Aspects of Abelian and Discrete Symmetries in F-Theory Compactification

Mirjam Cvetič

Penn
University of Pennsylvania

CAMTP
University of Maribor

F-Theory Compactifications with Abelian Symmetries

[Elliptic Fibrations with n-rational sections - $U(1)^n \leftarrow\rightarrow$ rnk n Mordell-Weil (MW) Group]

Based on:
arXiv:1303.6970 [hep-th]: M. C., Denis Klevers, Hernan Piragua
arXiv:1307.6425 [hep-th]: M. C., D. Klevers, H. Piragua
(rnk 2 MW on Calabi-Yau threefolds $\rightarrow U(1)^2$)
arXiv:1306.0236 [hep-th]: M. C., Antonella Grassi, D. Klevers, H. Piragua
(rnk 2 MW on Calabi-Yau fourfolds; inclusion of flux)
arXiv:1310.0463 [hep-th]: M.C., D. Klevers, H. Piragua, Peng Song
(rnk 3 MW)
arXiv:1503.02068 [hep-th]: M.C., D.Klevers, Damian. M.Peña, P. Konstantin Oehlmann , Jonas Reuter
(particle physics models)
arXiv:1505....[hep-th]: M.C., D. Klevers, H. Piragua, Wati Taylor
(rnk 2 MW and ``un-Higgsing'')

related

F-Theory Compactifications with Discrete Symmetries

[Elliptic Fibrations with n-section $Z_n \leftarrow\rightarrow$ rnk n Tate-Shafarevich (TS) Group]

Based on:
arXiv:1502.6970[hep-th]:
M. C., Ron Donagi, Denis Klevers, Hernan Piragua, Maximilian Poretschkin

(rnk 3 TS on Calabi-Yau threefolds $\rightarrow Z_3$)

Type IIB perspective

F-THEORY BASIC INGREDIENTS

F-theory

M-theory

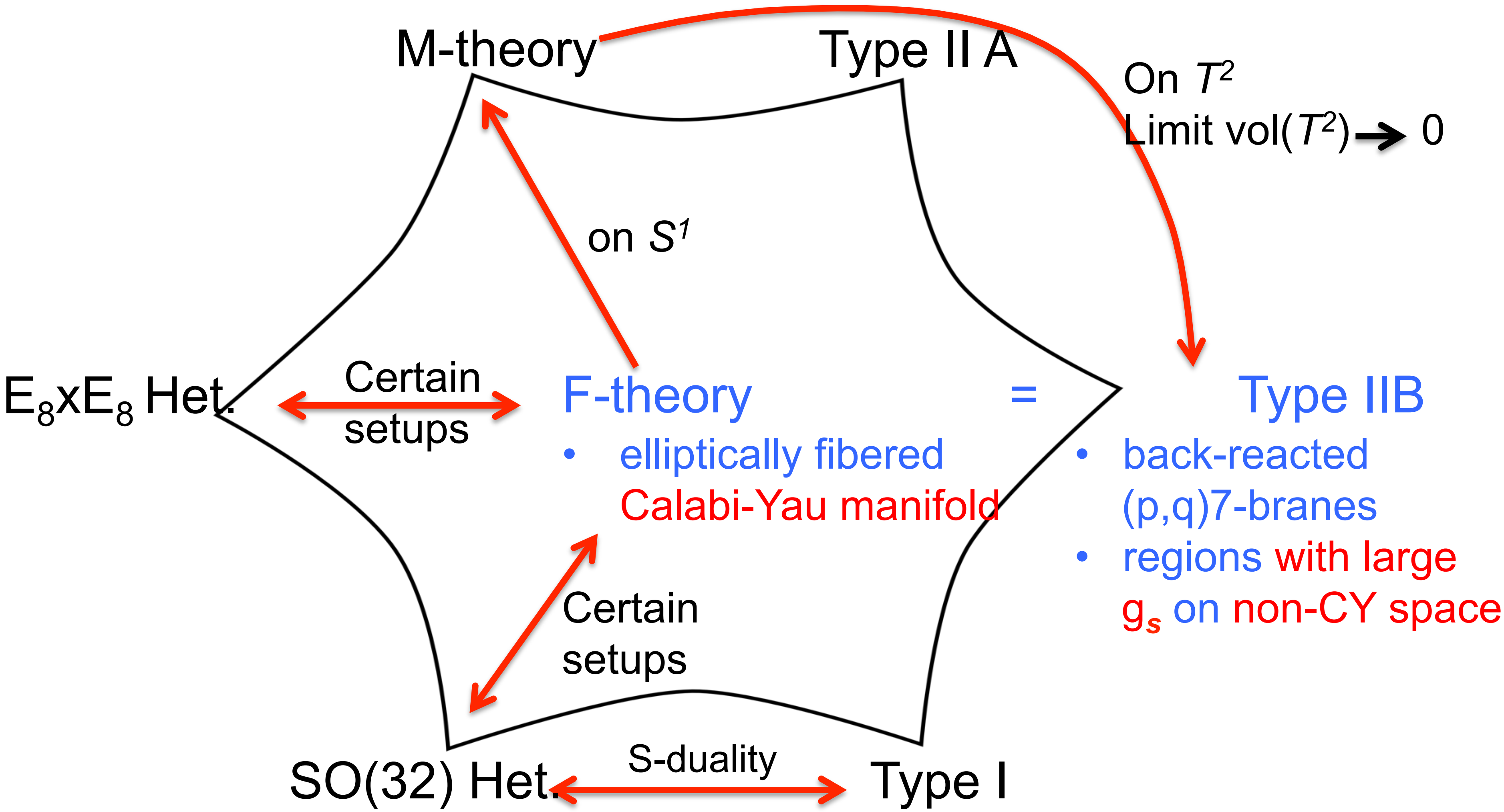
on S^1

F-theory = Type IIB

- elliptically fibered Calabi-Yau manifold

- back-reacted (p,q)7-branes
- regions with large g_s on non-CY space

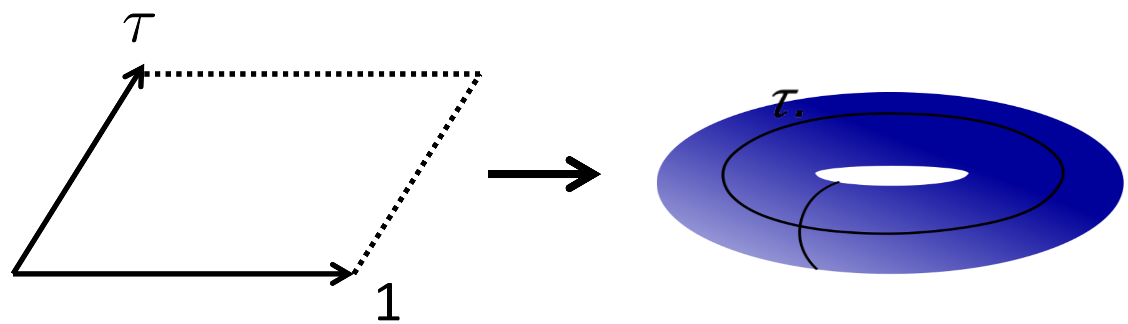
F-theory


M-theory
Type II A
On T^2
Limit vol(T^2) → 0
on S^1
E_8xE_8 Het.
Certain setups
F-theory
=
Type IIB
• elliptically fibered Calabi-Yau manifold
• back-reacted (p,q)7-branes
• regions with large g_s on non-CY space
Certain setups
SO(32) Het.
S-duality
Type I

