \xi t_\theta = \begin{pmatrix} 0 & e^{i\theta}\,\gamma^{-1}\Delta \\ e^{-i\theta}\,\gamma^{-1}\bar{\Delta} & 0 \end{pmatrix} c \tag{2.5}$$

$$\gamma^{-1} \equiv \xi e^{-\phi} \tag{2.6}$$

## 2.2 Cubic solution

We can start by searching for a solution to the cubic equation of motion

$$Q\Psi + \Psi^2 = 0, \tag{2.7}$$

where $\Psi$ has ghost number one and picture number zero. This equation has the obvious solution

$$\Psi_{\rm tv} = \begin{pmatrix} \psi_{\rm tv} & 0 \\ 0 & \psi_{\rm tv} \end{pmatrix} \tag{2.8}$$

Where $\psi_{\rm tv}$ is a tachyon vacuum solution on a single $D$-brane. A simple choice is the (superstring generalization of) the Erler-Schnabl solution

$$\psi_{\rm tv} = (c - Q(Bc))\frac{1}{1+K}. \tag{2.9}$$

We can compute the energy from known results in the literature, to get (the traces over the internal and external Chan-Paton's are normalized with 1/2)

$$E = -S^{(cubic)} = \frac{1}{6}\,\mathrm{tr}_{Y_{-2}}[\Psi Q \Psi] = -\frac{1}{2\pi^2}, \tag{2.10}$$

which is the total energy carried by the system, with respect to the empty closed background (the tachyon vacuum). We can also check that the solution has the correct (vanishing) boundary state by computig the Ellwood invariant

$$\mathrm{tr}_{V_{-2}}[\Psi_{\rm tv}] = -\frac{1}{4\pi i}\left(\langle V_{-2}(0)c(1)\rangle^{D}_{Disk} + \langle V_{-2}(0)c(1)\rangle^{\bar D}_{Disk}\right). \tag{2.11}$$

The cubic solution thus captures the bulk observables of the starting background. However this is not completely satisfactory: from the direct world-sheet perspective we expect that the process of condensation is driven by the stretched tachyons, while here it is really the zero-momentum GSO(+) "bosonic" tachyon $c_1|0\rangle$ which condenses. It is therefore useful to search for the corresponding solution in the WZW-like theory where, much more physically, the GSO(–) sector will play a crucial role.

## 2.3 Berkovits's solution

The action is given by

$$S = -\int_0^1 dt\ \mathrm{Tr}[(\eta\Psi_t(t))\Psi_Q(t)], \tag{2.12}$$

where Tr is the large Hilbert space BPZ trace form, and it is related to tr of the previous slide by

$$\mathrm{tr}[(\mathrm{small})] = \mathrm{Tr}[\xi(\mathrm{small})]. \tag{2.13}$$

The flat connections $\Psi_t(t)$ and $\Psi_Q(t)$ are defined by

$$\Psi_t(t) = g^{-1}(t)\,\partial_t g(t) \tag{2.14}$$

$$\Psi_Q(t) = g^{-1}(t)\,Qg(t), \tag{2.15}$$

and the interpolating group element is such that

$$g(0) = 1 \tag{2.16}$$

$$g(1) = g \equiv e^{\Phi}, \tag{2.17}$$

where $\Phi$ is the dynamical string field in the large Hilbert space, with picture and ghost number zero. The equation of motion is given by

$$\eta(g^{-1}Qg) = 0. \tag{2.18}$$

A strategy to solve this equation (Erler, 2008), goes through the following steps

1. Solve the cubic equation
$$Q\Psi + \Psi^2 = 0$$

2. Set
$$g^{-1}Qg = \Psi \tag{2.19}$$

3. Solve the more general equation
$$Qg - g\Psi \equiv Q_{0\Psi}g = 0 \quad \leftrightarrow \quad g = Q_{0\Psi}(\beta), \tag{2.20}$$

4. CHALLENGE: Determine $\beta$ such that $g^{-1}$ exists. This last step is the most delicate and, for example, it is responsible for the absence of a tachyon vacuum solution on a BPS brane (inside the GSO+ $\{K, B, c, \gamma^2, \gamma^{-2}\}$ algebra), despite the solution exists in the cubic theory.

A prominent example is the solution for the tachyon vacuum on a non-BPS D-brane (Erler 2013) which comes from the choice

$$\beta = (-\gamma^{-2}c) + q\,(\gamma^{-1}c)\frac{B}{1+K} \equiv \alpha + q\,\zeta\frac{B}{1+K} \tag{2.21}$$

where $q \neq 0$ is a gauge parameter and $\alpha = -\gamma^{-2}c$ is the trivializer of $Q$ in the large Hilbert space, $Q\alpha = 1$. The invertibility of $g = Q_{0\Psi}\beta$ is guaranteed by a non zero vev for the zero momentum tachyon $\zeta = \gamma^{-1}c$, which is why we need $q \neq 0$.

