The status of the strong coupling from tau decays in 2016

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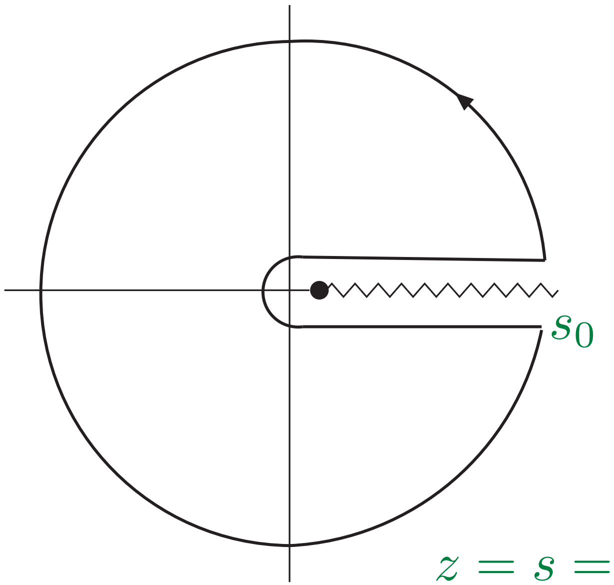
Controversy:

- Pich, Rodríguez-Sánchez (PRD94 (2016) 034027): $\alpha_s(m_\tau^2) = 0.328(12)$
  Davier *et al.* (EPJC74 (2014) 2803): $\alpha_s(m_\tau^2) = 0.332(12)$

- Boito *et al.* (PRD91 (2015) 034003): $\alpha_s(m_\tau^2) = 0.301(10)$

- What explains the difference? This talk:
  Flaws in P&R (and Davier *et al*.) analysis

- *Technical note:* these are averages between CIPT and FOPT values.
  In this talk we will not consider such averages, because averaging is not justified. Not related to controversy.



$z = s = q^2$ plane

$\rho_{V+A}(s)$ V+A non-strange spectral function, measured in tau decays (ALEPH)

$\Pi_{V+A}(z = q^2)$ V+A current two point function, gives access to strong coupling

polynomial

Cauchy:

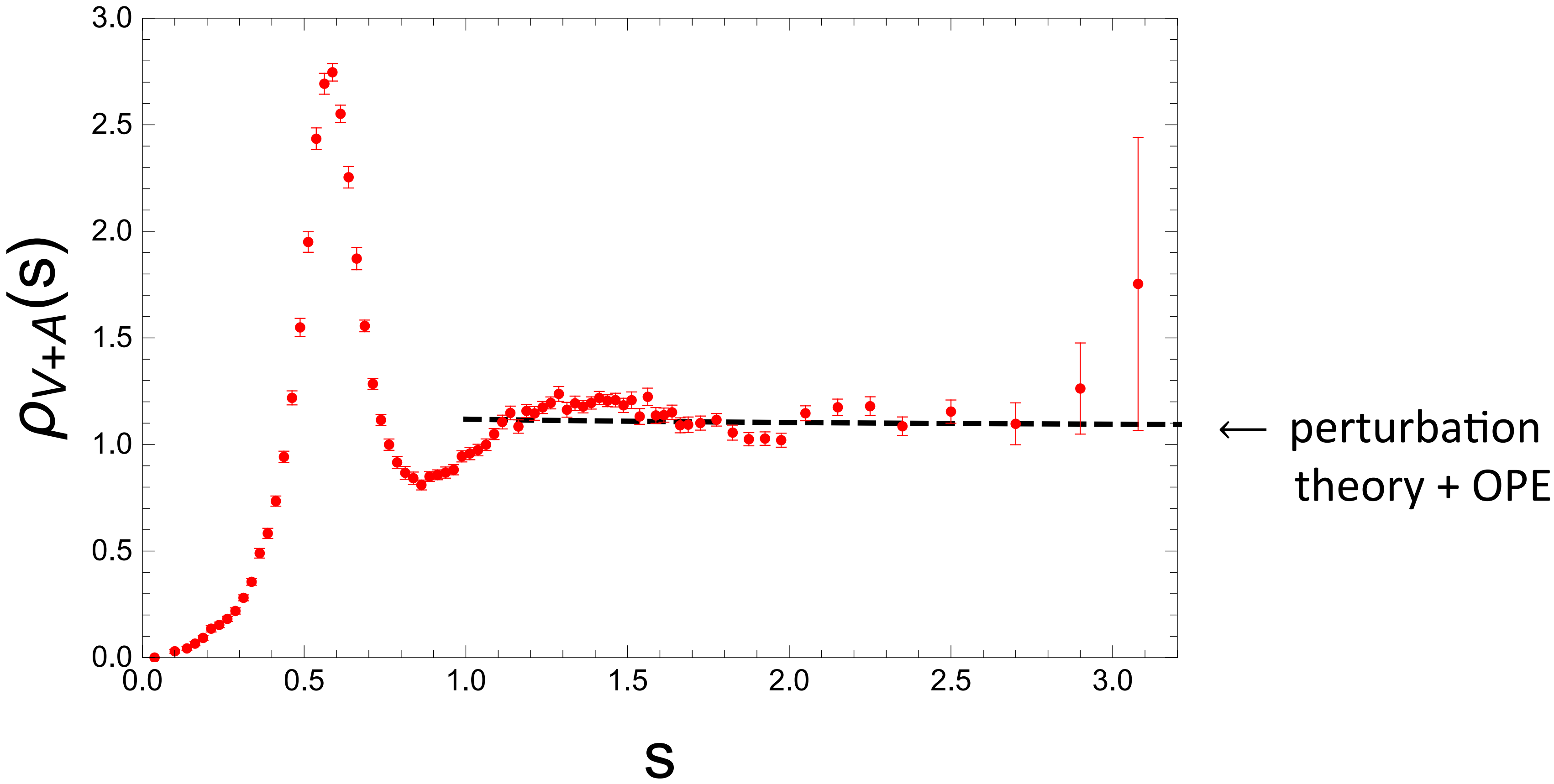
$$\int_0^{s_0} ds\, w(s)\, \rho_{V+A}(s) = -\frac{1}{2\pi i} \oint_{|z|=s_0} dz\, w(z)\, \Pi_{V+A}(z)$$

