Multibrane Solutions and Chan-Paton Factors

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This talk is based on the work in collaboration with

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"Open String Fields as Matrices",
I. Kishimoto, T. Masuda, T. Takahashi and S. Takemoto,
Prog. Theor. Exp. Phys. (2015) 033B05 [arXiv:1412.4855 [hep-th]].

Introduction

Open string field theory has the possibility of revealing non-perturbative aspects of string theory.

Recently, Erler and Maccaferri have proposed a method to construct classical solutions, which are expected to describe any open string background (2014).

Erler-Maccaferri's solutions provide correct vacuum energy and gauge invariant observables.

Is it possible to find open spectra in the background of the solution?

It is difficult to give a definite answer to this problem, because there are some subtleties concerning BRST cohomology in the background.

There is another question related to the degree of freedom of string fields in the background.

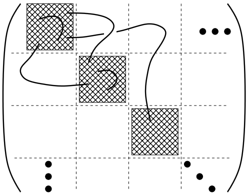
We have one string field in the theory on a single D-brane.

However, in the case that the multi-brane solution provides the background of the N D-branes, the number of string fields increases to N^2 around the solution.

Here, it is natural to ask how to generate N^2 string fields or Chan-Paton factors from one string field.

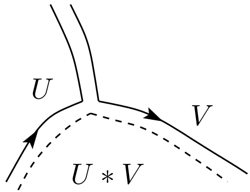
On the other hand, it is well-known that matrix theories are able to describe various D-branes (BFSS 1996, IKKT 1996).

In matrix theories, D-branes are created by classical solutions as block diagonal matrices. After expanding a matrix around the solution, block matrices can be understood as representing open strings connecting each D-brane.

$X^\mu =$ 

Here, it should be noted that there are similarities between the matrix and the open string field: the matrix is deeply tied to the open string degree of freedom, and an open string field is interpreted as a matrix in which the left and right indices correspond to the left and right half-strings (Witten 1985).

$$(U * V)[X_L, X_R] = \sum_Y U[X_L,\, Y]\, V[Y,\, X_R]$$



Then, it seems plausible that N^2 string fields on N D-branes are embedded like block matrices in a string field on a D-brane.

The purpose of our talk is to clarify the origin of the N^2 string fields in the background of an N D-brane solution.

We will show that the theory expanded around the solution is regarded as an open string field theory on $N+1$ D-branes, but in which a D-brane vanishes as a result of tachyon condensation.

Then, the N^2 string fields will be given as block matrices in a string field as an infinite-dimensional matrix.

Cubic bosonic open string field theory

Action

$$S[\Psi;\, Q_B] = -\frac{1}{g^2}\int\left(\frac{1}{2}\Psi * Q_B\Psi + \frac{1}{3}\Psi * \Psi * \Psi\right)$$

open string field: $\Psi[X(\sigma), \cdots]$
(functional of string coordinates $X^\mu(\sigma)$)

Q_B: Kato-Ogawa BRST operator

Gauge symmetry

The action is invariant under the gauge transformation

$$\Psi' = e^{-\Lambda} * Q_B e^{\Lambda} + e^{-\Lambda} * \Psi * e^{\Lambda}$$

$$e^{\Lambda} = I + \Lambda + \frac{1}{2!}\Lambda * \Lambda + \frac{1}{3!}\Lambda * \Lambda * \Lambda + \cdots$$

I : identity string field, $\quad I * A = A * I = A \ \text{ for } \forall \ A$

Equations of motion

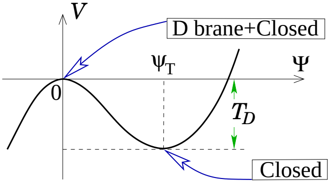
$$Q_B \Psi + \Psi * \Psi = 0$$

tachyon vacuum solution

Open bosonic SFT has a tachyon vacuum solution.

