Effective Operators, New Physics and CEPC400

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Outline

- Effective Operators beyond SM at CEPC
- Tree Level & Loop effects (The Covariant Derivative Expansion); the RG running and operator mixing between different operators.
- CEPC 400 and Higgs self couplings
- Summary:

Why Effective Operators at the lepton colldiers?

CEPC

Circular e^+ e^- collider with center of mass energy 240 GeV

What we use effective operators?

- Fixed energy: EFT is always valid.
- General model independent parametrization & categorization.
- Simply map to the lepton collider measurments

CEPC

Cicular e^+ e^- collider with center of mass energy 240 GeV

What can it go beyond the LEP?

- EW precision
- Tri-gauge boson precision
- Higgs precision

CEPC

Underlying Models	Simplified Models	Effective Operators	CEPC Observables
Models with new symmetries, dynamics to the interpret EWSB, naturalness, etc.	Just some new particles, or some strong dynamics	Many operators	Total cross section, angle distributions, etc

Effective Operators Beyond SM (TGC example)

Operators beyond SM

There are 81 operators up to dimension 6, including one dimension 5 operator which gives the neutrino mass (Weinberg operator)

Flavor diagonal, no B-violating.

For the 80 d=6 operators, e.o.m. and CP conserving would reduce this number to 52

Let's see what an electron collider can do for those operators before Higgs discovery

Operators beyond SM

Independent observables related to LEP I, II

bosonic fields	Higgs/fermions	4-fermion
$\mathcal{O}_{WB} = \left(H^\dagger \sigma^a H\right) W^a_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}^s_{hl} = i\left(H^\dagger D^\mu H\right)\left(\bar{L}_L \gamma_\mu L_L\right)$	$\mathcal{O}^s_{ll} = \frac{1}{2}\left(\bar{L}_L \gamma^\mu L_L\right)\left(\bar{L}_L \gamma_\mu L_L\right)$
$\mathcal{O}_h = \left(H^\dagger D_\mu H\right)^2$	$\mathcal{O}^s_{hq} = i\left(H^\dagger D^\mu H\right)\left(\bar{Q}_L \gamma_\mu Q_L\right)$	$\mathcal{O}^s_{lq} = \left(\bar{L}_L \gamma^\mu L_L\right)\left(\bar{Q}_L \gamma_\mu Q_L\right)$
$\mathcal{O}_W = \epsilon^{abc} W^{a\nu}_\mu W^{b\lambda}_\nu W^{c\mu}_\lambda$	$\mathcal{O}_{hu} = i\left(H^\dagger D^\mu H\right)\left(\bar{u}_R \gamma_\mu u_R\right)$	$\mathcal{O}_{le} = \left(\bar{L}_L \gamma^\mu L_L\right)\left(\bar{e}_R \gamma_\mu e_R\right)$
	$\mathcal{O}_{he} = i\left(H^\dagger D^\mu H\right)\left(\bar{e}_R \gamma_\mu e_R\right)$	$\mathcal{O}_{lu} = \left(\bar{L}_L \gamma^\mu L_L\right)\left(\bar{u}_R \gamma_\mu u_R\right)$
	$\mathcal{O}^t_{hl} = i\left(H^\dagger \sigma^a D^\mu H\right)\left(\bar{L}_L \gamma_\mu \sigma^a L_L\right)$	$\mathcal{O}_{ee} = \frac{1}{2}\left(\bar{e}_R \gamma^\mu e_R\right)\left(\bar{e}_R \gamma_\mu e_R\right)$
	$\mathcal{O}^t_{hq} = i\left(H^\dagger \sigma^a D^\mu H\right)\left(\bar{Q}_L \gamma_\mu Q_L\right)$	$\mathcal{O}^t_{ll} = \frac{1}{2}\left(\bar{L}_L \gamma^\mu \sigma^a L_L\right)\left(\bar{L}_L \gamma_\mu \sigma^a L_L\right)$
	$\mathcal{O}_{hd} = i\left(H^\dagger D^\mu H\right)\left(\bar{d}_R \gamma_\mu d_R\right)$	$\mathcal{O}^t_{lq} = \left(\bar{L}_L \gamma^\mu L_L\right)\left(\bar{Q}_L \gamma_\mu \sigma^a Q_L\right)$
		$\mathcal{O}_{qe} = \left(\bar{Q}_L \gamma^\mu Q_L\right)\left(\bar{e}_R \gamma_\mu e_R\right)$
		$\mathcal{O}_{ld} = \left(\bar{L}_L \gamma^\mu L_L\right)\left(\bar{d}_R \gamma_\mu d_R\right)$
		$\mathcal{O}_{eu} = \left(\bar{e}_R \gamma^\mu e_R\right)\left(\bar{u}_R \gamma_\mu u_R\right)$
		$\mathcal{O}_{ed} = \left(\bar{e}_R \gamma^\mu e_R\right)\left(\bar{d}_R \gamma_\mu d_R\right)$

Bosonic fields

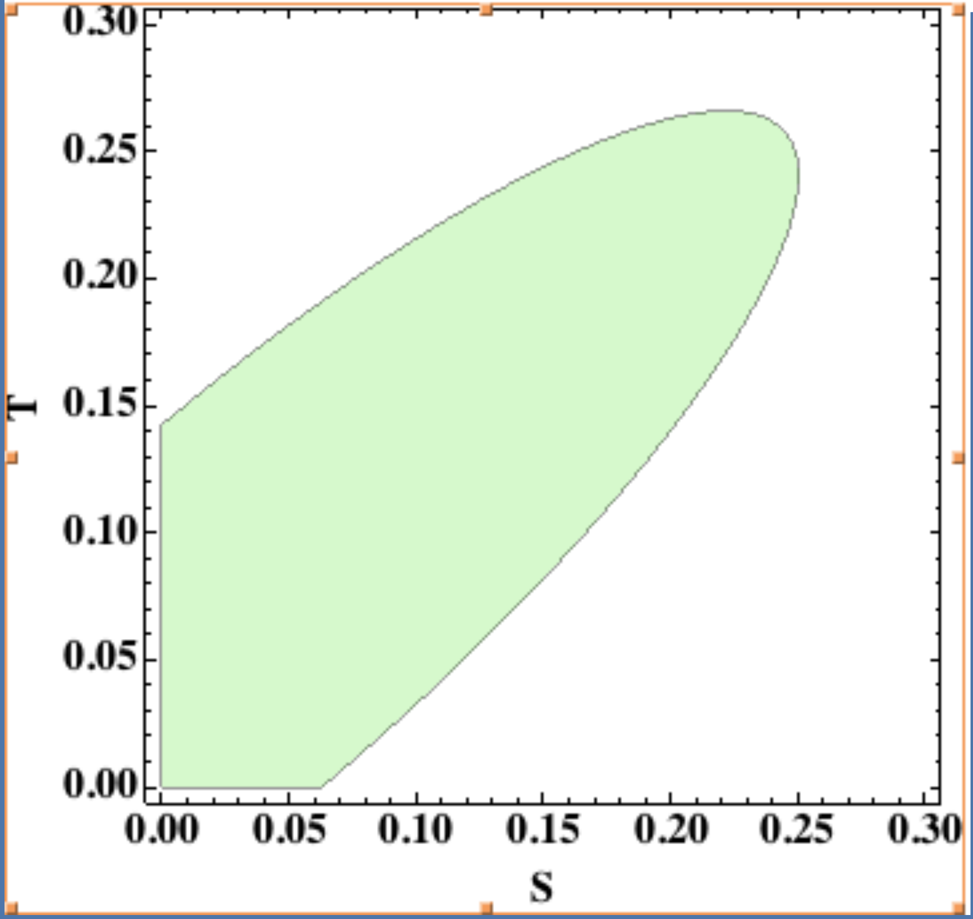
$$\mathcal{O}_{WB} = \left(H^\dagger \sigma^a H\right) W^a_{\mu\nu} B^{\mu\nu}$$
$$\mathcal{O}_h = \left(H^\dagger D_\mu H\right)^2$$
$$\mathcal{O}_W = \epsilon^{abc} W^{a\nu}_{\mu} W^{b\lambda}_{\nu} W^{c\mu}_{\lambda}$$

Famous S, T parameter

$$a_{WB} = \frac{1}{4sc}\frac{\alpha}{v^2}S, \;\; a_h = -2\frac{\alpha}{v^2}T,$$

Let's consider different BSM examples

95% C. L.



Operators beyond SM

Practically, this is more complicated since we need to consider redundant operators for convenience.

