# Constraining Natural SUSY at the HL-LHC, ILC and CEPC

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# References:

- T. Li, S. Raza and K. Wang, Phys. Rev. D **93**, no. 5, 055040 (2016) [arXiv:1601.00178 [hep-ph]].

# Outline

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Introduction

The MSSM with GmSUGRA

Numerical Results

Conclusion

# The Supersymmetric Standard Models:

- Solving the gauge hierarchy problem
- Gauge coupling unification
- Radiatively electroweak symmetry breaking
- Natural dark matter candidates
- Electroweak baryogenesis
- Electroweak precision: R parity

# Supersymmetry

- The most promising new physics beyond the Standard Model.
- Gauge coupling unification in the SSMs strongly suggests the Grand Unified Theories (GUTs)

  $SU(5)$, and $SO(10)$, etc.
- The SUSY GUTs can be constructed from superstring theory

  Heterotic $E_8 \times E_8$ string theory, D-brane models, free-fermionic string models, F-theory models, etc.

**Supersymmetry is a bridge between the low energy phenomenology and high-energy fundamental physics.**

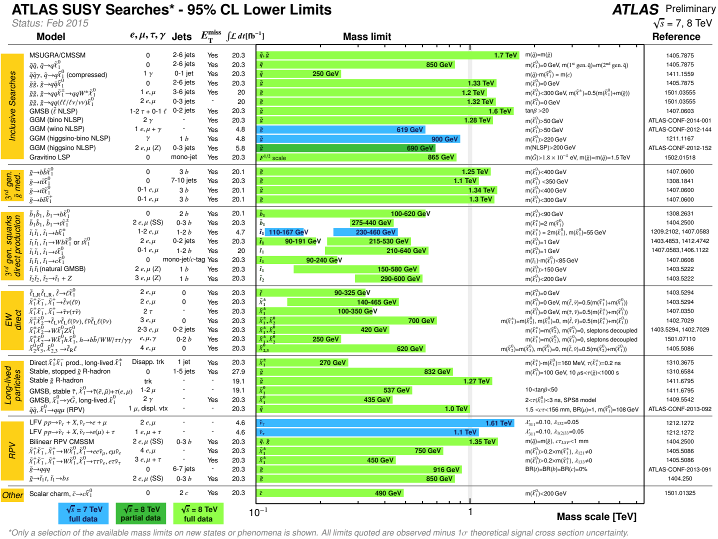
# Higgs boson mass in the MSSM:

- The SM-like Higgs boson mass is around 126 GeV.
- The tree-level Higgs boson mass is smaller than $M_Z$.
- The Higgs boson mass is enhanced by the top quarks/squarks loop corrections.
- The maximal stop mixing is needed to relax the fine-tuning.

# The LHC Supersymmetry Search Contraints:

- The gluino and squark mass low bounds are around 1.7 TeV in the CMSSM/mSUGRA
- The gluino mass low bound is around 1.3 TeV.
- The stop/sbottom mass low bounds are around 600 GeV.