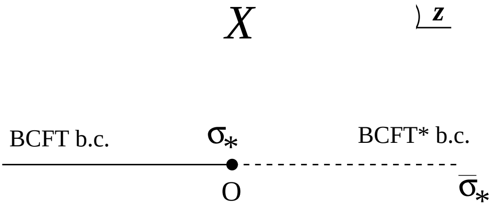
$\Psi = 0 \ \cdots$ perturbative vacuum:
the string field on D-branes
closed strings as quantum effects

$\Psi = \psi_{\mathrm{T}} \ \cdots$ tachyon vacuum:
D-branes disappear
on-shell modes are unphysical (BRST quartets).
closed strings as quantum effects (?)



boundary condition changing operators

We can introduce bcc operators:



$$\sigma = \sigma_* e^{i\sqrt{h}X^0}$$

$$\bar{\sigma} = \bar{\sigma}_* e^{-i\sqrt{h}X^0}$$

In general, we can construct the operators as

$$\lim_{s\to 0+\epsilon} \bar{\sigma}(s)\sigma(0) = 1$$

It is noted that $\lim_{s\to 0+\epsilon} \sigma(s)\bar{\sigma}(0) \neq 1$ in general.

We consider multi-brane solutions in the Minkowski background for simplicity.

For N D-brane solutions, the bcc operators can be given by

$$\sigma_a(z) = e^{ik_a \cdot X(z)}, \quad \bar{\sigma}_a(z) = e^{-ik_a \cdot X(z)} \quad (a = 1, 2, \cdots, N),$$

where k_a^μ satisfy $k_a^2 = 0$ and $k_a \cdot k_b < 0$ $(a \neq b)$. KMTT (2014)

$$\lim_{\epsilon \to 0} \bar{\sigma}_a(\epsilon)\sigma_b(0) = \lim_{\epsilon \to 0} \epsilon^{-k_a \cdot k_b} = \delta_{a,b}$$

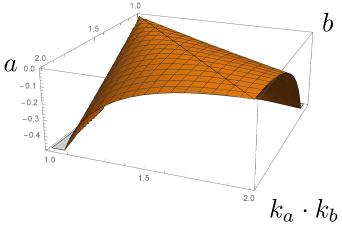
For example, we can take KMTT (2014)

$$k_a^\mu = (a, 1, \sqrt{a^2-1}, 0, \cdots, 0) \ (a = 1, 2, \cdots).$$

Then, we find

$$\begin{aligned} k_a \cdot k_b &= -ab + 1 + \sqrt{a^2-1}\sqrt{b^2-1} \\ &= -(ab-1) + \sqrt{(ab-1)^2 - (a-b)^2} \\ &\leq 0. \end{aligned}$$

The two sides are equal iff $a = b$.



Erler and Maccaferri introduced N pairs of regularized bcc operators:

$$\Sigma_a, \quad \bar{\Sigma}_a \quad (a = 1, \cdots, N).$$

These operators satisfy

$$\bar{\Sigma}_a \Sigma_b = \delta_{ab},$$

and

$$Q_{\rm T} \Sigma_a = Q_{\rm T} \bar{\Sigma}_a = 0,$$

where $Q_{\rm T}$ is a modified BRST operator on the tachyon vacuum.

We can construct Σ_a and $\bar{\Sigma}_a$ as

$$\Sigma_a = Q_{\rm T} \left(\frac{1}{\sqrt{1+K}} B \sigma_a \frac{1}{\sqrt{1+K}} \right), \quad \bar{\Sigma}_a = Q_{\rm T} \left(\frac{1}{\sqrt{1+K}} B \bar{\sigma}_a \frac{1}{\sqrt{1+K}} \right).$$

Erler-Maccaferri's solution

Using the tachyon vacuum solution $\psi_{\rm T}$, and Σ_a, $\bar{\Sigma}_a$, Erler-Maccaferri provided a multi-brane solution as

$$\Psi_0 = \psi_{\rm T} - \sum_{a=1}^{N} \Sigma_a \psi_{\rm T} \bar{\Sigma}_a$$

The solution Ψ_0 provides a correct vacuum energy for N D-branes:

$$S[\Psi_0; Q_{\rm B}] = -(N-1) S[\psi_{\rm T}; Q_{\rm B}].$$

Expanding the string field around the solution as $\Psi = \Psi_0 + \psi$, we can obtain the action for the fluctuation ψ:

$$S[\Psi; Q_{\rm B}] = S[\Psi_0; Q_{\rm B}] + S[\psi; Q_{\Psi_0}].$$

Does $S[\psi; Q_{\Psi_0}]$ describes the theory on the N D-branes?