Consider a simple case where one integrate out a vector SU(2)_L triplet in MCHM.

$$+\frac{1}{g_{\rho_L}^2 f^2}\mathcal{O}_W + \frac{1}{g_{\rho|_R}^2 f^2}\mathcal{O}_B + \frac{g^2}{g_{\rho_L}^2 m_{\rho_L}^2}\mathcal{O}_{2W} + \frac{g'^2}{g_{\rho_R}^2 m_{\rho_R}^2}\mathcal{O}_{2B}.$$

The first two terms are related with the S parameter,

$$\mathcal{O}_B = \mathcal{O}_{HB} + \mathcal{O}_{BB} + \mathcal{O}_{WB}\,,$$
$$\mathcal{O}_W = \mathcal{O}_{HW} + \mathcal{O}_{WW} + \mathcal{O}_{WB}\,,$$

W & Y can be rewrite using the e.o.m. of the gauge fields

$$(D_\nu W^{\nu\mu})^a = -\frac{1}{2}g(ih^\dagger \overleftrightarrow{D}^\mu \sigma^a h + \bar{l}\gamma^\mu\sigma^a l + \bar{q}\gamma^\mu\sigma^a q),$$
$$\partial_\nu B^{\nu\mu} = -\frac{1}{2}g'(ih^\dagger \overleftrightarrow{D}^\mu h) - g'\sum_f Y_f \bar{f}\gamma^\mu f,$$

Operators beyond SM

But certainly we can do those e.o.m. and generate too many independent operators to do the constrain:

Therefore, one should include all the redundant operators for the fits

$\mathcal{O}_{GG}$	$=$	$g_s^2\,\|H\|^2\,G^a_{\mu\nu}G^{a,\mu\nu}$	$\mathcal{O}_H$	$=$	$\frac{1}{2}(\partial_\mu\,\|H\|^2)^2$
$\mathcal{O}_{WW}$	$=$	$g^2\,\|H\|^2\,W^a_{\mu\nu}W^{a,\mu\nu}$	$\mathcal{O}_T$	$=$	$\frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$
$\mathcal{O}_{BB}$	$=$	$g'^2\,\|H\|^2\,B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_R$	$=$	$\|H\|^2\,\|D_\mu H\|^2$
$\mathcal{O}_{WB}$	$=$	$2gg'H^\dagger\tau^a H W^a_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_D$	$=$	$\|D^2 H\|^2$
$\mathcal{O}_W$	$=$	$ig(H^\dagger\tau^a\overleftrightarrow{D}^\mu H)D^\nu W^a_{\mu\nu}$	$\mathcal{O}_6$	$=$	$\|H\|^6$
$\mathcal{O}_B$	$=$	$ig'Y_H(H^\dagger\overleftrightarrow{D}^\mu H)\partial^\nu B_{\mu\nu}$	$\mathcal{O}_{2G}$	$=$	$-\frac{1}{2}(D^\mu G^a_{\mu\nu})^2$
$\mathcal{O}_{3G}$	$=$	$\frac{1}{3!}g_s f^{abc}G^{a\mu}_\rho G^{b\nu}_\mu G^{c\rho}_\nu$	$\mathcal{O}_{2W}$	$=$	$-\frac{1}{2}(D^\mu W^a_{\mu\nu})^2$
$\mathcal{O}_{3W}$	$=$	$\frac{1}{3!}g\epsilon^{abc}W^{a\mu}_\rho W^{b\nu}_\mu W^{c\rho}_\nu$	$\mathcal{O}_{2B}$	$=$	$-\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$

CP conserving bosonic operators

The CHM

In the CCWZ formulism of MCHM, integrating out the heavy spin one vector meson "rho" and axi-vector meson "a"

$$\Delta\mathcal{L} = -\frac{\Delta^2}{4g_a^2}(d_{\mu\nu}^{\hat{a}})^2 - \frac{1}{4g_{\rho_L}^2}(E_{\mu\nu}^{aL})^2 - \frac{1}{4g_{\rho_R}^2}(E_{\mu\nu}^{aR})^2 - \frac{1}{2}\frac{1}{m_{\rho_L}^2 g_{\rho_L}^2}D_\mu E^{aL\mu\nu} D_\rho E^{aL\rho}{}_\nu - \frac{1}{2}\frac{1}{m_{\rho_R}^2 g_{\rho_R}^2}D_\mu E^{aR\mu\nu} D_\rho E^{aR\rho}{}_\nu + \cdots ,$$

$$= -\frac{\Delta^2}{g_a^2 f^2}\left(\mathcal{O}_W + \mathcal{O}_B - (\mathcal{O}_{HW} + \mathcal{O}_{HB})\right) + \frac{1}{g_{\rho_L}^2 f^2}\mathcal{O}_W + \frac{1}{g_{\rho_R}^2 f^2}\mathcal{O}_B + \frac{g^2}{g_{\rho_L}^2 m_{\rho_L}^2}\mathcal{O}_{2W} + \frac{g'^2}{g_{\rho_R}^2 m_{\rho_R}^2}\mathcal{O}_{2B}.$$

rho contributes to S, W, Y

a contributes to -S, TGC

One loop diagram needed (large rho coupling)

Real singlet for EWPT

$$\Delta\mathcal{L} = \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{1}{2}m^2\Phi^2 - A\,|H|^2\,\Phi - \frac{1}{2}k\,|H|^2\,\Phi^2 - \frac{1}{3!}\mu\Phi^3 - \frac{1}{4!}\lambda_\Phi\Phi^4.$$

Tree Level:

$$\begin{aligned}\Delta\mathcal{L}_{\text{eff,tree}} &= -A\,|H|^2\,\Phi_c + \frac{1}{2}\Phi_c\left(-\partial^2 - m^2 - k\,|H|^2\right)\Phi_c - \frac{1}{3!}\mu\Phi_c^3 - \frac{1}{4!}\lambda_\Phi\Phi_c^4\\ &\approx \frac{1}{2m^2}A^2\,|H|^4 + \frac{A^2}{m^4}\mathcal{O}_H + \left(-\frac{kA^2}{2m^4} + \frac{1}{3!}\frac{\mu A^3}{m^6}\right)\mathcal{O}_6.\end{aligned}$$

One loop:

$$\begin{aligned}\Delta\mathcal{L}_{\text{eff,1-loop}} &= \frac{1}{2(4\pi)^2}\frac{1}{m^2}\left[-\frac{1}{12}(P_\mu U)^2 - \frac{1}{6}U^3\right]\\ &= \frac{1}{(4\pi)^2}\frac{1}{m^2}\left(\frac{k^2}{12}\mathcal{O}_H - \frac{k^3}{12}\mathcal{O}_6\right).\end{aligned}$$

EW scalar doublet (Stop)

$$\mathcal{L} \supset |D_\mu \Phi|^2 - m^2 |\Phi|^2 - \frac{\lambda_\Phi}{4} |\Phi|^4 + \left(\eta_H |H|^2 + \eta_\Phi |\Phi|^2\right)\left(\Phi \cdot H + \text{h.c}\right)$$
$$- \lambda_1 |H|^2 |\Phi|^2 - \lambda_2 |\Phi \cdot H|^2 - \lambda_3 \left[\left(\Phi \cdot H\right)^2 + \text{h.c.}\right],$$

Stop: One-loop

$c_H = \frac{1}{(4\pi)^2}\left[6\eta_\Phi\eta_H + \frac{1}{12}\left(4\lambda_1^2 + 4\lambda_1\lambda_2 + \lambda_2^2 + 4\lambda_3^2\right)\right]$	$c_{BB} = \frac{1}{(4\pi)^2}\frac{1}{12}Y_\Phi^2(2\lambda_1 + \lambda_2)$	$c_{3W} = \frac{1}{(4\pi)^2}\frac{1}{60}g^2$
$c_T = \frac{1}{(4\pi)^2}\frac{1}{12}\left(\lambda_2^2 - 4\lambda_3^2\right)$	$c_{WW} = \frac{1}{(4\pi)^2}\frac{1}{48}(2\lambda_1 + \lambda_2)$	$c_{2W} = \frac{1}{(4\pi)^2}\frac{1}{60}g^2$
$c_R = \frac{1}{(4\pi)^2}\left[6\eta_\Phi\eta_H + \frac{1}{6}\left(\lambda_2^2 + 4\lambda_3^2\right)\right]$	$c_{WB} = -\frac{1}{(4\pi)^2}\frac{1}{12}\lambda_2 Y_\Phi$	$c_{2B} = \frac{1}{(4\pi)^2}\frac{1}{60}4g'^2Y_\Phi^2$

$$c_6 = \eta_H^2 + \frac{1}{(4\pi)^2}\left[\frac{3}{2}\lambda_\Phi\eta_H^2 + 6\eta_\Phi(\lambda_1 + \lambda_2) - \frac{1}{6}\left(2\lambda_1^3 + 3\lambda_1^2\lambda_2 + 3\lambda_1\lambda_2^2 + \lambda_2^3\right) - 2(\lambda_1 + \lambda_2)\lambda_3^2\right]$$

A general coding:

All bosonic operators

$$\mathcal{O}_{GG} = g_s^2 (H^\dagger H) G^a_{\mu\nu} G^{a\mu\nu}, \quad \mathcal{O}_H = \frac{1}{2}(\partial_\mu |H|^2)^2$$

$$\mathcal{O}_{WW} = g^2 |H|^2 W^a_{\mu\nu} W^{a\mu\nu}, \quad \mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$$

$$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}, \quad \mathcal{O}_R = |H|^2 |D_\mu H|^2$$

$$\mathcal{O}_{WB} = 2gg' (H^\dagger \tau^a H) W^a_{\mu\nu} B^{\mu\nu}, \quad \mathcal{O}_D = |D^2 H|^2$$

$$\mathcal{O}_W = ig(H^\dagger \tau^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu}, \quad \mathcal{O}_6 = |H|^6$$

$$\mathcal{O}_B = ig' Y_H (H^\dagger \overleftrightarrow{D}^\mu H) \partial^\nu B_{\mu\nu}, \quad \mathcal{O}_{2G} = -\frac{1}{2}(D^\mu G^a_{\mu\nu})^2$$

$$\mathcal{O}_{3G} = \frac{1}{3!} g_s f^{abc} G^{a\nu}_\mu G^{b\lambda}_\nu G^{c\mu}_\lambda, \quad \mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2$$

$$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon^{abc} W^{a\nu}_\mu W^{b\lambda}_\nu W^{c\mu}_\lambda, \quad \mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$$

Han & Skiba basis

(1) Operators that modify gauge boson propagators

$$O_{WB} = (h^\dagger \sigma^a h) W^a_{\mu\nu} B^{\mu\nu}, O_h = (h^\dagger D^\mu h)(D_\mu h^\dagger h). \tag{6}$$

(2) Operators that affect tree level SM gauge-fermion couplings

$$O^s_{hl} = i(h^\dagger D_\mu h)(\bar{l}\gamma^\mu l) + h.c., O^t_{hl} = i(h^\dagger D_\mu \sigma^a h)(\bar{l}\gamma^\mu \sigma^a l) + h.c. \tag{7}$$

$$O_{he} = i(h^\dagger D_\mu h)(\bar{e}\gamma^\mu e) + h.c., O^s_{hq} = i(h^\dagger D_\mu h)(\bar{q}\gamma^\mu q q) + h.c. \tag{8}$$

$$O^t_{hq} = i(h^\dagger D_\mu \sigma^a h)(\bar{q}\gamma^\mu \sigma^a q) + h.c., O_{hu} = i(h^\dagger D_\mu h)(\bar{u}\gamma^\mu u) + h.c. \tag{9}$$

$$O_{hd} = i(h^\dagger D_\mu h)(\bar{d}\gamma^\mu d) + h.c.. \tag{10}$$

(3) Four-fermion opertors

$$O^t_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l), O^s_{lq} = \frac{1}{2}(\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q) \tag{11}$$

$$O^t_{lq} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q), O_{le} = (\bar{l}\gamma^\mu l)(\bar{e}\gamma_\mu e) \tag{12}$$

$$O_{qe} = (\bar{q}\gamma^\mu q)(\bar{e}\gamma_\mu e), O_{lu} = (\bar{l}\gamma^\mu l)(\bar{u}\gamma_\mu u) \tag{13}$$

$$O_{ld} = (\bar{l}\gamma^\mu l)(\bar{d}\gamma_\mu d), O_{ee} = \frac{1}{2}(\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e) \tag{14}$$

$$O_{eu} = (\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu u), O_{ed} = (\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu d) \tag{15}$$

$$O^s_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu l)(\bar{l}\gamma_\mu l). \tag{16}$$

(4) Operators that modify triple gauge boson couplings

$$O_W = \epsilon^{abc} W^{a\nu}_\mu W^{b\lambda}_\nu W^{c\mu}_\lambda, (O_{WB}). \tag{17}$$

Han & Skiba basis

Use E.O.M. to change basis

$$\mathcal{O}'_T = -2O_h^{(3)} - O_h^{(1)} + 3\lambda O_h - \frac{1}{2}(\Gamma_e O_{eh} + \Gamma_u O_{uh} + \Gamma_d O_{dh}) - m^2|h|^4 \quad (18)$$

$$\mathcal{O}'_W = g^2(\frac{3}{2}O_h^{(1)} + O_{hl}^{(3)} + O_{hq}^{(3)} - \frac{3}{2}\lambda O_h + \frac{1}{4}(\Gamma_e O_{eh} + \Gamma_u O_{uh} + \Gamma_d O_{dh}) + \frac{1}{2}m^2|h|^4) \quad (19)$$

$$\mathcal{O}'_B = \frac{1}{2}g'^2(2O_h^{(3)} + O_h^{(1)} - 3\lambda O_h + \frac{1}{2}(\Gamma_e O_{eh} + \Gamma_u O_{uh} + \Gamma_d O_{dh}) + m^2|h|^4$$
$$-\frac{1}{2}O_{hl}^{(1)} - O_{he} + \frac{1}{6}O_{hq}^{(1)} + \frac{2}{3}O_{hu} - \frac{1}{3}O_{hd}) \quad (20)$$

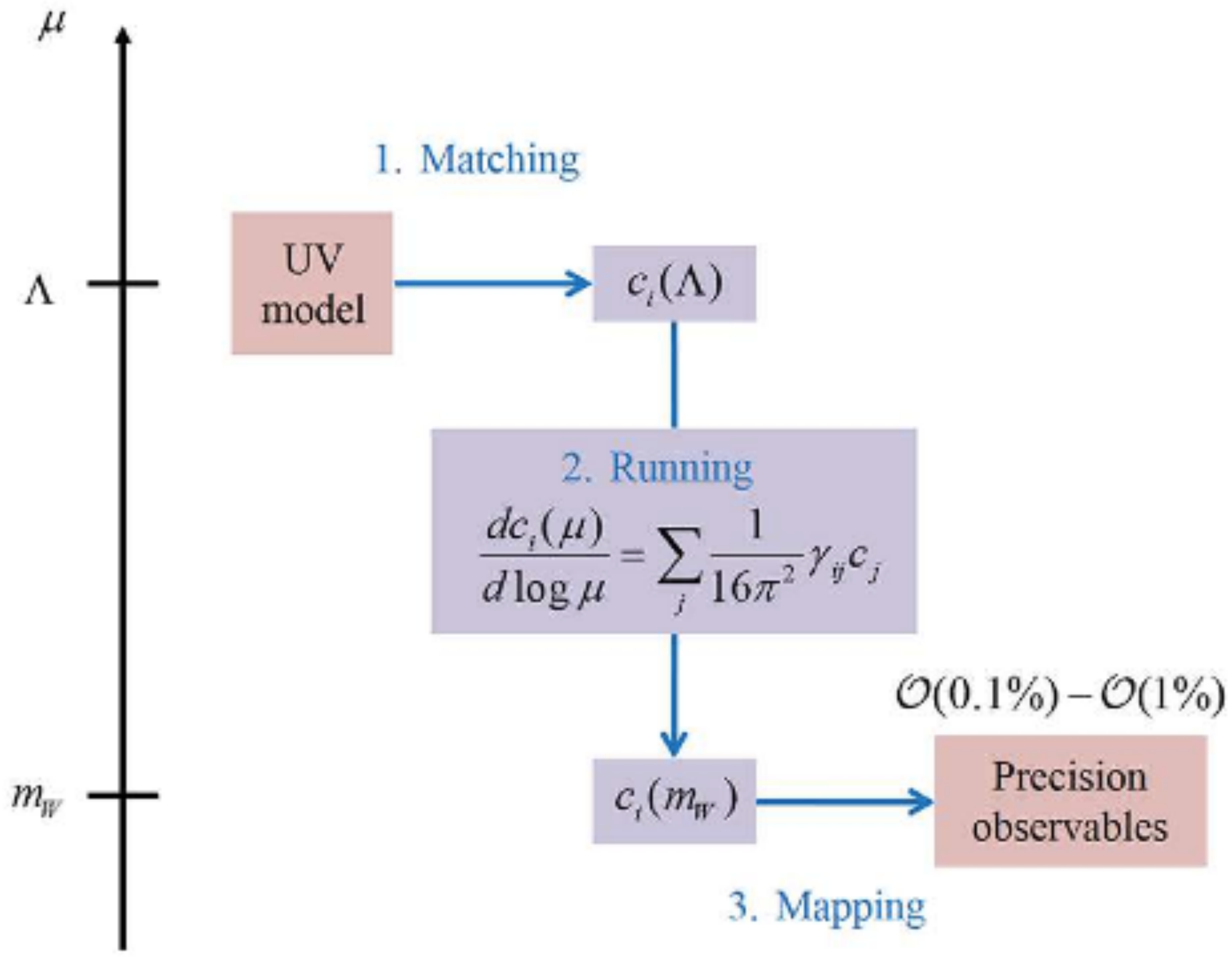
$$\mathcal{O}'_{2W} = -\frac{1}{2}g^2(\frac{3}{2}O_h^{(1)} + 2O_{hl}^{(3)} + 2O_{hq}^{(3)} + \frac{1}{2}O_{ll}^{(3)} + \frac{1}{2}O_{qq}^{(1,3)} + O_{lq}^{(3)} - \frac{3}{2}\lambda O_h$$
$$+\frac{1}{4}(\Gamma_e O_{eh} + \Gamma_u O_{uh} + \Gamma_d O_{dh}) + \frac{1}{2}m^2|h|^4) \quad (21)$$