**The SSMs are fine-tuned!**



## ATLAS SUSY Searches* - 95% CL Lower Limits

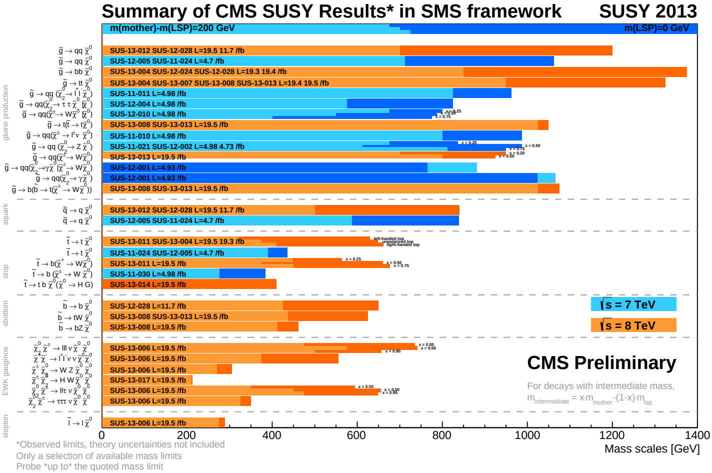
Status: Feb 2015

**ATLAS** Preliminary

$\sqrt{s}$ = 7, 8 TeV

| | Model | $e, \mu, \tau, \gamma$ | Jets | $E_T^{\rm miss}$ | $\int \mathcal{L}\, dt[\mathrm{fb}^{-1}]$ | Mass limit | | | Reference |
|---|---|---|---|---|---|---|---|---|---|
| Inclusive Searches | MSUGRA/CMSSM | 0 | 2-6 jets | Yes | 20.3 | $\tilde{q}, \tilde{g}$ | 1.7 TeV | $m(\tilde{q})=m(\tilde{g})$ | 1405.7875 |
| | $\tilde{q}\tilde{q}, \tilde{q}\rightarrow q\tilde{\chi}_1^0$ | 0 | 2-6 jets | Yes | 20.3 | $\tilde{q}$ | 850 GeV | $m(\tilde{\chi}_1^0)$=0 GeV, m(1st gen. $\tilde{q}$)=m(2nd gen. $\tilde{q}$) | 1405.7875 |
| | $\tilde{q}\tilde{q}\gamma, \tilde{q}\rightarrow q\tilde{\chi}_1^0$ (compressed) | 1 $\gamma$ | 0-1 jet | Yes | 20.3 | $\tilde{q}$ | 250 GeV | $m(\tilde{q})$-$m(\tilde{\chi}_1^0)$ = $m(c)$ | 1411.1559 |
| | $\tilde{g}\tilde{g}, \tilde{g}\rightarrow q\bar{q}\tilde{\chi}_1^0$ | 0 | 2-6 jets | Yes | 20.3 | $\tilde{g}$ | 1.33 TeV | $m(\tilde{\chi}_1^0)$=0 GeV | 1405.7875 |
| | $\tilde{g}\tilde{g}, \tilde{g}\rightarrow qq\tilde{\chi}_1^\pm\rightarrow qqW^\pm\tilde{\chi}_1^0$ | 1 $e, \mu$ | 3-6 jets | Yes | 20 | $\tilde{g}$ | 1.2 TeV | $m(\tilde{\chi}_1^0)$<300 GeV, $m(\tilde{\chi}^\pm)$=0.5($m(\tilde{\chi}_1^0)$+$m(\tilde{g})$) | 1501.03555 |
| | $\tilde{g}\tilde{g}, \tilde{g}\rightarrow qq(\ell\ell/\ell\nu/\nu\nu)\tilde{\chi}_1^0$ | 2 $e, \mu$ | 0-3 jets | - | 20 | $\tilde{g}$ | 1.32 TeV | $m(\tilde{\chi}_1^0)$=0 GeV | 1501.03555 |
| | GMSB ($\tilde{\ell}$ NLSP) | 1-2 $\tau$ + 0-1 $\ell$ | 0-2 jets | Yes | 20.3 | $\tilde{g}$ | 1.6 TeV | tan$\beta$ >20 | 1407.0603 |
| | GGM (bino NLSP) | 2 $\gamma$ | - | Yes | 20.3 | $\tilde{g}$ | 1.28 TeV | $m(\tilde{\chi}_1^0)$>50 GeV | ATLAS-CONF-2014-001 |
| | GGM (wino NLSP) | 1 $e, \mu + \gamma$ | - | Yes | 4.8 | $\tilde{g}$ | 619 GeV | $m(\tilde{\chi}_1^0)$>50 GeV | ATLAS-CONF-2012-144 |
| | GGM (higgsino-bino NLSP) | $\gamma$ | 1 $b$ | Yes | 4.8 | $\tilde{g}$ | 900 GeV | $m(\tilde{\chi}_1^0)$>220 GeV | 1211.1167 |
| | GGM (higgsino NLSP) | 2 $e, \mu$ ($Z$) | 0-3 jets | Yes | 5.8 | $\tilde{g}$ | 690 GeV | m(NLSP)>200 GeV | ATLAS-CONF-2012-152 |
| | Gravitino LSP | 0 | mono-jet | Yes | 20.3 | $F^{1/2}$ scale | 865 GeV | $m(\tilde{G})>1.8\times10^{-4}$ eV, $m(\tilde{g})=m(\tilde{q})$=1.5 TeV | 1502.01518 |
| $3^{rd}$ gen. $\tilde{g}$ med. | $\tilde{g}\rightarrow b\bar{b}\tilde{\chi}_1^0$ | 0 | 3 $b$ | Yes | 20.1 | $\tilde{g}$ | 1.25 TeV | $m(\tilde{\chi}_1^0)$<400 GeV | 1407.0600 |
| | $\tilde{g}\rightarrow t\bar{t}\tilde{\chi}_1^0$ | 0 | 7-10 jets | Yes | 20.3 | $\tilde{g}$ | 1.1 TeV | $m(\tilde{\chi}_1^0)$ <350 GeV | 1308.1841 |
| | $\tilde{g}\rightarrow t\bar{t}\tilde{\chi}_1^0$ | 0-1 $e, \mu$ | 3 $b$ | Yes | 20.1 | $\tilde{g}$ | 1.34 TeV | $m(\tilde{\chi}_1^0)$<400 GeV | 1407.0600 |
| | $\tilde{g}\rightarrow b\bar{t}\tilde{\chi}_1^+$ | 0-1 $e, \mu$ | 3 $b$ | Yes | 20.1 | $\tilde{g}$ | 1.3 TeV | $m(\tilde{\chi}_1^0)$<300 GeV | 1407.0600 |
| $3^{rd}$ gen. squarks direct production | $\tilde{b}_1\tilde{b}_1, \tilde{b}_1\rightarrow b\tilde{\chi}_1^0$ | 0 | 2 $b$ | Yes | 20.1 | $\tilde{b}_1$ | 100-620 GeV | $m(\tilde{\chi}_1^0)$<90 GeV | 1308.2631 |
| | $\tilde{b}_1\tilde{b}_1, \tilde{b}_1\rightarrow t\tilde{\chi}_1^\pm$ | 2 $e, \mu$ (SS) | 0-3 $b$ | Yes | 20.3 | $\tilde{b}_1$ | 275-440 GeV | $m(\tilde{\chi}_1^\pm)$=2 $m(\tilde{\chi}_1^0)$ | 1404.2500 |
| | $\tilde{t}_1\tilde{t}_1, \tilde{t}_1\rightarrow b\tilde{\chi}_1^\pm$ | 1-2 $e, \mu$ | 1-2 $b$ | Yes | 4.7 | $\tilde{t}_1$ | 110-167 GeV<br>230-460 GeV | $m(\tilde{\chi}_1^\pm)$ = 2$m(\tilde{\chi}_1^0)$, $m(\tilde{\chi}_1^0)$=55 GeV | 1209.2102, 1407.0583 |
| | $\tilde{t}_1\tilde{t}_1, \tilde{t}_1\rightarrow Wb\tilde{\chi}_1^0$ or $t\tilde{\chi}_1^0$ | 2 $e, \mu$ | 0-2 jets | Yes | 20.3 | $\tilde{t}_1$ | 90-191 GeV<br>215-530 GeV | $m(\tilde{\chi}_1^0)$=1 GeV | 1403.4853, 1412.4742 |
| | $\tilde{t}_1\tilde{t}_1, \tilde{t}_1\rightarrow t\tilde{\chi}_1^0$ | 0-1 $e, \mu$ | 1-2 $b$ | Yes | 20 | $\tilde{t}_1$ | 210-640 GeV | $m(\tilde{\chi}_1^0)$=1 GeV | 1407.0583,1406.1122 |
| | $\tilde{t}_1\tilde{t}_1, \tilde{t}_1\rightarrow c\tilde{\chi}_1^0$ | 0 | mono-jet/$c$-tag | Yes | 20.3 | $\tilde{t}_1$ | 90-240 GeV | $m(\tilde{t}_1)$-$m(\tilde{\chi}_1^0)$<85 GeV | 1407.0608 |
| | $\tilde{t}_1\tilde{t}_1$(natural GMSB) | 2 $e, \mu$ ($Z$) | 1 $b$ | Yes | 20.3 | $\tilde{t}_1$ | 150-580 GeV | $m(\tilde{\chi}_1^0)$>150 GeV | 1403.5222 |
| | $\tilde{t}_2\tilde{t}_2, \tilde{t}_2\rightarrow\tilde{t}_1 + Z$ | 3 $e, \mu$ ($Z$) | 1 $b$ | Yes | 20.3 | $\tilde{t}_2$ | 290-600 GeV | $m(\tilde{\chi}_1^0)$<200 GeV | 1403.5222 |
| EW direct | $\tilde{\ell}_{\rm L,R}\tilde{\ell}_{\rm L,R}, \tilde{\ell}\rightarrow\ell\tilde{\chi}_1^0$ | 2 $e, \mu$ | 0 | Yes | 20.3 | $\tilde{\ell}$ | 90-325 GeV | $m(\tilde{\chi}_1^0)$=0 GeV | 1403.5294 |