Projectors

To clarify the physical interpretation of $S[\psi; Q_{\Psi_0}]$, we introduce N projection states as follows: KMTT (2014)

$$P_a = \Sigma_a \bar{\Sigma}_a \quad (a = 1, \cdots, N).$$

In addition to P_a, we define the 0th projection as a complementary projector:

$$P_0 = 1 - \sum_{a=1}^{N} P_a,$$

where 1 denotes the identity string field.

By definition, these $N+1$ projections satisfy

$$\sum_{\alpha=0}^{N} P_\alpha = 1,$$

where the Greek indices are used for values $0, 1, \cdots, N$.

From $Q_{\rm T} P_\alpha = 0$, it follows that

$$Q_{\rm B} P_\alpha = P_\alpha \psi_{\rm T} - \psi_{\rm T} P_\alpha.$$

Moreover, we can find some relations among P_α, Σ_a and $\bar{\Sigma}_a$:

$$P_a \Sigma_b = \Sigma_a \delta_{ab}, \quad \bar{\Sigma}_a P_b = \bar{\Sigma}_a \delta_{ab}, \quad P_0 \Sigma_b = 0, \quad \bar{\Sigma}_a P_0 = 0.$$

With the help of these projectors, the string field Ψ can be partitioned into $(N+1) \times (N+1)$ blocks:

$$\Psi = \sum_{\alpha=0}^{N} \sum_{\beta=0}^{N} P_\alpha \Psi P_\beta = \begin{pmatrix} \Psi_{00} & \Psi_{01} & \cdots & \Psi_{0N} \\ \Psi_{10} & \Psi_{11} & \cdots & \Psi_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_{N0} & \Psi_{N1} & \cdots & \Psi_{NN} \end{pmatrix}$$

where $\Psi_{\alpha\beta}$ is defined as the (α, β) sector of Ψ; $\Psi_{\alpha\beta} \equiv P_\alpha \Psi P_\beta$.

The second term in Erler-Maccaferri's solution is a solution to the equation of motion at the tachyon vacuum.

Then, the second term is represented as

$$-\sum_{a=1}^{N}\Sigma_a\psi_{\rm T}\bar{\Sigma}_a=\begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & -\Sigma_1\psi_{\rm T}\bar{\Sigma}_1 & 0 & \cdots & 0 \\ 0 & 0 & -\Sigma_2\psi_{\rm T}\bar{\Sigma}_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -\Sigma_N\psi_{\rm T}\bar{\Sigma}_N \end{pmatrix}.$$

Accordingly, it turns out that the N D-brane solution at the tachyon vacuum is given as a block diagonal matrix.

This is a similar result to the case of matrix theories (BFSS, IKKT).

Background described by the solution

Now, we consider the fluctuation ψ around the N D-brane solution.

Using the projectors, ψ can be written by matrix representation:

$$\psi = \sum_{\alpha=0}^{N}\sum_{\beta=0}^{N}\tilde{\phi}_{\alpha\beta}.$$

Here, we consider change of variables of $\tilde{\phi}_{\alpha\beta}$.

$\tilde{\phi}_{ab}$ can be rewritten as

$$\tilde{\phi}_{ab} = P_a\tilde{\phi}_{ab}P_b = \Sigma_a(\bar{\Sigma}_a\tilde{\phi}_{ab}\Sigma_b)\bar{\Sigma}_b.$$

So, we can change the variables from $\tilde{\phi}_{ab}$ to $\phi_{ab} = \bar{\Sigma}_a\tilde{\phi}_{ab}\Sigma_b$.

Similarly, writing $\tilde{\phi}_{0a} = \chi_a \bar{\Sigma}_a$, $\tilde{\phi}_{a0} = \Sigma_a \bar{\chi}_a$, the fluctuation ψ is represented as

$$
\begin{aligned}
\psi &= \chi + \sum_{a=1}^{N} \chi_a \bar{\Sigma}_a + \sum_{a=1}^{N} \Sigma_a \bar{\chi}_a + \sum_{a=1}^{N} \sum_{b=1}^{N} \Sigma_a \phi_{ab} \bar{\Sigma}_b \\
&= \begin{pmatrix} \chi & \chi_b \bar{\Sigma}_b \\ \\ \Sigma_a \bar{\chi}_a & \Sigma_a \phi_{ab} \bar{\Sigma}_b \end{pmatrix},
\end{aligned}
$$

where we rewrite $\tilde{\phi}_{00}$ as χ.

