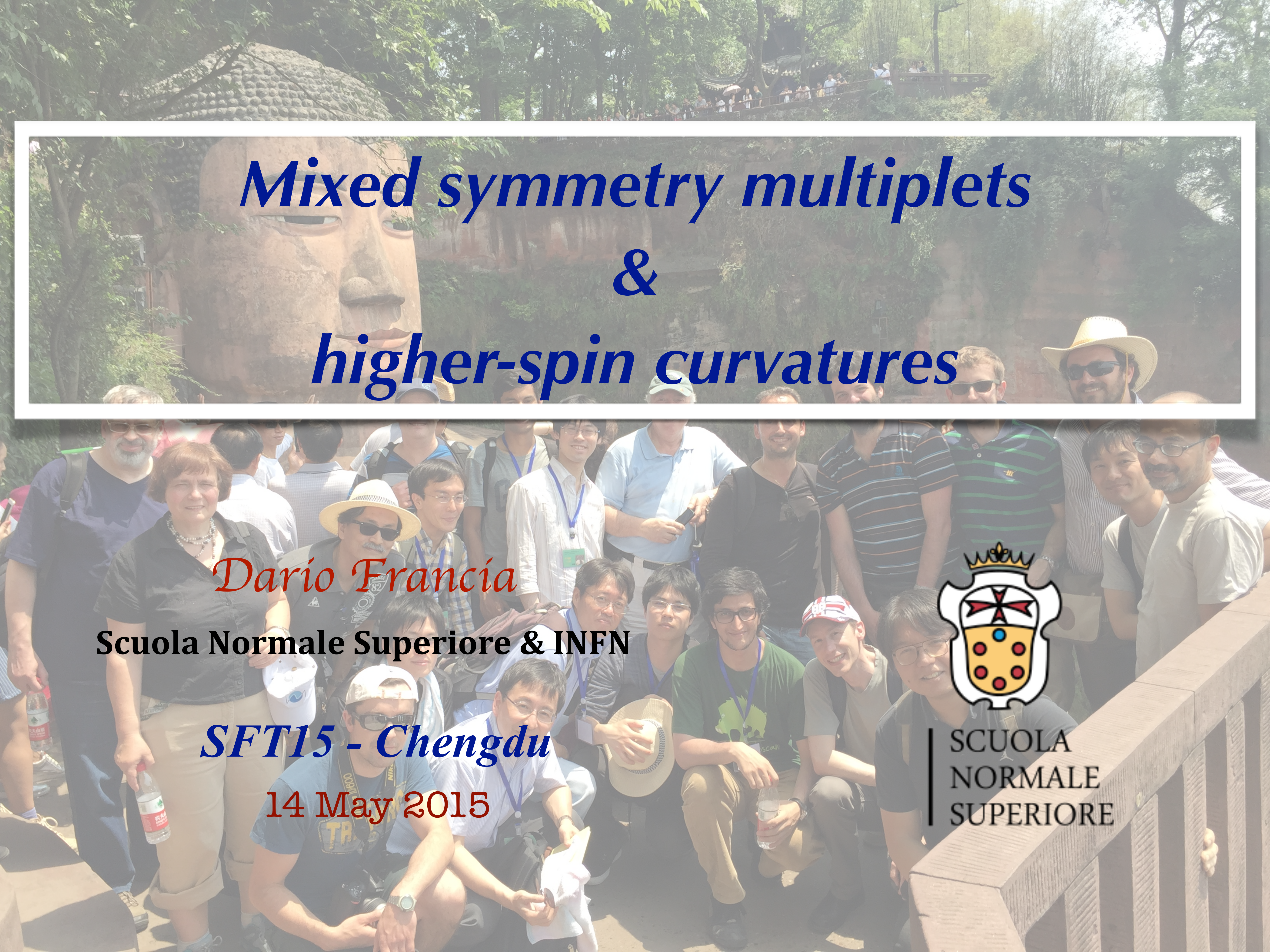

Mixed symmetry multiplets & higher-spin curvatures

Dario Francia

Scuola Normale Superiore & INFN

SFT15 - Chengdu

14 May 2015

SCUOLA NORMALE SUPERIORE

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➜ *true for free equations of motion as well,*
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we shall present an extension (completion?) of the Bargmann-Wigner program, covering multi-particle representations trying to justify our perspective connecting new and old results.