F-theory Compactification: Basic Ingredients

F-theory - geometric SL(2, Z) invariant formulation of Type IIB string theory:

invariant geometric object is two-torus $T^2(\tau)$

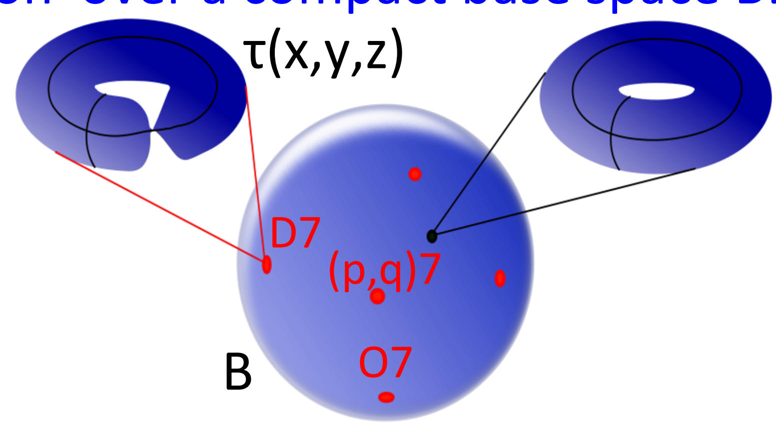


[Vafa]

The string coupling (axio-dilaton): $\tau \equiv C_0 + i g_s^{-1}$

modular parameter of two-torus $T^2(\tau)$ - SL(2,Z)

Compactifcation is a two-torus $T^2(\tau)$-fibration over a compact base space B:



Weierstrass form:

$$y^2 = x^3 + f x z^4 + g z^6$$

f, g- function fields on B

[z:x:y] homog. coords. on $\mathbf{P}^2(1,2,3)$

F-theory Compactification: Basic Ingredients

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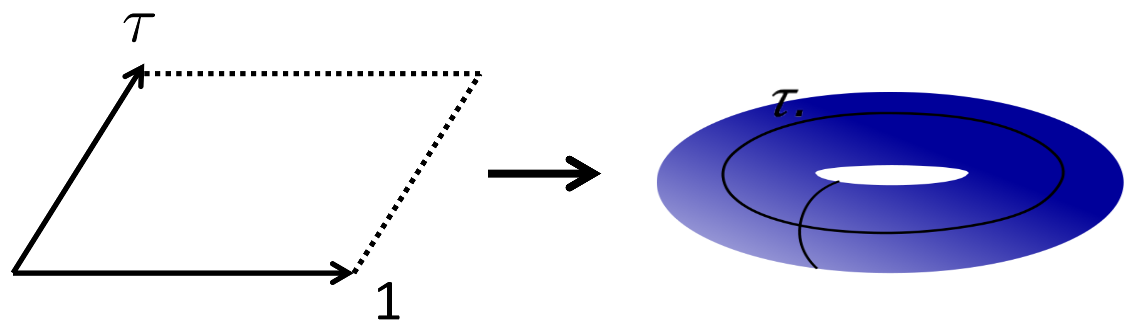

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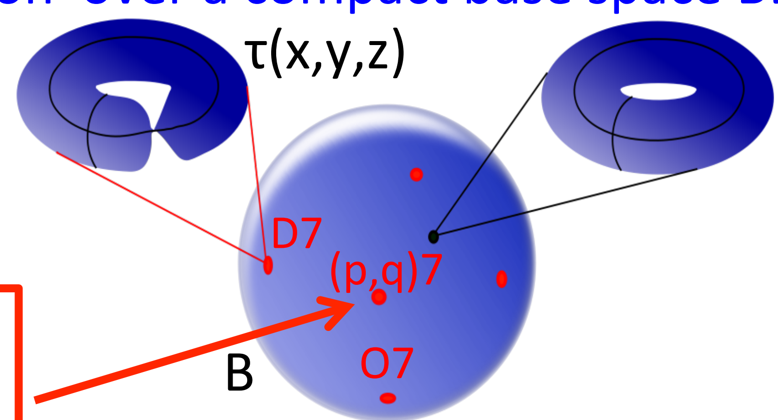


singular $T^2(\tau)$-fibr. → $g_s \to \infty$

location of 7-branes

F-theory Compactification: Basic Ingredients

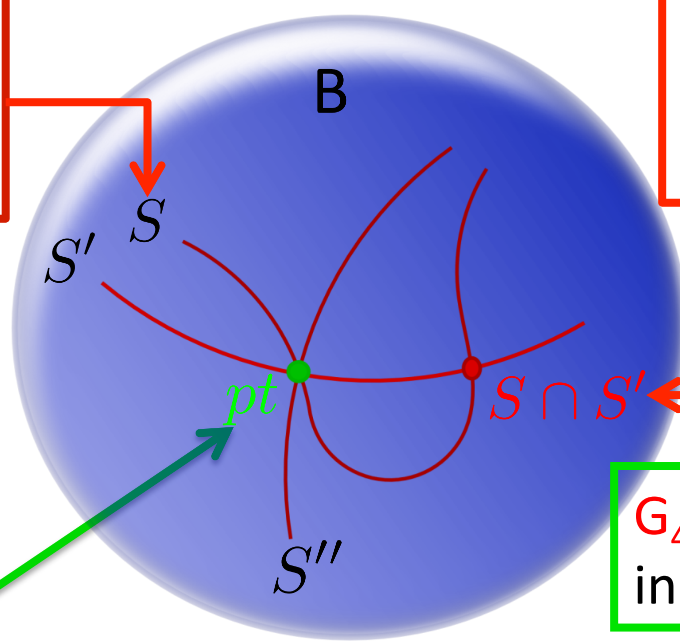
- Total space of $T^2(\tau)$-fibration: singular elliptic Calabi-Yau manifold X

  D=4, SUSY vacua: fourfold X_4 [D=6, SUSY vacua: threefold X_3]

- X_4-singularities encode complicated set-up of intersecting 7-branes:

Gauge theory (ADE…)
in co-dim. one in B_3
(7-branes) S

Matter in co-dim.
two in B_3 [Katz,Vafa]
(intersecting 7-branes)



Yukawa couplings
in co-dim. three in B_3
$pt = S \cap S' \cap S''$

G_4-flux at intersection
induces chiral 4D matter

F-theory & Particle Physics

MOTIVATION

F-theory and Particle Physics

A broad domain of non-perturbative string theory landscape with new promising particle physics & cosmology

Recent past, primary focus on [SU(5)] GUT's ('08….):

Local model building: [Donagi,Wijnholt;Beasley,Heckman,Vafa;… Font,Ibanez;… Hayashi,Kawano,Tsuchiya,Watari,Yamazaki;…Dudas,Palti;… Cecotti,Cheng,Heckman,Vafa;…]

Global model building: [Blumenhagen,Grimm,Jurke,Weigand; Marsano,Saulina,SchäferNameki;Grimm,Krause,Weigand;… M.C.,Halverson,Garcia-Etxebarria;…]

Standard Model [Lin,Weigand]

3-family Standard, Pati-Salam, Trinification Models [M.C., Klevers, Peña, Oehlmann, Reuter]

[Vafa; Vafa,Morrison,…]

Employing geometric techniques for elliptically fibered Calabi-Yau manifolds and/or dualities to determine

Gauge symmetries, matter repres.& multiplicities, Yukawa couplings,…

F-theory & Abelian Gauge Symmetries

MOTIVATION

Abelian Symmetries in F-theory

Physics: important ingredient of the Standard Model and beyond

➡ Multiple U(1)'s desirable

Formal developments: new CY elliptic fibrations with rational sections

While non-Abelian symmetries extensively studied ('96…)
[Kodaira; Tate;Morrison,Vafa; Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa; Candelas,Font,...]