$$\mathcal{O}'_{2B} = -\frac{1}{2}g'^2(O_h^{(3)} + \frac{1}{2}O_h^{(1)} - \frac{3}{2}\lambda O_h + \frac{1}{4}(\Gamma_e O_{eh} + \Gamma_u O_{uh} + \Gamma_d O_{dh}) + \frac{1}{2}m^2|h|^4$$
$$-\frac{1}{2}O_{hl}^{(1)} - O_{he} + \frac{1}{6}O_{hq}^{(1)} + \frac{2}{3}O_{hu} - \frac{1}{3}O_{hd}$$
$$+\frac{1}{2}O_{ll}^{(1)} + 2O_{ee} + \frac{1}{18}O_{qq}^{(1,1)} + \frac{8}{9}O_{uu}^{(1)} + \frac{2}{9}O_{dd}^{(1)} + O_{le} - \frac{1}{6}O_{lq}^{(1)}$$
$$-\frac{2}{3}O_{lu} + \frac{1}{3}O_{ld} - \frac{1}{3}O_{qe} - \frac{4}{3}O_{ue} + \frac{2}{3}O_{de} + \frac{2}{9}O_{qu}^{(1)} - \frac{1}{9}O_{qd}^{(1)} - \frac{4}{9}O_{ud}^{(1)}) \quad (22)$$

$$\mathcal{O}'_{BB} = 2g'^2 O_{hB}, \quad \mathcal{O}'_{WB} = gg' O_{WB}, \quad \mathcal{O}'_{WW} = 2g^2 O_{hW} \quad (23)$$

$$\mathcal{O}'_{3W} = \frac{1}{3!}g O_W, \quad \mathcal{O}'_H = O_{\partial h}, \quad \mathcal{O}'_6 = 3O_h \quad (24)$$

Operators beyond SM



Future CEPC makes it just like B, flavor physics

Can have both tree and loop results at the UV

RG running: one loop UV operators contribute to IR (weak scale) operators

Weak scale operators maps to the Observable.

RG Runnings

$$\{\mathcal{O}'_H, \mathcal{O}'_T, \mathcal{O}'_B, \mathcal{O}'_W, \mathcal{O}'_{2B}, \mathcal{O}'_{2W}, \mathcal{O}'_{BB}, \mathcal{O}'_{WW}, \mathcal{O}'_{WB}, \mathcal{O}'_{3W}\} \ .$$

Running effect

$$\begin{pmatrix} c_{BB}(m_W) \\ c_{WW}(m_W) \\ c_{WB}(m_W) \\ c_{3W}(m_W) \end{pmatrix} = \begin{pmatrix} 0.914758 & 0.000128298 & -0.0185242 & -3.25679e-05 \\ 3.10393e-06 & 0.90556 & -0.00165509 & -0.0154459 \\ -0.00332701 & -0.012287 & 0.875589 & 0.00314274 \\ 0 & 0 & 0 & 0.885251 \end{pmatrix} \begin{pmatrix} c_{BB}(\Lambda) \\ c_{WW}(\Lambda) \\ c_{WB}(\Lambda) \\ c_{3W}(\Lambda) \end{pmatrix},$$

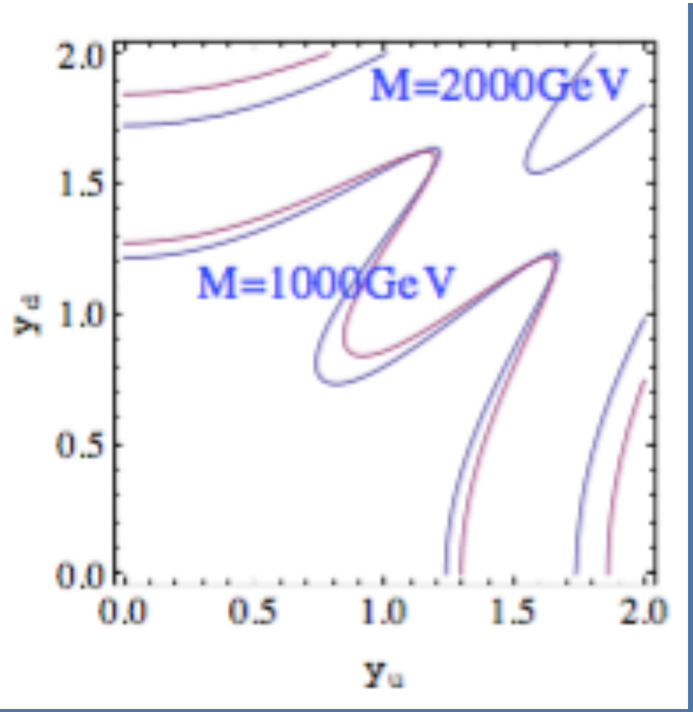
$$(c_H, c_T, c_B, c_W, c_{2B}, c_{2W})^t (m_W) \quad (3($$

$$= \begin{pmatrix} 0.817183 & 0.0232377 & -0.0014199 & 0.00975445 & 0.00106505 & -0.034927 \\ -0.00221894 & 0.78282 & 0.00274073 & 0.00146195 & -0.00199478 & -0.000736735 \\ 0.00455058 & 0.022526 & 0.909505 & -0.00299701 & -0.025421 & 0.00151002 \\ 0.00444872 & 0.00442166 & -0.000270172 & 0.857823 & -3.50986e-05 & -0.068742 \\ 2.91354e-06 & 1.4471e-05 & 0.00114785 & -1.90868e-06 & 0.94341 & 9.59124e-07 \\ 1.01829e-05 & 1.01354e-05 & -1.47944e-06 & 0.0038691 & -0.00138992 & 0.824636 \end{pmatrix} \begin{pmatrix} c_H(\Lambda) \\ c_T(\Lambda) \\ c_B(\Lambda) \\ c_W(\Lambda) \\ c_{2B}(\Lambda) \\ c_{2W}(\Lambda) \end{pmatrix}$$

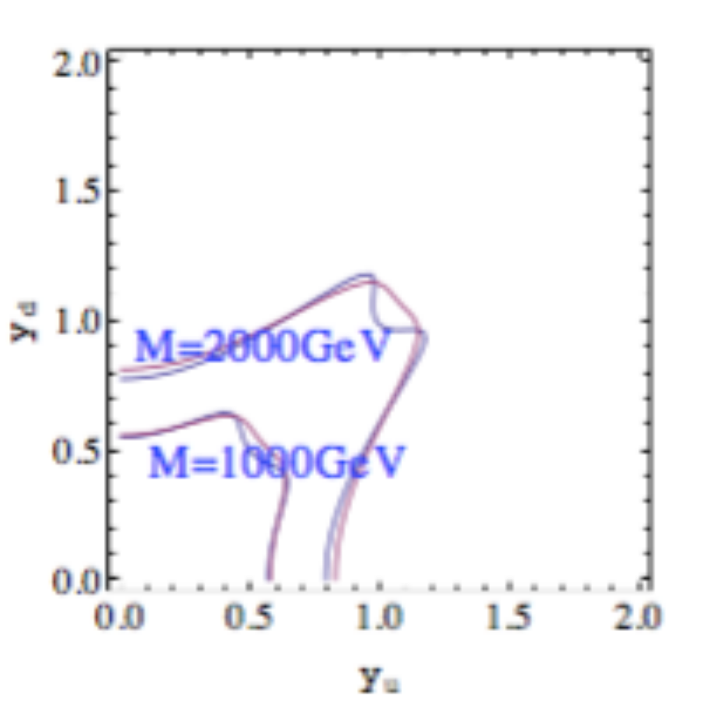
Vector fermion example

$$
\begin{aligned}
\mathcal{L} \supset \frac{m}{(4\pi)^2}\Big[& -\frac{2(|y_u|^6+|y_d|^6)}{15M^2}\mathcal{O}_6 \\
& -\frac{28(|y_u|^4+|y_d|^4)-12|y_u|^2|y_d|^2}{15M^2}\mathcal{O}_H+\frac{2(|y_u|^2-|y_d|^2)^2}{5M^2}\mathcal{O}_T-\frac{34(|y_u|^4+|y_d|^4)+24|y_u|^2|y_d|^2}{15M^2}\mathcal{O}_R \\
& -\frac{|y_u|^2+|y_d|^2}{48M^2}\mathcal{O}_{WW}-\frac{(1+16Y+32Y^2)|y_u|^2+(1-16Y+32Y^2)|y_d|^2}{48M^2}\mathcal{O}_{BB} \\
& +\frac{(3+8Y)|y_u|^2+(3-8Y)|y_d|^2}{24M^2}\mathcal{O}_{WB} \\
& +\frac{7(|y_u|^2+|y_d|^2)}{60M^2}\mathcal{O}_W+\frac{7(|y_u|^2+|y_d|^2)}{60M^2}\mathcal{O}_B \\
& +\frac{53(|y_u|^2+|y_d|^2)}{20M^2}\mathcal{O}_{HW}+\frac{53(|y_u|^2+|y_d|^2)}{20M^2}\mathcal{O}_{HB} \\
& +\frac{|y_u|^2+|y_d|^2}{15M^2}\mathcal{O}_D\Big] \\
+\frac{1}{(4\pi)^2} & \frac{f(m)(|y_u|^2+|y_d|^2)}{M^2}\mathcal{O}_{GG}\,, \qquad (31)
\end{aligned}
$$