| | $\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\rightarrow\tilde{\ell}\nu(\ell\tilde{\nu})$ | 2 $e, \mu$ | 0 | Yes | 20.3 | $\tilde{\chi}_1^\pm$ | 140-465 GeV | $m(\tilde{\chi}_1^0)$=0 GeV, $m(\tilde{\ell}, \tilde{\nu})$=0.5($m(\tilde{\chi}_1^\pm)$+$m(\tilde{\chi}_1^0)$) | 1403.5294 |
| | $\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\rightarrow\tilde{\tau}\nu(\tau\tilde{\nu})$ | 2 $\tau$ | - | Yes | 20.3 | $\tilde{\chi}_1^\pm$ | 100-350 GeV | $m(\tilde{\chi}_1^0)$=0 GeV, $m(\tilde{\tau}, \tilde{\nu})$=0.5($m(\tilde{\chi}_1^\pm)$+$m(\tilde{\chi}_1^0)$) | 1407.0350 |
| | $\tilde{\chi}_1^\pm\tilde{\chi}_2^0\rightarrow\tilde{\ell}_{\rm L}\nu\tilde{\ell}_{\rm L}\ell(\tilde{\nu}\nu), \ell\tilde{\nu}\tilde{\ell}_{\rm L}\ell(\tilde{\nu}\nu)$ | 3 $e, \mu$ | 0 | Yes | 20.3 | $\tilde{\chi}_1^\pm, \tilde{\chi}_2^0$ | 700 GeV | $m(\tilde{\chi}_1^\pm)=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)$=0, $m(\tilde{\ell}, \tilde{\nu})$=0.5($m(\tilde{\chi}_1^\pm)$+$m(\tilde{\chi}_1^0)$) | 1402.7029 |
| | $\tilde{\chi}_1^\pm\tilde{\chi}_2^0\rightarrow W\tilde{\chi}_1^0 Z\tilde{\chi}_1^0$ | 2-3 $e, \mu$ | 0-2 jets | Yes | 20.3 | $\tilde{\chi}_1^\pm, \tilde{\chi}_2^0$ | 420 GeV | $m(\tilde{\chi}_1^\pm)=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)$=0, sleptons decoupled | 1403.5294, 1402.7029 |
| | $\tilde{\chi}_1^\pm\tilde{\chi}_2^0\rightarrow W\tilde{\chi}_1^0 h\tilde{\chi}_1^0, h\rightarrow b\bar{b}/WW/\tau\tau/\gamma\gamma$ | $e, \mu, \gamma$ | 0-2 $b$ | Yes | 20.3 | $\tilde{\chi}_1^\pm, \tilde{\chi}_2^0$ | 250 GeV | $m(\tilde{\chi}_1^\pm)=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)$=0, sleptons decoupled | 1501.07110 |
| | $\tilde{\chi}_2^0\tilde{\chi}_3^0, \tilde{\chi}_{2,3}^0\rightarrow\tilde{\ell}_{\rm R}\ell$ | 4 $e, \mu$ | 0 | Yes | 20.3 | $\tilde{\chi}_{2,3}^0$ | 620 GeV | $m(\tilde{\chi}_2^0)=m(\tilde{\chi}_3^0)$, $m(\tilde{\chi}_1^0)$=0, $m(\tilde{\ell}, \tilde{\nu})$=0.5($m(\tilde{\chi}_2^0)$+$m(\tilde{\chi}_1^0)$) | 1405.5086 |
| Long-lived particles | Direct $\tilde{\chi}_1^+\tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^\pm$ | Disapp. trk | 1 jet | Yes | 20.3 | $\tilde{\chi}_1^\pm$ | 270 GeV | $m(\tilde{\chi}_1^\pm)$-$m(\tilde{\chi}_1^0)$=160 MeV, $\tau(\tilde{\chi}_1^\pm)$=0.2 ns | 1310.3675 |
| | Stable, stopped $\tilde{g}$ R-hadron | 0 | 1-5 jets | Yes | 27.9 | $\tilde{g}$ | 832 GeV | $m(\tilde{\chi}_1^0)$=100 GeV, 10 $\mu$s<$\tau(\tilde{g})$<1000 s | 1310.6584 |
| | Stable $\tilde{g}$ R-hadron | trk | - | - | 19.1 | $\tilde{g}$ | 1.27 TeV | | 1411.6795 |
| | GMSB, stable $\tilde{\tau}, \tilde{\chi}_1^0\rightarrow\tilde{\tau}(\tilde{e}, \tilde{\mu})+\tau(e, \mu)$ | 1-2 $\mu$ | - | - | 19.1 | $\tilde{\chi}_1^0$ | 537 GeV | 10<tan$\beta$<50 | 1411.6795 |
| | GMSB, $\tilde{\chi}_1^0\rightarrow\gamma\tilde{G}$, long-lived $\tilde{\chi}_1^0$ | 2 $\gamma$ | - | Yes | 20.3 | $\tilde{\chi}_1^0$ | 435 GeV | 2<$\tau(\tilde{\chi}_1^0)$<3 ns, SPS8 model | 1409.5542 |
| | $\tilde{q}\tilde{q}, \tilde{\chi}_1^0\rightarrow qq\mu$ (RPV) | 1 $\mu$, displ. vtx | - | - | 20.3 | $\tilde{q}$ | 1.0 TeV | 1.5 <$c\tau$<156 mm, BR($\mu$)=1, $m(\tilde{\chi}_1^0)$=108 GeV | ATLAS-CONF-2013-092 |
| RPV | LFV $pp\rightarrow\tilde{\nu}_\tau + X, \tilde{\nu}_\tau\rightarrow e + \mu$ | 2 $e, \mu$ | - | - | 4.6 | $\tilde{\nu}_\tau$ | 1.61 TeV | $\lambda'_{311}$=0.10, $\lambda_{132}$=0.05 | 1212.1272 |
| | LFV $pp\rightarrow\tilde{\nu}_\tau + X, \tilde{\nu}_\tau\rightarrow e(\mu) + \tau$ | 1 $e, \mu + \tau$ | - | - | 4.6 | $\tilde{\nu}_\tau$ | 1.1 TeV | $\lambda'_{311}$=0.10, $\lambda_{1(2)33}$=0.05 | 1212.1272 |
| | Bilinear RPV CMSSM | 2 $e, \mu$ (SS) | 0-3 $b$ | Yes | 20.3 | $\tilde{q}, \tilde{g}$ | 1.35 TeV | $m(\tilde{q})=m(\tilde{g})$, $c\tau_{LSP}$<1 mm | 1404.2500 |
| | $\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\rightarrow W\tilde{\chi}_1^0, \tilde{\chi}_1^0\rightarrow ee\tilde{\nu}_\mu, e\mu\tilde{\nu}_e$ | 4 $e, \mu$ | - | Yes | 20.3 | $\tilde{\chi}_1^\pm$ | 750 GeV | $m(\tilde{\chi}_1^0)$>0.2×$m(\tilde{\chi}_1^\pm)$, $\lambda_{121}\neq0$ | 1405.5086 |
| | $\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\rightarrow W\tilde{\chi}_1^0, \tilde{\chi}_1^0\rightarrow\tau\tau\tilde{\nu}_e, e\tau\tilde{\nu}_\tau$ | 3 $e, \mu + \tau$ | - | Yes | 20.3 | $\tilde{\chi}_1^\pm$ | 450 GeV | $m(\tilde{\chi}_1^0)$>0.2×$m(\tilde{\chi}_1^\pm)$, $\lambda_{133}\neq0$ | 1405.5086 |
| | $\tilde{g}\rightarrow qqq$ | 0 | 6-7 jets | - | 20.3 | $\tilde{g}$ | 916 GeV | BR($t$)=BR($b$)=BR($c$)=0% | ATLAS-CONF-2013-091 |
| | $\tilde{g}\rightarrow\tilde{t}_1 t, \tilde{t}_1\rightarrow bs$ | 2 $e, \mu$ (SS) | 0-3 $b$ | Yes | 20.3 | $\tilde{g}$ | 850 GeV | | 1404.250 |
| Other | Scalar charm, $\tilde{c}\rightarrow c\tilde{\chi}_1^0$ | 0 | 2 $c$ | Yes | 20.3 | $\tilde{c}$ | 490 GeV | $m(\tilde{\chi}_1^0)$<200 GeV | 1501.01325 |