:

Abelian sector rather unexplored

A lot of recent progress ('11-'15):

[...Morrison,Park; ....Lawrie,Schäfer-Nameki; Borchmann,Mayrhofer,Palti,Weigand; U(1)

M.C.,Klevers,Piragua; Grimm,Kapfer,Keitel;Braun,Grimm,Keitel; MC,Grassi,Klevers,Piragua; $U(1)^2$
Borchman,Mayrhofer,Palti,Weigand;...

M.C.,Klevers,Piragua,Song; $U(1)^3$

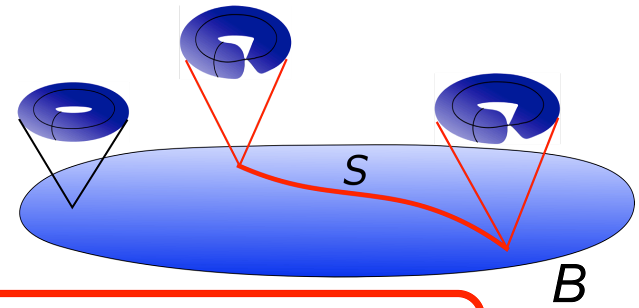
....Morrison,Taylor; ...]

Non-Abelian Gauge Symmetry

[Kodaira;Tate;Vafa;Morrison,Vafa; Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa;...]

1. Weierstrass form for elliptic fibration of *X*

$$y^2 = x^3 + fxz^4 + gz^6$$



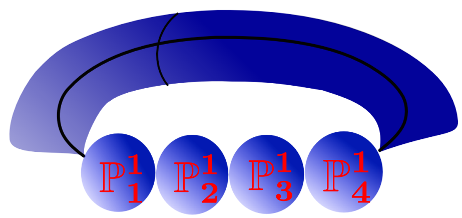
2. Severity of singularity along divisor S in B:

$[ord_S(f), ord_S(g), ord_S(\Delta)]$ ⟷ Singularity type of fibration of *X*

3. Resolution: singularity type ⟷ structure of a tree of $\mathbb{P}^1$'s over *S*

(←→ Dynkin diagram)

I_n-singularity (←→ SU(n)):



Recent refinements: [Esole,Yau;...;Hayashi,Lawrie,Schäfer-Nameki,Morrison;...]

- Cartan generators for A^i gauge bosons: in M-theory via Kaluza-Klein (KK) reduction of C_3 forms along (1,1)-forms $\omega_i \leftrightarrow \mathbb{P}^1_i$ on X

$$C_3 \supset A^i \omega_i$$

- Non-Abelian generators: light M2-brane excitations on $\mathbb{P}^1$'s [Witten]

U(1)'s-Abelian Symmetry

U(1) gauge bosons A^m should also arise via KK-reduction $C_3 \supset A^m \omega_m$

(1,1)-forms on X

Forbid non-Abelian enhancement (by M2's wrapping $\mathbb{P}^1$'s): only I_1-fibers

U(1)'s-Abelian Symmetry & Rational Sections

U(1) gauge bosons A^m should also arise via KK-reduction $C_3 \supset A^m \omega_m$

(1,1)-forms on X

[Morrison, Vafa]

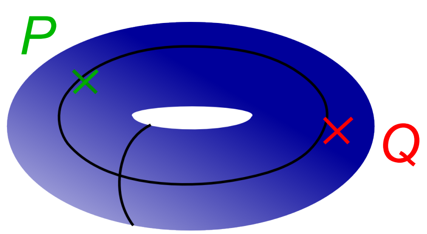
(1,1) - form ω_m ⟺ rational section

Rational point Q on elliptic curve E with zero point P

- is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

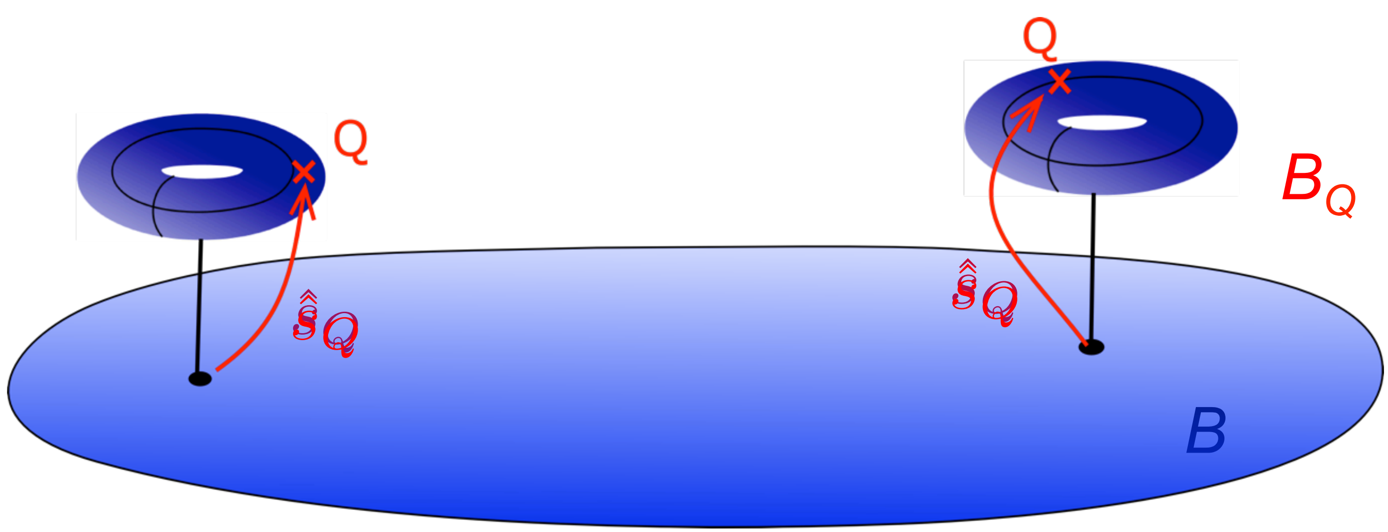
- Rational points form a group (under addition) on E