Vector fermion example



EW + Higgs fit



Future EW + Higgs fit

Blue is the one with RG running

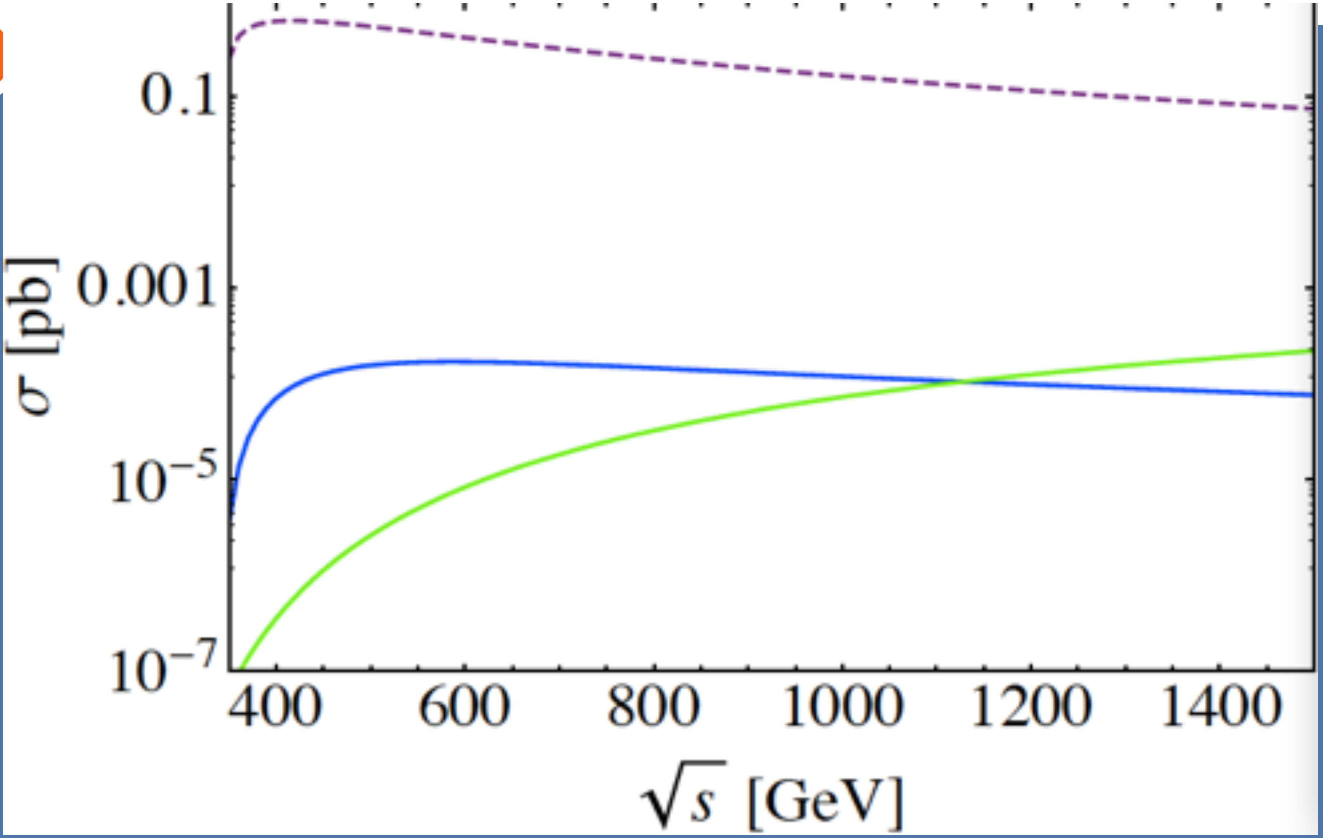
The one-loop from UV

There are also one-loop contributions can be obtained by directly using the Feymann diagram or the covariant derivative expansion.

Realistic calculation is still very difficult. (Huo Ran & John Ellis's group difference: Regulator)

CEPC 400

CEPC-400



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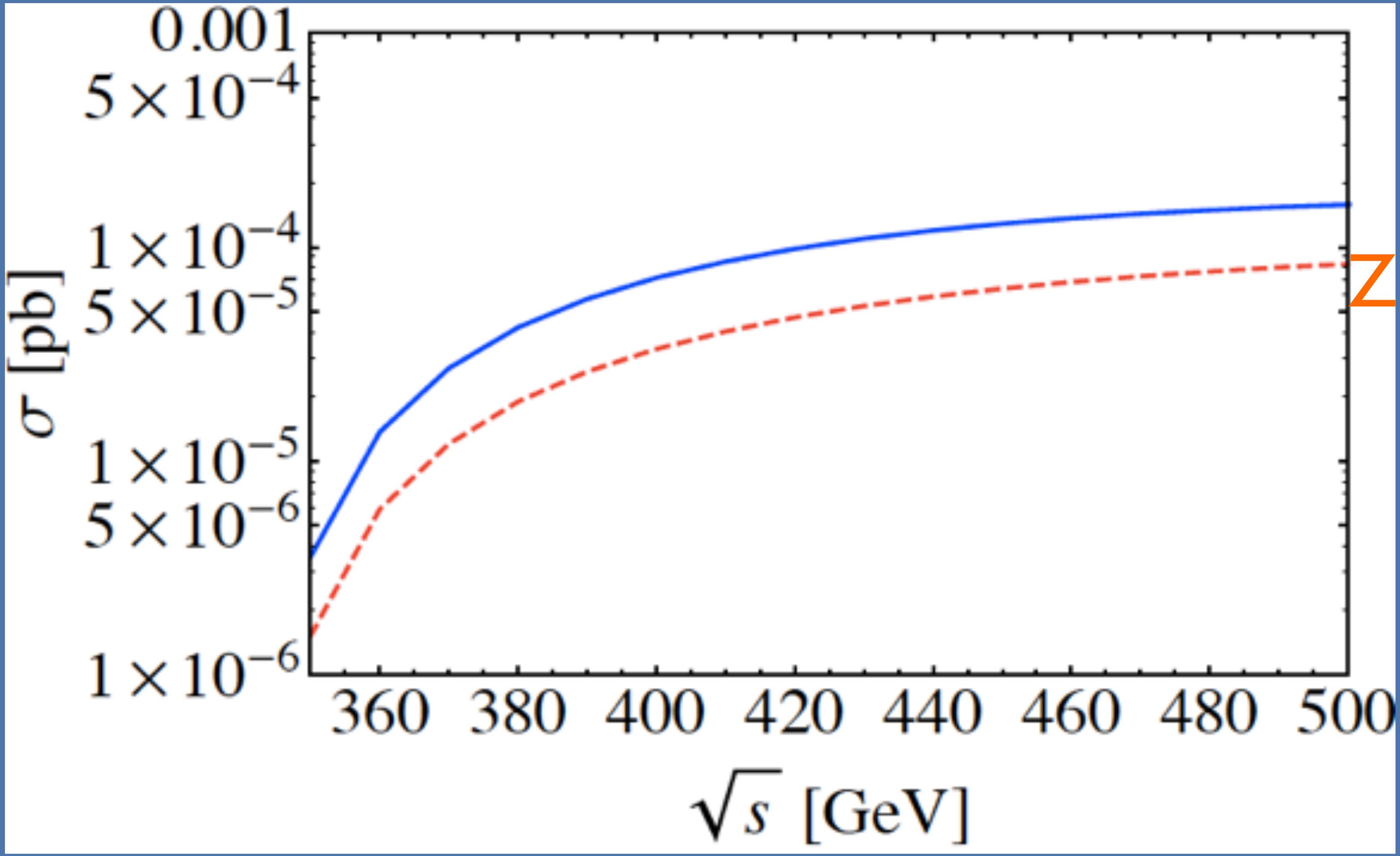
vhvh