*Only a selection of the available mass limits on new states or phenomena is shown. All limits quoted are observed minus 1$\sigma$ theoretical signal cross section uncertainty.


Summary of CMS SUSY Results* in SMS framework
SUSY 2013
m(mother)-m(LSP)=200 GeV
m(LSP)=0 GeV
gluino production
SUS-13-012 SUS-12-028 L=19.5 11.7 /fb
SUS-12-005 SUS-11-024 L=4.7 /fb
SUS-13-004 SUS-12-024 SUS-12-028 L=19.3 19.4 /fb
SUS-13-004 SUS-13-007 SUS-13-008 SUS-13-013 L=19.4 19.5 /fb
SUS-11-011 L=4.98 /fb
SUS-12-004 L=4.98 /fb
SUS-12-010 L=4.98 /fb
SUS-13-008 SUS-13-013 L=19.5 /fb
SUS-11-010 L=4.98 /fb
SUS-11-021 SUS-12-002 L=4.98 4.73 /fb
SUS-13-013 L=19.5 /fb
SUS-12-001 L=4.93 /fb
SUS-12-001 L=4.93 /fb
SUS-13-008 SUS-13-013 L=19.5 /fb
squark
SUS-13-012 SUS-12-028 L=19.5 11.7 /fb
SUS-12-005 SUS-11-024 L=4.7 /fb
stop
SUS-13-011 SUS-13-004 L=19.5 19.3 /fb
left-handed top
unpolarized top
right-handed top
SUS-11-024 SUS-12-005 L=4.7 /fb
SUS-13-011 L=19.5 /fb
SUS-11-030 L=4.98 /fb
SUS-13-014 L=19.5 /fb
sbottom
SUS-12-028 L=11.7 /fb
SUS-13-008 SUS-13-013 L=19.5 /fb
SUS-13-008 L=19.5 /fb
EWK gauginos
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-017 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
slepton
SUS-13-006 L=19.5 /fb
x = 0.25
x = 0.50
x = 0.75
x = 0.20
x = 0.05
x = 0.95
√s = 7 TeV
√s = 8 TeV
CMS Preliminary
For decays with intermediate mass,
m_intermediate = x·m_mother -(1-x)·m_lsp
0
200
400
600
800
1000
1200
1400
Mass scales [GeV]
*Observed limits, theory uncertainties not included
Only a selection of available mass limits
Probe *up to* the quoted mass limit

# Fine-Tuning Definition

- The minimization condition for electroweak symmetry breaking

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \ .$$

- The fine-tuning measure [1]

$$\Delta_{\rm EW} \equiv {\rm Max}\{\frac{2C_i}{M_Z^2}\} \ .$$

**We consider $\Delta_{\rm EW} < 100$.**

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[1] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, Phys. Rev. D **87**, no. 11, 115028 (2013) [arXiv:1212.2655 [hep-ph]].

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Introduction

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# Grand Unified Theories with Gravity Mediated Supersymmetry Breaking (mSUGRA/CMSSM): [2]

- The supersymmetry breaking soft terms: $M_{1/2}$, $M_0$, $A_0$, $\tan\beta$, $\mathrm{sgn}(\mu)$.

$$\frac{1}{\alpha_3} = \frac{1}{\alpha_2} = \frac{1}{\alpha_1} ~,$$

$$\frac{M_3}{\alpha_3} = \frac{M_2}{\alpha_2} = \frac{M_1}{\alpha_1} ~.$$

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[2] A. H. Chamseddine, R. L. Arnowitt and P. Nath, Phys. Rev. Lett. **49**, 970 (1982); H. P. Nilles, Phys. Lett. B **115**, 193 (1982); L. E. Ibanez, Phys. Lett. B **118**, 73 (1982); R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B **119**, 343 (1982); H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **120**, 346 (1983); J. R. Ellis, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **121**, 123 (1983); J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **125**, 275 (1983); L. J. Hall, J. D. Lykken and S. Weinberg, Phys. Rev. D **27**, 2359 (1983).

