Invariant Functionals

in

Higher-Spin Theory

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Plan

HS theory versus String Theory

Unfolded dynamics

AdS_4/CFT_3 HS holographic duality from unfolded formulation

Structure of HS equations

Supertrace versus Lagrangians in the extended HS equations

Invariants of the AdS_4 HS theory: actions of boundary conformal HS theory and generating functionals for correlators

Black hole charges in AdS_4 HS theory

Conclusion

HS Theory and String Theory

HS gauge theory: infinite towers of massless HS fields in AdS

String theory: infinite towers of massive HS fields:
spontaneously broken HS gauge theory?

String Theory $\gg$ HS theory

Known HS theories: first Regge trajectory of String Theory $s = 0, 1/2, 1 \ldots$

Full String-like HS theory is still unknown.
Interesting conjectures 2012, Gaberdiel and Gopakumar 2013

Both theories have interesting holographic duals:

String theory: $N4\ 4d\ SYM$

HS theory: $3d$ **vectorial sigma model**

Though HS theory contains gravity, it is analogous to open string: HS fields are valued in some associative algebra: A_∞ **structure**

The main theme of this talk is how to construct invariants in HS theory?

New method: invariants as central elements of the A_∞ **structure:**

Interesting to compare with SFT

Application to AdS_4/CFT_3 **HS holography and BH physics**

Unfolded equations

Covariant first-order differential equations

$$\mathrm{d}W^{\Omega}(x) = G^{\Omega}(W(x)), \qquad \mathrm{d} = dx^n \partial_n\,, \qquad G^{\Omega}(W) = \sum_{n=1}^{\infty} f^{\Omega}{}_{\Phi_1\ldots\Phi_n} W^{\Phi_1} \wedge \ldots \wedge W^{\Phi_n}$$

$G^{\Omega}(W)$: **function of "supercoordinates"** W^{Ω}

$d > 1$: **Nontrivial compatibility conditions**

$$G^{\Phi}(W) \wedge \frac{\partial G^{\Omega}(W)}{\partial W^{\Phi}} = 0\,, \qquad \mathrm{L}_{\infty}$$

Any solution to generalized Jacobi identities: FDA Sullivan (1968)

Gauge transformation

$$\delta W^{\Omega} = \mathrm{d}\varepsilon^{\Omega} + \varepsilon^{\Phi} \frac{\partial G^{\Omega}(W)}{\partial W^{\Phi}}\,,$$

where the gauge parameter $\varepsilon^{\Omega}(x)$ is a $(p_{\Omega} - 1)$-form

(No gauge parameters for zero-forms W^{Ω})

Properties

- General applicability
- Manifest (HS) gauge invariance
- Invariance under diffeomorphisms: exterior algebra formalism
- Lie algebra cohomology interpretation
- Independence of ambient space-time: Geometry is encoded by $G^{\Omega}(W)$:

unfolded equations make sense in any space-time

$$\mathrm{d}W^{\Omega}(x) = G^{\Omega}(W(x))\,, \quad x \to X = (x, z)\,, \quad \mathrm{d}_x \to \mathrm{d}_X = \mathrm{d}_x + \mathrm{d}_z\,, \quad \mathrm{d}_z = dz^u \frac{\partial}{\partial z^u}$$

X-dependence is reconstructed in terms of $W(X_0) = W(x_0, z_0)$ at any X_0

Classes of holographically dual models: different G 2012

Cartan formulation of gravity

Diffeomorphisms without distinguished metric tensor: exterior algebra

Vierbein one-form $e^{\alpha\dot\alpha} = dx^n e_n^{\alpha\dot\alpha}$ $\qquad \alpha,\beta = 1,2,\ \dot\alpha,\dot\beta = 1,2$

Lorentz connection $\omega^{\alpha\beta} = dx^n \omega_n^{\alpha\beta}$, $\bar\omega^{\dot\alpha\dot\beta} = dx^n \bar\omega_n^{\dot\alpha\dot\beta}$

$o(3,2) \sim sp(4)$ **connections** $w^{AB} = w^{BA}$, $A, B = 1, \dots 4$ **and curvatures**

$$R^{AB} = \mathrm{d}w^{AB} + w^{AC} w^{DB} C_{CD}\,, \qquad C_{AB} = -C_{BA}\,, \qquad A = (\alpha, \dot\alpha)$$

Torsion

$$R_{\alpha\dot\beta} = \mathrm{d}e_{\alpha\dot\beta} + \omega_\alpha{}^\gamma e_{\gamma\dot\beta} + \bar\omega_{\dot\beta}{}^{\dot\delta} e_{\alpha\dot\delta}$$

