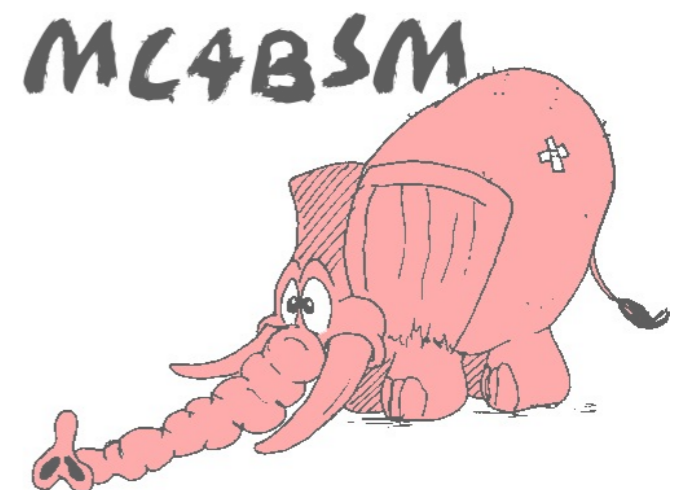


# MT2

Myeonghun Park (明勳 朴)

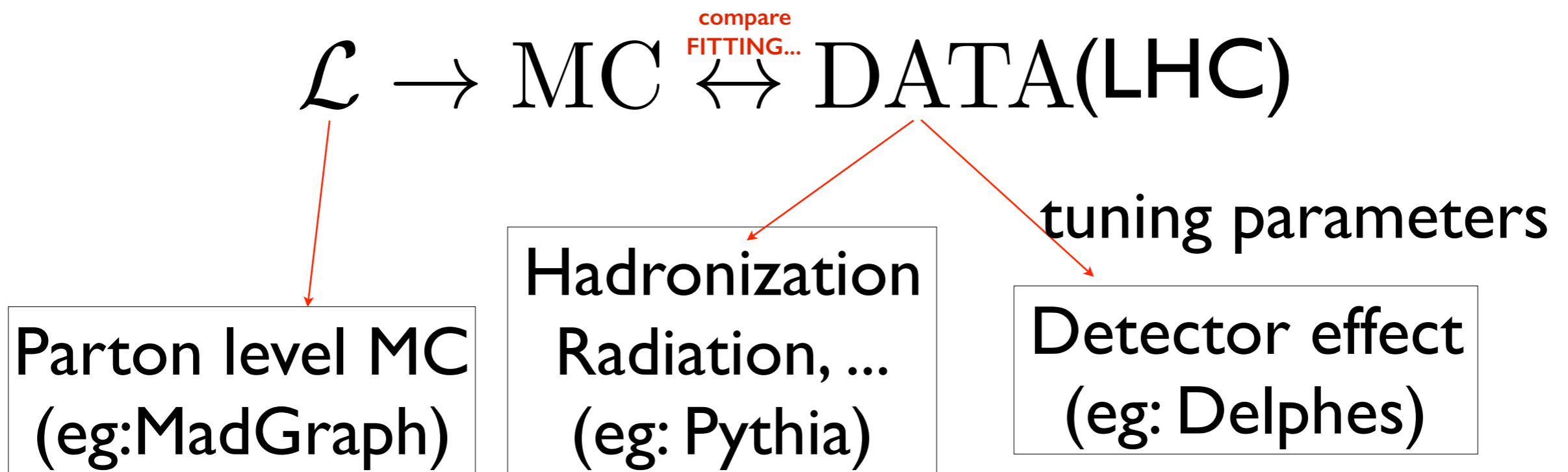
**C**enter for **T**heoretical **P**hysics of the **U**niverse

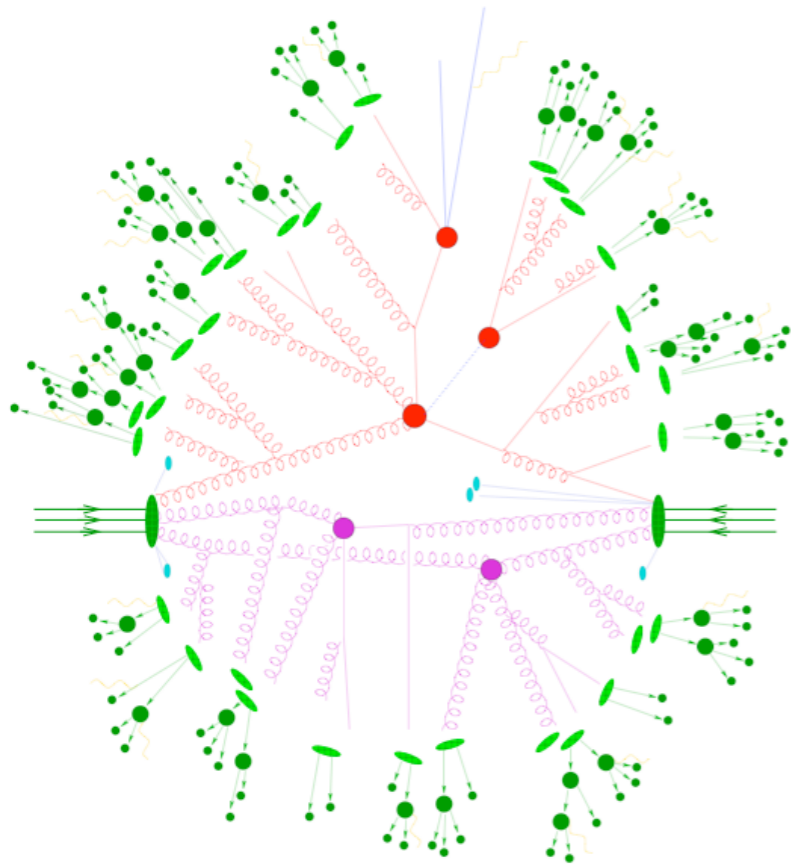


*MC4BSM 2016 Beijing*

# Understanding physics@LHC

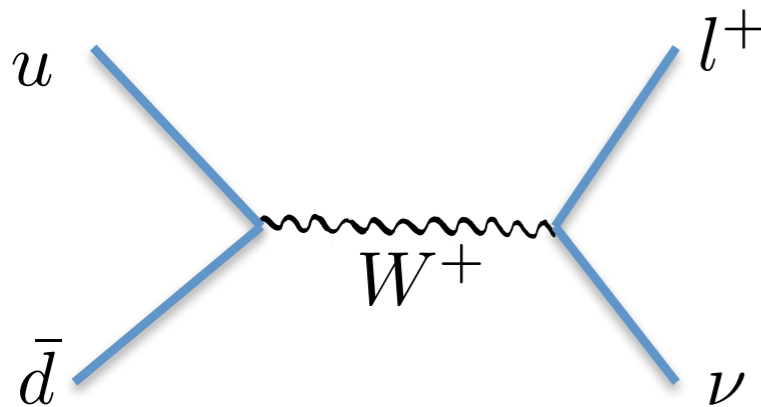
- If we know the physics model (theory [=Lagrangian]) and want to determine parameters, we can directly compare Monte Carlo (MC) simulations to data





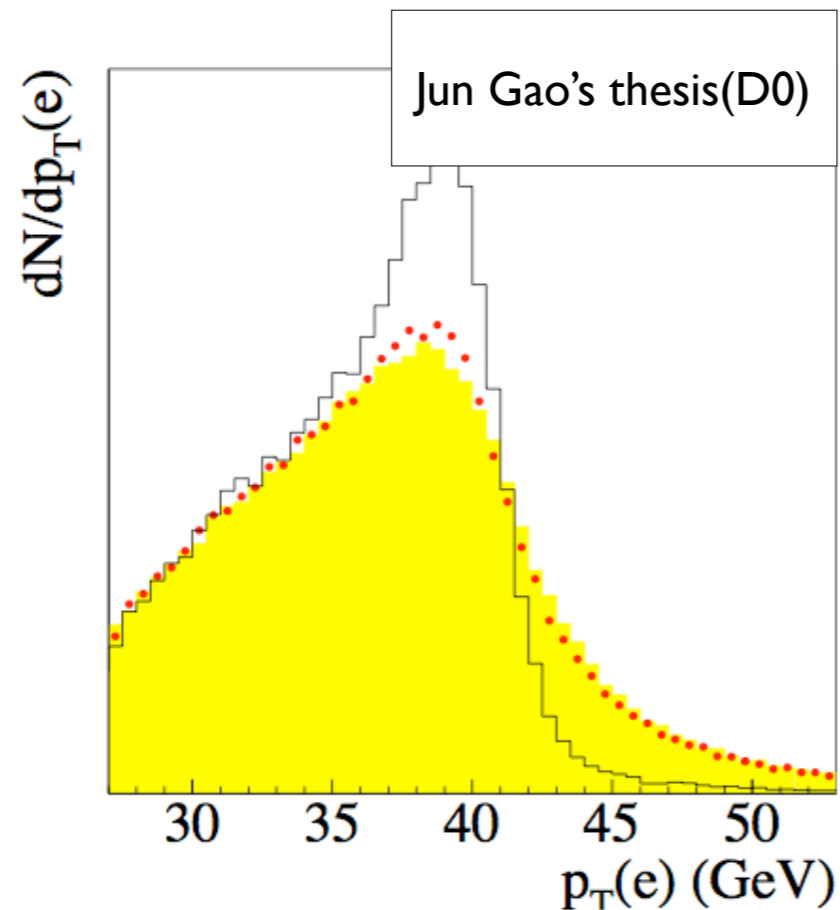
Sherpa ([Tanju Gleisberg, et.al.](#))  
**JHEP 0402 (2004) 056**

- Huge amount of community's efforts have been focused on MC to describe physics more precisely. (to remove systematic uncertainties from MC, DATA comparison)
- We need observables(histograms) to compare MC outputs with data.
- One example: W-boson mass measurement@Tevatron
- At LO, W boson's transverse momentum (orthogonal to beam direction)  $\sim 0$ .
- Using the change of variable, we have a well-known Jacobian peak in leptons' PT distribution.

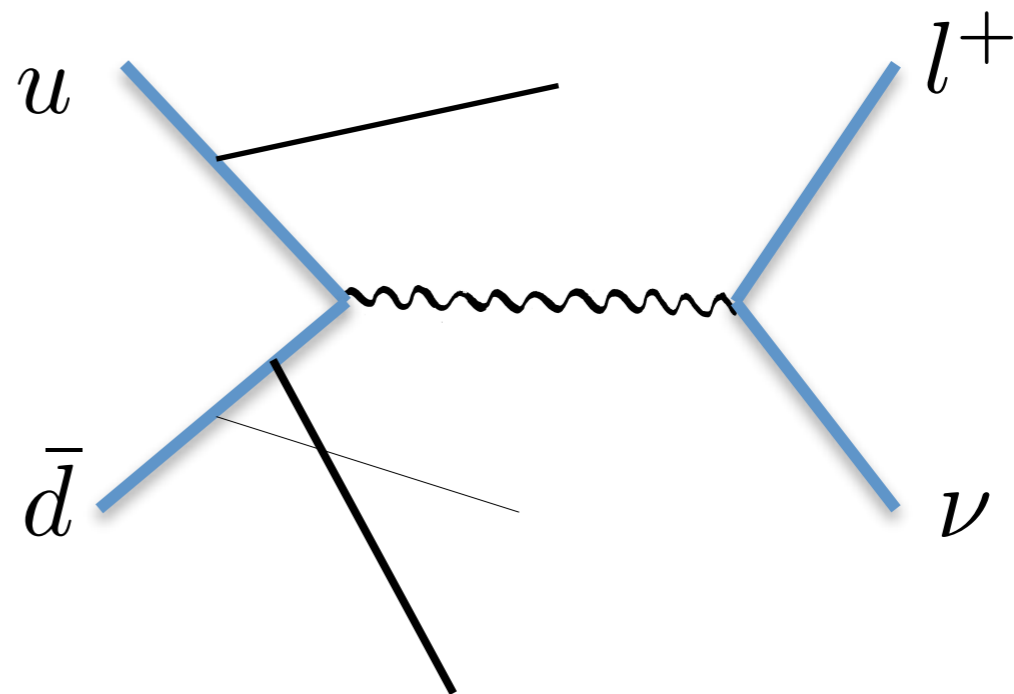


$$\frac{d\sigma}{dp_{\perp}} = \frac{d \cos \hat{\theta}}{dp_{\perp}} \frac{d\sigma}{d \cos \hat{\theta}} = \frac{p_{\perp}}{\sqrt{\left(\frac{M_W}{2}\right)^2 - p_{\perp}^2}} \frac{d\sigma}{d \cos \hat{\theta}}$$

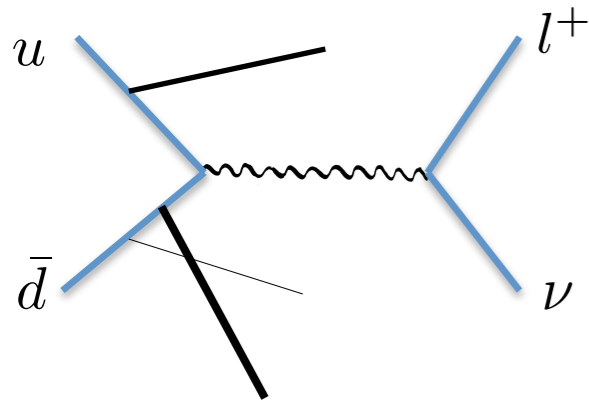
- In reality,  $W$  boson will be kicked off by extra jets!
- Thus, precise measurement will be highly dependent on the goodness of MC tools.
- Option: We can make some special variable to remove effects from this effect.