# Modification of Gauge Coupling Unification [3]

- The SM gauge couplings need not be unified at the GUT Scale due to the high-dimensional operators

$$\mathcal{L} \supset \frac{c}{M_*} \mathrm{Tr} \left( \Phi F_{\mu\nu} F^{\mu\nu} \right) ~.$$

$$\delta(1/g_3^2) ~:~ \delta(1/g_2^2) ~:~ \delta(1/g_1^2) ~=~ 2 ~:~ -3 ~:~ -1$$

- $M_*$ can be the reduced Planck scale, string scale, or compactification scale.

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[3] C. T. Hill, Phys. Lett. B **135**, 47 (1984); Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52**, 875 (1984). J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B **247**, 373 (1984); J. R. Ellis, K. Enqvist, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B **155**, 381 (1985); M. Drees, Phys. Lett. B **158**, 409 (1985).

- In the GUTs with large number of fields, the renormalization effects significantly decrease the scale at which quantum gravity (or fundamental scale) becomes strong [4]

$$M_* = \frac{M_{\rm Pl}}{\Delta}, \quad \Delta = \sqrt{1+\frac{N}{12\pi}}, \quad N = N_0 + N_{1/2} - 4N_1 .$$

- This can be realized in the $SU(5)$ Models with $U(1)_Y$ flux [5] and $SO(10)$ Models with $U(1)_{B-L}$ flux [6].

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[4] X. Calmet, S. D. H. Hsu and D. Reeb, Phys. Rev. Lett. **101**, 171802 (2008).

[5] R. Donagi and M. Wijnholt, Adv. Theor. Math. Phys. **15**, no. 6, 1523 (2011) [arXiv:0808.2223 [hep-th]]; R. Blumenhagen, Phys. Rev. Lett. **102**, 071601 (2009).

[6] T. Li, Phys. Rev. D **81**, 065018 (2010) [arXiv:0905.4563 [hep-th]].

# The Generalized mSUGRA (GmSUGRA): [7]

- Gauge coupling relation

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_3} = k\left(\frac{1}{\alpha_1} - \frac{1}{\alpha_3}\right) ,$$

- Gaugino mass relation

$$\frac{M_2}{\alpha_2} - \frac{M_3}{\alpha_3} = k\left(\frac{M_1}{\alpha_1} - \frac{M_3}{\alpha_3}\right) ,$$

- Index $k$ might be obtained from the LHC and ILC. We choose $k = 5/3$, and consider $M_1$ and $M_2$ as free parameters.

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[7] TL and D. V. Nanopoulos, Phys. Lett. B **692**, 121 (2010) [arXiv:1002.4183 [hep-ph]]; C. Balazs, TL, D. V. Nanopoulos and F. Wang, JHEP **1009**, 003 (2010) [arXiv:1006.5559 [hep-ph]].

# The GmSUGRA

- Choosing slepton masses as input parameters, we can parametrize the squark masses as follows

$$\begin{aligned} m^2_{\tilde{Q}_i} &= \frac{5}{6}(m_0^U)^2 + \frac{1}{6} m^2_{\tilde{E}_i^c} ~, \\ m^2_{\tilde{U}_i^c} &= \frac{5}{3}(m_0^U)^2 - \frac{2}{3} m^2_{\tilde{E}_i^c} ~, \\ m^2_{\tilde{D}_i^c} &= \frac{5}{3}(m_0^U)^2 - \frac{2}{3} m^2_{\tilde{L}_i} ~. \end{aligned}$$

- The Higgs soft masses $m_{\tilde{H}_u}$ and $m_{\tilde{H}_d}$, and the trilinear soft terms $A_U$, $A_D$ and $A_E$.

# The Higgs couplings

- The deviations from the SM Higgs couplings

$$k_i \equiv g_{hii}^{\rm SUSY}/g_{hii}^{\rm SM} ~,$$

where $i = W, Z, b, \tau, t, g, \gamma$.

- The Higgs couplings to gauge bosons

$$k_V \equiv \frac{g_{hVV}^{\rm SUSY}}{g_{hVV}^{\rm SM}} \sim 1 - \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\frac{2}{\tan^2\beta}\right), \text{ for V = W, Z.}$$

$$k_g \approx 1 + \frac{m_t^2}{4}\left[\frac{1}{m_{\tilde{t}_1}^2} + \frac{1}{m_{\tilde{t}_2}^2} - \frac{X_t^2}{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}\right] ~,$$

where $X_t = \mid A_t - \mu/\tan\beta \mid$ is the stop mixing parameter.

# The Higgs couplings

- The Higgs couplings to the SM fermions

$$
\begin{aligned}
k_{b,\tau} &\sim 1 + \mathcal{O}\left(2\frac{m_Z^2}{m_A^2}\right) ~, \\
k_t &\sim 1 - \mathcal{O}\left(\frac{m_Z^2}{m_A^2}\frac{2}{\tan^2\beta}\right) ~.
\end{aligned}
$$

Thus, $\delta k_{b,\tau}$ and $\delta k_t$ are predicted to have opposite sign.

# The anomalous magnetic moment of the muon

- The discrepancy of $a_\mu = (g-2)_\mu/2$

$$\Delta a_\mu \equiv a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) = (28.7 \pm 8.0) \times 10^{-10} ~.$$

- The neutralino-smuon loop contribution

$$\Delta a_\mu^{\tilde{\chi}^0 \tilde{\mu}} ~~\simeq \frac{1}{192\pi^2} \frac{m_\mu^2}{M_{SUSY}^2} \left( \mathrm{sgn}(\mu \mathrm{M}_1) g_1^2 - \mathrm{sgn}(\mu \mathrm{M}_2) g_2^2 \right) \tan\beta ~.$$

# The anomalous magnetic moment of the muon

- The chargino-sneutrino loop contribution

$$\Delta a_\mu^{\tilde{\chi}^\pm \tilde{\nu}} \simeq \mathrm{sgn}(\mu \mathrm{M}_2) \frac{1}{32\pi^2} \frac{m_\mu^2}{M_{SUSY}^2} g_2^2 \tan\beta \ .$$

- $\Delta a_\mu \sim 10^{-9} \left(\frac{100 \ \mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 \tan\beta$ for $\mathrm{sgn}(\mu M_2) > 0$.
- The $2\sigma$ bound on $\Delta a_\mu$ can be achieved for $\tan\beta = 10$ if four relevant sparticles are lighter than $600 - 700$ GeV. While for smaller $\tan\beta$ ($\sim$3), the lighter sparticles ($\lesssim$ 500 GeV) are needed.