➡ Mordell-Weil group of rational points

U(1)'s-Abelian Symmetry &Mordell-Weil Group

2. Q on E induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration

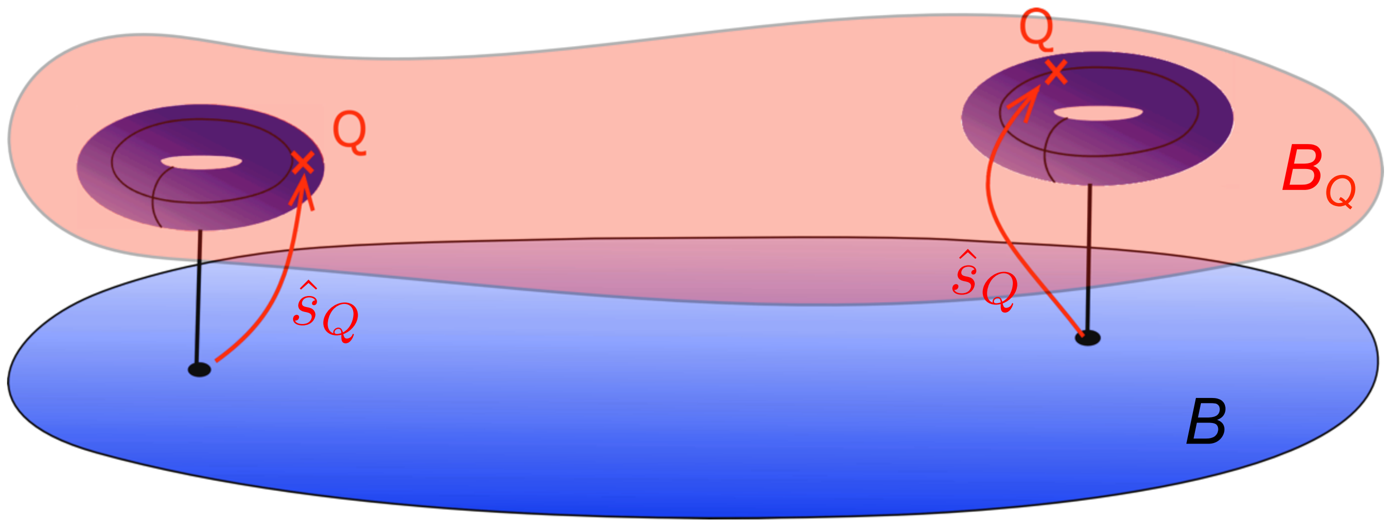


➡ $\hat{s}_Q$ gives rise to a second copy of B in X:

new divisor B_Q in X

U(1)'s-Abelian Symmetry &Mordell-Weil Group

2. *Q* on *E* induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

➡ (1,1)-form ω_m constructed from divisor B_Q (Shioda map)

indeed (1,1) - form ω_m ⬌ rational section

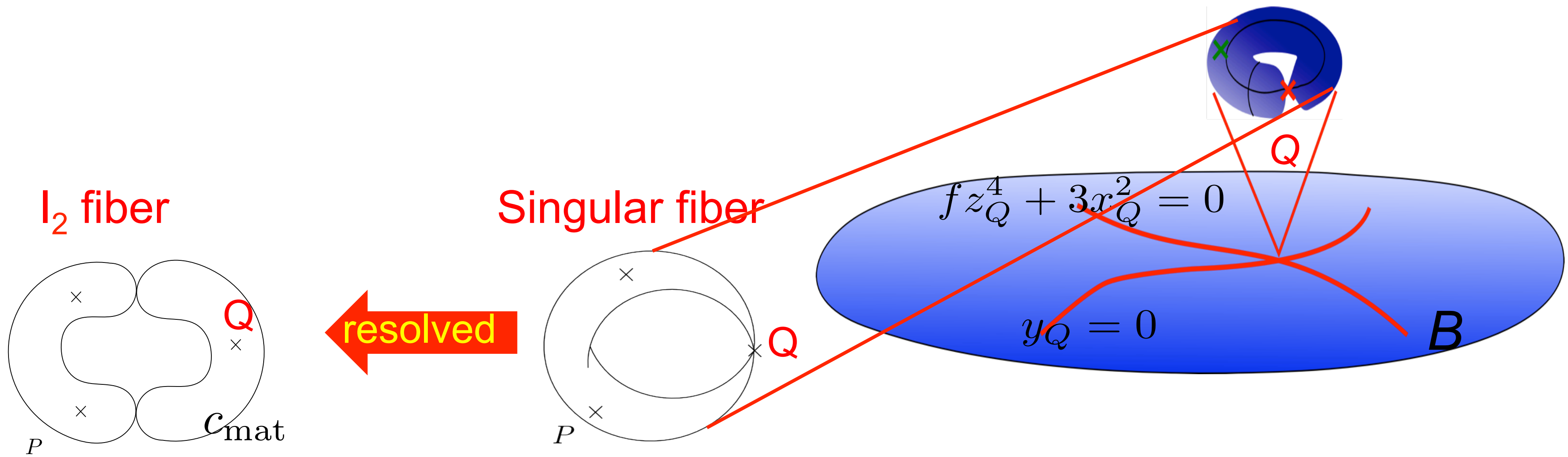
Structure of Elliptic Fibrations with Rational Sections

Weierstrass form (WSF) w/ rat. point Q= $[x_Q : y_Q : z_Q]$

Conifold singularity at codimension two in *B*:

-Factorization: $(y - y_Q)(y + y_Q) = (x - x_Q)(x^2 + x_Q x + f z_Q^4 + x_Q^2)$

- Singularity: $y_Q = f z_Q^4 + 3x_Q^2 = 0$ ➡ WSF singular at Q



w/isolated (matter) curve

➡ Section $\hat{s}_Q$ implies U(1)-charged matter

Elliptic curves with rank-2 Mordell-Weil group

EXAMPLE WITH $U(1)^2$ [TWO RATIONAL SECTIONS]

Rnk 2: Concrete example

[M.C., Klevers,Piragua]

Elliptic curve E with two rational points Q, R

Consider line bundle M=O(P+Q+R) of degree 3 on E (non-generic cubic in $\mathbf{P}^2$)

⇒ natural representation as hypersurface p=0 in del Pezzo dP_2

$$p = u(s_1 u^2 e_1^2 e_2^2 + s_2 uve_1 e_2^2 + s_3 v^2 e_2^2 + s_5 uwe_1^2 e_2 + s_6 vwe_1 e_2 + s_8 w^2 e_1^2) + s_7 v^2 we_2 + s_9 vw^2 e_1$$

$[u:v:w:e_1:e_2]$ –homogeneous coordinates of dP_2

(blow-up of $\mathbf{P}^2$ w/ $[u':v':w']$ at 2 points: $u'=ue_1e_2$, $v'=ve_2$, $w'=we_1$)

	u	v	w	e_1	e_2
$P:\ E_2\cap p=$	$[-s_9$	s_8	1	1	$0]$,
$Q:\ E_1\cap p=$	$[-s_7$	1	s_3	0	$1]$,
$R:\ D_u\cap p=$	$[0$	1	1	$-s_7$	$s_9]$.