Black: Parton level [ $PT(W)=0$ ]  
 RED: Parton level [ $PT(W) \neq 0$ ]  
 Yellow: Detector level



Like as invariant mass is boost-invariant, a “transverse mass” will be invariant under the boost along transverse direction.

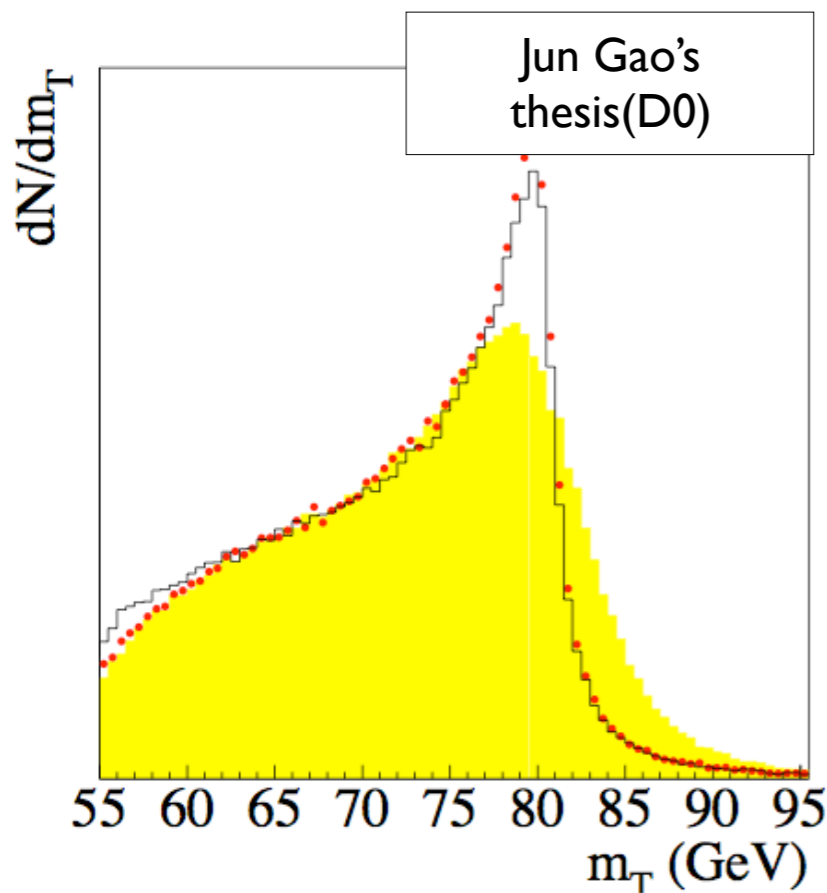


$$m_T^2 = (|\mathbf{p}_\perp^\nu| + |\mathbf{p}_\perp^\ell|)^2 - (\mathbf{p}_\perp^\nu + \mathbf{p}_\perp^\ell)^2$$

This variable is bounded by the mass of W boson, and have Jacobian peak just like lepton’s  $p_T$  distribution.

## What we learned from old days:

It is important to **design observables** that are **strong** under (complicated, uncontrolled) effects.



Black: Parton level [ $PT(W)=0$ ]  
 RED: Parton level [ $PT(W) \neq 0$ ]  
 Yellow: Detector level

D0(arxiv:0908.0766)

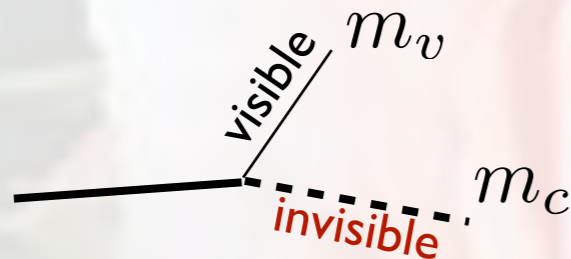
TABLE II: Systematic uncertainties of the  $M_W$  measurement.

Source	$\Delta M_W$ (MeV)		
	$m_T$	$p_T^e$	$\cancel{E}_T$
Electron energy calibration	34	34	34
Electron resolution model	2	2	3
Electron shower modeling	4	6	7
Electron energy loss model	4	4	4
Hadronic recoil model	6	12	20
Electron efficiencies	5	6	5
Backgrounds	2	5	4
Experimental Subtotal	35	37	41
PDF	10	11	11
QED	7	7	9
Boson $p_T$	2	5	2
Production Subtotal	12	14	14
Total	37	40	43

# Variable in Rosy dream before July. 2012

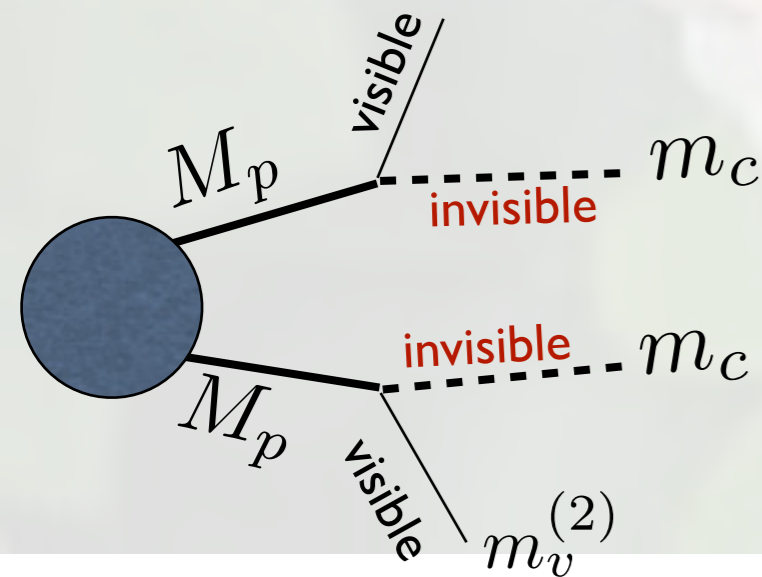
- A transverse mass  $M_T$ : 
$$M_T(m_c) = \sqrt{m_v^2 + m_c^2 + 2(e_v e_c - \vec{p}_T^v \cdot \vec{p}_T^{(c)})}$$

with a transverse energy,  $e_c = \sqrt{\vec{p}_T^{(c)} \cdot \vec{p}_T^{(c)} + m_c^2}$



- For double decay chain event: Let's use  $M_T$  twice.

C.Lester, D. Summers (hep-ph/9906349)

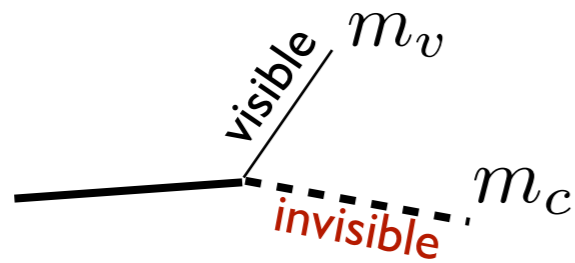


Assumptions:

- I. Decaying particle in both chain has a common mass :  $M_p$
- II. Invisible particle in both chain has a common mass :  $m_c$
- III. No invisible particles except LSP



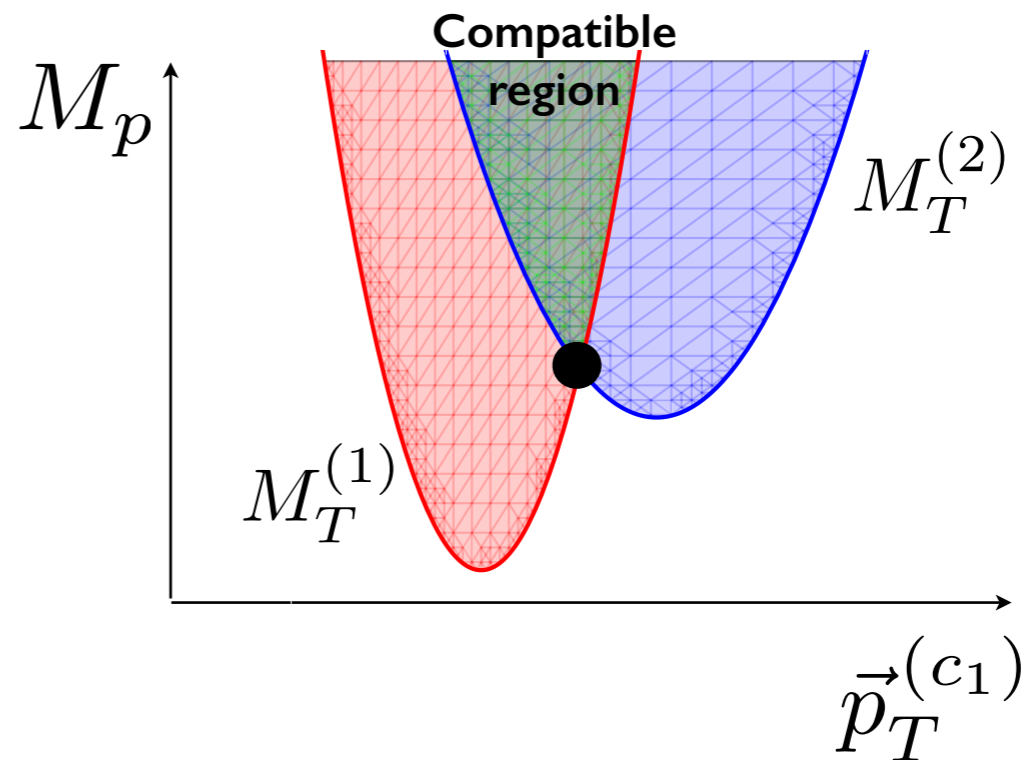
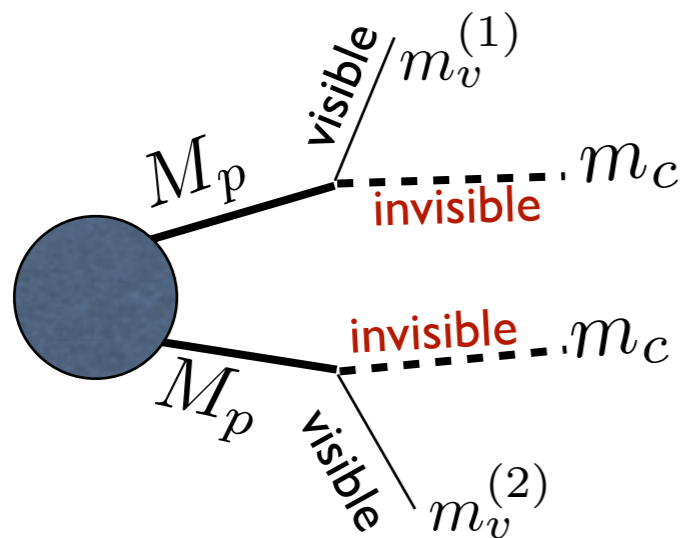
- A transverse mass  $M_T$ :



$$M_T(m_c) = \sqrt{m_v^2 + m_c^2 + 2(e_v e_c - \vec{p}_T^v \cdot \vec{p}_T^{(c)})}$$

with a transverse energy,  $e_c = \sqrt{\vec{p}_T^{(c)} \cdot \vec{p}_T^{(c)} + m_c^2}$

- For double decay chain event: Let's use  $M_T$  twice



- Minimum possible  $M_p$  with above kinematics constraints =  $M_{T2}$

$$M_T^{(1)} \leq M_p \ \& \ M_T^{(2)} \leq M_p$$

(Transverse mass is less than the actual mass.)