Lorentz curvature

$$R_{\alpha\beta} = \mathrm{d}\omega_{\alpha\beta} + \omega_\alpha{}^\gamma \omega_{\beta\gamma} + \lambda^2 e_\alpha{}^{\dot\delta} e_{\beta\dot\delta}\,, \qquad \mathcal{R}_{\alpha\beta} = \mathrm{d}\omega_{\alpha\beta} + \omega_\alpha{}^\gamma \omega_{\beta\gamma}$$

AdS_4**:** $\qquad R_{\alpha\beta} = 0\,, \quad \overline{R}_{\dot\alpha\dot\beta} = 0\,, \quad R_{\alpha\dot\alpha} = 0$

For nontrivial geometry some of components of the curvatures are nonzero being represented by new zero-form fields: Weyl tensor

Vacuum Geometry

h: **a Lie algebra.** $\omega = \omega^\alpha T_\alpha$: **a one-form valued in** h

$$G(\omega) = -\omega\,\omega \equiv -\frac{1}{2}\omega^\alpha \omega^\beta [T_\alpha, T_\beta]$$

Unfolded equation with $W = \omega$ **is the flatness condition**

$$\mathrm{d}\omega + \omega\,\omega = 0$$

Compatibility condition: Jacobi identity for h.

FDA gauge transformation: usual gauge transformation of ω.

The zero-curvature equation describes background geometry in a coordinate independent way.

If h **is Poincare or anti-de Sitter algebra it describes Minkowski or** AdS_d **space-time**

Free fields unfolded

Let W^{α} contain p-forms $\mathcal{C}^i$ (e.g. 0-forms) and G^i be linear in ω and $\mathcal{C}$

$$G^i = -\omega^{\alpha}(T_{\alpha})^i{}_j \mathcal{C}^j \,.$$

The compatibility condition implies that $(T_{\alpha})^i{}_j$ form some representation T of h V of $\mathcal{C}^i$. The unfolded equation is

$$D_{\omega}\mathcal{C} = 0$$

$D_{\omega} \equiv \mathsf{d} + \omega$: covariant derivative in the h-module V of $\mathcal{C}^i$

Linear equations in a chosen background: covariant constancy equation

h: global symmetry

Lagrangians via contractible systems

Contractible system

$$\mathrm{d}w = \mathcal{L}\,, \qquad \mathrm{d}\mathcal{L} = 0$$

is dynamically empty: gauge transformations

$$\delta w(x) = \varepsilon(x)\,, \qquad \delta\mathcal{L}(x) = \mathrm{d}\varepsilon(x)$$

Gauge fixing $w = 0 \quad \Longrightarrow \quad \mathcal{L} = 0$

For the system

$$\mathrm{d}w + L(W) = \mathcal{L}\,, \qquad \mathrm{d}\mathcal{L} = 0$$

where $L(W)$ **is some closed function of other fields** W.

In the *canonical gauge* $w = 0$

$$\mathcal{L} = L(W)\,, \qquad \mathrm{d}L(W) = 0\,.$$

The singlet (invariant) field L **becomes a Lagrangian giving rise to an invariant action**

HS AdS/CFT correspondence

General idea of HS duality   Sundborg (2001), Witten (2001)

AdS_4 HS theory is dual to $3d$ vectorial conformal models

Klebanov, Polyakov (2002), Petkou, Leigh (2005), Sezgin, Sundell (2005); Giombi and Yin (2009);

Maldacena, Zhiboedov (2011,2012); MV (2012); Koch, Jevicki, Jin, Rodrigues (2011-2014);

Giombi, Klebanov; Tseytlin (2013,2014) ...

AdS_3/CFT_2 correspondence   Gaberdiel and Gopakumar (2010)

Analysis of HS holography helps to uncover the origin of AdS/CFT ?!

Despite significant progress in the construction of actions during last thirty years: A.Bengtsson, I.Bengtsson, Brink (1983); Berends, Burgers, van Dam (1984); Fradkin, MV (1987), ... Boulanger, Sundell (2012) ...

construction of the generating functional for correlators and entropies was lacking

$3d$ conformal equations

Rank-one conformal massless equations Shaynkman, MV (2001)

$$\left(\frac{\partial}{\partial x^{\alpha\beta}} \pm i\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\right)C_j^\pm(y|x) = 0\,, \qquad \alpha,\beta = 1,2\,, \quad j = 1,\dots\mathcal{N}$$

Bosons (fermions) are even (odd) functions of y^α: $C_i(-y|x) = (-1)^{p_i}C_i(y|x)$