# Outline

- Introduction
- The MSSM with GmSUGRA
- Numerical Results
- Conclusion

# The parameter space for scan

$$\begin{aligned}
0\,\mathrm{GeV} &\leq& m_0^U \leq 9000\,\mathrm{GeV}, \\
100\,\mathrm{GeV} &\leq& M_1 \leq 2000\,\mathrm{GeV}, \\
100\,\mathrm{GeV} &\leq& M_2 \leq 2100\,\mathrm{GeV}, \\
100\,\mathrm{GeV} &\leq& m_{\tilde{L}} \leq 1200\,\mathrm{GeV}, \\
100\,\mathrm{GeV} &\leq& m_{\tilde{E}^c} \leq 1200\,\mathrm{GeV}, \\
100\,\mathrm{GeV} &\leq& \mu \leq 1500\,\mathrm{GeV}, \\
-16000\,\mathrm{GeV} &\leq& A_U = A_D \leq 18000\,\mathrm{GeV}, \\
-6000\,\mathrm{GeV} &\leq& A_E \leq 6000\,\mathrm{GeV}, \\
0\,\mathrm{GeV} &\leq& m_A \leq 9500\,\mathrm{GeV}, \quad 2 \leq \tan\beta \leq 60, \\
\mu &>& 0, \quad m_t = 173.3\ \mathrm{GeV}, \quad m_b^{\bar{\mathrm{DR}}}(M_Z) = 2.83\ \mathrm{GeV}.
\end{aligned}$$

# Basic constraints (I)

- The Radiative Electroweak Symmetry Breaking (REWSB).
- One of the neutralinos is the LSP.
- The Higgs boson mass

$$123\,\mathrm{GeV} \leq m_h \leq 127\,\mathrm{GeV}\,.$$

- The fine-tuning measure $\Delta_{\mathrm{EW}} \leq 100$.

# Basic constraints (I)

- The sparticle mass bounds

$$\begin{aligned} m_{\tilde{t}_1}, m_{\tilde{b}_1} &\geq 100\,\mathrm{GeV}, \\ m_{\tilde{\tau}_1} &\geq 105\,\mathrm{GeV}, \\ m_{\tilde{\chi}_1^{\pm}} &\geq 103\,\mathrm{GeV}. \end{aligned}$$

$$\begin{aligned} 1.7\,\mathrm{TeV} \leq m_{\tilde{g}}\ (\mathrm{for}\ m_{\tilde{g}} \sim m_{\tilde{q}}) &\quad , \\ 1.3\,\mathrm{TeV} \leq m_{\tilde{g}}\ (\mathrm{for}\ m_{\tilde{g}} \ll m_{\tilde{q}}) &\quad , \\ 300\,\mathrm{GeV} \leq m_A &\quad . \end{aligned}$$

# Basic constraints (I)

- B-physics

$$
\begin{aligned}
1.6 \times 10^{-9} \leq \mathrm{BR}(B_s \to \mu^+\mu^-) \leq 4.2 \times 10^{-9} \ (2\sigma) \quad &, \\
2.99 \times 10^{-4} \leq \mathrm{BR}(b \to s\gamma) \leq 3.87 \times 10^{-4} \ (2\sigma) \quad &, \\
0.70 \times 10^{-4} \leq \mathrm{BR}(B_u \to \tau\nu_\tau) \leq 1.5 \times 10^{-4} \ (2\sigma) \quad &.
\end{aligned}
$$

# Muon anomalous magnetic moment constraint (II)

$$4.7 \times 10^{-10} \leq \Delta a_{\mu} \leq 52.7 \times 10^{-10} \ (3\sigma) \ .$$

# Higgs coupling constraints (III)

| Collider | HL-LHC | ILC | CEPC (2 IP) | FCC-ee (4 IP) |
|---|---|---|---|---|
| $\sqrt{s}$ (GeV) | 14000 | 500 | 240 | 240 |
| $\mathcal{L}$ ($\mathrm{fb}^{-1}$) | 3000 | 500 | 5000 | 10000 |
| $(e^-, e^+)$ | - | (-0.8, +0.3) | (0, 0) | (0, 0) |
| $k_g$ | 9.1 | 2.3 | 1.5 | 1.1 |
| $k_W$ | 5.1 | 1.2 | 1.2 | 0.85 |
| $k_Z$ | 4.4 | 1.0 | 0.26 | 0.16 |
| $k_\gamma$ | 4.9 | 8.4 | 4.7 | 1.7 |
| $k_b$ | 12 | 1.7 | 1.3 | 0.88 |
| $k_\tau$ | 9.7 | 2.4 | 1.4 | 0.94 |
| $k_t$ | 11 | 14 | - | - |

Table: Summary for the precisions of Higgs boson coupling measurements in percentage at different colliders.

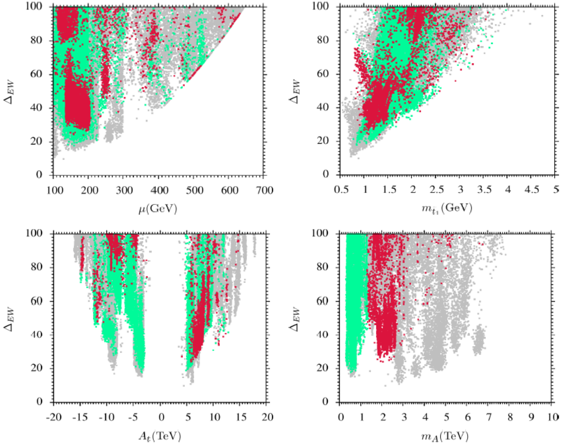


Figure: $\Delta_{EW}$ vs. $\mu$, $m_{\tilde{t}_1}$, $A_t$ and $m_A$: Grey points satisfy the basic constraints (**I**). Green points are a subset of grey points and satisfy the muon magnetic moment constraint (**II**). Red points are a subset of green points and satisfy the Higgs coupling constraint (**III**) from CEPC (2 IP) only.