$$\Pi(z) = \Pi^{\text{pert}}(z) + \Pi^{\text{nonpert}}_{\text{OPE}}(z) + \Pi^{\text{nonpert}}_{\text{DV}}(z)$$

α_s resonances

$$\Pi_{\text{OPE}}(z) = \frac{C_4}{z^2} - \frac{C_6}{z^3} + \frac{C_8}{z^4} - \dots$$

V+A non-strange spectral function (Davier *et al.*, 2014, ALEPH)



P&R: take $s_0 = m_\tau^2$ and weights ($x \equiv z/s_0$)

$$w_{00}(x) = (1-x)^2(1+2x)$$
$$w_{10}(x) = (1-x)^3(1+2x)$$
$$w_{11}(x) = (1-x)^3(1+2x)x$$
$$w_{12}(x) = (1-x)^3(1+2x)x^2$$
$$w_{13}(x) = (1-x)^3(1+2x)x^3$$

(ALEPH)

or

$$w^{(2,1)} = 1 - 3x^2 + 2x^3$$
$$w^{(2,2)} = 1 - 4x^3 + 3x^4$$
$$w^{(2,3)} = 1 - 5x^4 + 4x^5$$
$$w^{(2,4)} = 1 - 6x^5 + 5x^6$$
$$w^{(2,5)} = 1 - 7x^6 + 6x^7$$

("optimal")

Assume: $(C_{10} =)\ C_{12} = C_{14} = C_{16} = 0$ and resonance effects negligible

$\Rightarrow$ fit four parameters ($\alpha_s,\ C_6,\ C_8,\ C_{4/10}$) to five data spectral integrals

However: $$\frac{1}{2\pi i}\oint dz\, z^n \frac{C_{2k}}{z^k} = C_{2(n+1)}\delta_{k,n+1}$$

- need OPE coefficients up to C_{16}
- resonance oscillations around OPE clearly visible in V+A spectral function

P&R:

ALEPH	$\alpha_s(m_\tau^2)$	C_4 (10^{-3} GeV^4)	C_6 (10^{-3} GeV^6)	C_8 (10^{-3} GeV^8)
FOPT	$0.319^{+0.010}_{-0.006}$	$-0.8^{+1.6}_{-2.5}$	$1.3^{+1.4}_{-0.8}$	$-0.8^{+0.4}_{-0.7}$
CIPT	$0.339^{+0.011}_{-0.009}$	$-2.8^{+0.8}_{-0.8}$	$0.9^{+0.3}_{-0.4}$	$-1.0^{+0.5}_{-0.7}$

optimal	$\alpha_s(m_\tau^2)$	C_6 (10^{-3} GeV^6)	C_8 (10^{-3} GeV^8)	C_{10} (10^{-3} GeV^{10})
FOPT	$0.315^{+0.010}_{-0.005}$	$1.4^{+1.6}_{-0.9}$	$-0.9^{+0.8}_{-1.4}$	$0.3^{+0.8}_{-0.5}$
CIPT	$0.334^{+0.012}_{-0.008}$	$1.0^{+0.6}_{-0.4}$	$-1.0^{+0.5}_{-1.0}$	$0.2^{+0.4}_{-0.4}$

Our check:

ALEPH	$\alpha_s(m_\tau^2)$	C_4 (10^{-3} GeV^4)	C_6 (10^{-3} GeV^6)	C_8 (10^{-3} GeV^8)	χ^2/dof
FOPT	0.316(3)	−0.6(3)	1.2(3)	−0.8(3)	1.39/1
CIPT	0.336(3)	−2.6(4)	0.9(3)	−1.0(4)	0.89/1

optimal	$\alpha_s(m_\tau^2)$	C_6 (10^{-3} GeV^6)	C_8 (10^{-3} GeV^8)	C_{10} (10^{-3} GeV^{10})	χ^2/dof
FOPT	0.317(3)	1.4(4)	−1.0(5)	0.4(3)	1.27/1
CIPT	0.337(4)	1.0(4)	−1.1(4)	0.3(3)	0.83/1

(statistical errors only)

P&R: turn on C_{10} in ALEPH fit (not a fit, no degrees of freedom!):

ALEPH	$\alpha_s(m_\tau^2)$	C_4 $(10^{-3}\ \mathrm{GeV}^4)$	C_6 $(10^{-3}\ \mathrm{GeV}^6)$	C_8 $(10^{-3}\ \mathrm{GeV}^8)$
FOPT	$0.319^{+0.010}_{-0.006}$	$-0.8^{+1.6}_{-2.5}$	$1.3^{+1.4}_{-0.8}$	$-0.8^{+0.4}_{-0.7}$
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ALEPH	$\alpha_s(m_\tau^2)$	C_4 $(10^{-3}\ \mathrm{GeV}^4)$	C_6 $(10^{-3}\ \mathrm{GeV}^6)$	C_8 $(10^{-3}\ \mathrm{GeV}^8)$	C_{10} $(10^{-3}\ \mathrm{GeV}^{10})$
FOPT	$0.333^{+0.013}_{-0.012}$	$-1.5^{+1.6}_{-3.7}$	7^{+7}_{-4}	-5^{+4}_{-6}	12^{+12}_{-9}
CIPT	$0.355^{+0.016}_{-0.015}$	$-3.9^{+1.6}_{-1.2}$	5^{+3}_{-3}	-5^{+3}_{-3}	10^{+8}_{-8}

- OPE coefficients C_6 and C_8 grow by a factor 5 or so!
- OPE not convergent (asymptotic?) ⇒ expect growth of C_{2k} with k
- Assumption $C_{10} = C_{12} = C_{14} = C_{16} = 0$ only made to be able to do a fit with 5 data points
- Similar comment for “optimal” (and other) P&R fits

Choose for example $C_{10} = -0.0832 \text{ GeV}^{10}$

$$C_{12} = 0.161 \text{ GeV}^{12}$$

$$C_{14} = -0.17 \text{ GeV}^{14}$$

$$C_{16} = -0.55 \text{ GeV}^{16}$$

Reasonable values on the scale of QCD; we find (stat. errors only):

ALEPH	$\alpha_s(m_\tau^2)$	C_4 (GeV^4)	C_6 (GeV^6)	C_8 (GeV^8)	χ^2/dof
FOPT	0.295(3)	0.0043(3)	−0.0128(3)	0.0355(3)	0.99/1
CIPT	0.308(4)	0.0031(3)	−0.0129(3)	0.0354(3)	0.74/1

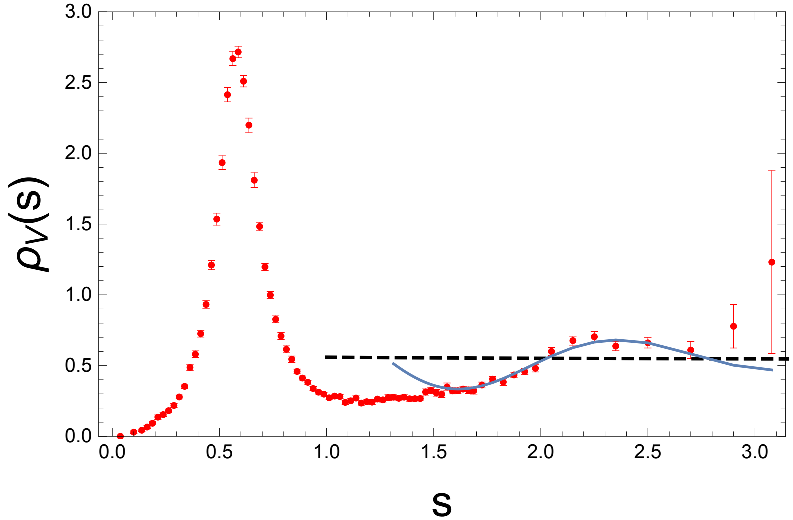
Compare with fits assuming $C_{10} = C_{12} = C_{14} = C_{16} = 0$:

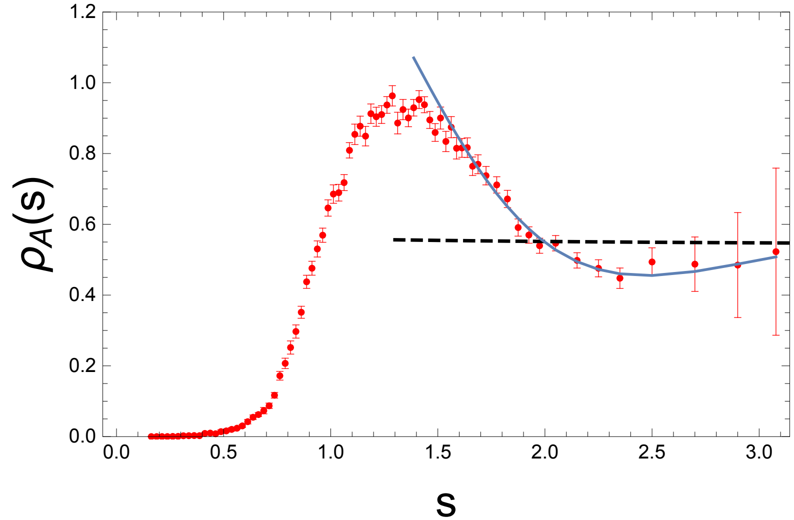
ALEPH	$\alpha_s(m_\tau^2)$	C_4 (GeV^4)	C_6 (GeV^6)	C_8 (GeV^8)	χ^2/dof
FOPT	0.316(3)	−0.0006(3)	0.0012(3)	−0.0008(3)	1.39/1
CIPT	0.336(3)	−0.0026(4)	0.0009(3)	−0.0010(4)	0.89/1

~8% shift in α_s , would increase total P&R error from ± 0.012 to ± 0.025
(similar for "optimal" and other fits)

A more stringent test: fake data

- Start from model having by construction lower $\alpha_s(m_\tau^2) = 0.312$ (CIPT) and non-negligible DVs compatible with the experimental spectral function. Test P&R/Davier approach

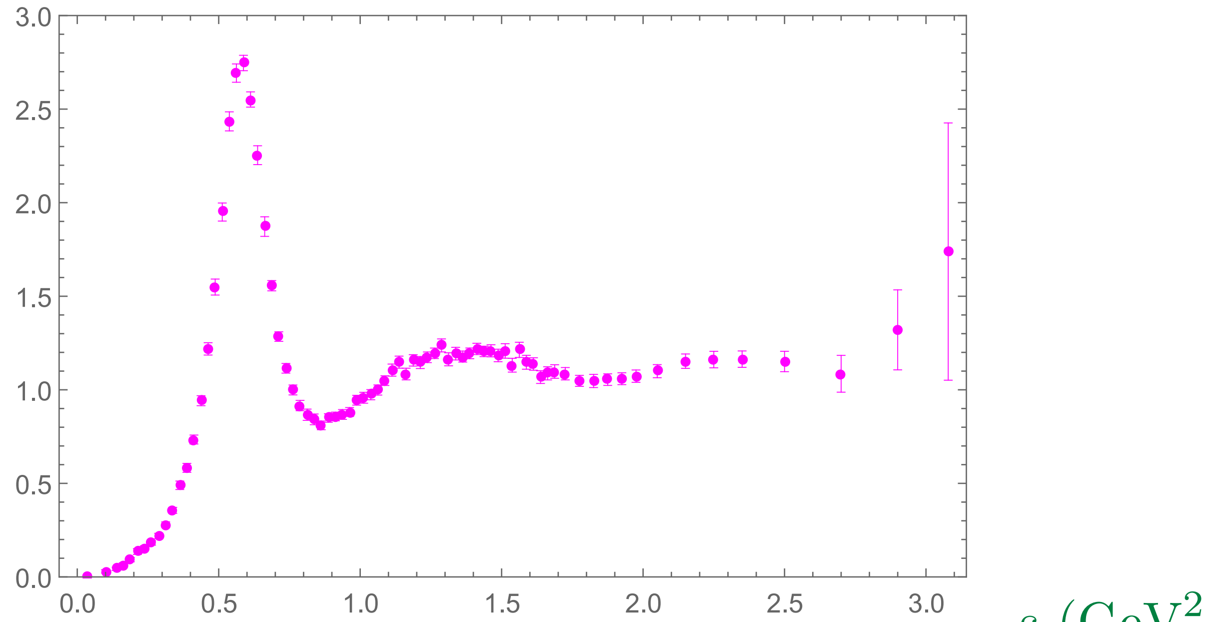




- Generate fake data from this model (using real-data covariances).
- Perform P&R type fits on these fake data.
- Compare the parameter values ($\alpha_s(m_\tau^2)$) obtained from these fits to the input value, $\alpha_s(m_\tau^2) = 0.312$.