Back to basics:

wave equations for particles with zero mass

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two options:

gauge dependent

gauge independent

Wave equations for m=0, s=2

~

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Wave equations for m=0, s=2

$\sim$

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$$h_{\mu\nu} \sim \boxed{\mu}\boxed{\nu}_{GL(D)}$$

s.t.

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gauge equivalence: finite spin

same tensor as
for massive irreps

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Fierz 1939

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Wave equations for m=0, s=2

Connecting the two descriptions:

$$\partial_{[\lambda}\mathcal{R}_{\mu\nu],\,\rho\sigma} = 0 \quad \longrightarrow \quad \mathcal{R}_{\mu\nu,\,\rho\sigma}(h) = \partial_\mu \partial_\rho h_{\nu\sigma} + \dots$$

Poincaré Lemma

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$\rightarrow$

- $\partial_{[\lambda} \mathcal{R}_{\mu\nu],\, \rho\sigma}(h) \equiv 0$
- $\eta^{\mu\rho} \mathcal{R}_{\mu\nu,\rho\sigma}(h) = 0$ corresponds to the vanishing of the linearised Ricci tensor, that can be written

$$\Box h_{\mu\nu} = \partial_{(\mu} \Lambda_{\nu)}(h)$$

so as to stress that it reduces to $P^2 = 0$ upon partial gauge fixing

Wave equations for $m = 0$, spin s

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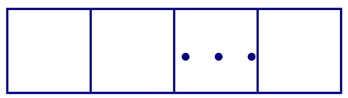
$\varphi \equiv \varphi_{\mu_1 \ldots \mu_s} \sim$ 

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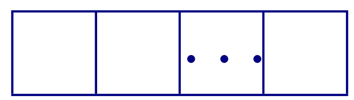
$\varphi \equiv \varphi_{\mu_1 \ldots \mu_s} \sim$ 

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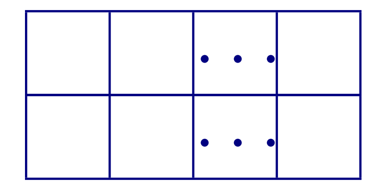
$\mathcal{R} \equiv \mathcal{R}_{\mu_1 \nu_1, \ldots, \mu_s \nu_s} \sim$ 

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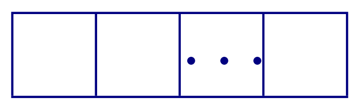

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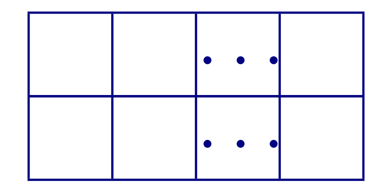

s.t.

$$d\,\mathcal{R} = 0$$

$$\mathcal{R}' = 0$$

Wave equations for spin s

Connecting the two descriptions:

$$d\mathcal{R} = 0 \longrightarrow \mathcal{R}_{\mu_1\nu_1,\ldots,\mu_s\nu_s} = \partial_{\mu_1}\ldots\partial_{\mu_s}\varphi_{\nu_1\ldots\nu_s} + \ldots$$

Generalised Poincaré Lemma

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Wave equations for spin s

Bekaert Boulanger
2002, 2003

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Generalised Poincaré Lemma

→

- $d\mathcal{R}(\varphi) \equiv 0$

- The higher-derivative equation $\mathcal{R}' = 0$ can be proven to be equivalent to the wave equation

$$\Box\varphi = \partial\Lambda(\varphi)$$

where the r.h.s. can be gauge fixed to zero. (! *Note*: this is not the Fronsdal equation)

Goal of this talk

~

To summarise:

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Curvatures generalise to all spins:

$$\mathcal{R}_{\,\mu\nu,\,\rho\sigma}\,(h)$$

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standard hsp theories are ``Ricci-like"

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Spin one (and p-forms)

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s.t.