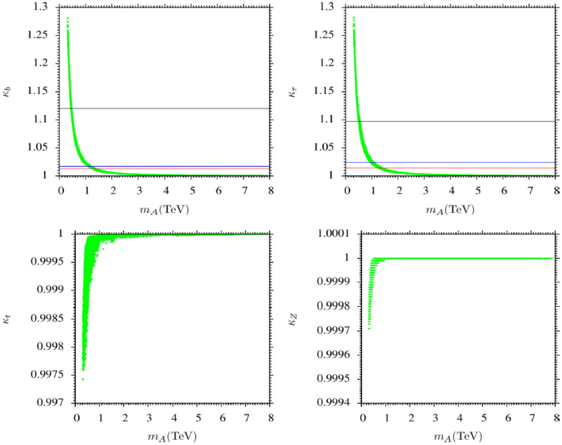


Figure: $k_b$, $k_\tau$, $k_t$ and $k_Z$ vs. $m_A$: Green points satisfy the basic constraints (**I**). The horizontal lines (if shown) in these plots label the precisions at different colliders: HL-LHC (black), ILC (blue) and CEPC (2 IP) (red) respectively.

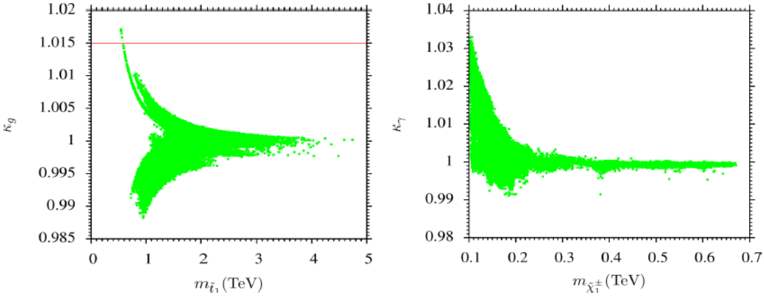


Figure: $k_g$ vs. $m_{\tilde{t}_1}$ and $k_\gamma$ vs. $m_{\tilde{\chi}_1^\pm}$.

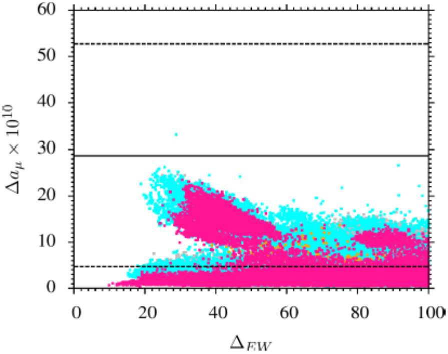


Figure: $\Delta a_\mu$ vs. $\Delta_{EW}$: Grey points satisfy the basic constraints (**I**). Cyan, orange and red points are subsets of grey points and satisfy the Higgs coupling constraint (**III**) from HL-LHC, ILC, and CEPC (2 IP), respectively. The solid horizontal line displays the central value of $\Delta a_\mu$ from the muon magnetic moment experiment, while the dash lines show the 3-$\sigma$ values.

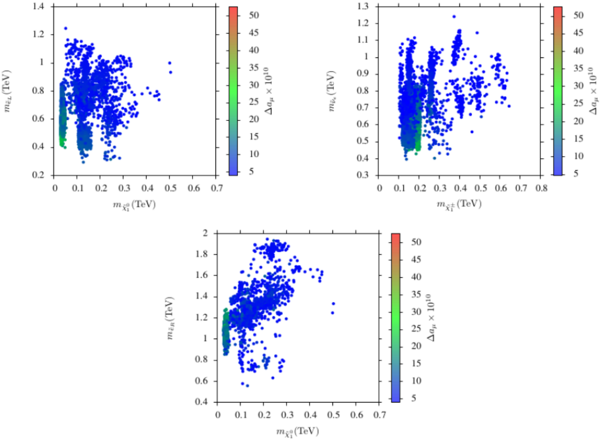


Figure: $m_{\tilde{e}_{R,L}}$ vs. $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\nu}_e}$ vs. $m_{\tilde{\chi}_1^\pm}$: Points satisfy both the basic constraints (**I**), the muon magnetic moment constraint (**II**) and the Higgs coupling constraint (**III**) from the CEPC (2 IP) only. Color represents the $\Delta a_\mu$ values.

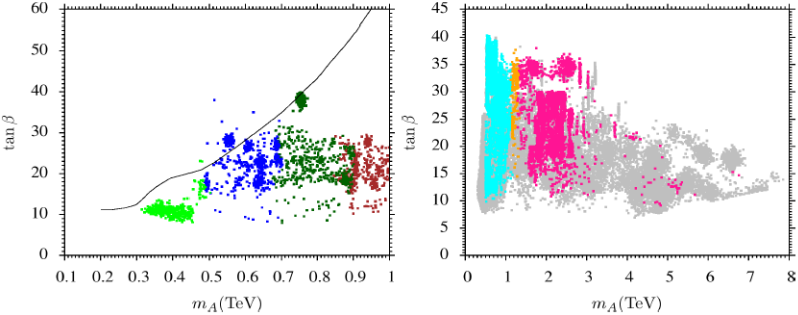
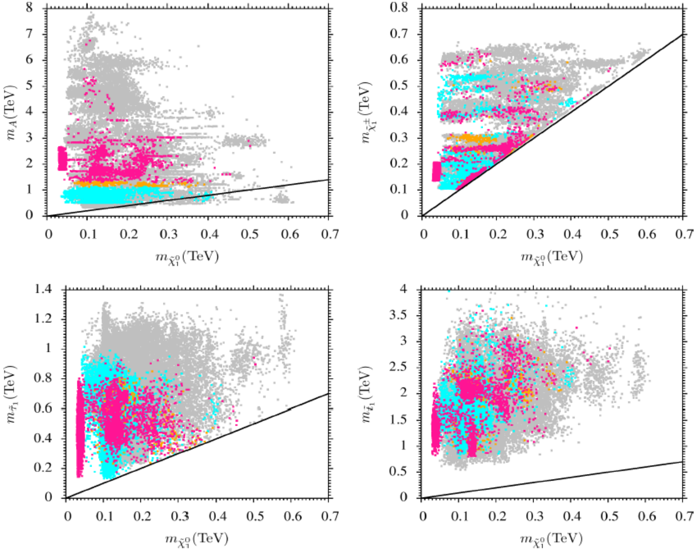


Figure: $\tan\beta$ vs. $m_A$: In the left plot, all points satisfy the basic constraints (**I**) and $m_A \leq 2\mu$; green, blue, dark green and brown points represent $k_b \geq 1.1$, $1.05 \leq k_b \leq 1.1$, $1.03 \leq k_b \leq 1.05$ and $k_b \leq 1.03$ respectively; black curve shows the $1-\sigma$ bound from the ATLAS direct heavy Higgs search. In the right plot, grey points satisfy the basic constraints (**I**); cyan, orange and red points are subsets of grey points, and satisfy both the muon magnetic moment constraint (**II**) and the Higgs coupling constraint (**III**) from HL-LHC, ILC, and CEPC (2 IP), respectively.