Points represented by intersections of different divisors in dP_2 with p

[generalizations: M.C., Klevers, Piragua, Taylor]

Construction of CY Elliptic Fibrations

[M.C.,Klevers,Piragua;M.C.,Grassi,Klevers,Piragua]

CY-fibrations *X* with rank 2 curve in dP_2

Related work: [Borchmann,Mayrhofer,Palti,Weigand]

$$\hat{E} \subset dP_2 \longrightarrow \hat{X}$$
$$\downarrow$$
$$B$$

three sections $\hat{s}_P, \hat{s}_Q, \hat{s}_R$

p=0 in dP_2

- dP_2 coordinates $[u:v:w:e_1:e_2]$ and coefficients s_i lifted to sections on *B*
- CY manifold *X* topologically determined by divisors $\mathcal{S}_7 = (s_7) \quad \mathcal{S}_9 = (s_9)$

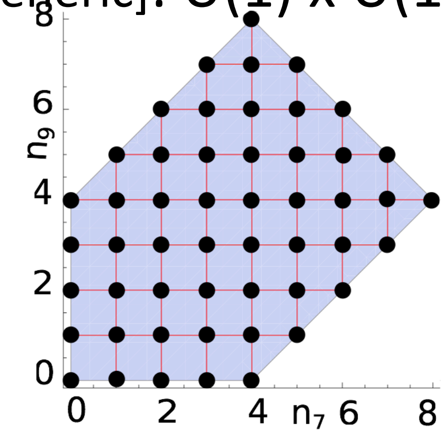
Construction of CY Elliptic Fibrations

Classify all vacua with fixed E in dP_2 & chosen base B in D=6 and D=4

Example: D=4, $B = \mathbb{P}^3$

1. X generic [all s_i exist, generic]: U(1) x U(1)

$$\mathcal{S}_7 = n_7 H_{\mathbb{P}^3}$$
$$\mathcal{S}_9 = n_9 H_{\mathbb{P}^3}$$

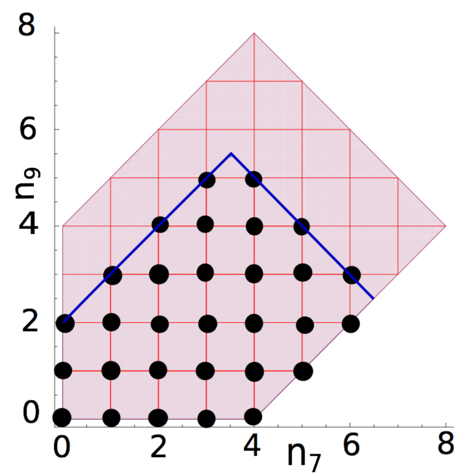


Can construct and study entire family of CY's explicitly

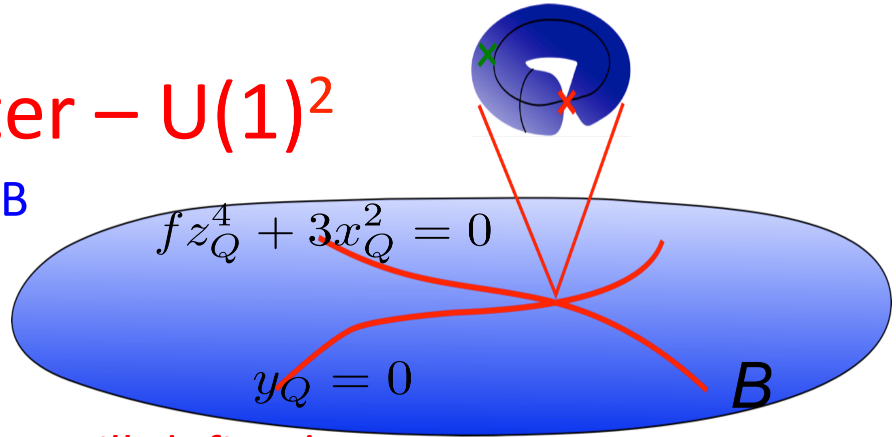
:

2. X non-generic [s_i, realize SU(5) at t=0]: SU(5) x U(1) x U(1)

$$s_1 = t^3 s_1'$$
$$s_2 = t^2 s_2'$$
$$s_3 = t^2 s_3'$$
$$s_5 = t s_5'$$



Charged Matter – U(1)^2



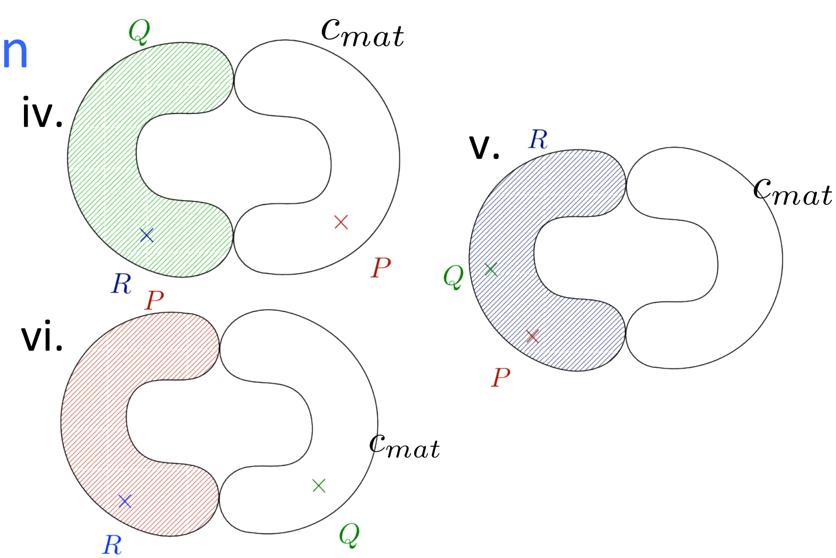
Charged matter at codim-two singularities in B

Matter locus at : $y_Q = fz_Q^4 + 3x_Q^2 = 0$

- Type A: matter at loci in *B* where sections are ill-defined
- Type B: matter at loci characterized by additional constraints

Type A: loci in B where the sections are ill-defined sections no longer points in E, wrap entire **P^1** in smooth X

Representation	Ill-defined section
iv. $(q_1,q_2)=(-1,1)$	Q
v. $(q_1,q_2)=(0,-2)$	R
vi. $(q_1,q_2)=(-1,-2)$	zero section P



F-theory & Discerete Gauge Symmetries

MOTIVATION

F-theory and Discrete Symmetries

Physics: important ingredient of the beyond the Standard Model physics

➡ forbid terms for fast proton decay and other R-parity violating terms,
e.g., R-parity (Z_2), baryon triality (Z_3) and proton hexality (Z_6)

Discrete symmetries must be realized as discrete gauge symmetries, typically arising as a remnant of a broken U(1) gauge symmetry

Geometry: new Calabi-Yau geometries with genus-one fibrations

These geometries do not admit a section, but only a multi-section

Earlier work: [Witten; deBoer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi;..]
Recent extensive efforts ('14-'15): [Braun, Morrison; Morrison, Taylor; Klevers, Pena, Oehlmann, Piragua, Reuter; Anderson, Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand;...]