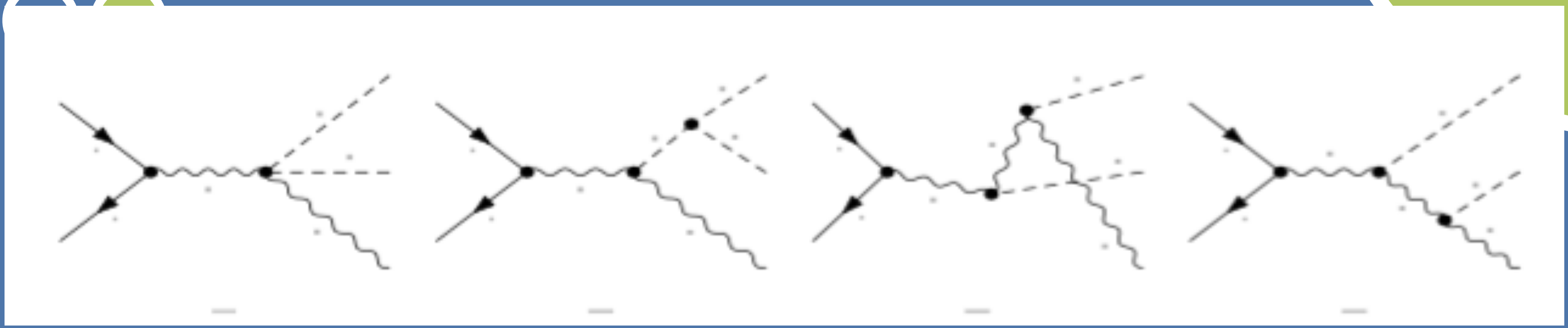
Zhh

Notice the CEPC 400 only lose a few (roughly a factor of two) cross section, but I believe it makes the circular electron collider much easier

CEPC-High


0.001
5×10^{-4}
1×10^{-4}
5×10^{-5}
1×10^{-5}
5×10^{-6}
1×10^{-6}
σ [pb]
360
380
400
420
440
460
480
500
$\sqrt{s}$ [GeV]
ZHH total
ZHH (no 3H)

Why hhh in lepton collider



VVhh VVh + hhh Vhh

All tree level, no new physics particles in the loop like 100 TeV SPPC

VVhh is essentially connected with VVh (very well measured), especially when VVh deviation is small

Determine the hhh

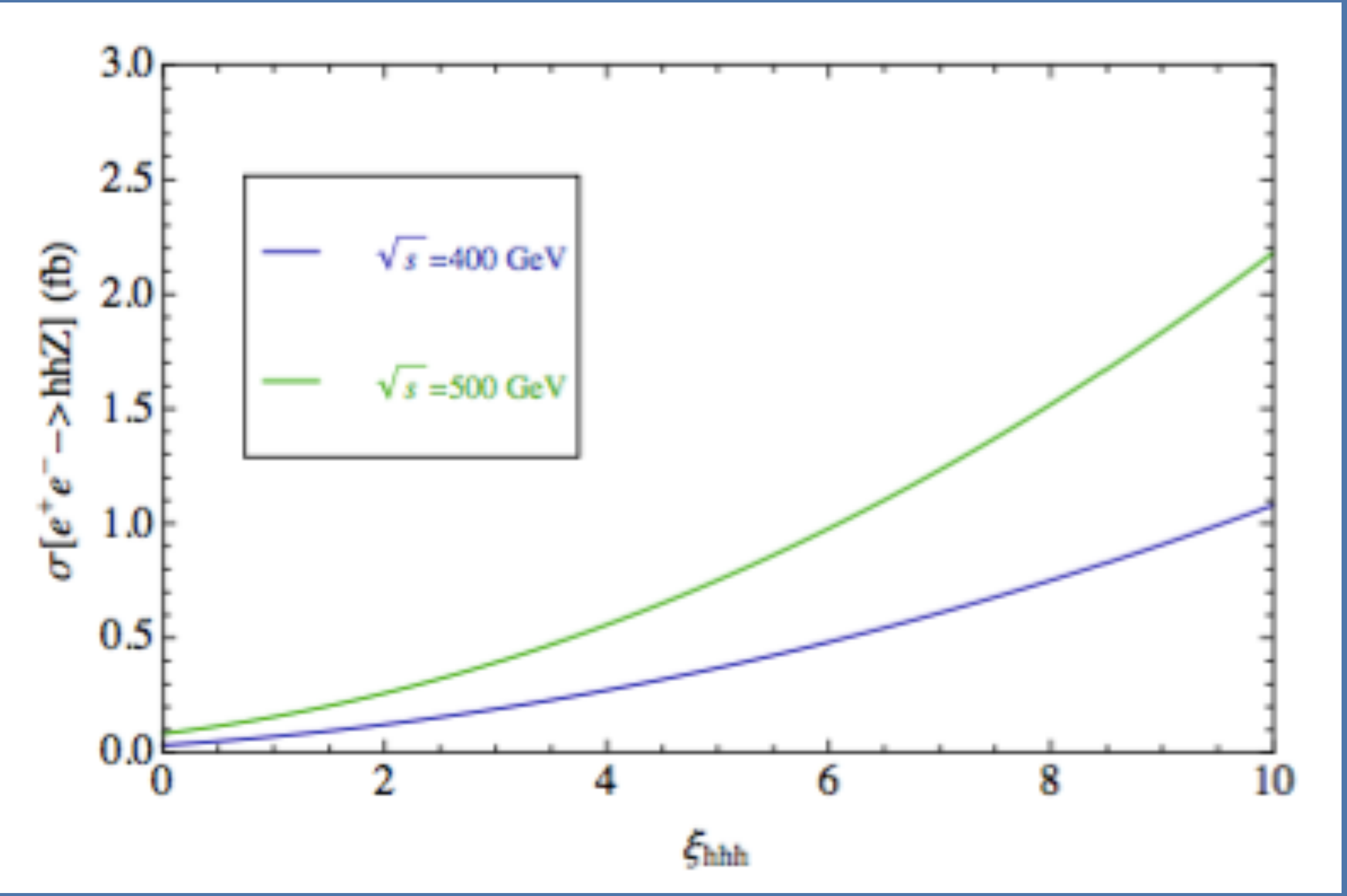
$$\frac{\sigma[e^+e^- \to hhZ]}{\sigma[e^+e^- \to hhZ]_{\rm SM}} \equiv \alpha\,\xi_{hhZZ}^2 + \beta\,\xi_{hZZ}^2\xi_{hhh}^2 + \gamma\,\xi_{hZZ}^4 + \delta\,\xi_{hhZZ}\xi_{hZZ}\xi_{hhh} + \epsilon\,\xi_{hhZZ}\xi_{hZZ}^2 + \zeta\,\xi_{hZZ}^3\xi_{hhh}\,,$$

$$\xi_{hZZ} \equiv \frac{g_{hZZ}}{g_{hZZ}^{\rm SM}}\,, \qquad \xi_{hhZZ} \equiv \frac{g_{hhZZ}}{g_{hhZZ}^{\rm SM}}\,, \qquad \xi_{hhh} \equiv \frac{\lambda_{hhh}}{\lambda_{hhh}^{\rm SM}}\,.$$

	α	β	γ	δ	ϵ	ζ
$\sqrt{s} = 400$ GeV	0.174	0.105	0.079	0.267	0.212	0.157
$\sqrt{s} = 500$ GeV	0.276	0.097	0.105	0.311	0.153	0.059

TABLE I: The numerical fits to the total cross section of $\sigma[e^+e^- \to hhZ]$.

Determine the hhh



For large higgs self-couplings, the sensitivity is very good!

Possible Channels

$h_1 \to$	$h_2 \to$	$Z \to$	Br	N_{event} (ILC)	N_{event} (CEPC)
$b\bar{b}$	$b\bar{b}$	$q\bar{q}$	23.3 %	73/101	83
$b\bar{b}$	WW^*	$q\bar{q}$	17.3 %	54/75	62
$b\bar{b}$	$W_h W_h^*$	$q\bar{q}$	7.9 %	25/35	29
$b\bar{b}$	$V_h V_h^*$	$q\bar{q}$	8.9 %	28/39	32
$b\bar{b}$	$b\bar{b}$	$\nu\bar{\nu}$	6.7 %	21/29	24
$b\bar{b}$	$b\bar{b}$	$\ell^+\ell^-$	2.2 %	7/10	8

TABLE II: Possible decay modes and their branching fractions. We also estimated the number of events for the SM triple Higgs self-couplings $\lambda_{hhh}^{\text{SM}}$ at the ILC (500 GeV , 2 ab^{-1}) for unpolarized/polarized cases and the CEPC (400 GeV , 5 ab^{-1}) runs.