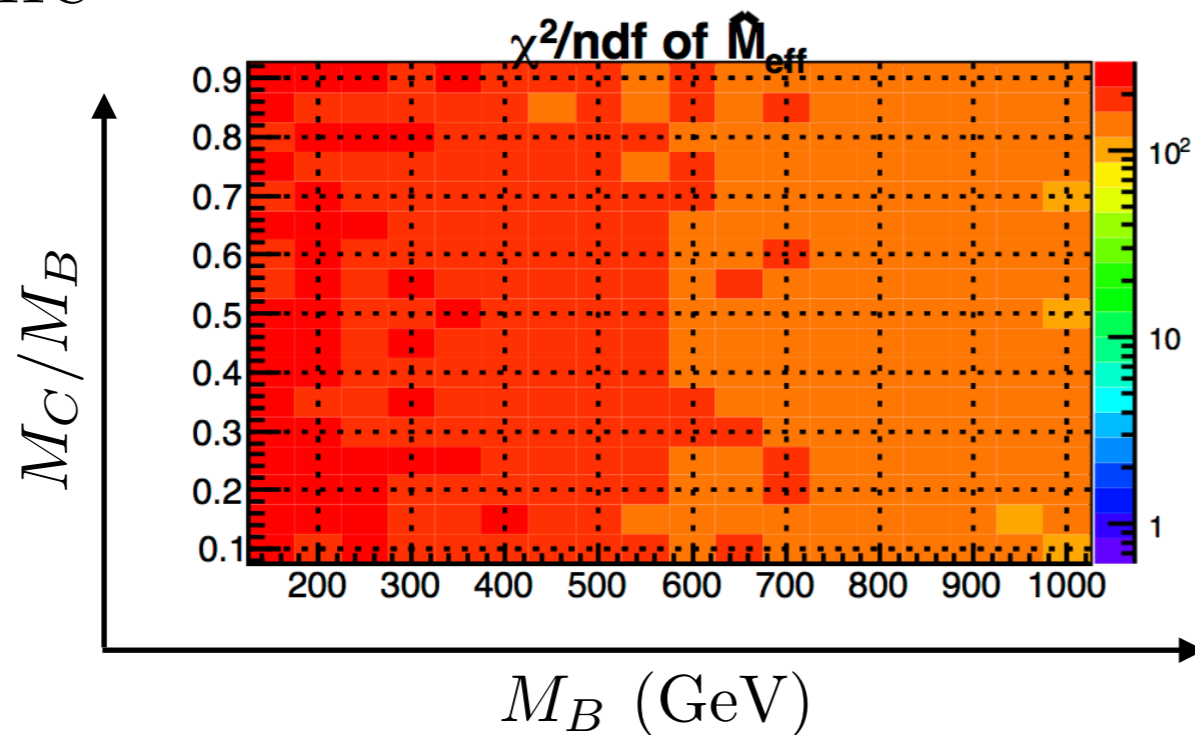
by H.-C. Cheng and Z. Han (hep-ph:0810.5178)

$$M_{T2}(m_c) = \min \left( \max[M_T^{(1)}(m_c), M_T^{(2)}(m_c)] \right)$$

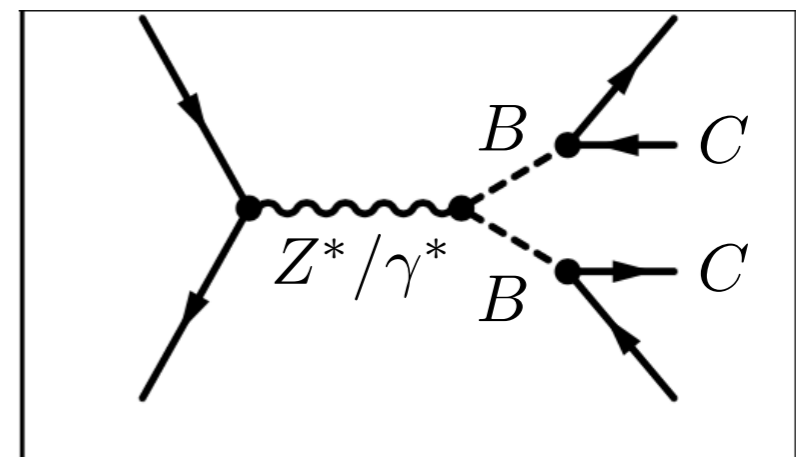
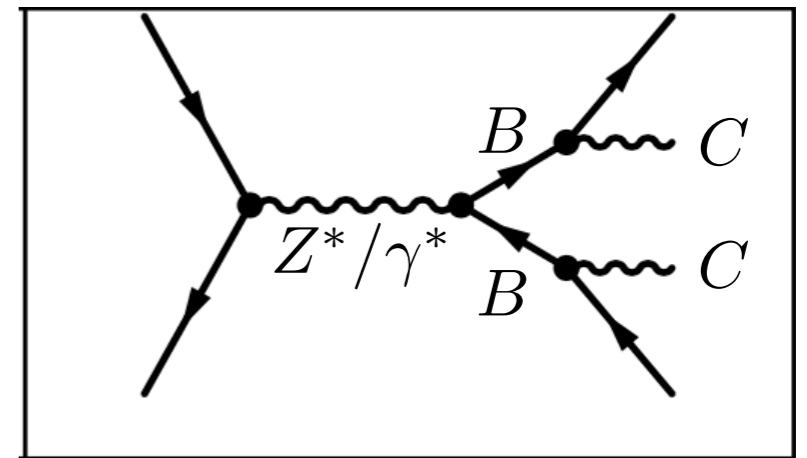
# Goodness of MT2

- It is insensitive to helicity structures:
  - Your analyses are **independent** to BSM scenarios
  - Easy to recast analysis in specific BSM to others.
  - Only depends on the kinematical structure.

8Tev LHC



$$M_{\text{eff}} = P_{1T} + P_{2T} + \cancel{E}_T$$

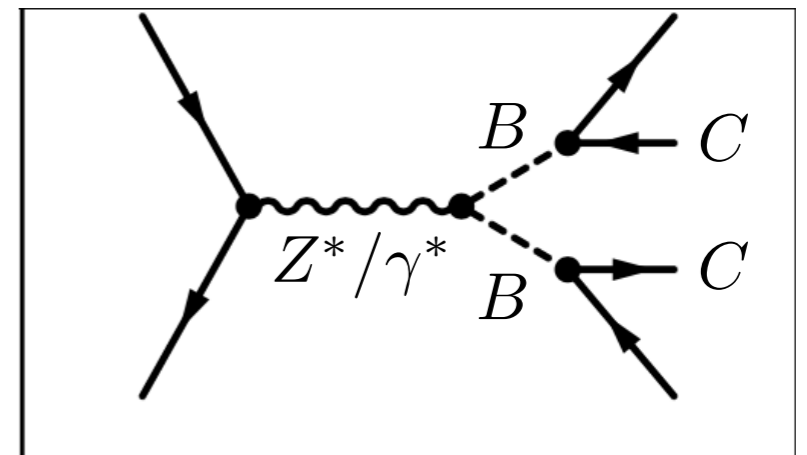
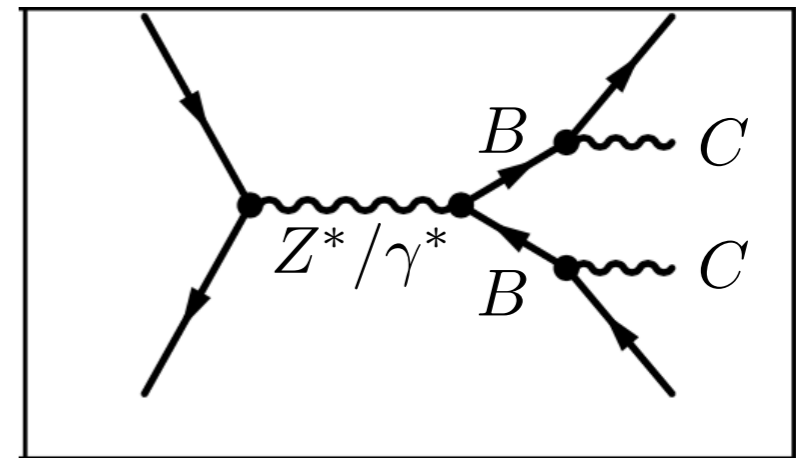
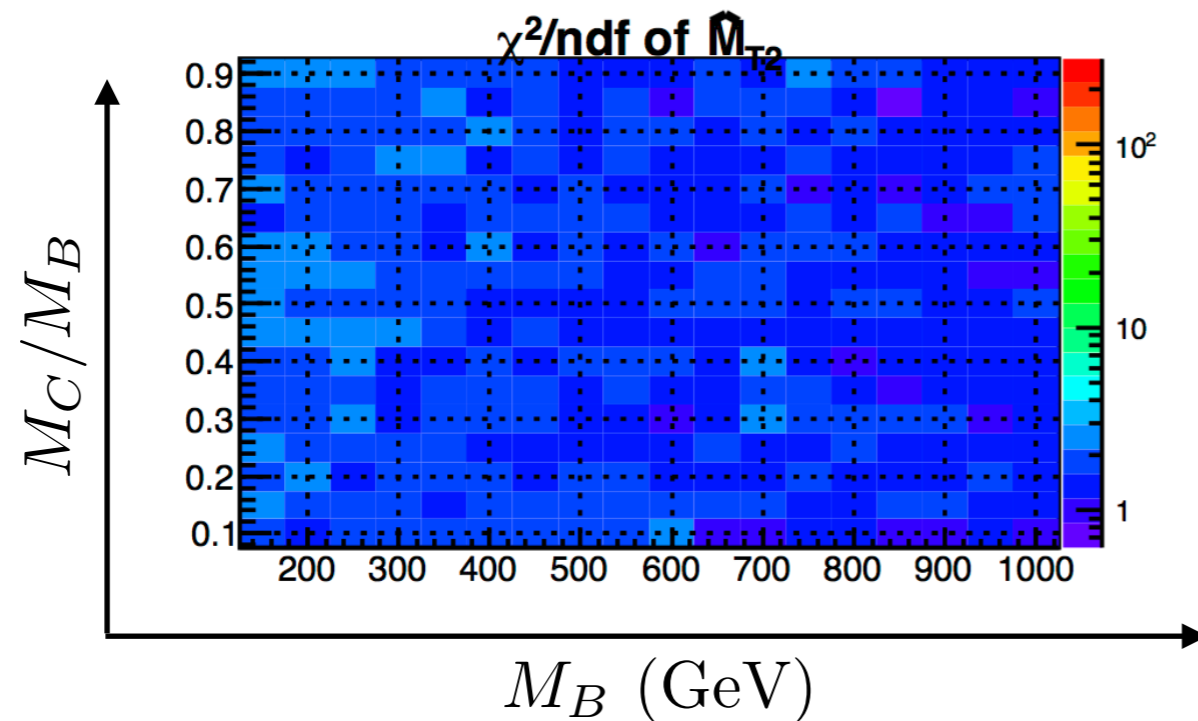




# Goodness of MT2

- It is insensitive to helicity structures:
  - Your analyses are **independent** to BSM scenarios
  - Easy to recast analysis in specific BSM to others.
  - Only depends on the kinematical structure.