& This talk!
[M.C., Donagi, Klevers, Piragua, Poretschkin]

F-theory and Discrete Symmetries

- A natural object attached to such F-theory compactifications is the Tate-Shafarevich group, a discrete group that organizes inequivalent genus-one geometries which share the same Jacobian.

  Expected that the Tate-Shafarevich group coincides with the resulting discrete symmetry, though, F-theory effective action is the same for all these geometries.

- In contrast, M-theory can distinguish between these geometries, due to a discrete flux that can be turned on the compactification circle from, say, 6D F-theory compactification to 5D. [default 6D→5D]

  Expected that the different M-theory vacua are in one to one correspondence with the elements of the TS group.

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

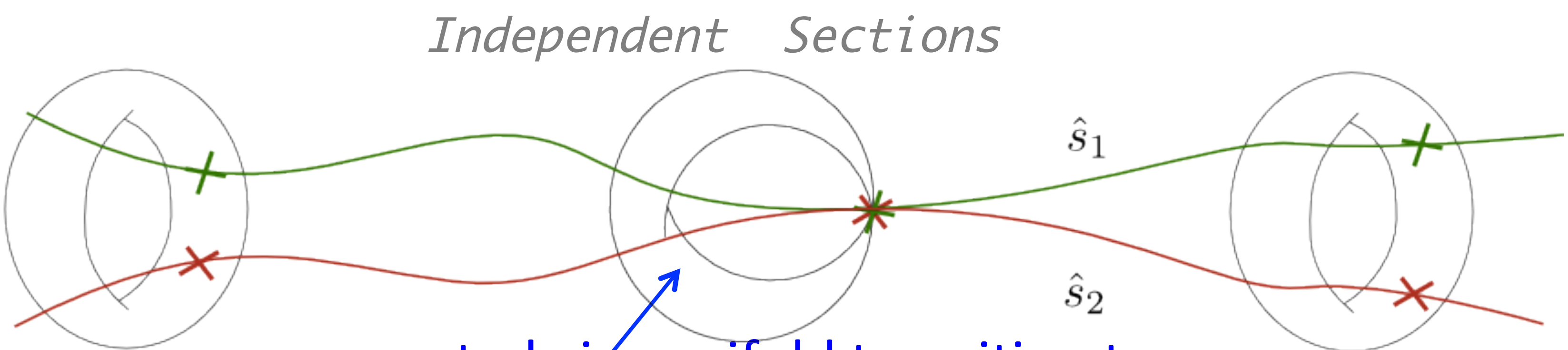
[Morrison, Taylor;
Anderson, García-Etxebarria, Grimm, Keitel;
Braun, Grimm, Keitel]

connected via conifold transition

F-theory compactification with an n-section (discrete Z_n symmetry)

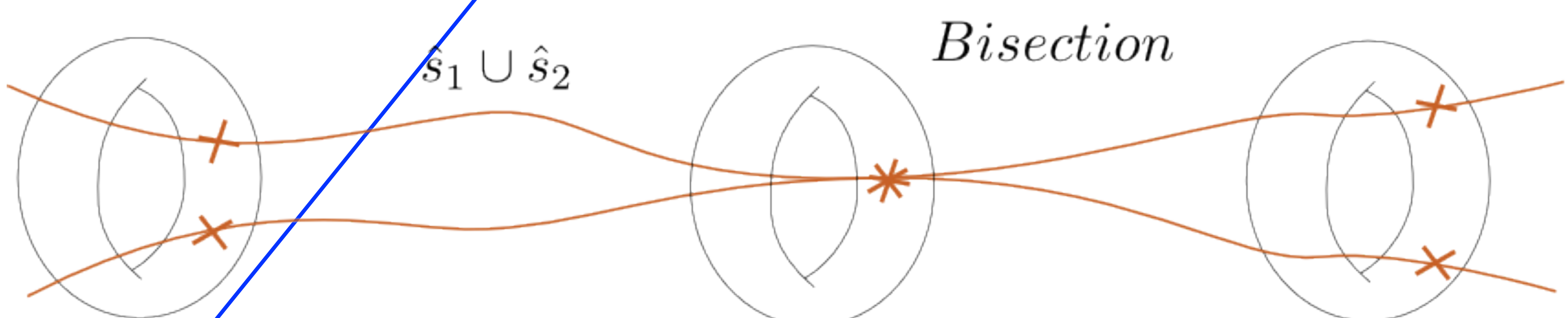
Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)



connected via conifold transition to

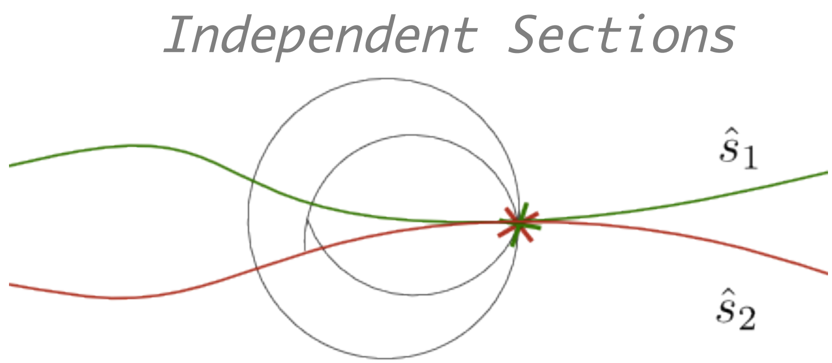
F-theory compactification with an n-section (discrete Z_n symmetry)



I_2 –fibers occur at certain co-dimension two loci in the base of the elliptic fibration

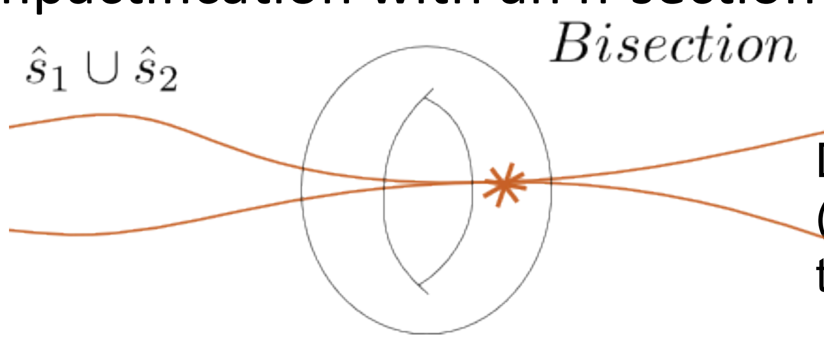
Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)



blow-down
(P^1 in the geometry with
multiple sections collapses)

F-theory compactification with an n-section (discrete Z_n symmetry)