Primaries are usual scalar and spinor

$$C(x) = C(0|x)\,, \qquad C_\alpha(x) = \frac{\partial}{\partial y^\alpha}C(y|x)\Big|_{y=0}$$

Higher components in

$$C(y\mid x) = i\sum_{n=0}^{\infty}\frac{1}{n!}y^{\alpha_1}\dots y^{\alpha_n}C_{\alpha_1\dots\alpha_n}(x)$$

are descendants expressed via x **derivatives of the primaries**

Conserved currents

Rank-two equations

$$\left\{ \frac{\partial}{\partial x^{\alpha\beta}} - \frac{\partial^2}{\partial y^{(\alpha}\partial u^{\beta)}} \right\} J(u,\, y|x) = 0$$

Gelfond, MV (2003)

$J(u,\, y|x)$: **generalized stress tensor. Rank-two equation is obeyed by**

$$J(u,\, y\,|x) = \sum_{i=1}^{\mathcal{N}} C_i^{-}(u+y|x)\, C_i^{+}(y-u|x)$$

Primaries: $3d$ **currents of all integer and half-integer spins**

$$J(u,0|x) = \sum_{2s=0}^{\infty} u^{\alpha_1}\dots u^{\alpha_{2s}} J_{\alpha_1\dots\alpha_{2s}}(x)\,, \qquad \tilde{J}(0,y|x) = \sum_{2s=0}^{\infty} y^{\alpha_1}\dots y^{\alpha_{2s}} \tilde{J}_{\alpha_1\dots\alpha_{2s}}(x)$$

$$J^{asym}(u,y|x) = u_\alpha y^\alpha J^{asym}(x)$$

$$\Delta J_{\alpha_1\dots\alpha_{2s}}(x) = \Delta \tilde{J}_{\alpha_1\dots\alpha_{2s}}(x) = s+1 \qquad \Delta J^{asym}(x) = 2$$

Conservation equation:

$$\frac{\partial}{\partial x^{\alpha\beta}} \frac{\partial^2}{\partial u_\alpha \partial u_\beta} J(u,0|x) = 0$$

Free massless fields in AdS_4

Infinite set of spins $s = 0, 1/2, 1, 3/2, 2\ldots$

Fermions require field doubling: $\omega^{ii}(y,\bar{y}\mid x)\,,\quad C^{i1-i}(y,\bar{y}\mid x)\,,\qquad i=0,1$

$$\bar{\omega}^{ii}(y,\bar{y}\mid x)=\omega^{ii}(\bar{y},y\mid x)\,,\qquad \bar{C}^{i\,1-i}(y,\bar{y}\mid x)=C^{1-i\,i}(\bar{y},y\mid x)$$

$$A(y,\bar{y}\mid x)=i\sum_{n,m=0}^{\infty}\frac{1}{n!m!}y_{\alpha_1}\ldots y_{\alpha_n}\bar{y}_{\dot{\beta}_1}\ldots\bar{y}_{\dot{\beta}_m}A^{\alpha_1\ldots\alpha_n}{}_{,}{}^{\dot{\beta}_1\ldots\dot{\beta}_m}(x)$$

The unfolded system for free massless fields is **(1989)**

- $\star$ $$R_1^{ii}(y,\bar{y}\mid x)=\eta\,\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\bar{y}^{\dot{\alpha}}\partial\bar{y}^{\dot{\beta}}}\,C^{1-i\,i}(0,\bar{y}\mid x)+\bar{\eta}\,H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\,C^{i\,1-i}(y,0\mid x)$$
- $\star$ $$\tilde{D}_0C^{i\,1-i}(y,\bar{y}\mid x)=0$$

$$R_1(y,\bar{y}\mid x):=D_0^{ad}\omega(y,\bar{y}\mid x)\qquad H^{\alpha\beta}:=e^{\alpha}{}_{\dot{\alpha}}e^{\beta\dot{\alpha}}\,,\quad \overline{H}^{\dot{\alpha}\dot{\beta}}:=e_{\alpha}{}^{\dot{\alpha}}e^{\alpha\dot{\beta}}$$

$$D_0^{ad}\omega:=D^L-\lambda e^{\alpha\dot{\beta}}\left(y_{\alpha}\frac{\partial}{\partial\bar{y}^{\dot{\beta}}}+\frac{\partial}{\partial y^{\alpha}}\bar{y}_{\dot{\beta}}\right),\qquad \tilde{D}_0:=D^L+\lambda e^{\alpha\dot{\beta}}\left(y_{\alpha}\bar{y}_{\dot{\beta}}+\frac{\partial^2}{\partial y^{\alpha}\partial\bar{y}^{\dot{\beta}}}\right)$$

$$D^L:=\mathsf{d}_x-\left(\omega^{\alpha\beta}y_{\alpha}\frac{\partial}{\partial y^{\beta}}+\bar{\omega}^{\dot{\alpha}\dot{\beta}}\bar{y}_{\dot{\alpha}}\frac{\partial}{\partial\bar{y}^{\dot{\beta}}}\right)$$