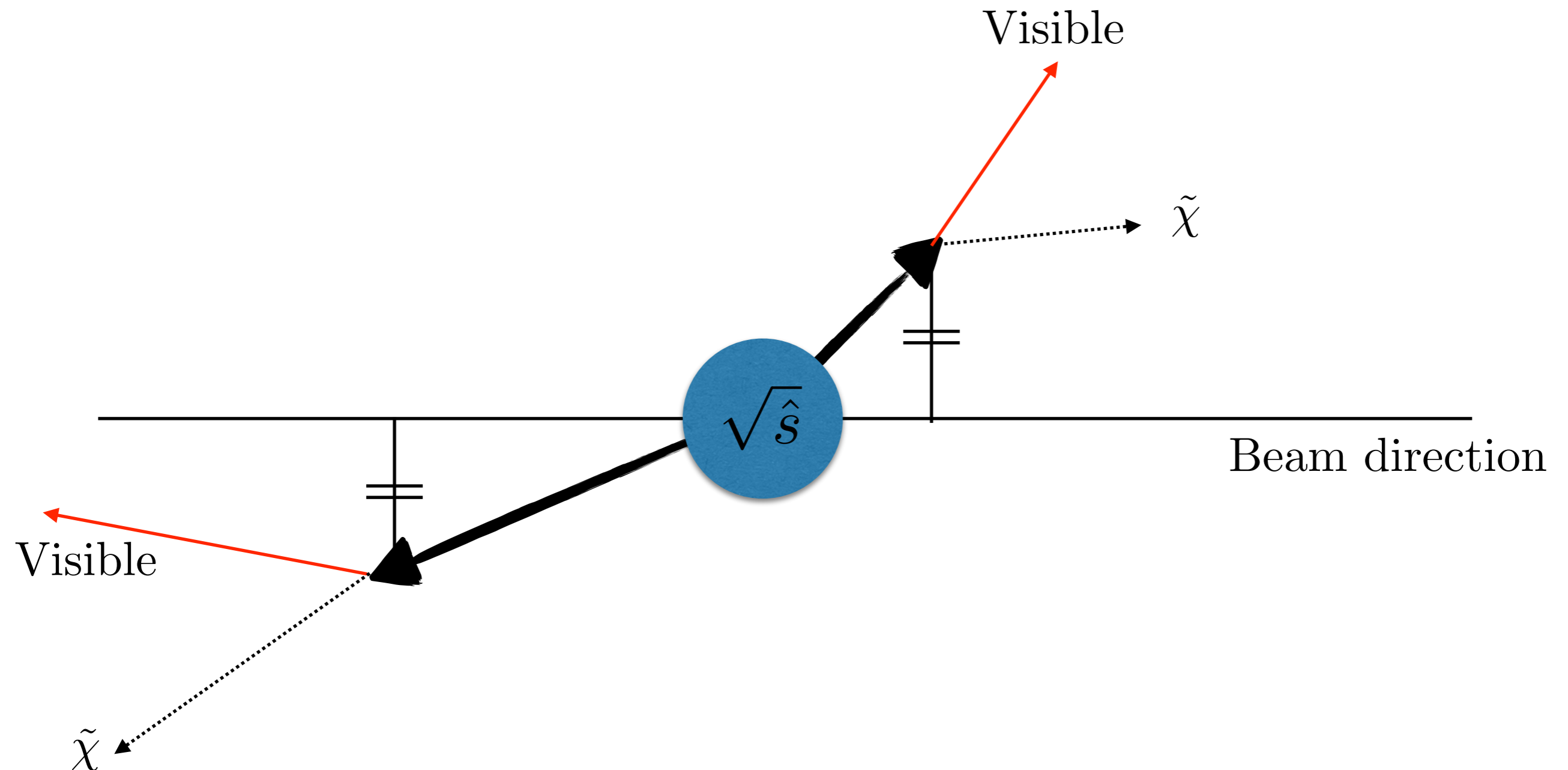
8Tev LHC



MT2 has a **Back to back boost Invariance**

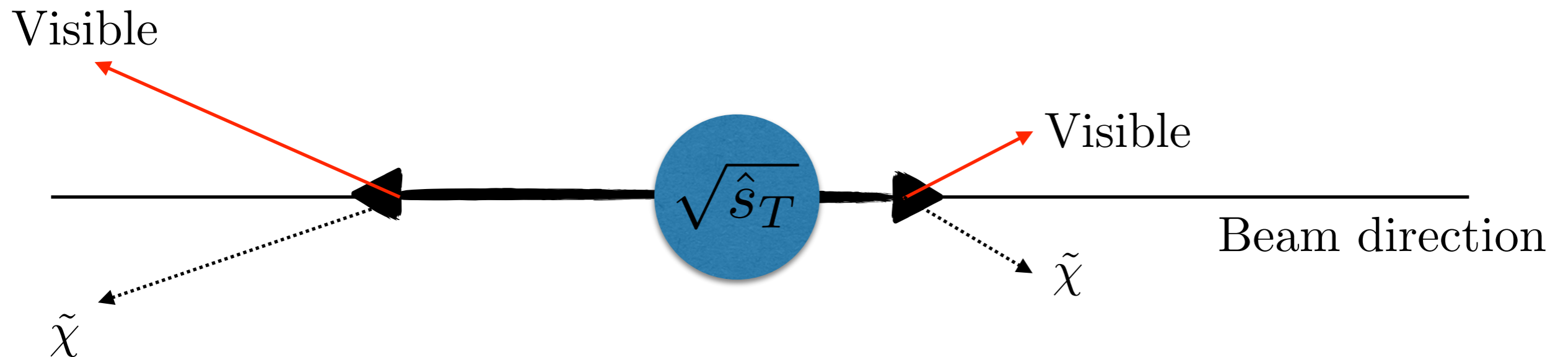
# Back to back “Transverse” BOOST INV

- Just like MT variable



# Back to back “Transverse” BOOST INV

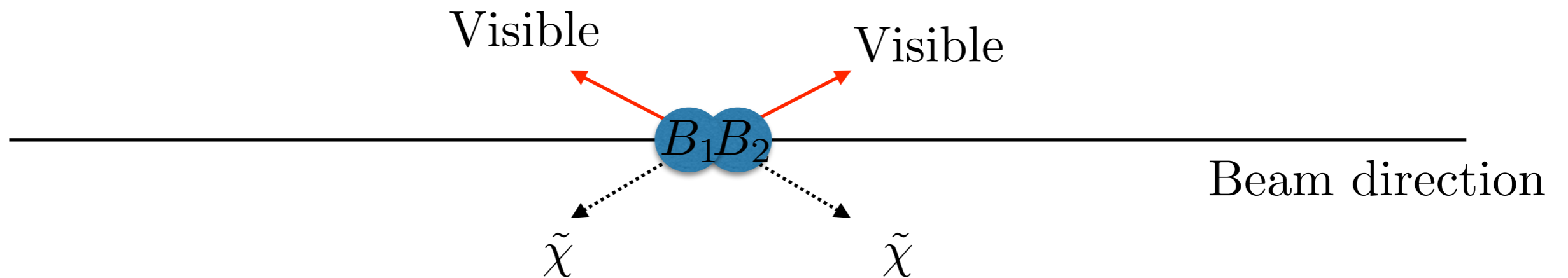
- Just like MT variable



# Back to back

## “Transverse” BOOST INV

- Just like MT variable

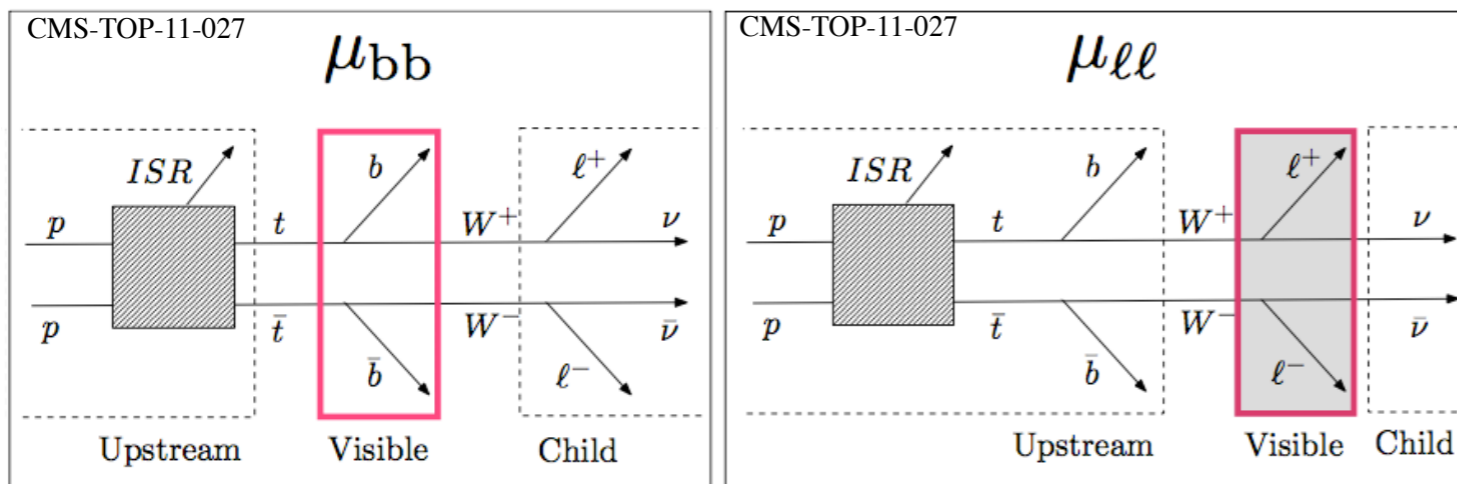


Very easy to **estimate** MT2 behavior for **Know kinematics**  
(Standard Model Back grounds)

# MT2 as a CUT variable

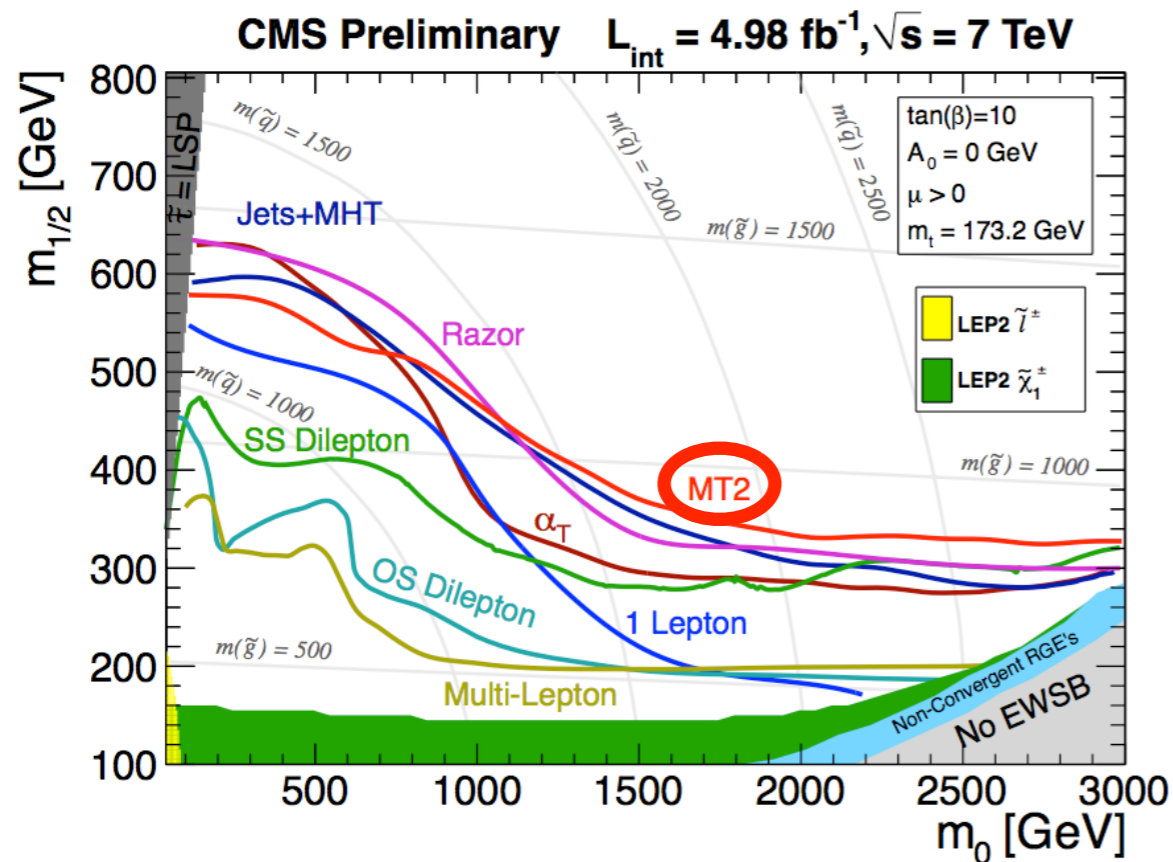
- $M_{T2}$  inherits the good property of  $M_T$ : Transverse boost invariance!  
(But only when you put the the right value for  $m_c$ )
- Since we don't know the true mass of LSP, we only can get the constraint of decayed particle in terms of LSP mass.
- But we know what will be missing particles of background (Standard Model) : neutrino  
- Thus, experimentalists started to use this variable as one of cuts.  
(by Alan Barr, Claire Gwenlan : arxiv:0907.2713)

example cartoon to describe MT2 in ttbar events



Process	$m_{T2}(v_1, v_2, \cancel{p}_T, 0, 0)$	Comments
QCD di-jet $\rightarrow$ hadrons	$= \max m_j$ by Lemmas 1,4	
QCD multi jets $\rightarrow$ hadrons	$= \max m_j$ by Lemma 4	
$t\bar{t}$ production	$= \max m_j$ by Lemma 4	fully hadronic decays
Single top / $tW$	$\leq m_t$ by Lemmas 1,7	any leptonic decays
Multi jets: "fake" $\cancel{p}_T$	$= \max m_j$ by Lemma 4	fully hadronic decays
Multi jets: "real" $\cancel{p}_T$	$\leq m_t$ by Lemmas 2,7	any leptonic decays
$Z \rightarrow \nu\bar{\nu}$	$= \max m_j$ by Lemma 5	single mismeasured jet <sup>a</sup>
$Z j \rightarrow \nu\bar{\nu} j$	$= \max m_j$ by Lemma 6	two mismeasured jets <sup>a</sup>
$W \rightarrow \ell\nu$	$= \max m_j$ by Lemma 5	single jet with leptonic $b$ decay <sup>a</sup>
$W j \rightarrow \ell\nu j$	$= \max m_j$ by Lemma 6	two jets with leptonic $b$ decays <sup>a</sup>
$WW \rightarrow \ell\nu\ell\nu$	$= 0$ by Lemma 3	
$ZZ \rightarrow \nu\bar{\nu}\nu\bar{\nu}$	$= m_j$ by Lemma 3	one ISR jet <sup>a</sup>
$LQ \bar{L}\bar{Q} \rightarrow q\nu\bar{q}\bar{\nu}$	$= m_\ell$ by Lemma 3	one ISR jet <sup>a</sup>
$\bar{q}q \rightarrow q\bar{\chi}_1^0 \bar{q}\bar{\chi}_1^0$	$\leq m_W$ by Lemma 2	
$q_1, \bar{q}_1 \rightarrow q\gamma_1, \bar{q}\gamma_1$	$\leq m_W$ by Lemma 1	
	$= 0$ by Lemma 3	also $= m_j$ for one ISR jet <sup>a</sup>
	$\leq m_{LQ}$	} i.e. can take large values
	$\leq m_{\bar{q}}$	
	$\leq m_{q_1}$	

# MT2 as a CUT variable



$$H_T = \sum E_T,$$

$$H_{ET} = \left| - \sum \vec{P}_T \right|,$$

$$\alpha_T = \frac{E_J^{2nd}}{\sqrt{H_T^2 - H_{ET}^2}} = \frac{E_J^{2nd}}{M_T},$$

$$R = \frac{M_T^R}{M_R},$$

$$M_{T2} = \min(\max\{M_{T1}, M_{T2}\}).$$

- $\alpha_T$  and Razor are good to suppress QCD multi jets corruptions to MET events. (No finite endpoint structure, Some characteristic # to cut backgrounds.)
- MT2 has a finite endpoint for SM backgrounds. (A.Barr arXiv:0907.2713)