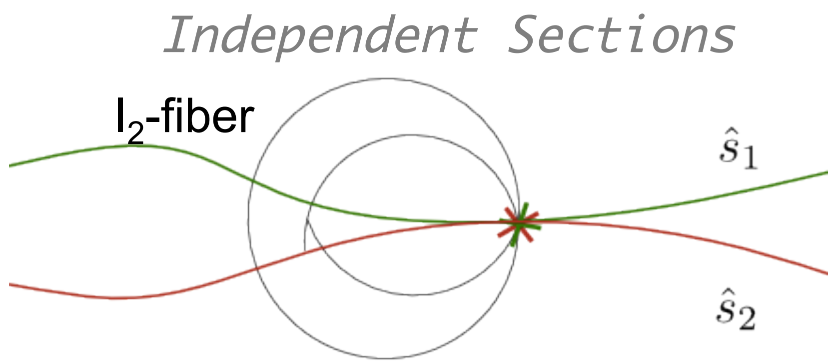


Deformation
(S^3 glues several sections
to a multi-section)

Conifold transition - Geometry

Abelian & Discrete Gauge Symmetry in F-theory

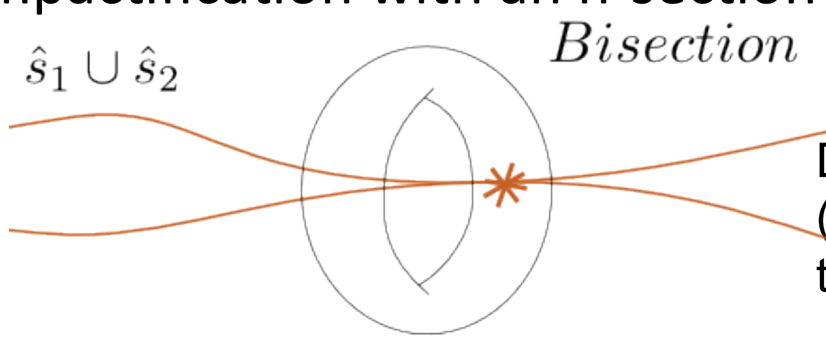
F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)



blow-down
(P^1 in the geometry with multiple sections collapses)

appearance of massless field

F-theory compactification with an n-section (discrete Z_n symmetry)



Deformation
(S^3 glues several sections to a multi-section)

massless field acquires VEV

Conifold transition - Effective theory

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

↓ blow-down — Geometry

connected via conifold transition to

↓ deformation — Effective field theory

F-theory compactification with an n-section (discrete Z_n symmetry)

Since Abelian symmetries better understood (c.f., recent works) most efforts focus on the geometry and spectrum of $U(1)^{(n-1)}$, to deduce, primarily via effective field theory, implications for Z_n.

Approach also taken here

Why Z_n symmetry with n>2?

Recent work primarily focused on the simplest case of Z_2.

[The Tate-Shafarevich group consists only of two elements associated with one genus-one fibration and its Jacobian.]

Goal: to extend the analysis beyond Z_2. Focus primarily on Z_3.

Expect these new findings will play a key role for Z_n (n>3), too.

Focus on obtaining different M-theory vacua by geometrically identifying (on the multiple section side) the scalar fields whose VEV's give rise to the three inequivalent field theories in 5D.

Tate-Shafarevich Group and F-Theory

The Tate-Shafarevich group is defined as

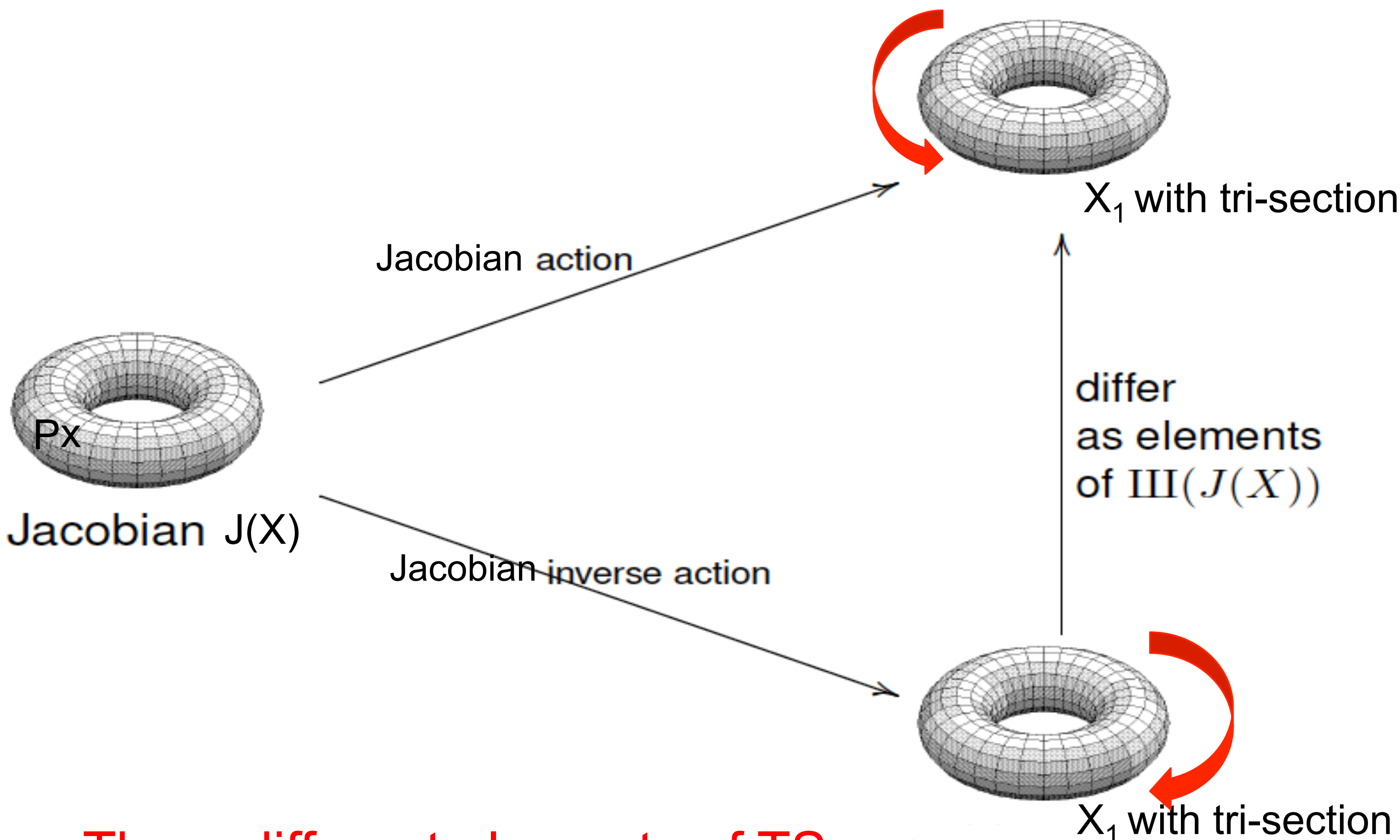
$Ш(J(X))$ = {Genus-one fibered Calabi-Yau manifolds X_i;
so that the Jacobian of X_i is J(X) }

An element $\mathbf{X_i}$ of $Ш(J(X))$ consists of a triple: $\mathbf{X_i} = (X_i; f_i; a_i)$:

X_i is a genus-one fibration over a base B
$f_i : X_i \longrightarrow B$ is the fibration map
a_i: $J(X) \times X_i \longrightarrow X_i$ is the Jacobian action

➡ Two geometries with different actions of the Jacobian differ as elements in $Ш(J(X))$

Tate-Shafarevich group and Z_3 in F-theory



Three different elements of TS group

But only two geometries J(X) and X_1!