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Our goal:

we wish to extend the Bargmann-Wigner program

to encompass the Maxwell-like equations

$$\partial \cdot \mathcal{R}\,(\varphi) \,=\, 0$$

for all spins, in any D, i.e. including tensors with mixed symmetry

Plan

§ Maxwell-like equations à la Bargmann-Wigner

§ Curvatures & wave operators for gauge potentials

§ Reducible multiplets and tensionless strings

Based on

- ***J.Phys.A: Math.Theor. 48 (2015)*** ***(with X. Bekaert and N. Boulanger)***
- ***Class.Quant.Grav. 29 (2012)***

see also

- ***Nucl.Phys. B881 (2014) 248-268*** ***(with S. Lyakhovic and A. Sharapov)***
- ***JHEP 1303 (2013) 168*** ***(with A. Campoleoni)***
- ***Prog.Theor.Phys.Suppl. 188 (2011)***
- ***Phys.Lett. B690 (2010)***
- ***J.Phys.Conf. Ser. 222 (2010)***

Maxwell-like equations à la Bargmann-Wigner

~

spin 2

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$$\left.\begin{array}{l} \partial_{[\lambda}\mathcal{R}_{\mu\nu],\rho\sigma} = 0 \\ \partial^{\mu}\mathcal{R}_{\mu\nu,\rho\sigma} = 0 \end{array}\right. \longrightarrow \Box\,\mathcal{R}_{\mu\nu,\rho\sigma} = 0$$

$$P^2 = 0 \longrightarrow p_{\mu} = (p_+, 0, \ldots, 0)$$

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$$\partial_{[\lambda}\mathcal{R}_{\mu\nu],\,\rho\sigma} = 0 \longrightarrow \mathcal{R}_{ij,\,kl} = 0$$

spin 2

➞ *The only non-vanishing components of* $\mathcal{R}_{\mu\nu,\,\rho\sigma}$ *are*

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i.e. they define a symmetric tensor of GL(D-2)

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one particle with $m = 0$, $s=2$

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Maxwell-like eqs propagate reducible multiplets

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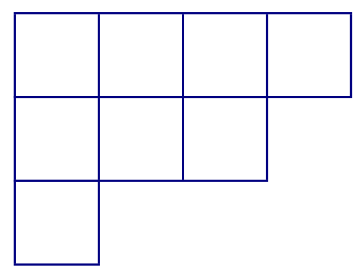
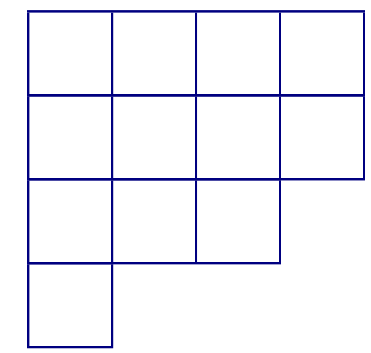
General case: consider an arbitrary tableau in GL(D-2) and build its Bargmann-Wigner counterpart, by adding a row on its top

$Y_{GL(D-2)} =$  $\longrightarrow$ $\mathcal{R}_{GL(D)} =$ 

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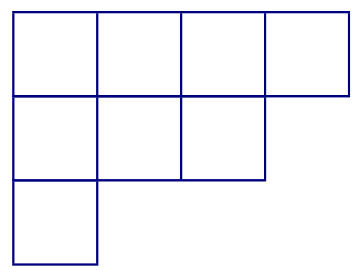
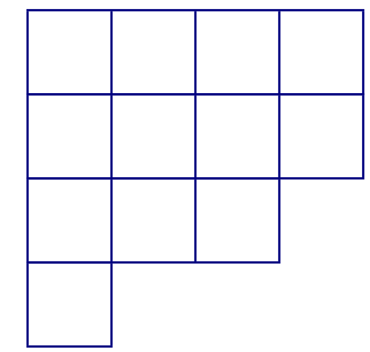
$Y_{GL(D-2)} =$  $\longrightarrow$ $\mathcal{R}_{GL(D)} =$ 