Similar to the equation $Q_{\Psi_0}(\Sigma_a A\bar{\Sigma}_b) = \Sigma_a(Q_{\mathrm{B}}A)\bar{\Sigma}_b$ given by Erler-Maccaferri, we find that

$$\begin{aligned}
Q_{\Psi_0}(P_0AP_0) &= Q_B(P_0AP_0) + \Psi_0(P_0AP_0) - (-1)^{|A|}(P_0AP_0)\Psi_0 \\
&= Q_B(P_0AP_0) + \psi_T P_0AP_0 - (-1)^{|A|}P_0AP_0\psi_T \\
&\qquad (\because\ \Psi_0 = \psi_T - \Sigma\psi_T\bar{\Sigma}, \quad \bar{\Sigma}P_0 = P_0\Sigma = 0) \\
&= P_0(Q_BA)P_0 + P_0\psi_T AP_0 - (-1)^{|A|}P_0A\psi_T P_0 \\
&\qquad (\because\ Q_BP_0 = P_0\psi_T - \psi_T P_0) \\
&= P_0(Q_{\mathrm{T}}A)P_0.
\end{aligned}$$

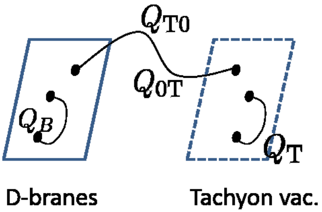
Similarly, we have

$$\begin{aligned}
Q_{\Psi_0}(P_0A\bar{\Sigma}_a) &= P_0(Q_{\mathrm{T}0}A)\bar{\Sigma}_a, \\
Q_{\Psi_0}(\Sigma_a AP_0) &= \Sigma_a(Q_{0\mathrm{T}}A)P_0,
\end{aligned}$$

where $Q_{\mathrm{T}0}A = Q_BA + \psi_T A$ and $Q_{0\mathrm{T}}A = Q_BA - (-1)^{|A|}A\psi_T$.

Then, we find a matrix representation of $Q_{\Psi_0}\psi$:

$$Q_{\Psi_0}\psi = \begin{pmatrix} P_0(Q_{\mathrm{T}}\chi)P_0 & P_0(Q_{\mathrm{T}0}\chi_b)\bar{\Sigma}_b \\ \Sigma_a(Q_{0\mathrm{T}}\bar{\chi}_a)P_0 & \Sigma_a(Q_{\mathrm{B}}\phi_{ab})\bar{\Sigma}_b \end{pmatrix}.$$



As a result, the action expanded around Ψ_0 can be rewritten as

$$S[\psi; Q_{\Psi_0}] \quad = \quad S[\phi_{ab}; Q_{\mathrm{B}}] + S'[\chi, \chi_a, \bar{\chi}_a, \phi_{ab}],$$

where each action is given by

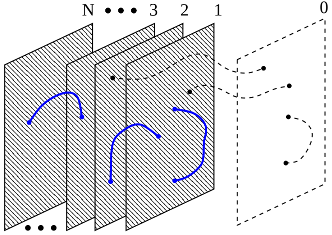
$$S[\phi_{ab}; Q_{\mathrm{B}}] = -\frac{1}{g^2}\int\left(\frac{1}{2}\sum_{a=1}^{N}\sum_{b=1}^{N}\phi_{ba}Q_{\mathrm{B}}\phi_{ab} + \frac{1}{3}\sum_{a=1}^{N}\sum_{b=1}^{N}\sum_{c=1}^{N}\phi_{ab}\phi_{bc}\phi_{ca}\right)$$

and

$$\begin{aligned} S'[\chi, \chi_a, \bar{\chi}_a, \phi_{ab}] \quad = \quad & -\frac{1}{g^2}\int\left(\frac{1}{2}\chi Q_{\mathrm{T}}\chi + \sum_{a=1}^{N}\bar{\chi}_a Q_{\mathrm{T}0}\chi_a + \frac{1}{3}\chi^3\right. \\ & \left. + \sum_{a=1}^{N}\bar{\chi}_a\chi\chi_a + \sum_{a=1}^{N}\sum_{b=1}^{N}\chi_a\phi_{ab}\bar{\chi}_b\right). \end{aligned}$$