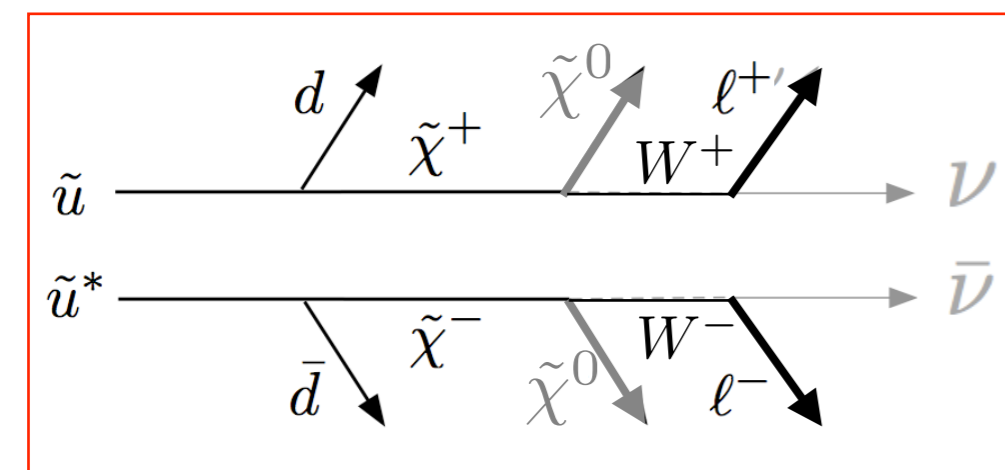
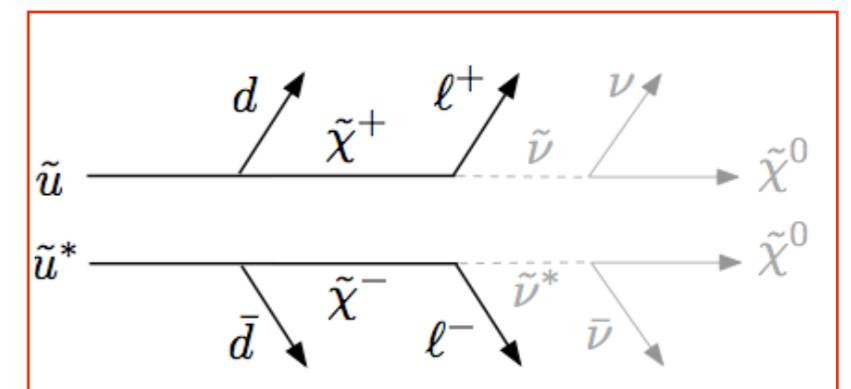
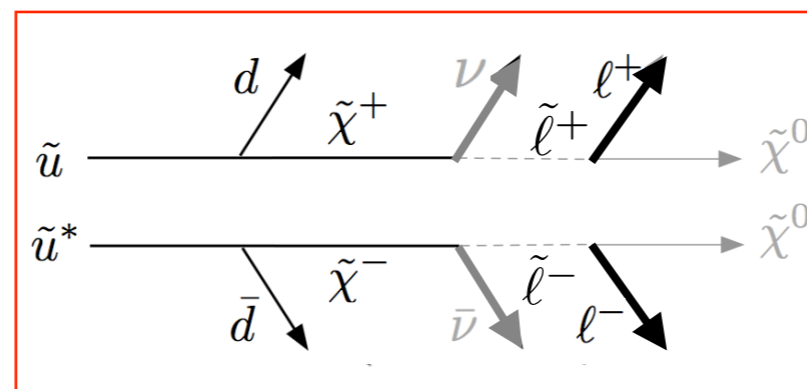
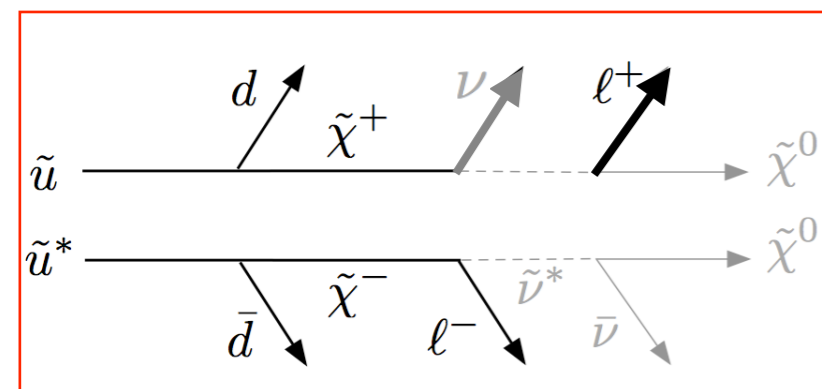
- Various contributions from all over the world.
  - Cambridge: Parents of MT2
  - Oxford: In detail study of MT2
  - ATLAS: Analyses
  - KAIST : Realization of “kink” feature of MT2, MAOS
  - KEK: Various in-depth phenomenological studies of MT2
  - U.C.Davis : New interpretation of MT2 as kinematical bound
  - U.Florida : Generalizations, link to CMS
  - CMS: Analyses
  - Cornell :  $T\bar{T}$  di-leptonic analysis@LHC
  - ETHZ : CMS MT2 analysis
  - CDF : Top quark measurement@Tevatron
  - D0
  - Even more vivid contributions so far
- Now, MT2 is the one of the **standard** variables in MET channels.

# MT2 under BIG assumptions

- I would like to remind you that  $M_{T2}$  was based on three big assumptions.
- Thus if most of signals (the new physics) violate at least one of these assumptions,  
is there any chance for signals **can hide behind Backgrounds?**
- I would like to study the behavior of  $M_{T2}$  when signals break some (all) of these assumptions.

# Various possibilities

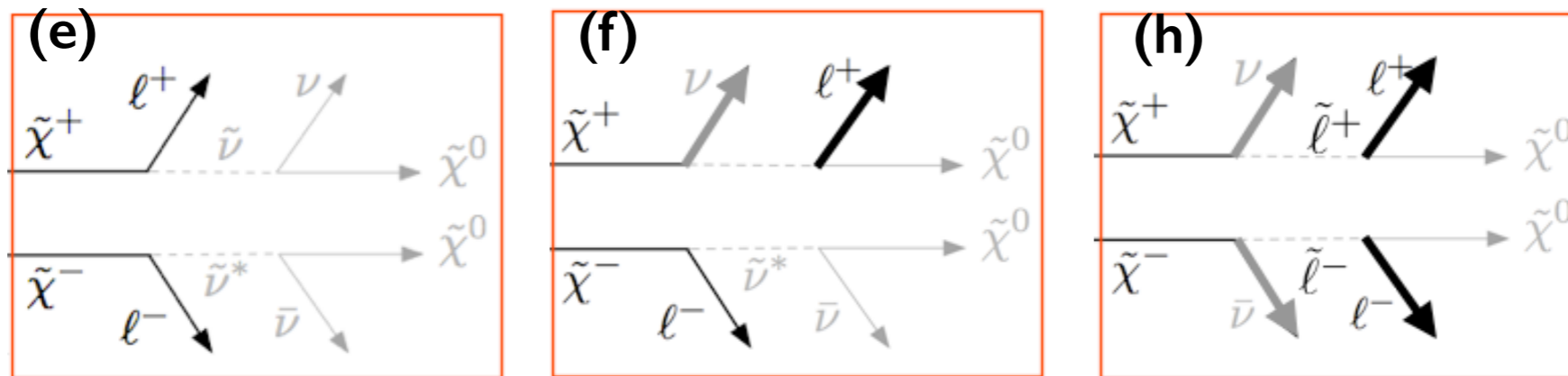
- There may be more than one diagram in the BSM with the same signature. Some can violate assumptions of MT2



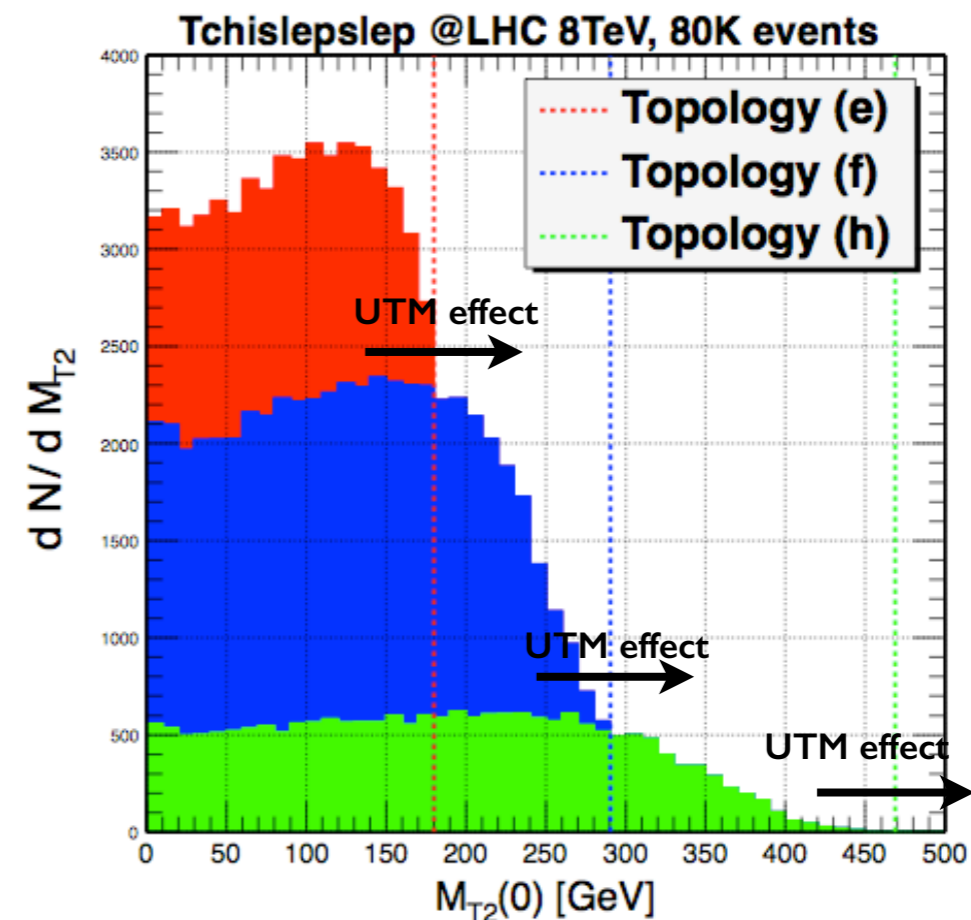
+ more if we consider different signals:  
squarks decays through long cascade  
(four leptons signals)

# Various possibilities

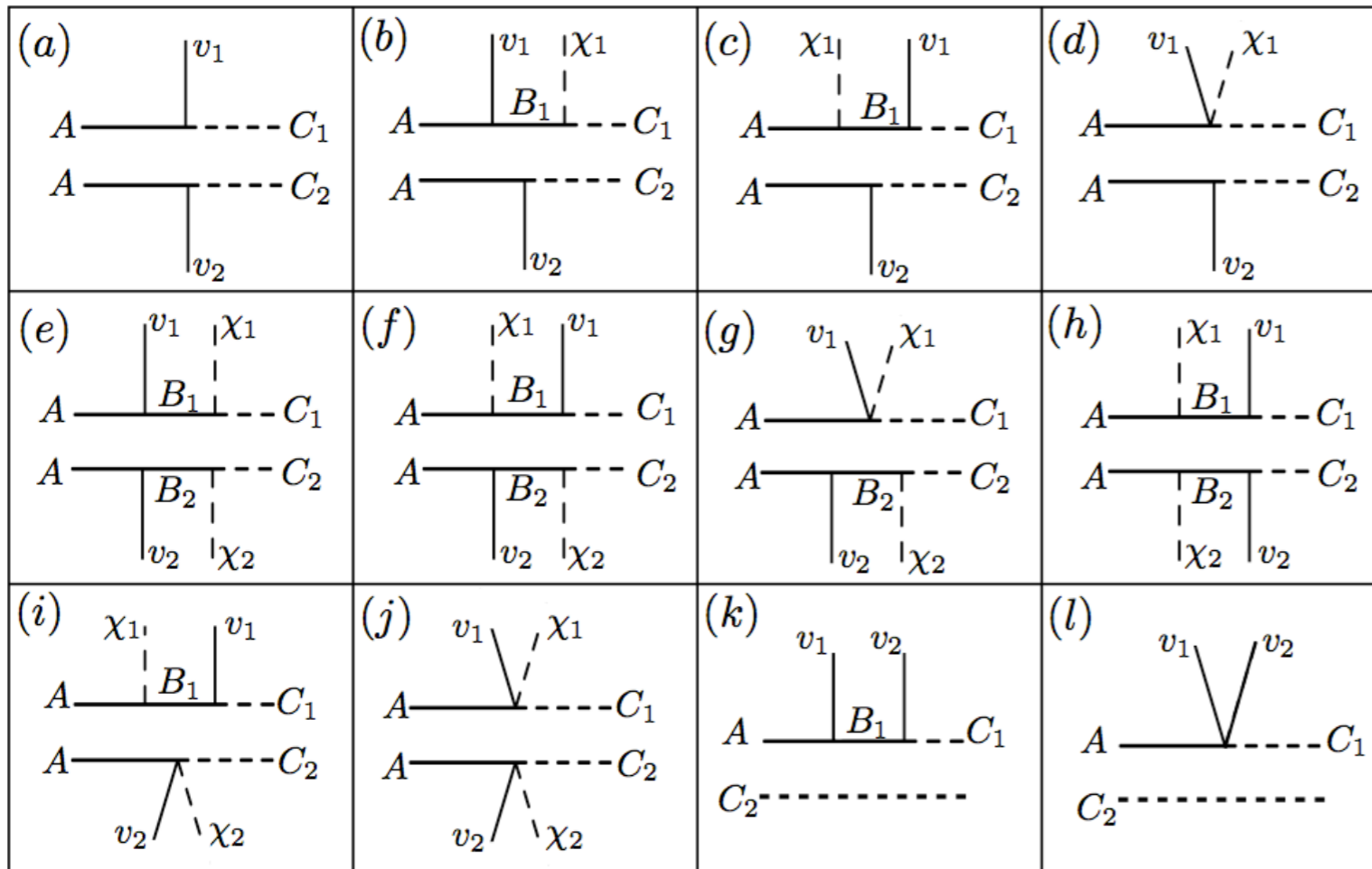
- As an example, we generated CMS Tchislepslep simplified model with  $M_{\tilde{\chi}^+} > M_{\tilde{\nu}_L} \simeq M_{\tilde{\ell}_L^+} > M_{\tilde{\chi}^0}$