Zero-forms $C(Y|x)$ **form a Weyl module** $\sim$ **boundary current module**

Holography at complex infinity

For manifest conformal invariance introduce

$$y^+_\alpha = \frac{1}{2}(y_\alpha - i\bar{y}_\alpha)\,, \qquad y^-_\alpha = \frac{1}{2}(\bar{y}_\alpha - iy_\alpha)\,, \qquad [y^-_\alpha\,, y^{+\beta}]_\star = \delta_\alpha^\beta$$

AdS_4 foliation: $x^n = (\mathbf{x}^a, \mathbf{z})$: $\mathbf{x}^a$ are coordinates of leaves $(a = 0, 1, 2,)$

Poincaré coordinate $\mathbf{z}$ is a foliation parameter. AdS infinity is at $\mathbf{z} = 0$

$$W = \frac{i}{\mathbf{z}} d\mathbf{x}^{\alpha\beta} y^-_\alpha y^-_\beta - \frac{d\mathbf{z}}{2\mathbf{z}} y^-_\alpha y^{+\alpha}$$

$$e^{\alpha\dot{\alpha}} = \frac{1}{2\mathbf{z}} dx^{\alpha\dot{\alpha}}\,, \qquad \omega^{\alpha\beta} = -\frac{i}{4\mathbf{z}} d\mathbf{x}^{\alpha\beta}\,, \qquad \bar{\omega}^{\dot{\alpha}\dot{\beta}} = \frac{i}{4\mathbf{z}} d\mathbf{x}^{\dot{\alpha}\dot{\beta}}$$

Vacuum connection can be extended to the complex plane of $\mathbf{z}$ with all components containing $d\bar{\mathbf{z}}$ being zero.

Generating functional for the boundary correlators

$$S = \frac{1}{2\pi i} \oint_{\mathbf{z}=0} \mathcal{L}(\phi)$$

An on-shell closed $(d+1)$-form $\mathcal{L}(\phi)$ for a d-dimensional boundary

$$\mathrm{d}\mathcal{L}(\phi) = 0\,, \qquad \mathcal{L} \neq \mathrm{d}M$$

Field equations at the boundary

Rescaling

$$C^{i\,1-i}(y,\bar{y}|\mathbf{x},\mathbf{z}) = \mathbf{z}\exp(y_\alpha\bar{y}^\alpha)T^{i\,1-i}(w,\bar{w}|\mathbf{x},\mathbf{z}) \qquad \mathbf{w}^\alpha = \mathbf{z}^{1/2}\mathbf{y}^\alpha \qquad \bar{\mathbf{w}}^\alpha = \mathbf{z}^{1/2}\bar{\mathbf{y}}^\alpha$$

$$W^{jj}(y^\pm|\mathbf{x},\mathbf{z}) = \Omega^{jj}(v^-,w^+|\mathbf{x},\mathbf{z}) \qquad \mathbf{v}^\pm = \mathbf{z}^{-1/2}\mathbf{y}^\pm \quad \mathbf{w}^\pm = \mathbf{z}^{1/2}\mathbf{y}^\pm$$

In the limit $\mathbf{z}\to 0$ free HS equations take the form of current conservation equations

$$\left[\mathsf{d}_\mathbf{x} - i d\mathbf{x}^{\alpha\beta}\frac{\partial^2}{\partial w^{+\alpha}\partial w^{-\beta}}\right]\mathcal{T}_\pm^{j\,1-j}(w^+,w^-|\mathbf{x},0) = 0$$

$$\mathcal{T}_\pm^{\mathbf{jj}}(\mathbf{w}^+,\mathbf{w}^-|\mathbf{x},\mathbf{0}) = \eta\mathbf{T}^{\mathbf{j}\,\mathbf{1-j}}(\mathbf{w}^+,\mathbf{w}^-\mid\mathbf{x},\mathbf{0}) \pm \bar{\eta}\mathbf{T}^{\mathbf{1-j}\,\mathbf{j}}(-\mathbf{i}\mathbf{w}^-,\mathbf{i}\mathbf{w}^+\mid\mathbf{x},\mathbf{0})$$