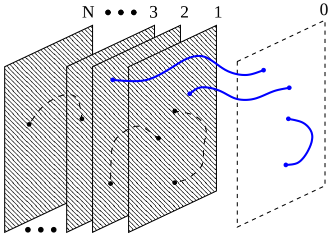
$$S[\phi_{ab};Q_{\mathrm{B}}]=-\frac{1}{g^2}\int\left(\frac{1}{2}\sum_{a=1}^{N}\sum_{b=1}^{N}\phi_{ba}Q_{\mathrm{B}}\phi_{ab}+\frac{1}{3}\sum_{a=1}^{N}\sum_{b=1}^{N}\sum_{c=1}^{N}\phi_{ab}\phi_{bc}\phi_{ca}\right)$$

Obviously, $S[\phi_{ab};Q_{\mathrm{B}}]$ represents the action for N D-branes; namely, ϕ_{ab} is a string field of an open string attached on the ath and bth D-branes.



$$S'[\chi,\chi_a,\bar{\chi}_a,\phi_{ab}] = -\frac{1}{g^2}\int\left(\frac{1}{2}\chi Q_{\mathrm{T}}\chi+\sum_{a=1}^{N}\bar{\chi}_a Q_{\mathrm{T0}}\chi_a+\frac{1}{3}\chi^3 + \sum_{a=1}^{N}\bar{\chi}_a\chi\chi_a+\sum_{a=1}^{N}\sum_{b=1}^{N}\chi_a\phi_{ab}\bar{\chi}_b\right).$$

χ is a string field on a D-brane with tachyon condensation, and χ_a and $\bar{\chi}_a$ represent string fields of an open string attaching on a D-brane with tachyon condensation and on one of the N D-branes.



Accordingly, these actions describe the theory for $N+1$ D-branes in which a D-brane vanishes due to tachyon condensation.

This system is physically equivalent to the N D-brane system and therefore this result is consistent with the expectation that the solution Ψ_0 is regarded as an N D-brane solution.

Chan-Paton factors are generated by Erler-Maccaferri's solution!

Let us consider an on-shell closed string coupling to an open string field.

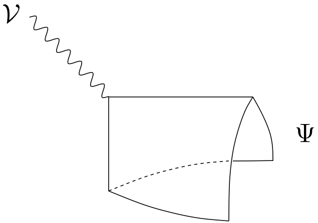
For the open string field Ψ, an interaction term with the closed string vertex is given as a gauge invariant overlap:

$$O_{\mathcal{V}}(\Psi) = \int V\Psi.$$

Here, V is a BRST invariant state

$$V = \mathcal{V}(i, -i)I,$$

where $\mathcal{V}$ is a closed string vertex operator.



In the background of the N D-brane solution, we can easily find couplings of the fluctuation fields to the closed string as

$$
\begin{aligned}
O_{\mathcal{V}}(\psi) &= \int V\left(\chi + \sum_{a=1}^{n} \chi_a \bar{\Sigma}_a + \sum_{a=1}^{N} \Sigma_a \bar{\chi}_a + \sum_{a,b=1}^{N} \Sigma_a \phi_{ab} \bar{\Sigma}_b\right) \\
&= O_{\mathcal{V}}(\chi) + \sum_{a=1}^{N} O_{\mathcal{V}}(\phi_{aa}).
\end{aligned}
$$

($\int AB = (-1)^{|A||B|} \int BA$, $VA = AV$, $\bar{\Sigma}_a \Sigma_b = \delta_{ab}$, $\bar{\Sigma}_a \chi_a = \bar{\chi}_a \Sigma_a = 0$)

This correctly provides a closed string interaction to open strings on the $N+1$ D-branes.

We consider the correspondence between gauge symmetries in the original action and the expanded action.