➞ *Require* $\mathcal{R}_{GL(D)}$ *to satisfy the closure and co-closure conditions*

$$d\,\mathcal{R} = 0$$
$$d^{\dagger}\mathcal{R} = 0$$

(w.r.t all rectangular blocks)

$\longrightarrow$ $P^2 = 0$ $\longrightarrow$ $p_{\mu} = (p_+, 0, \ldots, 0)$

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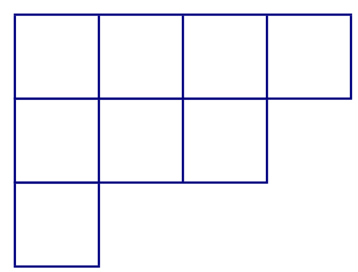
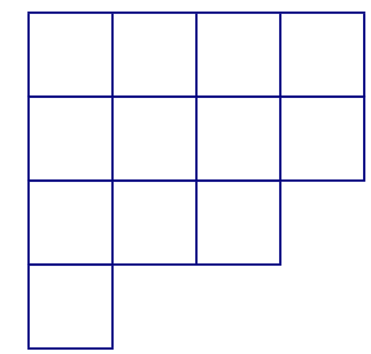
$Y_{GL(D-2)} =$  $\longrightarrow$ $\mathcal{R}_{GL(D)} =$ 

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- *The non-vanishing components,* $\mathcal{R}_{+\,j^1_1\dots j^1_{l_1},\,\dots,\,+\,j^i_1\dots j^i_{l_i},\,\dots,\,+\,j^s_1\dots j^s_{l_s}}$ *,*

correspond to a multiplet of massless particles:

branching of the GL(D-2)-irrep in terms of its O(D-2)-components.

Curvatures & wave operators for gauge potentials

High-derivative equations from curvatures

We make contact with gauge potentials solving for the closure conditions via the Generalised Poincaré Lemma:

High-derivative equations from curvatures

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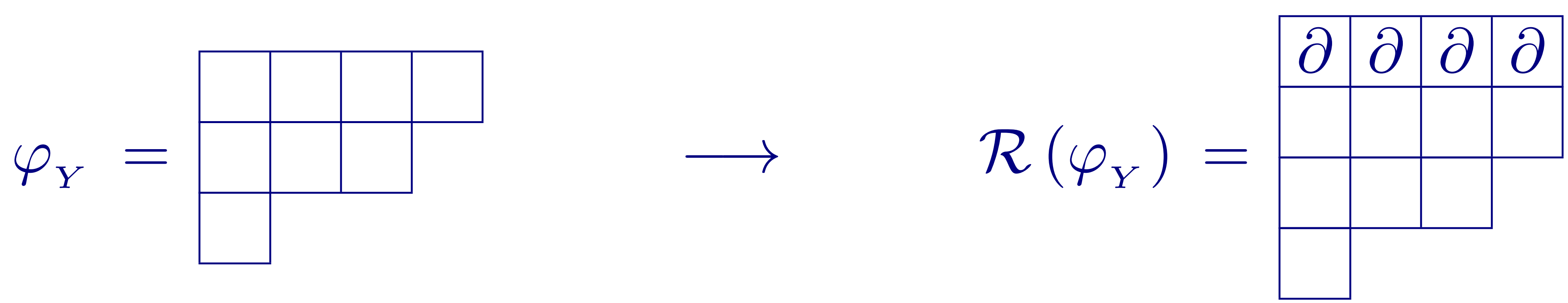
where $\mathcal{R}(\varphi)$ corresponds to the irrep of *GL(D)* obtained from a given tableau *Y* by adding an extra row on top of it:



Higher-spin curvatures

- more explicitly, for ***multi-forms potentials and curvatures***

$$\varphi = \begin{array}{|c|}\hline 1 \\ \hline \vdots \\ \hline s \\ \hline \end{array} \otimes \begin{array}{|c|}\hline 1 \\ \hline \vdots \\ \hline r \\ \hline \end{array} \otimes \ldots \qquad \longrightarrow \qquad \mathcal{R}\,(\varphi) = \begin{array}{|c|}\hline \partial \\ \hline 1 \\ \hline \vdots \\ \hline s \\ \hline \end{array} \otimes \begin{array}{|c|}\hline \partial \\ \hline 1 \\ \hline \vdots \\ \hline r \\ \hline \end{array} \otimes \ldots$$

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Example:

gauge potential

$$\varphi_{\,[\mu_1\mu_2\mu_3\mu_4],\,[\nu_1\nu_2\nu_3]} = \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array} \otimes \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array}$$

gauge parameters

$$\Lambda_1 = \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array} \otimes \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array} \qquad \Lambda_2 = \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array} \otimes \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$$

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gauge transformation

$$\delta\,\varphi = d^{\,1}\,\Lambda_1 + d^{\,2}\,\Lambda_2 \quad \Longrightarrow \quad \delta\,\mathcal{R}(\varphi) \equiv 0$$

High-derivative equations from curvatures

We go through the Bargmann-Wigner analysis again,
but now for high-derivative functions of gauge potentials

$$\mathcal{R}(\varphi) \equiv d^1 d^2 \cdots d^s \varphi$$

computing the divergence of $\mathcal{R}$

$$d_1 \mathcal{R}(\varphi) = d^2 \cdots d^s (\Box - d^i d_i) \varphi \sim \mathcal{O}(d) M = 0$$

where

$$M = (\Box - d^i d_i) \varphi$$

is a sort of second-order
Maxwell-like wave operator

From high- to 2nd-order equations

$\sim$

Problem: determine the kernel of the operator $\mathcal{O}\,(d)$

two main steps:

From high- to 2nd-order equations

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Getting an equation for M
via the Generalised Poincaré Lemma

$$d^{\,2} \cdots d^{\,s}\,(\Box - d^{\,i} d_{\,i})\,\varphi = 0$$

→

$$M = d^{\,i}\, d^{\,j}\, D_{\,ij}\,(\varphi)$$

From high- to 2nd-order equations

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two main steps:

1. Getting an equation for M via the Generalised Poincaré Lemma

$$d^2 \cdots d^s (\Box - d^i d_i)\varphi = 0$$

$$\longrightarrow$$

$$M = d^i d^j D_{ij}(\varphi)$$

2. Show that the resulting equation can be gauge fixed to $P^2 = 0$:

$$\Box\varphi = d^i \Lambda_i(\varphi)$$

$$\longrightarrow$$

$$\Box\varphi = 0 \qquad d^\dagger\varphi = 0$$

The Fronsdal-Labastida case

Same analysis when applying trace conditions:

$$T_{12}\,\mathcal{R}\,(\varphi) \;=\; d^{\,3}\,\cdots\,d^{\,s}\,\mathcal{F}\;\sim\;\hat{\mathcal{O}}\,(d)\,\mathcal{F}\;=\;0$$

$$\mathcal{F}\;:=\;\Box\varphi\;-\;d^{\,i}\,d_{\,i}\,\varphi\;+\;\frac{1}{2}\,d^{\,i}\,d^{\,j}\,T_{ij}\,\varphi$$

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Solving for the kernel of $\hat{\mathcal{O}}\,(d)$:

$$\mathcal{F}\,=\,\tfrac{1}{2}\,d^{\,i}\,d^{\,j}\,d^{\,k}\,\mathcal{H}_{\,ijk}\,(\varphi)$$

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Show that the resulting equation can be gauge fixed to $P^2 = 0$:

$$\Box\,\varphi = 0,\ \ d^{\dagger}\varphi = 0,\ \ T_{ij}\,\varphi = 0$$

$$M = d^i d^j D_{ij}(\varphi)$$

still higher-derivative eqs!