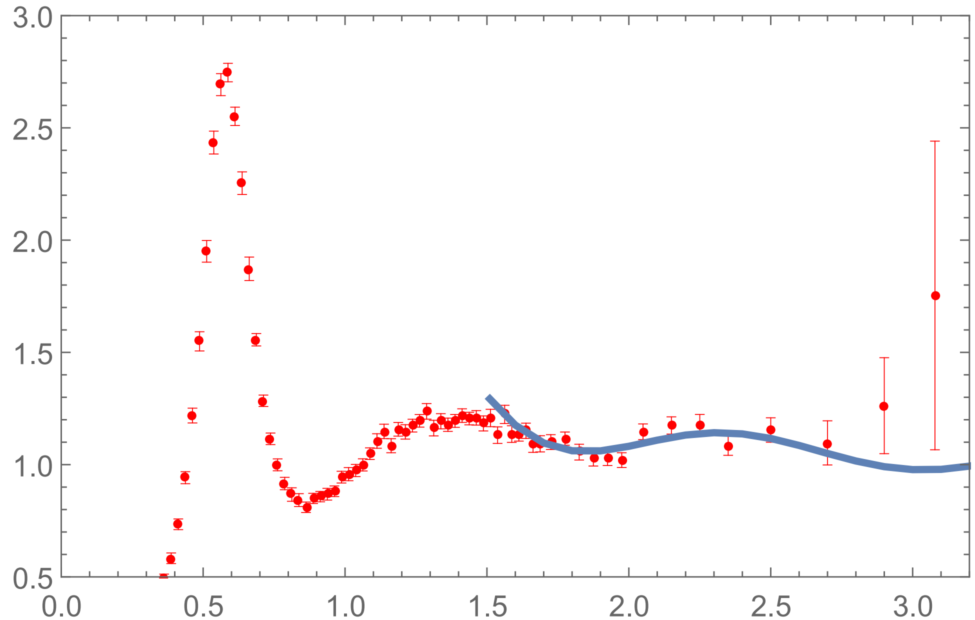
Fake data:

(fake above $s = 1.55\ \mathrm{GeV}^2$)



Real data:

(V+A spectral functions)



Results of this test:

ALEPH	$\alpha_s(m_\tau^2)$	$C_{4,V+A}$ (GeV^4)	$C_{6,V+A}$ (GeV^6)	$C_{8,V+A}$ (GeV^8)	χ^2/dof
true values	0.312	0.0027	-0.013	0.035	
fake data fit	0.334(3)	-0.0024(4)	0.0007(3)	-0.0008(4)	0.95/1

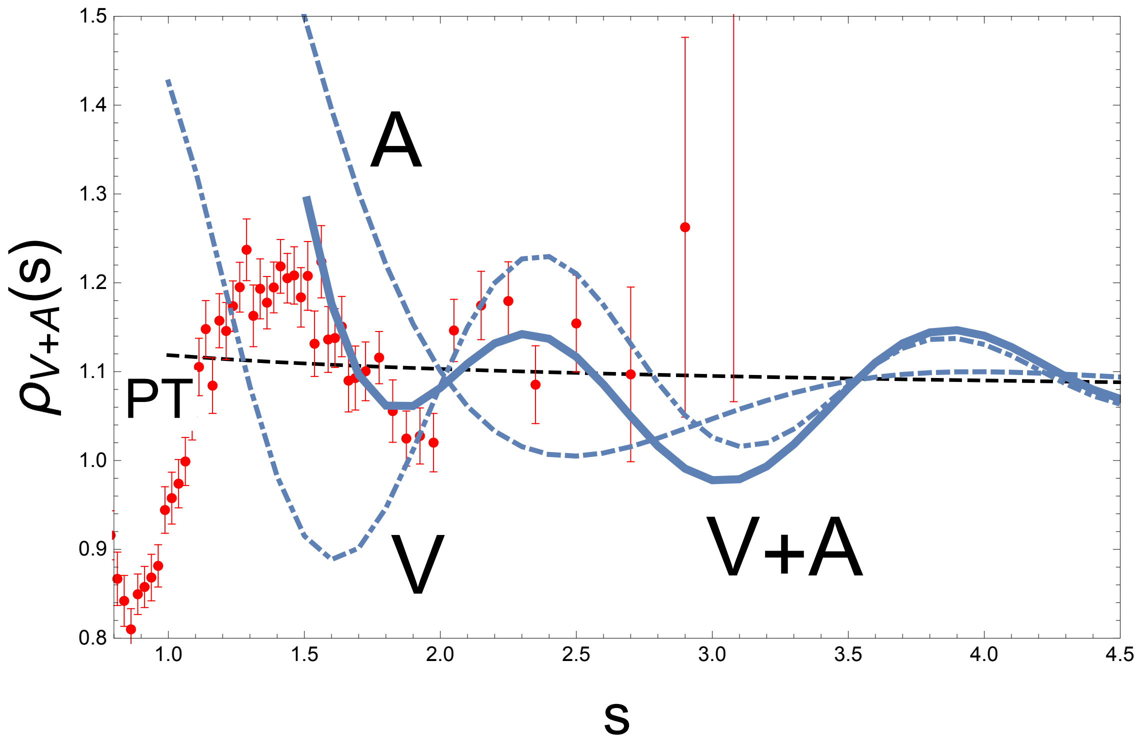
optimal	$\alpha_s(m_\tau^2)$	$C_{6,V+A}$ (GeV^6)	$C_{8,V+A}$ (GeV^8)	$C_{10,V+A}$ (GeV^{10})	χ^2/dof
true values	0.312	-0.013	0.035	-0.083	
fake data fit	0.334(4)	0.0008(4)	-0.0008(5)	0.0001(3)	0.92/1