### 2.3.1 Naive attempt

A physically motivated choice for $\beta$ is pretty obvious

$$\beta = \alpha + q\,\zeta'_\theta\frac{B}{1+K} \quad (?), \tag{2.22}$$

since this just substitutes the zero momentum tachyon on a coincident $D$-$\bar{D}$ system, with the zero momentum tachyon on a separated system. Setting $q = 1$ (for simplicity), this choice of "gauge fixing fermion" gives

$$g'_{\rm tv} = 1 + \zeta'_\theta + (Q\zeta'_\theta - c)\frac{B}{1+K} \quad (?) \tag{2.23}$$

By construction this proposed group element obeys

$$Qg' = \Psi_{\rm tv} g' \tag{2.24}$$

$$\Psi_{\rm tv} = (c - Q(Bc))\frac{1}{1+K}. \tag{2.25}$$

However, to have a genuine inverse, we need (at least) the string fields

$$\zeta_\theta^2, \qquad \zeta_\theta Q\zeta_\theta, \qquad (Q\zeta_\theta)\zeta_\theta$$

to be finite. But the contact term behaviour of the bcc operators $\Delta$ and $\bar{\Delta}$ , makes the $\zeta\zeta$ OPE divergent for $d > 1/\sqrt{2}$, which is where the stretched tachyons stops being tachyonic

$$\zeta'_\theta(s)\zeta'_\theta(0) \sim s^{1/2-d^2}\gamma^{-2}c\partial c(0) + (\text{less singular}). \tag{2.26}$$

This might be in a sense acceptable (no tachyons, so no tachyon condensation!). However it is not satisfactory, for at least two reasons

1. In the sigma model description an arbitrarily separated $D$-$\bar{D}$ system can undergo an exactly marginal boundary deformation which brings the branes together[2], then the stretched tachyons can trigger a boundary RG flow to the tachyon vacuum.

2. In the level truncation analysis by Bagchi and Sen, after including in the string field the marginal field controlling the D-branes' distance, the tachyon potential had a non trivial minimum also for $d > \frac{1}{\sqrt{2}}$.

Therefore we need something more.

[2]This marginal direction is lifted quantum mechanically (i.e. when closed strings are taken into account by open strings loops) and this results in the well known effective attraction between branes of opposite charge.

### 2.3.2 Resolution of contact singularities, algebra and solution

We can resolve the contact term singularity by substituting

$$\Delta \;\to\; e^{idX^0}\Delta = e^{id(X^0+\tilde{Y})} \;\equiv \sigma \tag{2.27}$$

$$\bar{\Delta} \;\to\; e^{-idX^0}\bar{\Delta} = e^{-id(X^0+\tilde{Y})} \;\equiv \bar{\sigma} \tag{2.28}$$

These are weight-zero matter superconformal primaries obeying

$$\sigma\bar{\sigma} = \bar{\sigma}\sigma = 1. \tag{2.29}$$

Their BRST variation is given by

$$Q\sigma \;=\; c\partial\sigma - \frac{d}{\sqrt{2}}\gamma\psi^+\sigma \tag{2.30}$$

$$Q\bar{\sigma} \;=\; c\partial\bar{\sigma} + \frac{d}{\sqrt{2}}\gamma\psi^+\bar{\sigma}, \tag{2.31}$$

where $\psi^+$ is the light-cone world sheet fermion

$$\psi^+ = \psi^0 + \psi^Y. \tag{2.32}$$

It is then natural to define a new (time-dependent) tachyon as

$$\zeta_\theta \;\equiv\; \gamma_\theta^{-1}c, \tag{2.33}$$

$$\gamma_\theta^{\pm 1} \;\equiv\; \begin{pmatrix} 0 & e^{i\theta}\,\gamma^{\pm 1}\sigma \\ e^{-i\theta}\,\gamma^{\pm 1}\bar{\sigma} & 0 \end{pmatrix}. \tag{2.34}$$

Computing the BRST variation we find

$$Q\zeta_\theta = cV_\theta + \gamma_\theta, \tag{2.35}$$

where

$$V_\theta = \frac{1}{2}\gamma_\theta^{-1}\partial c - \frac{d}{\sqrt{2}}\begin{pmatrix} 0 & e^{i\theta}\,\psi^+\sigma \\ -e^{-i\theta}\,\psi^+\bar{\sigma} & 0 \end{pmatrix} \equiv \frac{1}{2}\gamma_\theta^{-1}\partial c - \frac{d}{\sqrt{2}}\psi_\theta^+. \tag{2.36}$$