Charge leptons

The $4b + \ell^+\ell^-$ channel

Very Loose (or no Higgs) reconstruction

Lepton + jets

More than 3 b taggings

Z invariant mass

	zhh	zhh(lam=10)	tt	wwz	zzz
total	782	10900	5w*57	189400	5055
cut1	12	180	406	1089	109
cut2	5	115	111	0	0
cut3	4	101	2	0	0

hZZ included in the later analysis

Difficult Higgs reconstruction

The $4b + \nu\bar{\nu}$ channel

Unfortunately, the Z invisible invariant mass window is not sharp

For the ttbar and hZZ backgrounds, they all could fake the first higgs mass window very well (just like the ttbar h channel)

Only the 2nd Higgs mass window and the aggressive b tagging can help

A naive no showering BDT results show: zhh (12.9), tt(4.2), zzh(4.17)

WW*bbjj channel

The $4b + q\bar{q}$ channel

Just need more work

What needs to be done

- Full universal one-loop effective action (notice top Yukawa is large, the RG running effect would violate the universality, have to deal with it)
- Need to implement more future CEPC expected sensitivity and measurements
- Apply to more realistic models: Covariant Derivative Expansion for SUSY, composite Higgs, etc.
- More refined analysis for CEPC 400. (parton level + Delphes)

Back-up slice

Tri-gauge boson at LEP

In the Hagiwara-Peccei-Zeppenfeld-Hikasa basis

$$\mathcal{L}_{\rm TGC}/g_{WWV} = ig_{1,V}\left(W^+_{\mu\nu}W^-_\mu V_\nu - W^-_{\mu\nu}W^+_\mu V_\nu\right) + i\kappa_V W^+_\mu W^-_\nu V_{\mu\nu} + \frac{i\lambda_V}{M_W^2}W^+_{\lambda\mu}W^-_{\mu\nu}V_{\nu\lambda}$$
$$+g_5^V\varepsilon_{\mu\nu\rho\sigma}\left(W^+_\mu \overleftrightarrow{\partial}_\rho W_\nu\right)V_\sigma - g_4^V W^+_\mu W^-_\nu\left(\partial_\mu V_\nu + \partial_\nu V_\mu\right)$$
$$+i\tilde{\kappa}_V W^+_\mu W^-_\nu \tilde{V}_{\mu\nu} + \frac{i\tilde{\lambda}_V}{M_W^2}W^+_{\lambda\mu}W^-_{\mu\nu}\tilde{V}_{\nu\lambda}\,. \quad (1)$$

Only the 1st line is C and P conserving

In the SM, $g_{1,V} = \kappa_V = 1$

The W boson charge suggest $g_{1,\gamma} = 1$.

Five independent variables:

$$\Delta g_{1,Z}\,,\quad \Delta\kappa_\gamma\,,\quad \Delta\kappa_Z\,,\quad \lambda_\gamma\,,\quad \lambda_Z\,,$$

Unfortunately, poorly measured at LEP because the lack of data

Tri-gauge boson at LEP

Up to D=6 level, in the SILH basis,

$$\Delta\mathcal{L} = \frac{ic_W\, g}{2M_W^2}\left(H^\dagger\sigma^i \overleftrightarrow{D^\mu} H\right)(D^\nu W_{\mu\nu})^i + \frac{ic_{HW}\, g}{M_W^2}(D^\mu H)^\dagger\sigma^i(D^\nu H)W^i_{\mu\nu} + \frac{ic_{HB}\, g'}{M_W^2}(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu} + \frac{c_{3W}\, g}{6M_W^2}\,\epsilon^{ijk}W^{i\,\nu}_{\mu}W^{j\,\rho}_{\nu}W^{k\,\mu}_{\rho}$$

The first one is constrained by the S parameter,

$$\Delta g_{1,Z} = -\cot^2\theta_W c_{HW}\,,$$
$$\Delta\kappa_\gamma = -(c_{HW}+c_{HB})\,,$$
$$\lambda_\gamma = -c_{3W}\,,$$

$$\lambda_\gamma = \lambda_Z,\ \Delta\kappa_Z = \Delta g_{1,Z} - \tan^2\theta_W\Delta\kappa_\gamma.$$

Three independent variables:

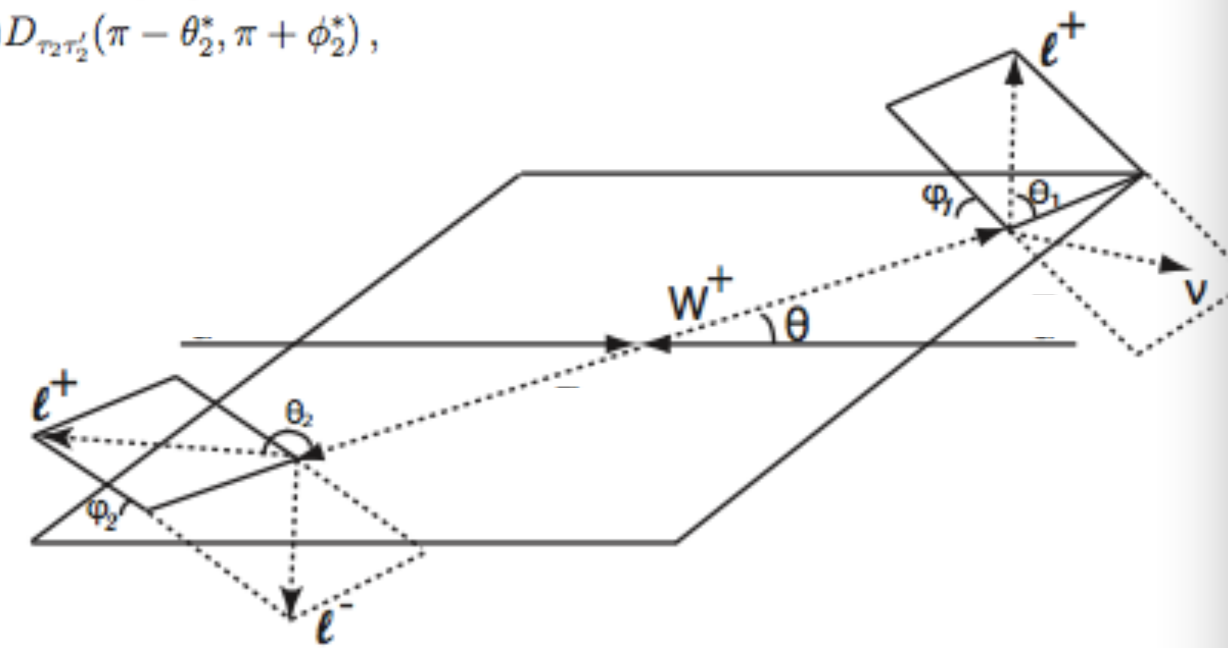
$$\Delta g_{1,Z}\,,\quad \Delta\kappa_\gamma\,,\quad \lambda_\gamma\,.$$

Kinematics

$$\frac{d\sigma(e^+e^- \to W^+W^- \to f_1\bar{f}_2\bar{f}_3 f_4)}{d\cos\theta\, d\cos\theta_1^* d\phi_1^* d\cos\theta_2^* d\phi_2^*} = \mathrm{BR}\cdot\frac{\beta}{32\pi s}\left(\frac{3}{8\pi}\right)^2 \sum_{\lambda\tau_1\tau_1'\tau_2\tau_2'} F^{(\lambda)}_{\tau_1\tau_2} F^{(\lambda)*}_{\tau_1'\tau_2'} \times D_{\tau_1\tau_1'}(\theta_1^*,\phi_1^*) D_{\tau_2\tau_2'}(\pi-\theta_2^*, \pi+\phi_2^*),$$

D: W decay matrix

C: Coupling coefficients



Production amplitude

$$F^\lambda_{\tau\tau'}(s,\cos\theta) = -\frac{\lambda e^2 s}{2}\Big[C^{(\nu)}(\lambda,t)\mathcal{M}^{(\nu)}_{\lambda\tau\tau'}(s,\cos\theta) + \sum_{i=1}^{7}\big(C_i^{(\gamma)}(\lambda,s,\alpha_j^{(\gamma)}) + C_i^{(Z)}(\lambda,s,\alpha_j^{(Z)})\big)\mathcal{M}^{(\nu)}_{i,\lambda\tau\tau'}(s,\cos\theta)\Big],$$

Five differential variables

$$(\theta, \theta_1, \theta_2, \phi_1, \phi_2)$$

Sensitivity:

In principle, one would get five independent histograms to discriminate S and Bs:

At the lepton collider, the reducible backgrounds of WW is less than 5% after cuts

leptonic or semi-leptonic

Multi-variable methods:

BDT methods (will be used soon)

Previous LEP only use theta

$$\chi^2 \equiv \sum_i \left(\frac{N_i^{\mathrm{aTGC}} - N_i^{\mathrm{SM}}}{\sqrt{N_i^{\mathrm{SM}}}} \right)^2 ,$$

Summing over different bins for 5 distributions

Linear Differential Sensitivity

5 ab^-1

TABLE I: estimations of the reaches of sensitivities ($\times 10^{-4}$) at CEPC

channels	$\Delta g_{1,Z}$	$\Delta\kappa_\gamma$	$\Delta\kappa_Z$	λ_γ	λ_Z
leptonic	14.49	8.02	9.82	12.70	12.00
semileptonic	5.52	2.71	3.59	4.32	4.63
hadronic	6.56	2.74	4.00	4.40	5.65
all	4.06	1.87	2.58	3.00	3.44

channels	$\Delta g_{1,Z}$	$\Delta\kappa_\gamma$	λ_γ	c_{HW}	c_{HB}	c_{3W}
leptonic	5.90	9.87	6.57	3.36	9.91	6.58
semileptonic	2.19	3.33	2.35	1.18	3.34	2.35
hadronic	2.51	3.37	2.54	1.26	3.37	2.54
all	1.59	2.30	1.67	0.84	2.31	1.67