(CIPT, statistical errors only)

- P&R fits get it wrong (similar conclusion for FOPT)
- big difference in behavior of OPE

Why do P&R fits get it wrong?

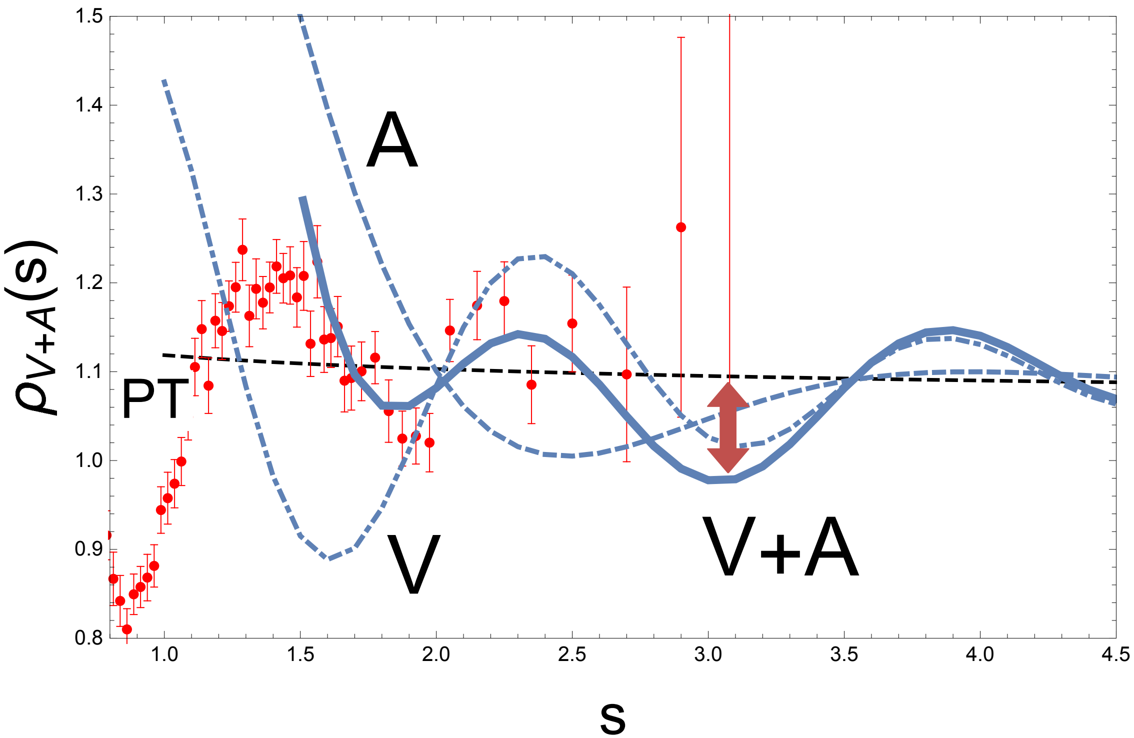
- Rely on uncontrolled assumption about the OPE in higher orders.
- Assume that duality violations (resonance effects) can be neglected, at least in V+A, *without testing this*.



Why do P&R fits get it wrong?

- Rely on uncontrolled assumption about the OPE in higher orders.
- Assume that duality violations (resonance effects) can be neglected, at least in V+A, *without testing this*.