Then one can easily show that all the properties listed in eq (3.10) of Erler's paper 1308.4400 are satisfied by the substitution

$$\text{there} \;\to\; \text{here}$$

$$\zeta \;\to\; \zeta_\theta \tag{2.37}$$

$$V \;\to\; V_\theta \tag{2.38}$$

$$\gamma^{\pm 1} \;\to\; \gamma_\theta^{\pm 1}. \tag{2.39}$$

Which is quite remarkable and not obviously expected!
We can now immediately write down the searched-for tachyon vacuum solution as

$$g_{\theta,q} = Q_{0\Psi_{\rm tv}}\left(\alpha + q\zeta_\theta \frac{B}{1+K}\right) = (1+q\zeta_\theta)\left[1-\big((1-q^2)c - qQ\zeta_\theta\big)\frac{B}{1+K}\right] \quad (2.40)$$

$$g^{-1}_{\theta,q} = \left[1+\big((1-q^2)c - qQ\zeta_\theta\big)\frac{B}{q^2+K+qV_\theta}\right](1-q\zeta_\theta) \quad (2.41)$$

Notice that $\theta$ is a phase in the two-dimensional tachyon space while $q>0$ is the modulus. The tachyon $q\zeta_\theta$ lives in the whole tachyon-plane, but the point $q=0$ (the origin) is a singularity which corresponds to the disappearance of the GSO(−) sector. It is elementary to show that

$$g^{-1}_{\theta,q} g_{\theta,q} = g_{\theta,q} g^{-1}_{\theta,q} = 1, \quad (2.42)$$

and

$$g^{-1}_{\theta,q} Q g_{\theta,q} = (c - Q(Bc))\frac{1}{1+K}, \quad (2.43)$$

and therefore

$$\eta\left(g^{-1}_{\theta,q} Q q_{\theta,q}\right) = 0. \quad (2.44)$$

It is also worth noting that, just as in 1308.4400, the solution is part of a GL(2) subgroup of the star algebra, which is spanned by string fields of the form

$$M_\theta = -\gamma_\theta B\ X_1\ \zeta_\theta + cB\ X_2 + \gamma_\theta B\ Y_1 - cB\ Y_2\ \zeta_\theta \sim \begin{pmatrix} X_1 & Y_1 \\ Y_2 & X_2 \end{pmatrix}, \quad (2.45)$$

where

$$[B, X_i] = [B, Y_i] = 0. \quad (2.46)$$

### 2.3.3 Time dependence is a gauge artifact

One curious property of the solution we are considering is that it is explicitly time dependent. To check this consider the time-translation operator, which is given by the following topological defect operator

$$\mathcal{D}_\tau = e^{i\tau\mathcal{P}_0}, \quad (2.47)$$

where $\mathcal{P}_0$ is the even derivation

$$\mathcal{P}_0 = \oint \frac{dz}{2\pi i} i\partial X^0 \tag{2.48}$$

$$[\mathcal{P}_0, Q] = [\mathcal{P}_0, \eta] = 0. \tag{2.49}$$

This operator distributes over the star product

$$\mathcal{D}_\tau(A * B) = (\mathcal{D}_\tau A) * (\mathcal{D}_\tau B), \tag{2.50}$$

and it has the effect of changing

$$X^0 \to \mathcal{D}_\tau X^0 = X^0 + \tau. \tag{2.51}$$

It is not difficult to see that a time translation changes our solution in the following way

$$\mathcal{D}_\tau \, g_{\theta,t} = g_{\theta+d\tau,t}, \tag{2.52}$$

so the (opposite) phases of the two stretched tachyons (2.5) are translated by an (opposite) amount proportional to the time translation. Therefore time dependence of our solution can be interpreted as a rotation (in opposite directions for the the two stretched sectors) along the orbit spanned by $\theta$. But the orbit spanned by $\theta$ is a *gauge* orbit. This can be made explicit by noticing that ($q = 1$, for simplicity)

$$g_{\theta_1} = U_{12} \, g_{\theta_2} \tag{2.53}$$

$$U_{12} = (1+\zeta_1)\left(1 + Q(\zeta_1 - \zeta_2)\frac{B}{1+K+V_2}\right)(1-\zeta_2) = U_{21}^{-1} \tag{2.54}$$

$$QU_{12} = 0. \tag{2.55}$$

Therefore the time dependence of the solution is a motion at constant angular velocity (depending linearly on the distance between the branes) along a circular gauge orbit, around the singularity at $q = 0$. In particular time dependence is a gauge artifact.

### 2.3.4 Energy

Because of the identical $GL(2)$ structure, the analytic computation of the action can be read-off from 1308.4400, as far as the algebraic part is concerned. When we come to the the evaluation of world-sheet correlators we find two sources of difference wrt 1308.4400.