- Simulated[parton level] with masses:  
 chargino 500GeV  
 slepton(sneutrino) 400GeV  
 LSP: 100GeV



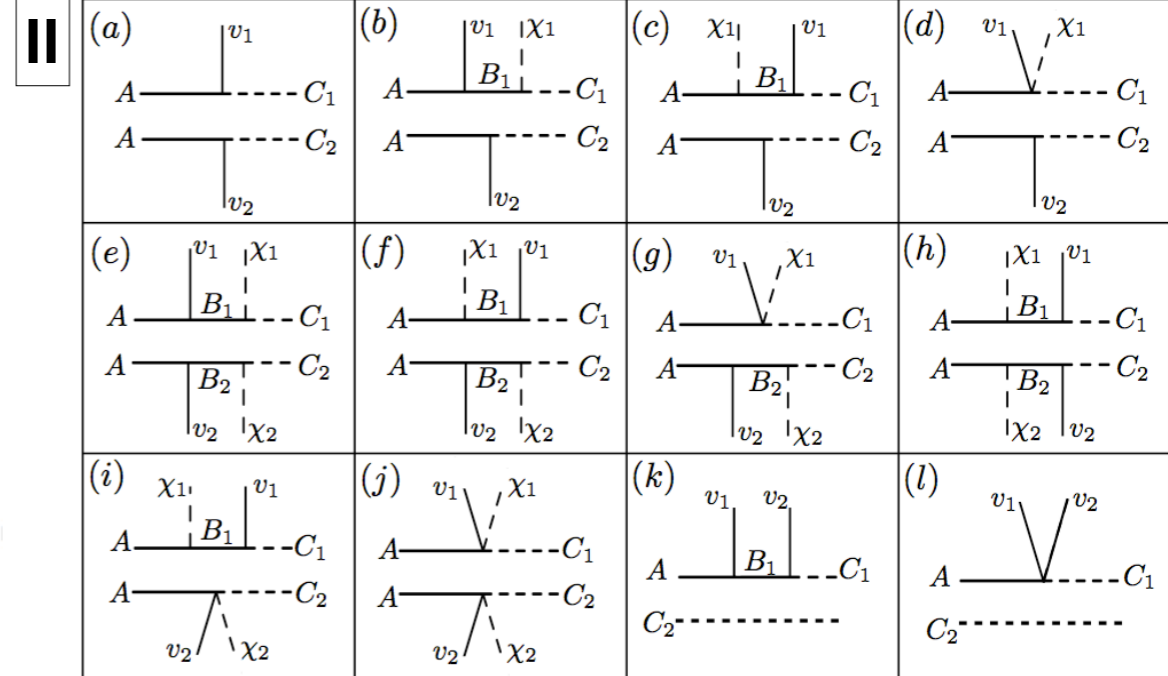
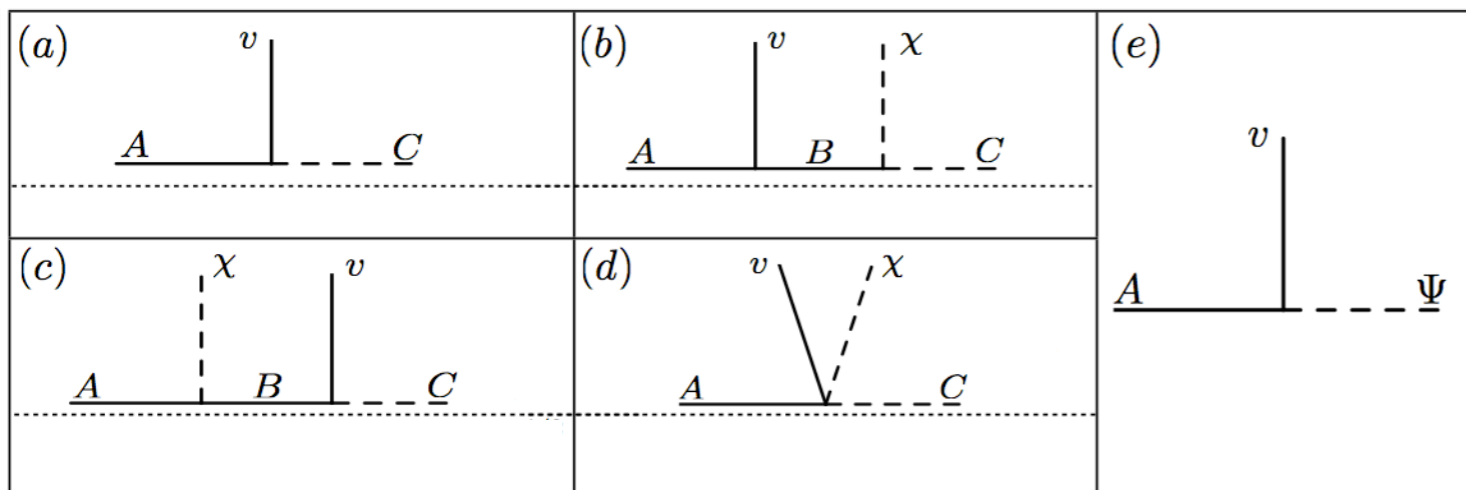
# Various possibilities



- There are 12 (sub) diagrams that have two visible particles and up to four invisible particles.
- We have options:
  1. we need to invent new observables based on each event-topology.
  2. And/Or we need to understand how to interpret a result of existing observables (e.g. MT2) for each event-topology case.

# Effective event-topology

## I. Number of invisible particle: Introduce Equivalent event-topology method



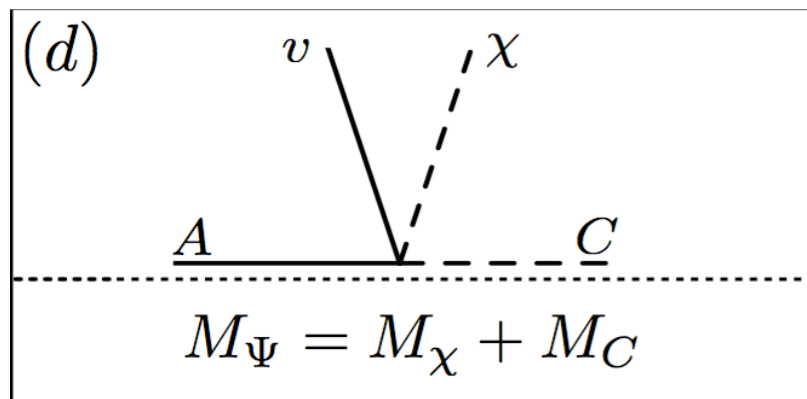
- We apply an observable that was motivated initially for the II (a) assumptions, and want to interpret results (endpoint of distributions) in various cases.
- Diagrams in II (except k,l) are combinations of a basic decaying leg I (a), (b), (c), and (d).
- For example, in I (b), we can treat B that decays invisibly as invisible particle.
- The only non-trivial case will be I (c).



- We are interested in the **endpoint** of distributions.  
Thus we need to focus on the range of a (transverse) momentum of visible particle  $v$  (at the rest frame of  $A$ .) **(Back to back Boost Inv.)**

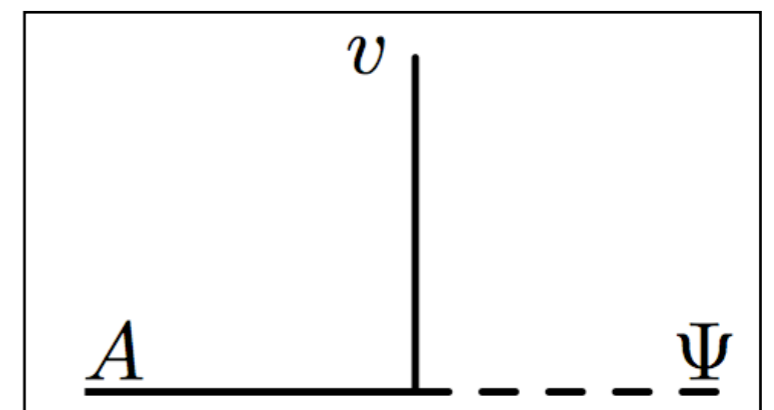
At  $A$ 's rest frame, a range of transverse momentum of  $v$

$$0 \leq P_T \leq \frac{M_A}{2} \left( 1 - \frac{M_{C\chi}^2}{M_A^2} \right)$$

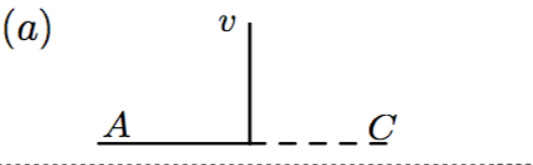
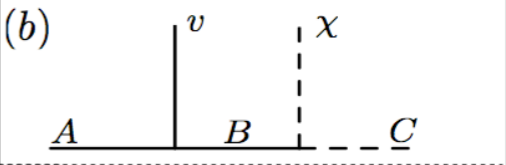
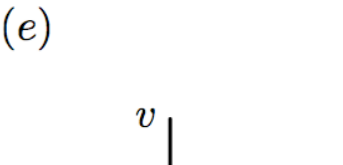
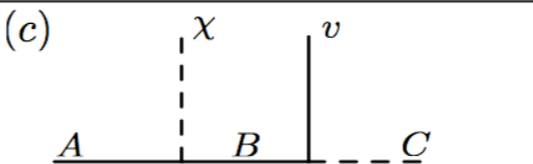
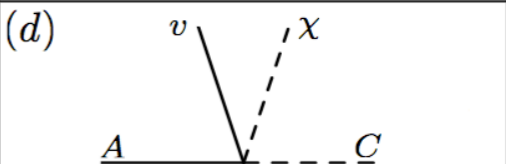


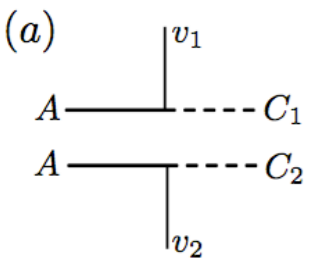
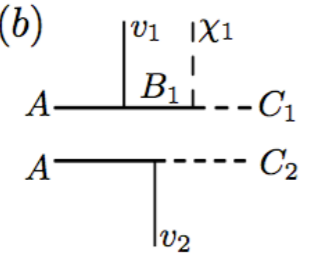
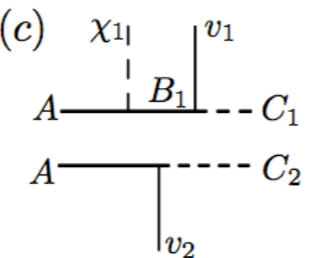
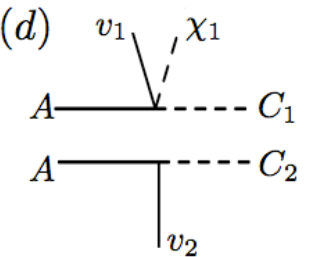
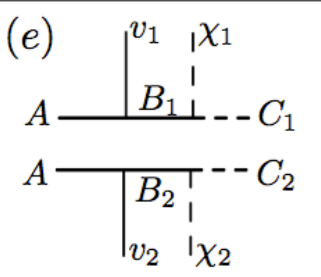
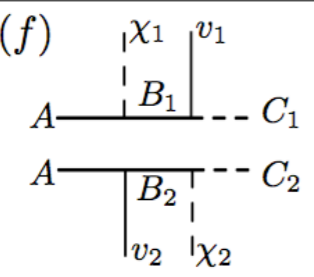
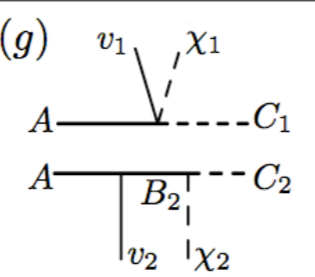
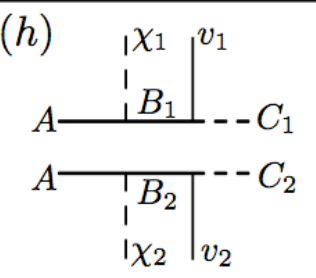
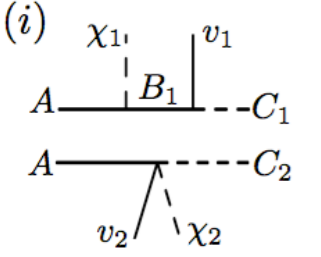
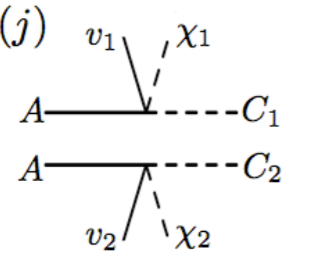
Thus,  $P_T$  will have a maximum when the invariant mass  $M_{C\chi}$  (of  $C$  and  $\chi$ ) has a minimum value  $= M_C + M_\chi$

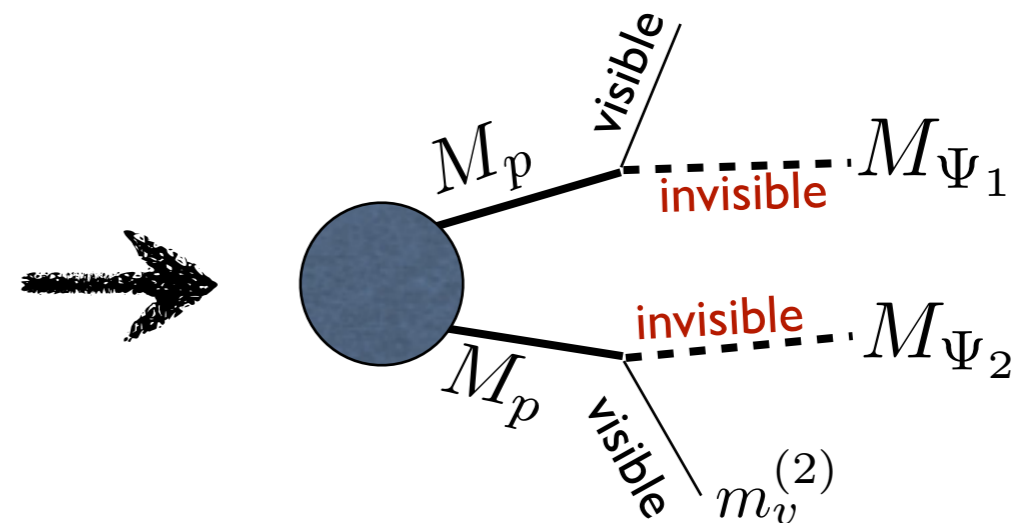
This range of  $P_T$  also come from the right diagram where a particle  $\Psi$  with a mass of  $M_\Psi = M_C + M_\chi$ . Thus we can replace (d) with a right diagram for the endpoint of transverse observables.