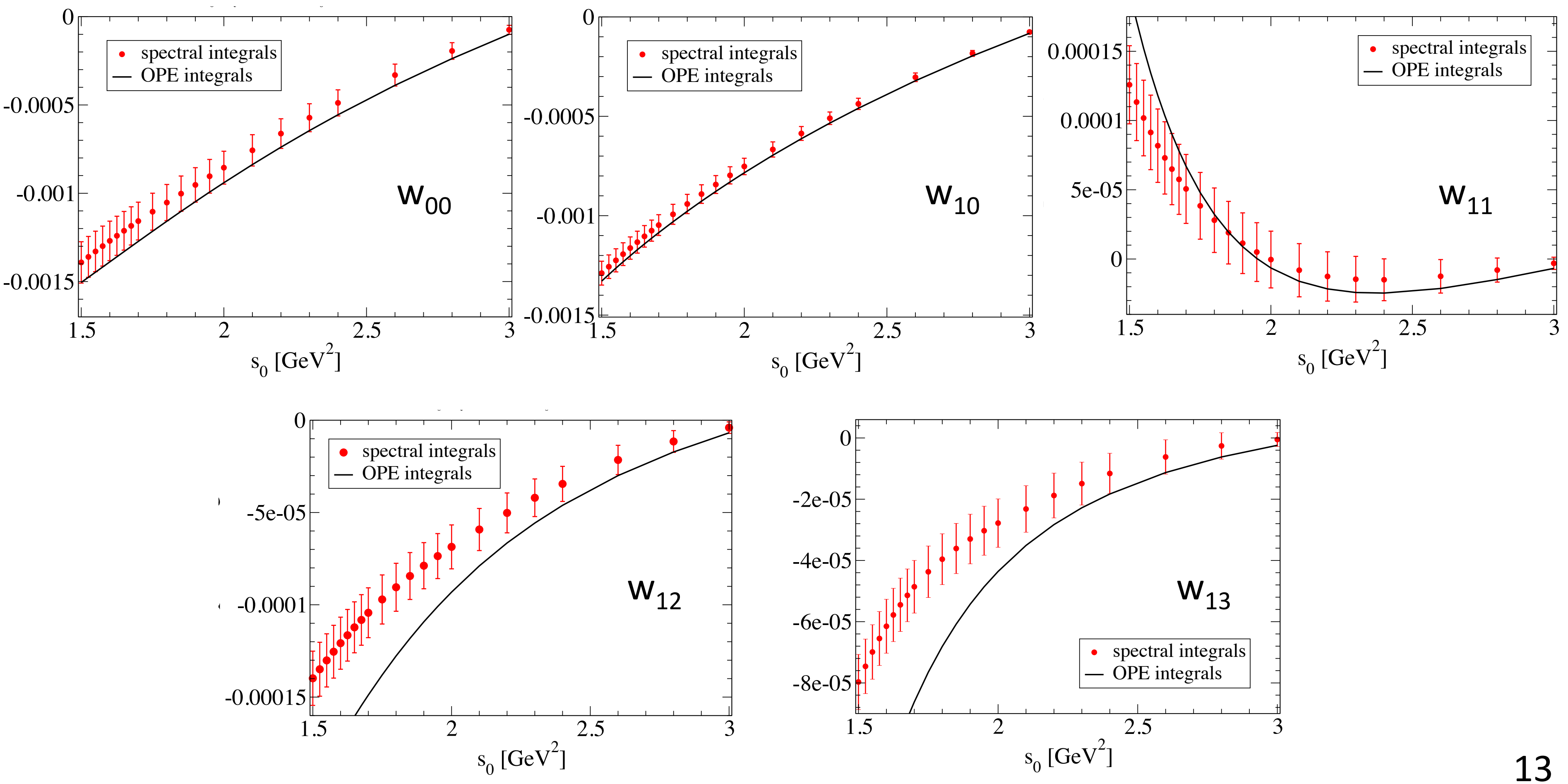
- Potentially large effect at $s_0 = m_\tau^2$!
  Not excluded by data.



How do we distinguish with the real data?