- The string field $V_\theta$ contains the world sheet fermion $\psi_\theta^+$. However, thanks to its light-cone nature, we have

$$\langle \psi_\theta^+(s_1)...\psi_\theta^+(s_n)\rangle = 0, \quad n > 0, \tag{2.56}$$

so the $\psi^+$ part of $V$ decouples from the energy computation.

- The ghost correlators of 1308.4400 will always be accompanied by $(X^0, \tilde{Y})$ correlators of the form

$$\langle e^{id\tilde{X}^+}(s_1)e^{-id\tilde{X}^+}(s_2)...e^{id\tilde{X}^+}(s_{2n-1})e^{-id\tilde{X}^+}(s_{2n})\rangle, \tag{2.57}$$

where

$$\tilde{X}^+ = X^0 + \tilde{Y}, \tag{2.58}$$

which is again a light-cone coordinate which makes the previous correlator trivially equal to one.

Therefore we obtain

$$S = \frac{1}{2\pi^2}. \tag{2.59}$$

The Ellwood invariant also (trivially) works, because it can be computed from the cubic solution $\Psi_{\rm tv}$.

# 3 Conclusion and comments

- The Berkovits tachyon vacuum solution on a separated $D$-$\bar{D}$ system is not universal, but it can be expressed algebraically in a way which is identical to the universal case of a non-BPS or a coincident $D$-$\bar{D}$ pair. In particular the solution exists for any initial value of the separation. The solution has a periodic time dependence which is however a gauge artifact, since time translations correspond to gauge transformations which don't change the (time independent) cubic solution.

- This is yet another example which shows that OSFT deals well in handling large changes in the BCFT moduli. Another interesting example is given by the solutions for marginal deformations in the bosonic string, described in 1402.3546 (CM). The solution is directly described in terms of the BCFT modulus $\lambda_{\rm BCFT}$ and one can explicitly compute the coefficient of the marginal field in the solution (typically called $\lambda_{\rm SFT}$) in terms of $\lambda_{\rm BCFT}$[3]. This was summarized in some detail in my talk at

[3]This work, in collaboration with M. Schnabl will appear soon.

SFT2104,
http://www.sissa.it/tpp/activity/conferences/SFT2014/talks/Maccaferri.pdf.

- The previous item suggests that we can directly get a time independent tachyon vacuum solution by taking inspiration from my previous work (1402.3546), defining the Takahashi-Tanimoto (TT) bcc- operators

$$\sigma = e^{i\,d\int_0^{i\infty}\frac{dz}{2\pi i}f(z)\tilde{Y}(z,\bar{z})}, \tag{3.1}$$

$$\bar{\sigma} = e^{-i\,d\int_0^{i\infty}\frac{dz}{2\pi i}f(z)\tilde{Y}(z,\bar{z})}, \tag{3.2}$$

where $f(z)$ is a regulating function, which spread the chiral current $\partial Y$ into the bulk and thus automatically regulates the contact term divergences. However this meets with a subtle open problem that is not present in the bosonic string. The BRST variation of $\sigma$ generates (by construction) the TT-solution $\sigma Q\bar{\sigma}$ which is in general not so well-behaved in the presence of $\frac{1}{1+K}$, which has non-vanishing support on the identity. As shown 1402.3546, this renders the computation of the observables quite ambiguous. In the bosonic case we can easily create solutions which have support on strictly positive wedge state, but this is not known in the Berkovits superstring and it actually turns out to be quite a challenge.

- The difficulty in defining a Berkovits' tachyon vacuum based on strictly positive wedge states also prevents to write down a fully regular solution describing the transition between two superstring background $\mathrm{BSCFT}_0$ and $\mathrm{BSCFT}_*$. A cubic solution can be immediately written as 1406.3021 (Erler, CM)

$$\Psi_{0\to *} = \Psi_{\rm tv} - \Sigma_{0*}\Psi_{\rm tv}\bar{\Sigma}_{0*} \tag{3.3}$$

However the choice of the Erler-Schnabl tachyon vacuum turns out to be too much identity-like, because of the superstring correction $B\gamma^2$. This is not a real problem in the cubic theory, however, because it is enough to choose a $K, B, c$ tachyon vacuum with strictly positive security strips, and define the $\Sigma$'s accordingly.
When we go to the Berkovits theory we can write in principle

$$g_{0\to *} = g_{\rm tv}^{(0)}(1 - \Sigma U_{\rm tv}^{(*)}\bar{\Sigma}), \tag{3.4}$$

where

$$U_{\rm tv}^{(*)} = 1 - (g_{\rm tv}^{(*)})^{-1}.$$

However $U_{\rm tv}^{(*)}$ still contains identity based pieces (for example $\zeta$) and the collision $\Sigma$-$\bar\Sigma$ is not protected by a finite piece of world-sheet.

Therefore further study is needed to improve our analytic techniques in the Berkovits theory, if we want to cleanly describe the superstring landscape.