[Anderson,Garcia-Etxebarria, Grimm; Braun,Grimm,Keitel; Mayrhofer, Palti, Till, Weigand]

F-Theory/M-Theory Fluxed Circle Compactification Diagram → Z_3

[M.C.,Donagi,Klevers,Piragua,Poretschkin]

F/M-theory Fluxed Circle Compactification (Z_3)

6D F-theory $\mathrm{U}(1)_{6d}$ —— Higgsing of Φ_3 ——→ $\mathbb{Z}_3$

S^1 comp. with $\xi = \int A_1$ [flux-Wilson line] ↓

S^1 comp. with $\xi = \int A_1^m$ [massive flux] ↓

M-theory $\mathrm{U}(1)_{5d} \times \mathrm{U}(1)_{KK}$ —— Higgsing of $\begin{cases} \phi_0(\xi = 0) \\ \phi_1(\xi = -\frac{1}{3}) \\ \phi_2(\xi = -\frac{2}{3}) \end{cases}$ ——→ $\begin{cases} \mathrm{U}(1) \times \mathbb{Z}_3 & [J(X)] \\ \mathrm{U}(1) & [X_1 \circlearrowright] \\ \mathrm{U}(1) & [X_1 \circlearrowleft] \end{cases}$

6D matter w/ charge 3

$$\Phi_3(x, y) = \sum_{n \in \mathbb{Z}} \phi_n(x) e^{2\pi i n y}, \qquad m(n, \xi) = |3\xi + n|$$

KK expanded tower w/ KK mass

F-THEORY with U(1) & CHARGE 3 MATTER

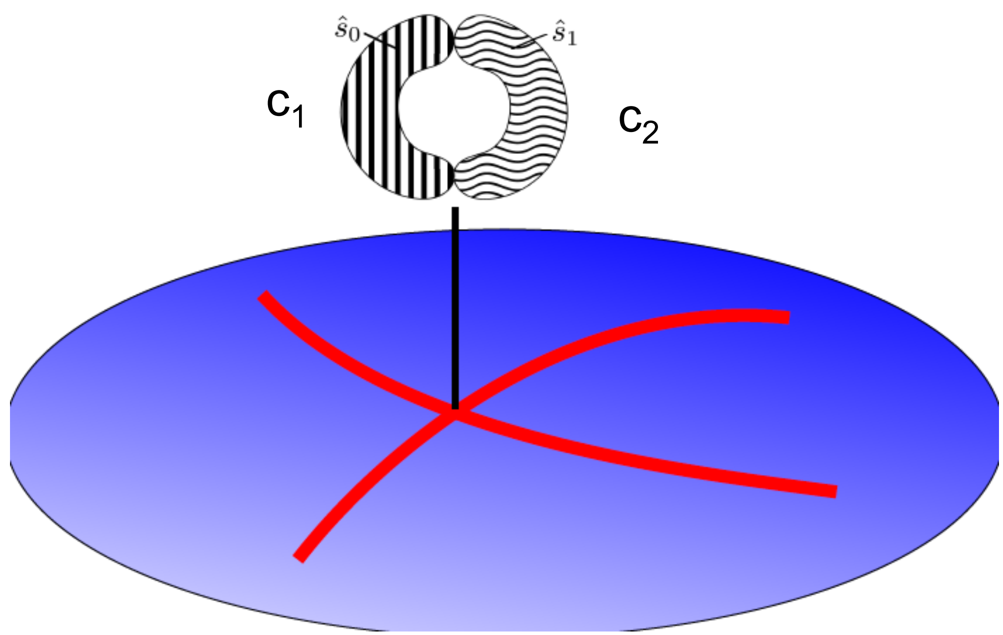
F-THEORY with U(1) & Charge 3 matter

[Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter]

Toric blow-up of singular dP_1 [cubic in P^2]

		c_1	c_2
$U(1)_{KK}$	$\tilde{S}_0$	-1	2
$U(1)_{5D}$	$\tilde{S}_1$	2	-1
$U(1)_{6D}$	$\tilde{S}_1 - \tilde{S}_0$	3	-3

- ambient space dP_1
- rational zero-section
- additional rational section
- Curves that lead to Higgs field of charge (2,-1) (-1,2) are manifest



In M-theory two correct matter charges for Higgsing (with flux ξ= -1/3; -2/3)

Investigating all M-theory phases

Problem:

- 3 curves needed, but I2 -fiber only has two rational components

  Only two curves, c_1 and c_{2}, with charges (2,-1) and (-1, 2) manifest
- curve with charge (3,0) is not visible in the toric resolution

  [curve in the class [$c_1 + T^2$] would have the right charges]

➡ Computation of the Gromov-Witten invariant reveals

$$N_{c1+T^2} = 1$$

Where is the geometry with the charge (3,0) curve manifest?

Investigating all M-theory phases

Strategy:

- Look at different phases where zero section is holomorphic, i.e. look at all different resolution phases (related to the original one by flop transition). [Witten]

- Exemplify here for Z_3.

- This strategy is expected to make all curves manifest and should apply to Z_n (w/ n>3).

Finding the Third Curve

Resolutions of the singular dP_1 via complete Intersection

		c_+	c_-
$U(1)_{KK}$	S'_0	1	0
$U(1)_{5D}$	S'_1	-2	3
$U(1)_{6D}$	$S'_1 - S'_0$	-3	3

- three-dimensional toric ambient space
- holomorphic zero section
- additional rational section
- Curves that lead to Higgs field of charge (1,-2) (3,0) are manifest

In M-theory, correct matter charges for Higgsing (with flux $\xi=0$) to $U(1) \times Z_3$

Summary

Specific Technical Results:

- Explicitly construct the three different curves that yield the fields with desired charges and consider the blow-down; crucial to consider two different resolutions of the singular dP_1 geometry, related by flop transition.

- Whereas one can expect to see at most two vacua in one resolution explicitly, we can detect the hidden one by computing the Gromov-Witten invariant of the corresponding curve class (which was done).

- We also demonstrate that the three different M-theory vacua do not lead to three different geometries but must be distinguished by the action of the Jacobian on the remaining two non-trivial elements of the Tate-Shafarevich group.

Open Questions

- Physical interpretation of the Jacobian action on the genus-one geometries? [Crucial to differentiate the two elements of the Tate-Shafarevich group which are geometrically identical.]

  Tempting to identify the sum operation of TS group to the addition of the discrete fluxes on the compactification circle.

- As the third M-theory vacuum corresponds to deformation via VEV of the mode with charges (3, 0), it exhibits a Z_3 in 5D

  → expected that the corresponding geometry has nontrivial torsional homology; geometrical insights still lacking

  [work in progress: M.C.,Donagi,Poretschkin]

  Generalization to Z_n (n>3); expect different n-section geometries; Field theory of higher charge fields (Abelian $U(1)^3$ constr. → Z_4)

  [M.C.,Klevers,Piragua,Song]