and

$$\left(\mathsf{d}_\mathbf{x} + 2i d\mathbf{x}^{\alpha\beta}v_\alpha^-\frac{\partial}{\partial w^{+\beta}}\right)\Omega^{jj}(v^-,w^+|\mathbf{x},0) = d\mathbf{x}^{\alpha\gamma}d\mathbf{x}^{\beta\gamma}\frac{\partial^2}{\partial w^{+\alpha}\partial w^{+\beta}}\mathcal{T}_-^{jj}(w^+,0\mid\mathbf{x},0)$$

$$D_\mathbf{x}\Omega_\mathbf{z}^{jj}(v^-,w^+|\mathbf{x},0)+D_\mathbf{z}\Omega_\mathbf{x}^{jj}(v^-,w^+|\mathbf{x},0) = -\frac{i}{2}d\mathbf{x}^{\alpha\beta}d\mathbf{z}\frac{\partial^2}{\partial w^{+\alpha}\partial w^{+\beta}}\mathcal{T}_+^{jj}(w^+,0\mid\mathbf{x},0)$$

Structure of the functional

The residue at $\mathbf{z} = 0$ gives the boundary functional of the following structure analogous to $\phi_{n_1 \ldots n_s} J^{n_1 \ldots n_s}$

$$S_{M^3}(\omega) = \int_{M^3} \mathcal{L} \,, \qquad \mathcal{L} = \frac{1}{2} \omega_{\mathbf{x}}^{\alpha_1 \ldots \alpha_{2(s-1)}} e_{\mathbf{x}}^{\alpha_{2s-1}}{}_{\beta} e_{\mathbf{x}}^{\alpha_{2s}\beta} (a C_{\alpha_1 \ldots \alpha_{2s}}(\omega) + \bar{a} C_{\dot{\alpha}_1 \ldots \dot{\alpha}_{2s}}(\omega))$$

$C_{\alpha_1 \ldots \alpha_{2s}}(\omega)$ has conformal properties of currents. Using that

$$a C_{\alpha_1 \ldots \alpha_{2s}}(\omega) + \bar{a} C_{\dot{\alpha}_1 \ldots \dot{\alpha}_{2s}}(\omega) = a_- \mathcal{T}_{-\alpha_1 \ldots \alpha_{2s}}(\omega) + a_+ \mathcal{T}_{+\dot{\alpha}_1 \ldots \dot{\alpha}_{2s}}(\omega)$$

$\mathcal{T}_-$ describes local boundary terms

$\mathcal{T}_+$ describes nontrivial correlators via the variation of S_{M_3} over the HS gauge fields $\omega_{\mathbf{x}}^{\alpha_1 \ldots \alpha_{2(s-1)}}$

$$\langle J(\mathbf{x}_1) J(\mathbf{x}_2) \ldots \rangle = \left. \frac{\delta^n \exp\left[-S_{M^3}(\omega, C(\omega))\right]}{\delta\omega(x_1)\delta\omega(x_2) \ldots} \right|_{\omega=0}$$

Computation of a_+: work in progress

Nonlinear HS equations

$$\mathcal{W}(Z;Y;k,\bar{k}|x) = (\mathsf{d} + W) + S\,, \qquad W = dx^n W_n\,, \qquad S = dz^\alpha S_\alpha + d\bar{z}^{\dot{\alpha}}\bar{S}_{\dot{\alpha}}$$

$$\mathcal{W}\star\mathcal{W} = i(dZ^A dZ_A + \eta dz^\alpha dz_\alpha B\star k\star\kappa + \bar{\eta} d\bar{z}^{\dot{\alpha}} d\bar{z}_{\dot{\alpha}} B\star\bar{k}\star\bar{\kappa})$$

$$\mathcal{W}\star B = B\star\mathcal{W}\,, \qquad B = B(Z;Y;k,\bar{k}|x)$$

HS star-product

$$(f\star g)(Z;Y) = \frac{1}{(2\pi)^4}\int d^4U\, d^4V \exp\left[iU_A V^A\right] f(Z+U;Y+U) g(Z-V;Y+V)$$

Manifest gauge invariance

$$\delta\mathcal{W} = [\varepsilon,\mathcal{W}]_\star\,, \qquad \delta B = \varepsilon\star B - B\star\varepsilon\,, \qquad \varepsilon = \varepsilon(Z;Y;K|x)$$