$$\mathcal{F} = \frac{1}{2} d^i d^j d^k \mathcal{H}_{ijk}(\varphi)$$

Our analysis shows that the two ``compensator'' structures

$D_{ij}(\varphi)$ and $\mathcal{H}_{ijk}(\varphi)$

can be consistently gauge fixed to zero, leading to

$$M = 0$$

$$d^i d^j d_{(i} \Lambda_{j)} = 0$$

D.F., A. Campoleoni 2013

$$\mathcal{F} = 0$$

$$T_{(ij} \Lambda_{k)} = 0$$

Fronsdal-Labastida '78, '89

Maxwell-like equations: mixed-symmetry

$\sim$

Maxwell-like Lagrangians can be defined for tensors of any symmetry;
let us get a feeling of the simplification obtained:

Maxwell-like, N families:

(multi-particle spectrum)

$$\mathcal{L} = \frac{1}{2}\,\varphi\, M\, \varphi$$

$$M = (\Box - \partial^{\,i}\,\partial_{\,i})$$

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Fronsdal-Labastida, N families:

$$\mathcal{L} \,=\, \frac{1}{2}\,\varphi\, \left\{ \mathcal{F} \,+\, \sum_{p\,=\,1}^{N} \frac{(-1)^p}{p!\,(p+1)!}\, \eta^{i_1 j_1}\, \ldots\, \eta^{i_{\,p} j_{\,p}}\, Y_{\{2^p\}}\, T_{i_1 j_1}\, \ldots\, T_{i_p j_p}\, \mathcal{F} \right\}\,,$$

$$\mathcal{F} \,=\, \left(M \,+\, \partial^{\,i}\partial^{\,j}\, T_{ij}\right)\, \varphi$$

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$$\mathcal{F} = \left(M + \partial^{\,i}\partial^{\,j}\,T_{ij}\right)\,\varphi$$

$$\begin{cases} T_{(\,ij}\,\Lambda_{\,k\,)} = 0 \\ T_{(\,ij}\,T_{kl\,)}\,\varphi = 0 \end{cases}$$

Reducible multiplets and tensionless strings

~

Massless higher spins from tensionless strings

$\sim$

Open bosonic string oscillators

$$[\alpha_k^\mu, \alpha_l^\nu] \; = \; k\,\delta_{k+l,0}\,\eta^{\mu\nu}$$

Massless higher spins from tensionless strings

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Virasoro generators and their rescaling limit:

$$L_k \;=\; \frac{1}{2}\sum_{l=-\infty}^{+\infty} \alpha^\mu_{k-l}\,\alpha_{\mu l}\ , \;\rightarrow\; \begin{cases} \tilde{L}_{k\neq 0} \;=\; \dfrac{1}{\sqrt{\alpha'}}\,L_k \\[2ex] \tilde{L}_0 \;=\; \dfrac{1}{\alpha'}\,L_0 \end{cases} \xrightarrow[\alpha' \to \infty]{} \begin{array}{l} l_k \;=\; p_\mu\,\alpha^\mu{}_k \\[2ex] l_0 \;=\; p_\mu\,p^\mu \end{array}$$

"tensionless" limit

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"tensionless" limit

$$[l_k, l_l] = k\,\delta_{k+l,0}\,l_0$$

Algebra with no central charge ⟶ identically nilpotent BRST charge $\mathcal{Q}$

same charge from tensionless limit of open string BRST charge, after rescaling of ghosts

Massless higher spins from tensionless strings

$$\mathcal{L} = \frac{1}{2} \langle \psi | \, Q \, | \psi \rangle \quad \xrightarrow[\alpha' \to \infty]{\sim} \quad \text{decomposes in diagonal blocks}$$

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for ``diagonal blocks'' associated to symmetric, rank-s tensors $\varphi_{\mu_1 \cdots \mu_s}$, (states generated by powers of α^{μ}_{-1}) the corresponding Lagrangian is

$$\mathcal{L}_{triplet} = \frac{1}{2} \varphi \Box \varphi - \frac{1}{2} s C^2 - \binom{s}{2} D \Box D + s \, \partial \cdot \varphi \, C + 2 \binom{s}{2} D \, \partial \cdot C$$