$m_A$(TeV)
8
7
6
5
4
3
2
1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$m_{\tilde{\chi}_1^0}$(TeV)
$m_{\tilde{\chi}_1^\pm}$(TeV)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$m_{\tilde{\chi}_1^0}$(TeV)
$m_{\tilde{\tau}_1}$(TeV)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$m_{\tilde{\chi}_1^0}$(TeV)
$m_{\tilde{t}_1}$(TeV)
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$m_{\tilde{\chi}_1^0}$(TeV)

| | Point 1 | Point 2 | Point 3 | Point 4 |
|---|---|---|---|---|
| $m_{0^U}$ | 1037 | 2582 | 5057 | 5327 |
| $m_{\tilde{Q}}$ | 1003.3 | 2366.4 | 4631 | 4879.2 |
| $m_{\tilde{U}^c}$ | 1162.1 | 3306.7 | 6487 | 6830.6 |
| $m_{\tilde{D}^c}$ | 1311.7 | 3326.3 | 6474.9 | 6841.3 |
| $m_{\tilde{L}}$ | 328.1 | 264.8 | 1023 | 858.2 |
| $m_{\tilde{E}^c}$ | 814.1 | 514.8 | 900.9 | 978.6 |
| $M_1$ | 857.1 | 350.9 | 602.7 | 686.5 |
| $M_2$ | 656 | 993 | 634.3 | 828.8 |
| $M_3$ | 1158.8 | -612.25 | 555.3 | 473.05 |
| $A_t = A_b$ | -3292 | 5390 | -9095 | -9684 |
| $A_{\tilde{\tau}}$ | -691.6 | 1192 | -1570 | -2937 |
| $\tan\beta$ | 11.6 | 33.8 | 19.7 | 21.4 |
| $\mu$ | 183 | 172.5 | 377.5 | 168.2 |
| $m_A$ | 338.9 | 754.9 | 603 | 846 |
| $\Delta_{HS}$ | 1883 | 3151 | 10873 | 11993.67 |
| $\Delta_{EW}$ | 27 | 23 | 87 | 87 |
| $\Delta a_\mu$ | $4.731 \times 10^{-10}$ | $16.63 \times 10^{-10}$ | $7.683 \times 10^{-10}$ | $12.161 \times 10^{-10}$ |
| $m_h$ | 123 | 123 | 125 | 125 |
| $m_H$ | 342 | 751 | 607 | 856 |
| $m_{H^\pm}$ | 348 | 750 | 608 | 850 |
| $\kappa_b, \kappa_t$ | 1.22541, 0.99817 | 1.04295, 0.99996 | 1.06459, 0.99983 | 1.03296, 0.99992 |
| $\kappa_\tau$, $\kappa_W = \kappa_Z$ | 1.22756, 0.99981 | 1.04787, 0.99999 | 1.06644, 0.99999 | 1.03454, 0.99999 |
| $\kappa_g, \kappa_\gamma$ | 0.99083, 1.00369 | 1.00473, 0.99763 | 0.99954, 1.00008 | 1.00038, 0.99714 |
| $m_{\tilde{\chi}^0_{1,2}}$ | 171, 191 | 131, 183 | 260, 376 | 161, 178 |
| $m_{\tilde{\chi}^0_{3,4}}$ | 368, 541 | 195, 834 | 390, 567 | 312, 717 |
| $m_{\tilde{\chi}^\pm_{1,2}}$ | 185, 534 | 179, 823 | 378, 555 | 175, 702 |
| $m_{\tilde{g}}$ | 2548 | 1539 | 1498 | 1317 |
| $m_{\tilde{u}_{L,R}}$ | 2419, 2558 | 2705, 3476 | 4721, 6513 | 4942, 6832 |
| $m_{\tilde{t}_{1,2}}$ | 1036, 1798 | 1151, 1762 | 2311, 3401 | 2326, 3543 |
| $m_{\tilde{d}_{L,R}}$ | 2421, 2532 | 2706, 3547 | 4722, 6595 | 4943, 6938 |
| $m_{\tilde{b}_{1,2}}$ | 1771, 2468 | 1212, 3089 | 2380, 6340 | 2401, 6630 |
| $m_{\tilde{\nu}_{1,2}}$ | 726 | 521 | 698 | 507 |
| $m_{\tilde{\nu}_3}$ | 718 | 343 | 645 | 308 |
| $m_{\tilde{e}_{L,R}}$ | 737, 525 | 514, 787 | 651, 1409 | 398, 1487 |
| $m_{\tilde{\tau}_{1,2}}$ | 518, 728 | 353, 568 | 627, 1338 | 185, 1333 |
| $\Omega_{CDM} h^2$ | 0.0014 | 0.0846 | 0.1017 | 0.0099 |

# Outline

Introduction

The MSSM with GmSUGRA

Numerical Results

Conclusion

# The Natural Supersymmetry from GmSUGRA at the CEPC

- $\Delta_{\rm EW} \sim 30$, consistence with all constraints, and having supersymmetric contributions to the muon anomalous magnetic moment within $1\sigma$ can be achieved.
- The precision of $k_b$ and $k_\tau$ measurements at CEPC can bound $m_A$ to be above 1.2 TeV and 1.1 TeV respectively.
- The combination of the Higgs coupling measurement and muon anomalous magnetic moment measurement constrain $\tilde{e}_R$ mass to be in the range from 0.6 TeV to 2 TeV. The range of both $\tilde{e}_L$ and $\tilde{\nu}_e$ masses is 0.4 TeV $\sim$ 1.2 TeV. And the $\tilde{\chi}_1^0$ mass needs to be small (mostly $\leq$ 400 GeV).
- The comparison of bounds in the $\tan\beta - m_A$ plane shows that the Higgs coupling measurement is complementary to the direct collider searches for heavy Higgs.

Thank You Very Much
for Your Attention!