Vacuum solution with $B = 0$

$$\mathcal{W}_0 = \mathcal{W}_0^{1,0} + \mathcal{W}_0^{0,1}\,, \qquad \mathcal{W}_0^{1,0} = dZ^A Z_A\,, \qquad \mathcal{W}_0^{0,1} = W_0(Y|x): \qquad \mathrm{AdS}_4$$

Resolution for Z reconstructs A_∞ structure of the HS nonlinear system

Klein operators and Supertrace

Klein operator

$$\kappa = \exp iz_\alpha y^\alpha\,, \qquad \kappa \star \kappa = 1$$

$$\kappa \star f(z,y) = f(-z,-y) \star \kappa$$

Supertrace

$$str(f(z,y)) = \frac{1}{(2\pi)^2}\int d^2u\, d^2v \exp\,[-iu_\alpha v^\beta]\, f(u,v)$$

$$str(f \star g) = str(g \star f)$$

Klein operators are well-defined with respect to the star product but have divergent supertrace

$$str(\kappa) \sim \delta^4(0)$$

In our construction invariant functionals have divergent supertrace.

HS equations have a form of de Rham cohomology in the twistor space

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Extended system

HS equations leave no room for an invariant action as a space-time p-form built from $\mathcal{W}$ and B since $str(\mathcal{W} \star f(B) \star \mathcal{W} \star g(B)) = 0$.

Zero-forms $str(f(B))$ suffer from divergencies of the supertrace (suggested to be regularized by Colombo, Iazeolla, Sezgin and Sundell).

$$- \times - = +$$

The new proposal is to consider Lagrangians that are not of the form $str(L)$ via the following extension of the HS unfolded equations

$$\mathcal{W} \star \mathcal{W} = F(c, \mathcal{B}) + \mathcal{L}_i \, c^i \,, \qquad \mathcal{W} \star \mathcal{B} = \mathcal{B} \star \mathcal{W} \,, \qquad \mathsf{d}\mathcal{L} = 0$$

$\mathcal{W} = \mathsf{d} + W$ and $\mathcal{B}$ are differential forms of odd and even degrees, respectively (both in dx and dZ).

c are x- and dx-independent central elements like $dZ_A dZ^A$, $\delta^2(dz) k \star \kappa \ldots$

Lagrangians $\mathcal{L}$ are x-dependent space-time differential forms of even degrees valued in the center of the algebra. In this talk: $c_i = I \;\; i = 1$

$$\mathcal{L}_i \, c^i = \mathcal{L} \, I$$

Symmetries

The system is consistent because $\mathcal{B}$ commutes with itself and with all c and $\mathcal{L}$. The gauge transformations are

$$\delta\mathcal{W} = [\mathcal{W}\,, \varepsilon]_\star\,, \qquad \delta\mathcal{B} = [\mathcal{B}\,, \varepsilon]_\star\,, \qquad \varepsilon = \varepsilon(dx, x, dZ, \dots)$$

$$\delta\mathcal{B} = \{\mathcal{W}\,, \xi\}\,, \qquad \delta\mathcal{W} = \xi^A \frac{\partial F(c, \mathcal{B})}{\partial \mathcal{B}^A}\,, \qquad \xi = \xi(dx, x, dZ, \dots)$$

$$\delta\mathcal{L}(dx, x) = \mathsf{d}\chi(dx, x)\,, \qquad \delta\mathcal{W} = \chi I\,, \qquad \chi(dx, x)$$

χ- transformation implies equivalence of $\mathcal{L}$ up to exact forms

allowing to choose canonical gauge $\mathcal{W}_I := \pi\mathcal{W} = 0$

π is the projection to I

$$\pi(f(Y, Z|x))) = f(0, 0|x)\,, \qquad \pi(f \star g) \neq \pi(g \star f)$$

Gauge transformation preserving canonical gauge

$$\delta\mathcal{L} = \mathsf{d}\chi\,, \qquad \chi = -\pi\Big([\mathcal{W}\,, \varepsilon]_\star + \xi^A \frac{\partial F(c, \mathcal{B})}{\partial \mathcal{B}^A}\Big)$$

$\mathcal{L}$ is on-shell closed and gauge invariant modulo exact forms

Actions versus supertrace

Gauge invariant action

$$S = \int_{\Sigma} \mathcal{L}$$

Since $\mathcal{L}$ is closed, it should be integrated over non-contractible cycles

For AdS/CFT the singularity is at infinity

BH invariants (entropies) are associated with $(d-2)$-forms

If the HS algebra possesses a supertrace

$$\mathcal{L} = str(\mathrm{d}\mathcal{W} + \mathcal{W} \star \mathcal{W})\Big|_{dZ=0}$$

This suggests that the second term vanishes and hence $\mathcal{L}$ is exact.