Using “Equivalent event-topology method”, we can change event-topologies with multi-invisible particles into an event-topology with two invisible particles.

(a) 	(b) 	(e) 
$M_\Psi = M_C$	$M_\Psi = M_B$	
(c) 	(d) 	
$M_\Psi = M_A \left\{ 1 - \frac{M_B}{M_A} \left( 1 - \frac{M_C^2}{M_B^2} \right) e^\eta \right\}^{1/2}$	$M_\Psi = M_\chi + M_C$	

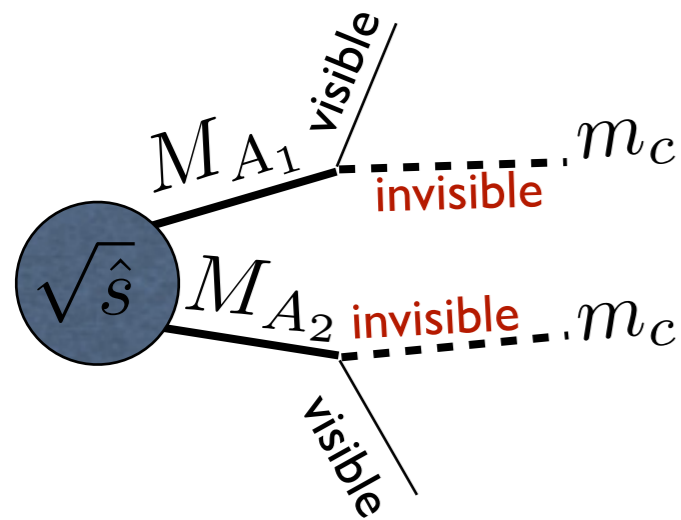
(a) 	(b) 	(c) 	(d) 
(e) 	(f) 	(g) 	(h) 
(i) 	(j) 		



- But, now we need to deal with the case with different types of invisible particle (  $M_{\Psi_1} \neq M_{\Psi_2}$  ): Studied by P.Konar, K.Matchev, K.Kong. MP [arxiv:0911.4126]

# When decaying particles are different

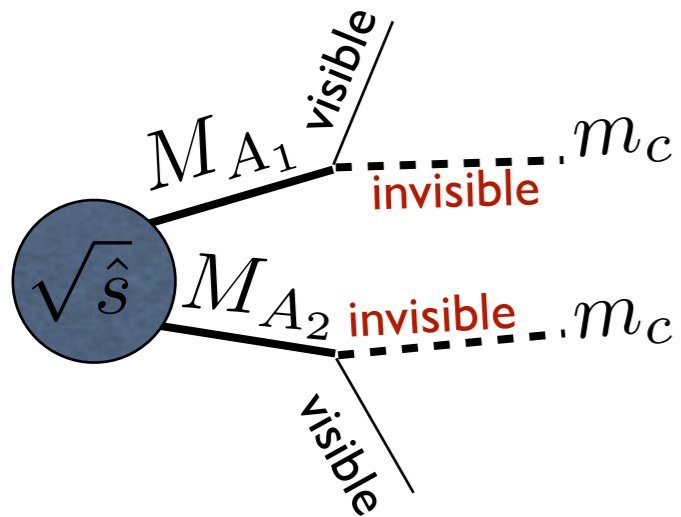
If  $M_{A_2} > M_{A_1}$ , then  $A_1$  get the additional boost  $\delta\eta(\sqrt{\hat{s}})$  from  $E_{CM}(\sqrt{\hat{s}})$  compared to  $A_2$ .



$$\eta_1(\sqrt{\hat{s}}) = \cosh^{-1} \left( \frac{\hat{s} + M_{A_1}^2 - M_{A_2}^2}{2\hat{s} M_{A_1}} \right) \quad \eta_2(\sqrt{\hat{s}}) = \cosh^{-1} \left( \frac{\hat{s} + M_{A_2}^2 - M_{A_1}^2}{2\hat{s} M_{A_2}} \right)$$

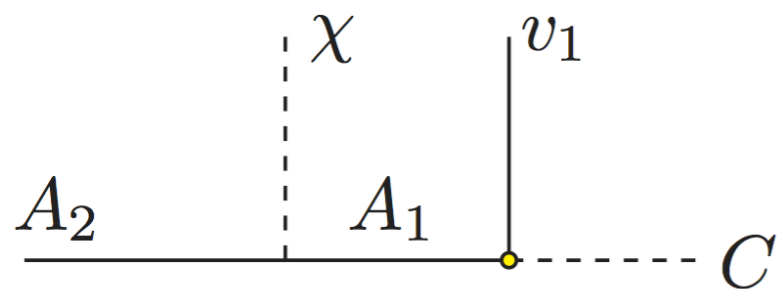
$$\delta\eta(\sqrt{\hat{s}}) \equiv \eta_1(\sqrt{\hat{s}}) - \eta_2(\sqrt{\hat{s}}) = \cosh^{-1} \left[ \frac{M_{A_2}^2 + M_{A_1}^2 - \left( \frac{M_{A_2}^2 - M_{A_1}^2}{\sqrt{\hat{s}}} \right)^2}{2M_{A_2}M_{A_1}} \right]$$

This additional boost will give effect on the visible part on  $A_1$ .  
We can mimic this situation by inserting “GHOST” particle in front of  $A_1$



$$\delta\eta(\sqrt{\hat{s}}) \equiv \eta_1(\sqrt{\hat{s}}) - \eta_2(\sqrt{\hat{s}}) = \cosh^{-1} \left[ \frac{M_{A_2}^2 + M_{A_1}^2 - \left( \frac{M_{A_2}^2 - M_{A_1}^2}{\sqrt{\hat{s}}} \right)^2}{2M_{A_2}M_{A_1}} \right]$$

This additional boost will give effect on the visible part on  $A_1$ . We can mimic this situation by inserting “GHOST” particle in front of  $A_1$

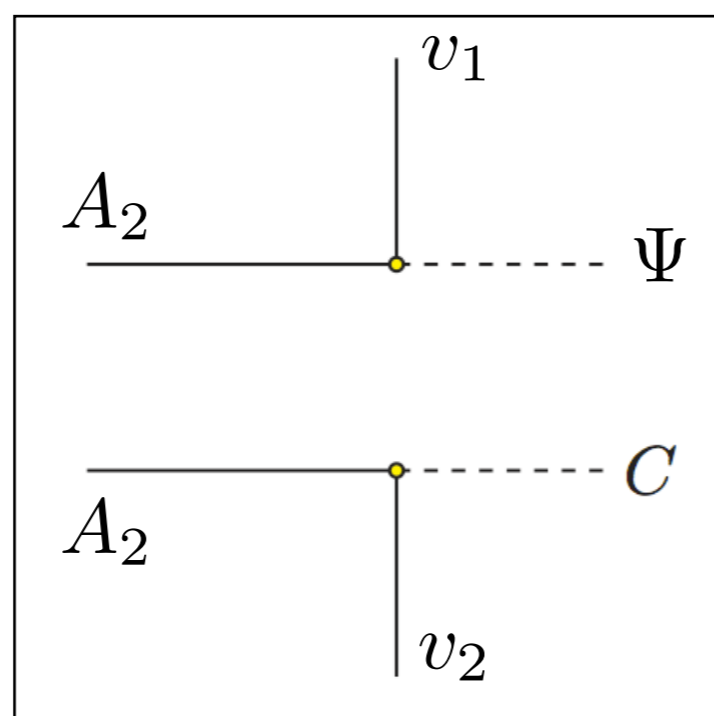
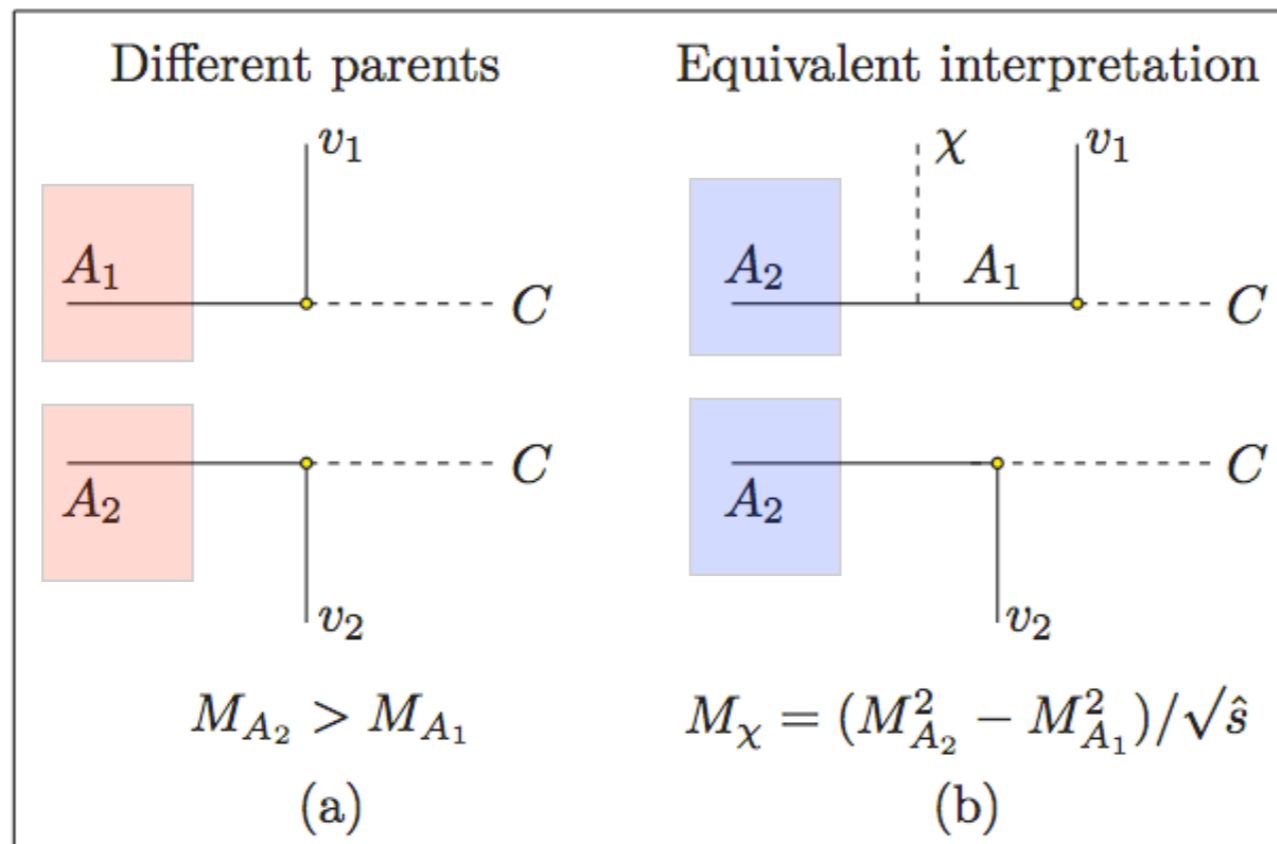


$$\eta = \cosh^{-1} \left( \frac{M_{A_2}^2 + M_{A_1}^2 - M_{\chi}^2}{2M_{A_2}M_{A_1}} \right)$$

Thus we can re-interpret this situation by putting invisible particle with mass

$$M_{\chi}(\sqrt{\hat{s}}) \equiv \frac{M_{A_2}^2 - M_{A_1}^2}{\sqrt{\hat{s}}}$$

$$M_{\Psi} = M_{A_2} \left\{ 1 - \frac{M_{A_1}}{M_{A_2}} \left( 1 - \frac{m_c^2}{M_{A_1}^2} \right) e^{\eta} \right\}^{1/2}$$