The original gauge transformation is given by

$$\delta_\Lambda \Psi = Q_{\rm B}\Lambda + \Psi\Lambda - \Lambda\Psi.$$

Here, we decompose Λ into $\tilde{\Lambda}_{\alpha\beta} = P_\alpha \Lambda P_\beta$ by the projectors. Then, changing variables as

$$\tilde{\Lambda}_{ab} = \Sigma_a \Lambda_{ab} \bar{\Sigma}_b, \quad \tilde{\Lambda}_{0a} = \lambda_a \bar{\Sigma}_a, \quad \tilde{\Lambda}_{a0} = \Sigma_a \bar{\lambda}_a,$$

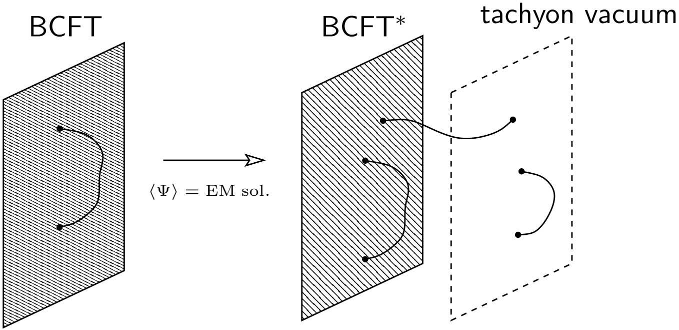
the gauge transformations for the components are given as

$$\begin{aligned}
\delta_\Lambda \phi_{ab} &= Q_{\rm B}\Lambda_{ab} + \phi_{ac}\Lambda_{cb} - \Lambda_{ac}\phi_{cb} + \bar{\chi}_a \lambda_b - \bar{\lambda}_a \chi_b, \\
\delta_\Lambda \chi_a &= P_0(Q_{\rm T0}\lambda_a) + \chi\lambda_a + \chi_b \Lambda_{ba} - \lambda\chi_a - \lambda_b \phi_{ba}, \\
\delta_\Lambda \bar{\chi}_a &= (Q_{0\rm T}\bar{\lambda}_a)P_0 + \bar{\chi}_a \lambda + \phi_{ab}\bar{\lambda}_b - \bar{\lambda}_a \chi - \Lambda_{ab}\bar{\chi}_b, \\
\delta_\Lambda \chi &= P_0(Q_{\rm T}\lambda)P_0 + \chi\lambda + \chi_a \bar{\lambda}_a - \lambda\chi - \lambda_a \bar{\chi}_a.
\end{aligned}$$

Conclusions

- The theory expanded around the N D-brane solution given by Erler-Maccaferri describes an $N+1$ D-brane system with a vanishing D-brane due to the tachyon condensation.
- By projectors made of regularized bcc operators, an open string field in the original theory is divided into multi-string fields with matrix indices.
- These indices can be regarded as Chan-Paton factors in the N D-brane background.
- N^2 string fields on N D-branes are embedded in a string field as block matrices.
- Similarly, gauge transformation parameters in the expanded theory are represented as block elements of a gauge parameter string field in the original theory.

In a general SFT given by a BCFT, the theory expanded Erler-Maccaferri's solution provides string fields on a D-brane given by a BCFT* and on a vanishing D-brane.



We should comment on the multiplicative ordering of Σ_a and $\bar{\Sigma}_a$ in the projectors.

The bcc operators break associativity, as discussed by Erler-Maccaferri.

Then, we should separate these states by some worldsheet.
In the case that $\psi_{\rm T}$ is given by the Erler-Schnabl solution, one possible choice for regularization is

$$P_a = \Sigma_a \, Q_{\rm T} \left(\frac{B}{1+K} e^{-\epsilon K} \right) \bar{\Sigma}_a,$$

where ϵ is a positive infinitesimal parameter.
$B/(1+K)$ is a homotopy operator for $Q_{\rm T}$ and it can easily be seen that $P_a P_b = \delta_{ab} P_a$ and $Q_{\rm T} P_a = 0$.

Future work

- Is it possible to connect any background with classical solutions and projectors in string field theory?
- What is a space of open string fields?
- Is it possible to formulate a kind of matrix theories, if we approximate a string field by a finite dimensional matrix?.