Not applicable if $str(\mathcal{W} \star \mathcal{W})$ is ill-defined:

$\mathcal{L}$ with well-defined $str(\mathcal{W} \star \mathcal{W})$ are exact.

$\mathcal{L}$ with ill-defined $str(\mathcal{W} \star \mathcal{W})$ have a chance to be nontrivial.

Invariants of the AdS_4 HS theory

$\mathcal{W}(dZ, dx; Z; Y; \mathcal{K}|x)$ **contains all one- and three-forms in** dZ **and** dx

$\mathcal{B}(dZ, dx; Z; Y; \mathcal{K}|x)$ **contains all zero- and two-forms in** dZ **and** dx

Lagrangians $\mathcal{L}(dx|x)$ **depend on space-time coordinates and differentials.**

Lagrangian relevant to the generating functional of correlators in

AdS_4/CFT_3 **HS holography is a four-form** $\mathcal{L}^4$

Lagrangian relevant to BH entropy is a two-form $\mathcal{L}^2$ **?!**

Extended HS system is

$$i\mathcal{W}\star\mathcal{W} = dZ_A dZ^A + \eta\delta^2(dz)\mathcal{B}\star k\star\kappa + \bar{\eta}\delta^2(d\bar{z})\mathcal{B}\star\bar{k}\star\bar{\kappa} + G(\mathcal{B})\delta^4(dZ)k\star\bar{k}\star\kappa\star\bar{\kappa} + \mathcal{L}I$$

$$\mathcal{L} = \mathcal{L}^2 + \mathcal{L}^4, \qquad G = g + O(\mathcal{B})$$

The g**-dependent term represents de Rham cohomology in the** Z**-space.**

Klein operators give rise to divergent traces and, hence, to nontrivial $\mathcal{L}$

Boundary functionals, parity, and conformal HS theory

Parity transformation $\mathbf{z} \to -\mathbf{z}$, $\mathbf{x} \to \mathbf{x}$

$$\theta^{\alpha}, z^{\alpha}, y^{\alpha}, k \quad \overset{P}{\Longleftrightarrow} \quad \bar{\theta}^{\dot{\alpha}}, \bar{z}^{\dot{\alpha}}, \bar{y}^{\dot{\alpha}}, \bar{k}\,.$$

For general η HS equations are not P-invariant.

The A-model $(\eta = 1)$ and B-model $(\eta = i)$ are P-invariant

Since $\mathbf{z}^{-1}d\mathbf{z}$ is P- even for A and B models $S = S^{loc}$ only contains boundary derivatives giving some gauge invariant boundary functional.

Original bulk Lagrangian is invariant under reflection of all coordinates.

Since $\mathbf{z}$ integration takes away one power of $\mathbf{z}$ the boundary Lagrangian is odd hence being of Chern-Simons type.

Actions $S^{A,B}$ of $3d$ conformal HS theory differ by the parity properties of the scalar field.

Nonlocal boundary functional

Naively, $S^{nloc} = 0$ in A and B-models.

For general η it is not difficult to see that

$$\mathcal{L} \sim \omega(cos(2\varphi)R_{\mathbf{xx}} - sin(2\varphi)R_{\mathbf{zx}})\,, \qquad \eta = \exp i\varphi$$

$$R_{\mathbf{xx}} \sim \eta e_{\mathbf{x}}e_{\mathbf{x}}C + \bar{\eta}e_{\mathbf{x}}e_{\mathbf{x}}\bar{C}\,, \qquad R_{\mathbf{xz}} \sim i\eta e_{\mathbf{z}}e_{\mathbf{x}}C - i\bar{\eta}e_{\mathbf{z}}e_{\mathbf{x}}\bar{C}$$

$S^{loc} \sim cos(2\varphi)$, $S^{nloc} \sim sin(2\varphi)$. $S^{nloc} = 0$ **for A, B models.**

Proper definition: factors in front of $\cos(2\varphi)$ **and** $\sin(2\varphi)$

$$S^{loc}_{A,B} = S(\varphi)\Big|_{\varphi=0,\frac{\pi}{2}}\,, \qquad S^{nloc}_{A,B} = \frac{1}{2}\frac{\partial S(\varphi)}{\partial\varphi}\Big|_{\varphi=0,\frac{\pi}{2}}$$

For general η it is impossible to separate S^{loc} and S^{nloc}

$S^{loc}+S^{nloc}$ **is gauge invariant: δS^{nloc} can contain local terms compensating** δS^{nloc}.