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equations of motion

$$\Box \, \varphi = \partial \, C$$
$$C = \partial \cdot \varphi - \partial \, D$$
$$\Box \, D = \partial \cdot C$$

$\varphi \to$ spin s

$C \to$ spin $s-1$

$D \to$ spin $s-2$

gauge transformations

$$\delta \, \varphi = \partial \, \Lambda$$
$$\delta \, C = \Box \, \Lambda$$
$$\delta \, D = \partial \cdot \Lambda$$

Massless higher spins from tensionless strings

➙ the field C is purely auxiliary (no kinetic term) and can be directly integrated away from the Lagrangian

Massless higher spins from tensionless strings

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- the field D is *pure gauge,* and as such contains no physical polarisations

$\longrightarrow$

Massless higher spins from tensionless strings

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the eom for the physical field from the tensionless string

$$M\,\varphi = 2\partial^2\,\mathcal{D}$$

are just the Maxwell-like equations with a ``compensator''

Massless higher spins from tensionless strings

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Bengtsson, Ouvry-Stern '86 Henneaux-Teitelboim '88
D.F.-Sagnotti '02, Sagnotti-Tsulaia '03
Fotopoulos-Tsulaia '08 . . .

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the eom for the physical field from the tensionless string

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[also valid for mixed-symmetry fields]

Conclusions

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$$\mathcal{R}^{\alpha}{}_{\alpha\,\mu_3\ldots\mu_s\,,\,\nu_1\ldots\nu_s} = 0$$

``Ricci = 0'' provides the backbone of gauge theories…

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Alternative option:

reducible, multi-particle theories

Conclusions

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Alternative option:

reducible, multi-particle theories

``Maxwell = 0'' seems to provide the proper model to this end

$$\partial^{\alpha}\mathcal{R}_{\alpha\,\mu_2\ldots\mu_s\,,\,\nu_1\ldots\nu_s} \;=\; 0$$

Why?

~

- ➞ They might provide a better choice of field variables; for instance for the spin-2 case the self-interactions of a single field would encompass all the vertices of a scalar-tensor theory

- ➞ A promising observation: These are the variables chosen by SFT

Maxwell-like geometric Lagrangians

~

- the field C is purely auxiliary

- the field D is pure gauge

how does the Lagrangian would look in terms of the physical field only?

Maxwell-like geometric Lagrangians

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- the field D is pure gauge

how does the Lagrangian would look in terms of the physical field only?

Integrating over the fields C and D we find

$$\mathcal{L}_{eff}(\varphi) = \frac{1}{2}\varphi(\Box - \partial\partial\cdot)\varphi + \frac{1}{2}\binom{s}{2}\partial\cdot\partial\cdot\varphi(\Box + \frac{1}{2}\partial\partial\cdot)^{-1}\partial\cdot\partial\cdot\varphi$$

Maxwell-like geometric Lagrangians

The inverse of the operator $\mathcal{O} = \Box + \frac{1}{2}\,\partial\,\partial\cdot$ on rank-*k* tensors is

$$\mathcal{O}^{-1}_{(k)} = \frac{1}{\Box}\{1 + \sum_{m=1}^{k} (-1)^m \frac{m!}{2^m \prod_{l=1}^{m}(1 + \frac{l}{2})} \frac{\partial^m}{\Box^m} \partial\cdot^m\}$$

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and the resulting Lagrangian is

$$\mathcal{L}_{eff}(\varphi) = \frac{(-1)^s}{2(s+1)}\mathcal{R}^{(s)}_{\mu_1\cdots\mu_s,\,\nu_1\cdots\nu_s}\frac{1}{\Box^{s-1}}\mathcal{R}^{(s)\,\mu_1\cdots\mu_s,\,\nu_1\cdots\nu_s}$$

Lagrangians $\sim$ *squares of curvatures*