10^-3 ~ 10^-4

Two orders improvements

Individual sensitivity

contributions		$\cos\theta$	$\cos\theta^*_\ell$	ϕ^*_ℓ	$\cos\theta^*_J$	ϕ^*_J
leptonic	$\Delta g_{1,Z}$	0.525	0.051	0.425	-	-
	$\Delta\kappa_\gamma$	0.523	0.272	0.205	-	-
	λ_γ	0.617	0.044	0.339	-	-
semi-leptonic	$\Delta g_{1,Z}$	0.650	0.032	0.261	0.031	0.027
	$\Delta\kappa_\gamma$	0.532	0.138	0.108	0.119	0.102
	λ_γ	0.709	0.025	0.192	0.024	0.050
hadronic	$\Delta g_{1,Z}$	0.850	-	-	0.080	0.070
	$\Delta\kappa_\gamma$	0.546	-	-	0.244	0.210
	λ_γ	0.827	-	-	0.056	0.118
all	$\Delta g_{1,Z}$	0.722	0.020	0.167	0.048	0.042
	$\Delta\kappa_\gamma$	0.538	0.081	0.065	0.170	0.147
	λ_γ	0.755	0.015	0.117	0.036	0.076

$$\frac{\Delta\chi^2(\Omega_k)}{\sum_k \Delta\chi^2(\Omega_k)}$$

In most cases, scattering angle and azimuthal angles are most sensitive

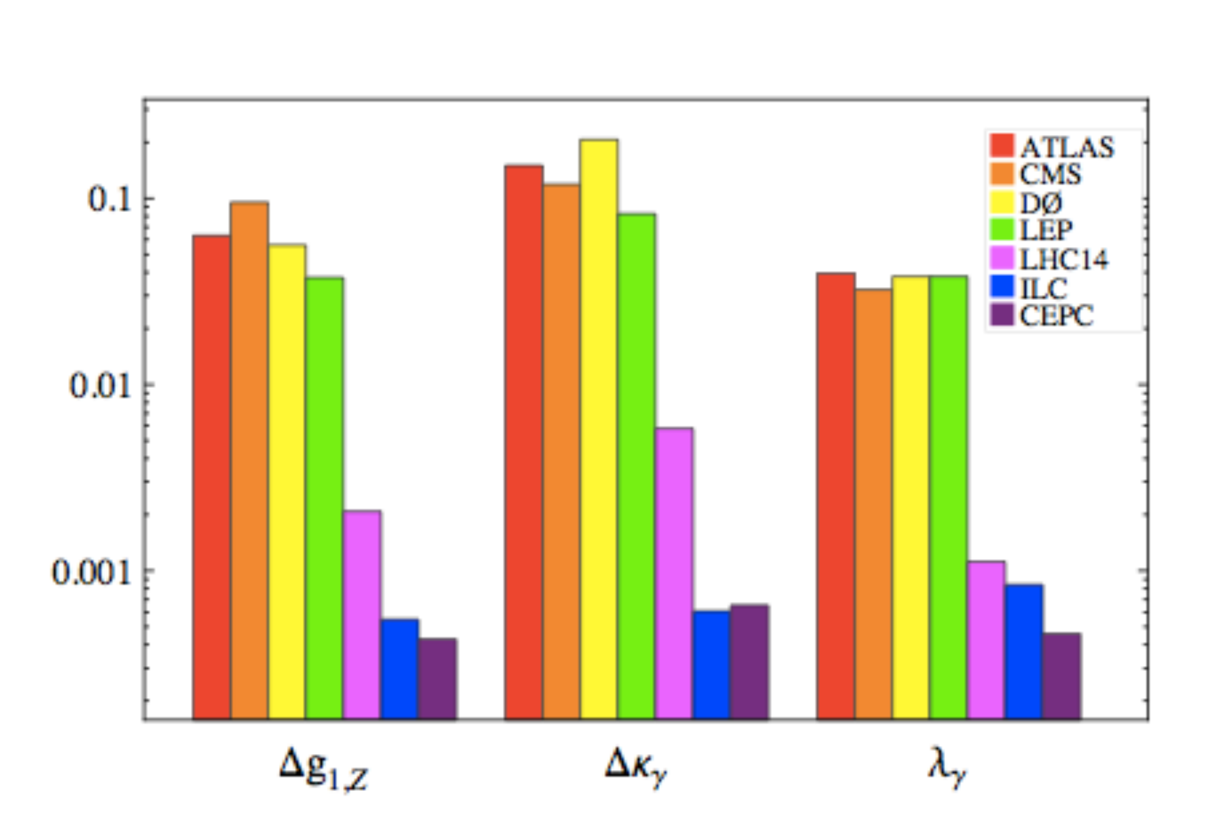
Systematics?

- Leptonic and semi-leptonic backgrounds are small

(full backgrounds simulation in semi-leptonic using whizard)

- Precision W mass. 3 MeV at CEPC
- Beam energy uncertainty. 10ppm ~ 1 MeV
- Detector simulation and radiative corrections are roughly at the same order. (ILC notes)
- < 10^{-5} in general, OK!

TGC Comparision



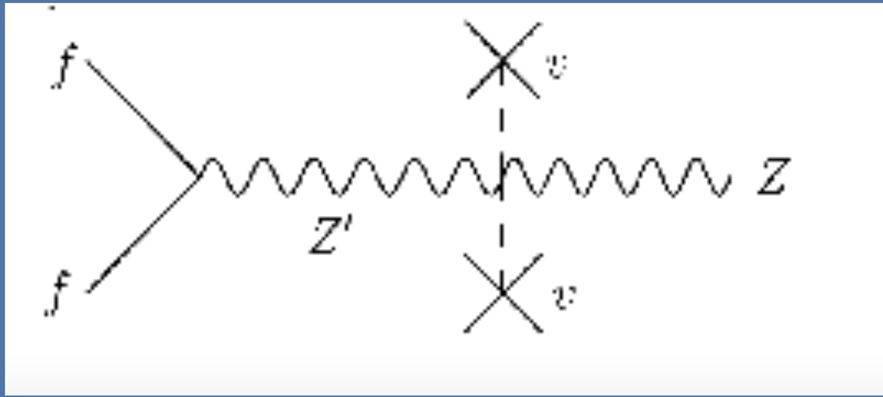
Improve more than two orders of magnitude at the CEPC

Why tri-gauge boson ?

Why learning the tri-gauge boson coupling is important?

Our current super-simplified EW constraints (S,T) are based on the facts that tri-gauge boson coupling are poorly measured!

Fermion gauge boson corrections arise very common in new physics models (a Z' model)



$$S = \frac{s}{2\pi} + \frac{a}{2\pi}$$
$$T = \frac{t}{8\pi e^2} + \frac{a g'^2}{8\pi e^2}$$

EW & TGC Interplay

$$-\frac{2gscv^2}{\alpha}\mathcal{O}_S - \frac{g'v^2}{\alpha}\mathcal{O}_T + g'\mathcal{O}^Y_{hf} = 2g'\mathcal{O}_{HB} - g'\mathcal{O}_{h2} + \frac{g'}{2}\mathcal{O}_{BB} - \frac{g'}{2}\mathcal{O}_{h3},$$

$$-\frac{4g'scv^2}{\alpha}\mathcal{O}_S + g(\mathcal{O}^t_{hl} + \mathcal{O}^t_{hq}) = 4g\mathcal{O}_{HW} - 6g\mathcal{O}_{h2} + g\mathcal{O}_{WW} - g\mathcal{O}_{h3},$$

$$c_{HB} \sim \frac{\alpha g^2}{4c^2}\Delta S \sim \frac{\alpha g^2}{2}\Delta T \sim 2c_{h2} \sim g^2\Delta g_{hZZ}/g_{hZZ},$$

$$c_{HW} \sim \frac{\alpha g^2}{4s^2}\Delta S \sim \frac{2}{3}c_{h2} \sim \frac{g^2}{3}\Delta g_{hZZ}/g_{hZZ},$$

EW & TGC Interplay

	future prospects	c_{HW}	c_{HB}
HL-LHC	-	6.3×10^{-4}	3×10^{-3}
CEPC	-	1.2×10^{-4}	3.3×10^{-4}
S: HL-LHC	0.13	5×10^{-4}	1.4×10^{-4}
T: HL-LHC	0.09	–	1.6×10^{-4}
$\frac{\Delta g_{hZZ}}{g_{hZZ}}$: HL-LHC	0.03	4.5×10^{-3}	1.3×10^{-2}
S: CEPC	0.04	1.6×10^{-4}	4.2×10^{-5}
T: CEPC	0.03	–	5.3×10^{-5}
$\frac{\Delta g_{hZZ}}{g_{hZZ}}$: CEPC	0.002	3×10^{-4}	9×10^{-4}

one sigma

Examples of how CEPC observables constraint operators