Only P-invariant A and B models allow gauge invariant local boundary functionals $S^{loc}_{A,B}$ = actions of the boundary conformal HS theory.

$S^{nloc}_{A,B}$ **are gauge invariant up to local terms.**

Black holes

$4d$ GR BH is characterized by a spin-one Papapetrou field 1966.

Papapetrou two-form $\mathcal{F}$ obeys the sourceless Maxwell equations

$$\mathrm{d}_x \mathcal{F} = 0\,, \qquad \mathrm{d}_x \tilde{\mathcal{F}} = 0\,, \qquad x \neq 0\,.$$

For Schwarzschild BH

$$\mathcal{F} = \frac{4}{r^2} dt dr\,, \qquad \tilde{\mathcal{F}} = d\Omega$$

t and r are the time and radial coordinates. $d\Omega$ is the angular two-form.

$M\tilde{\mathcal{F}}$ supports the BH charge. At the horizon

$$\tilde{\mathcal{F}} = (2M)^{-2} V_H\,,$$

where V_H is the horizon volume form.

BH charge

The spin-one sector of linearized HS equations

$$\mathrm{d}\omega(x) = \left(\eta \overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial\bar y^{\dot\alpha}\partial\bar y^{\dot\beta}}C^0(Y|x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}C^0(Y|x)\right)\Big|_{Y=0} + \mathcal{L}^2$$

Relation to Papapetrou field

$$\overline{H}^{\dot\alpha\dot\beta}\bar C_{\dot\alpha\dot\beta} + H^{\alpha\beta}C_{\alpha\beta} = M\mathcal{F}\,, \qquad H^{\alpha\beta} := e^{\alpha}{}_{\dot\alpha}e^{\beta\dot\alpha}\,, \quad \overline{H}^{\dot\alpha\dot\beta} := e_{\alpha}{}^{\dot\alpha}e^{\alpha\dot\beta}$$

M **is the BH mass, zero-forms** $C_{\alpha\beta}$ **and** $\bar C_{\dot\alpha\dot\beta}$ **are (anti)self-dual components of the spin-one field strength. The Hodge dual two-form is**

$$i\left(H^{\alpha\beta}C_{\alpha\beta} - \overline{H}^{\dot\alpha\dot\beta}\bar C_{\dot\alpha\dot\beta}\right) = M\,\tilde{\mathcal{F}}\,.$$

$C(Y|x)$ **extends the spin-two BH solution to HS fields**

For $\eta = \exp[i\varphi]$ **this gives in the canonical gauge** $\omega(x) = 0$

$$-\mathcal{L}^2 = \frac{\sin(\varphi)}{4M}V_H + M\cos(\varphi)\mathcal{F}\,.$$

The second term does not contribute since $\mathcal{F}$ **is the electric field of a point charge:** $\omega(x)$ **is the Coulomb field regular at infinity: its contribution to** $\mathcal{L}^2$ **is exact.**

$\omega(x)$ for $\tilde{\mathcal{F}}$ describes a monopole solution singular at infinity due to the Dirac string: $\mathcal{L}^2$ in the canonical gauge $\omega(x)=0$, is closed but not exact.
For the A-model with $\varphi=0$ the proper definition is

$$Q(0) = -\frac{\partial \mathcal{L}^2(\varphi)}{\partial \varphi}\Big|_{\varphi=0}.$$

$\mathcal{L}^2$ supports BH charges.

$\mathcal{L}^2$ is closed on-shell with no Killing symmetry of a particular solution?!
No on-shell closed local $\mathcal{L}^2$ is expected in a nonlinear $4d$ field theory.
$\mathcal{L}^2$ in HS theory are in a certain sense nonlocal involving infinitely many derivatives of fields with inverse powers of Λ (flat limit is obscure).
Being independent of local variations of Σ^2, $Q=\int_{\Sigma^2}\mathcal{L}^2(\phi)$ effectively // depends on fields away from Σ^2
For asymptotically free theory at infinity $\mathcal{L}^2$ is asymptotically local, reproducing usual asymptotic charges.

Conclusions

Invariant functionals are associated with central elements of the field algebra

Proposed formulation is coordinate-independent and applicable to any boundaries and bulk solutions

Manifest holographic duality at the level of the generating functional from the unfolded formulation of HS equations

Invariant functionals for singular solutions
BH entropy follow from the same construction via the $\mathcal{L}^2$-form

AdS_3/CFT_2: Invariant functional is a two-form: boundary functional is an integral of a one-form: holomorphicity of CFT_2