Check s_0 dependence, should work above $\sim 2\ \mathrm{GeV}^2$

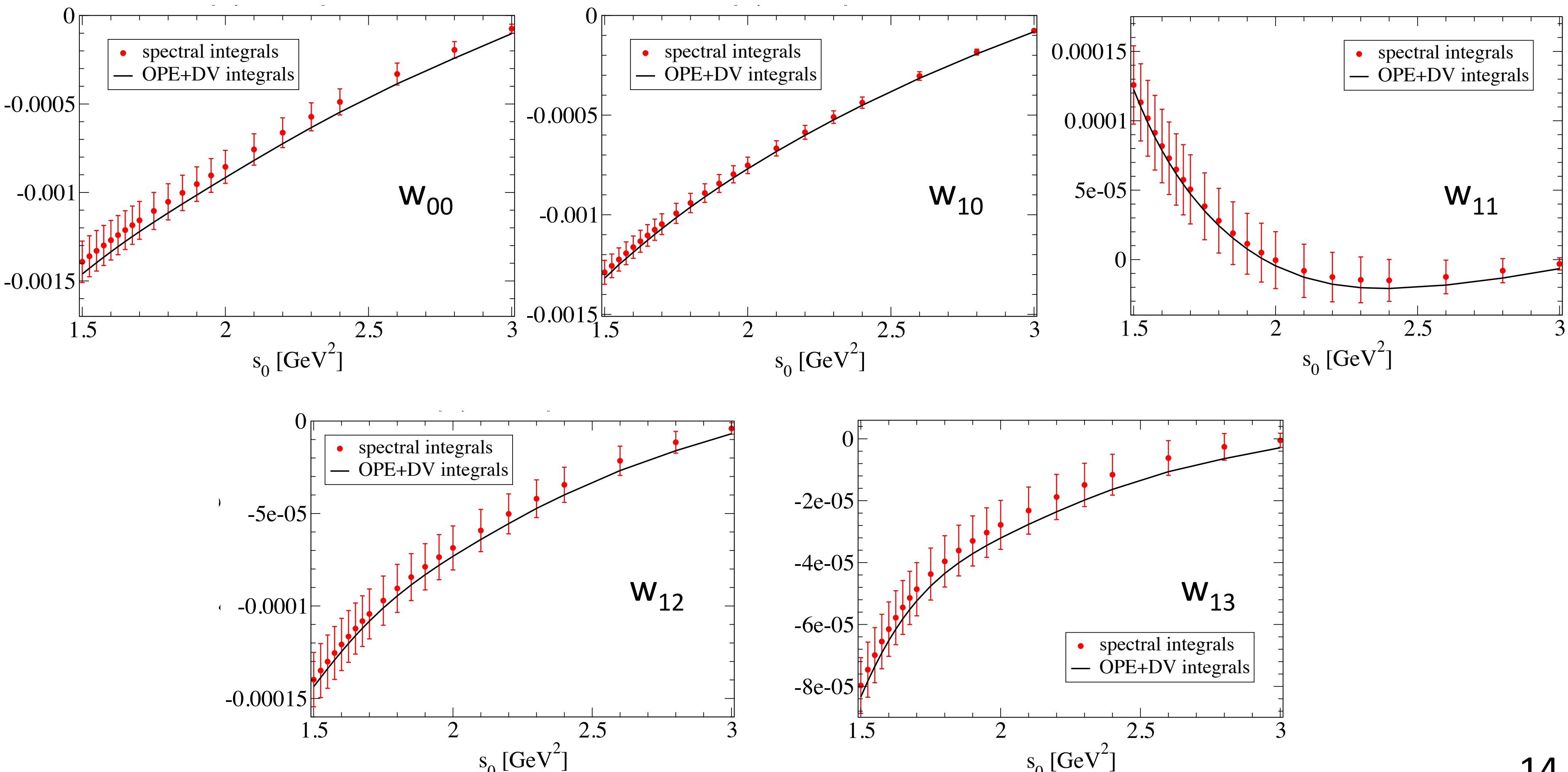
ALEPH moments for P&R (all used in fit, $s_0 = m_\tau^2$ minus s_0 differences):



How do we distinguish with the real data?

Check s_0 dependence, should work above $\sim 2\ \mathrm{GeV}^2$

ALEPH moments for Boito *et al.* (only w_{00} used in fit, others predictions):



Our solution (three papers and various conference proceedings)

- Use moments that probe OPE only to order C_8 .
- Compelled to allow for duality violations (resonance effects clearly seen!). Need to look at V and A channels separately (different resonances!).
- Our results: P&R:

$$\alpha_s(m_\tau^2) = 0.296(10) \text{ (FOPT)} \qquad \alpha_s(m_\tau^2) = 0.319(12) \text{ (FOPT)}$$
$$= 0.310(14) \text{ (CIPT)} \qquad = 0.335(13) \text{ (CIPT)}$$

- Even if you do not accept our model for duality violations, this implies an additional 0.024 (8%) spread in the value of $\alpha_s(m_\tau^2)$.
  Larger effect than the difference between FOPT and CIPT.
  ⇒ P&R analysis not competitive.

Conclusion

- P&R 2016 type analyses do not hold up:

- 1) No basis for their treatment of the OPE: arbitrary neglect of higher dimension condensates.
- 2) Neglect resonance effects ("duality violations"):
  - Dangerous: clearly visible even at the tau mass.
  - P&R fit strategy not capable of detecting residual DVs.
  - Need to estimate their quantitative effect – no escape!

- Theoretical and statistical errors in P&R 2016 criticism of our analysis to be detailed in forthcoming paper.