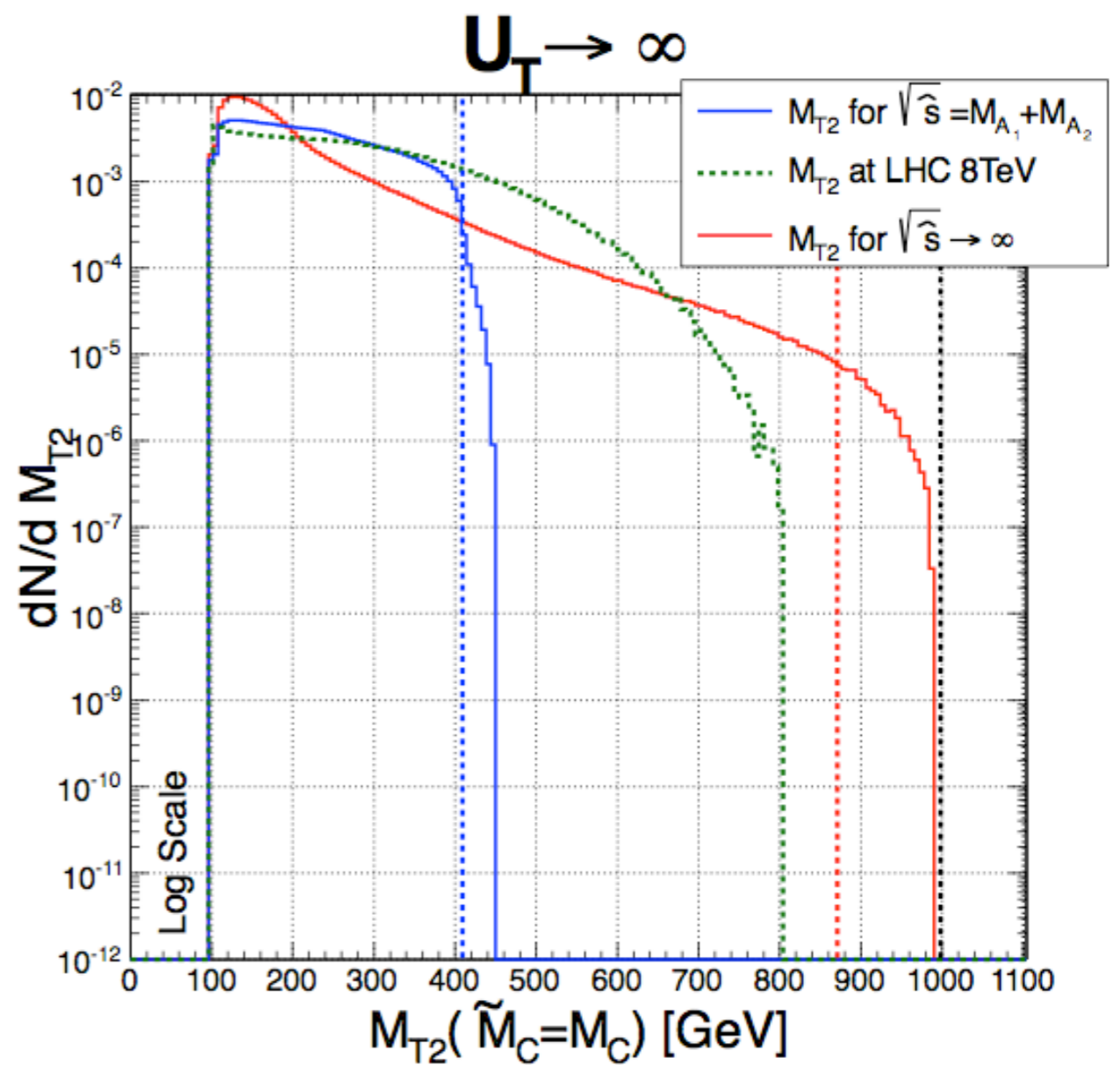
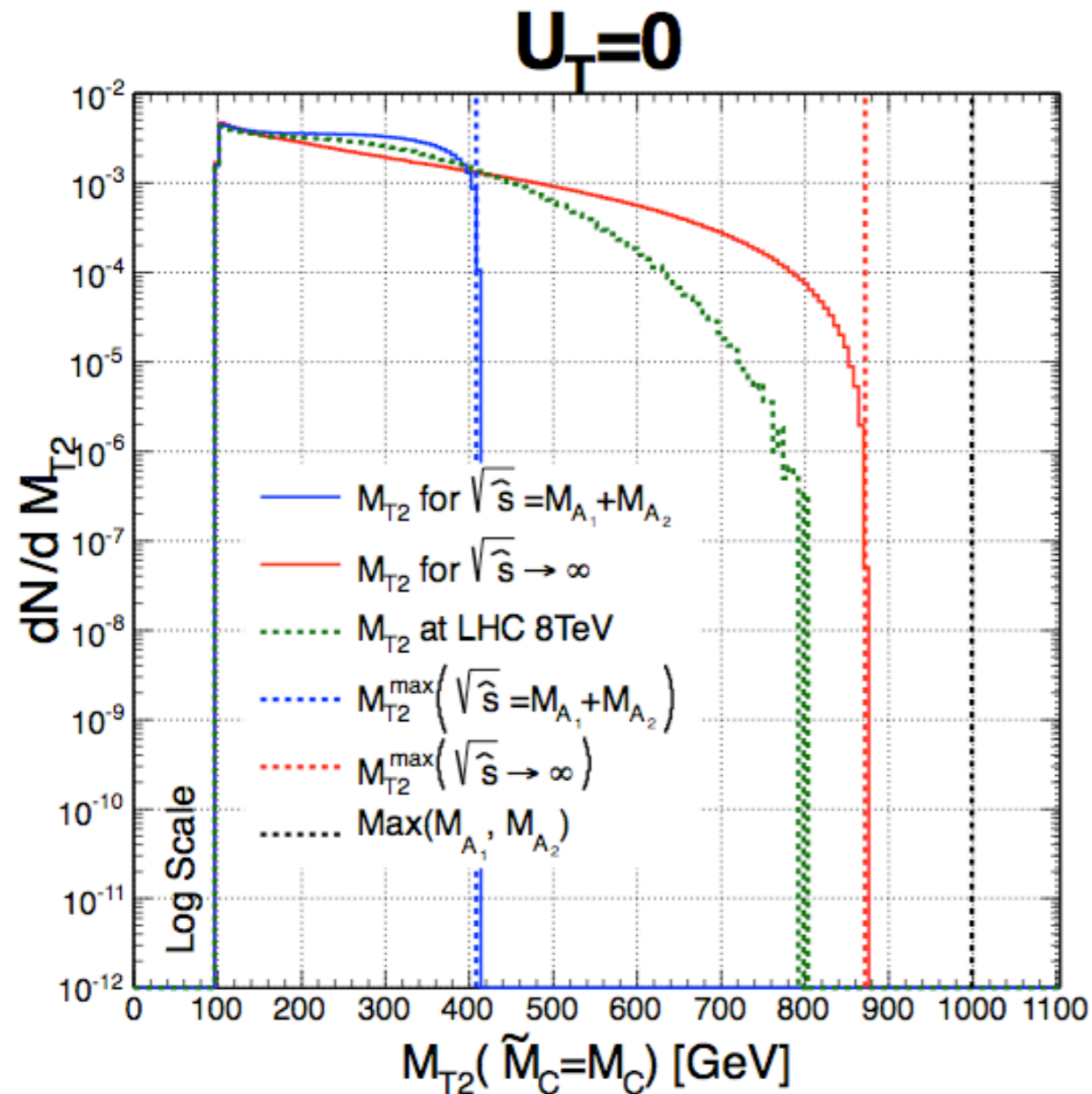
$$\eta = \cosh^{-1} \left( \frac{M_{A_2}^2 + M_{A_1}^2 - M_\chi^2}{2M_{A_2}M_{A_1}} \right)$$

$$M_\Psi = M_{A_2} \left\{ 1 - \frac{M_{A_1}}{M_{A_2}} \left( 1 - \frac{m_c^2}{M_{A_1}^2} \right) e^\eta \right\}^{1/2}$$

$$0 \leq M_{\chi}(\sqrt{\hat{s}}) \leq M_{A_2} - M_{A_1} \quad \text{with} \quad M_{A_1} + M_{A_2} \leq \sqrt{\hat{s}} < \infty$$

resulting in the effective particle  $\Psi$ 's mass dependency on the  $\sqrt{\hat{s}}$

$$(M_{A_2}, M_{A_1}, m_C) = (1\text{TeV}, 200\text{GeV}, 100\text{GeV})$$

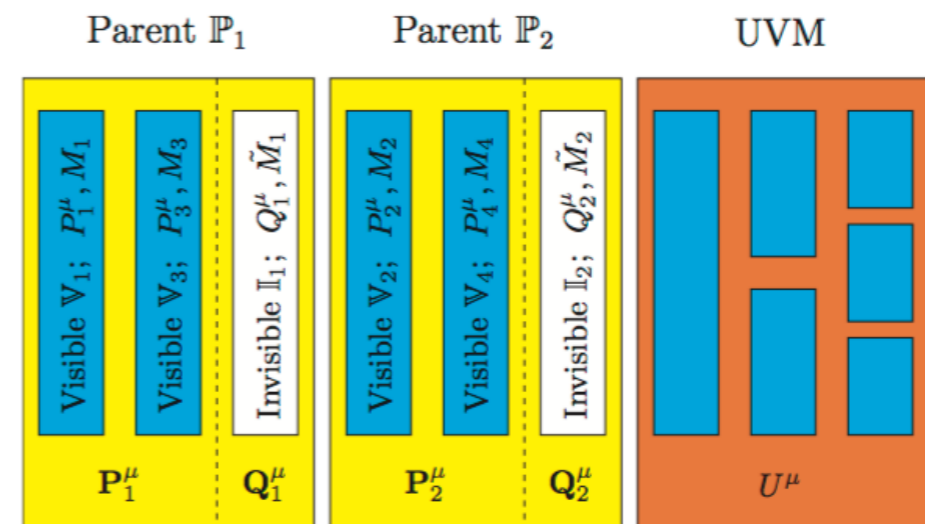
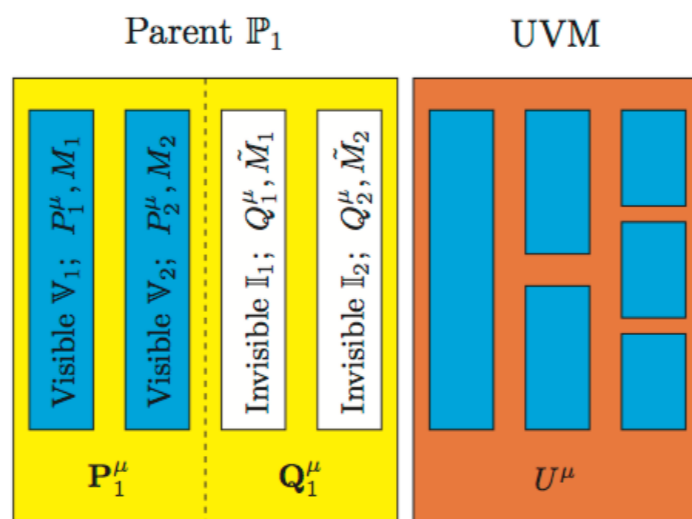
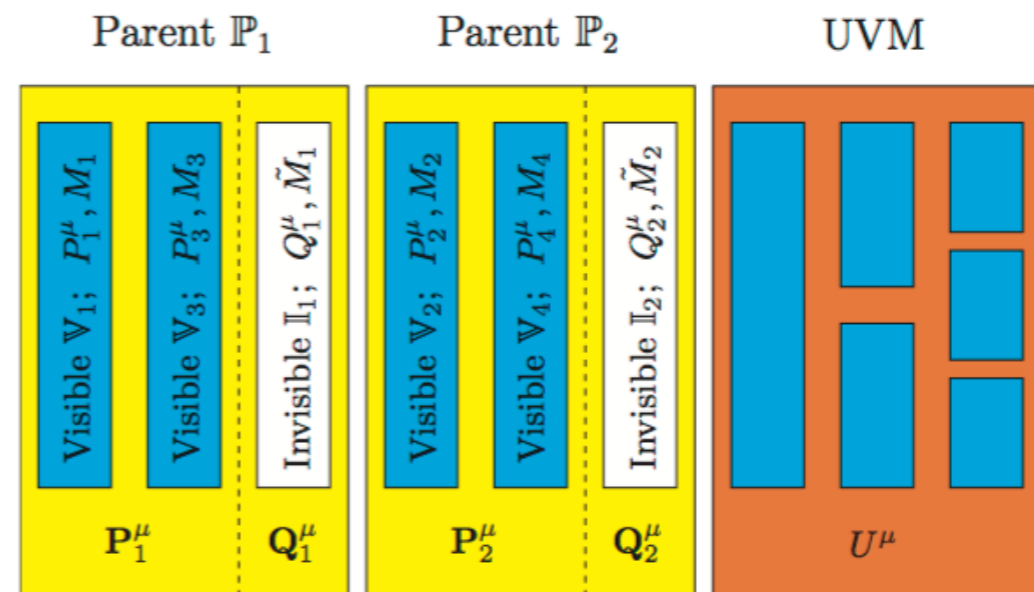
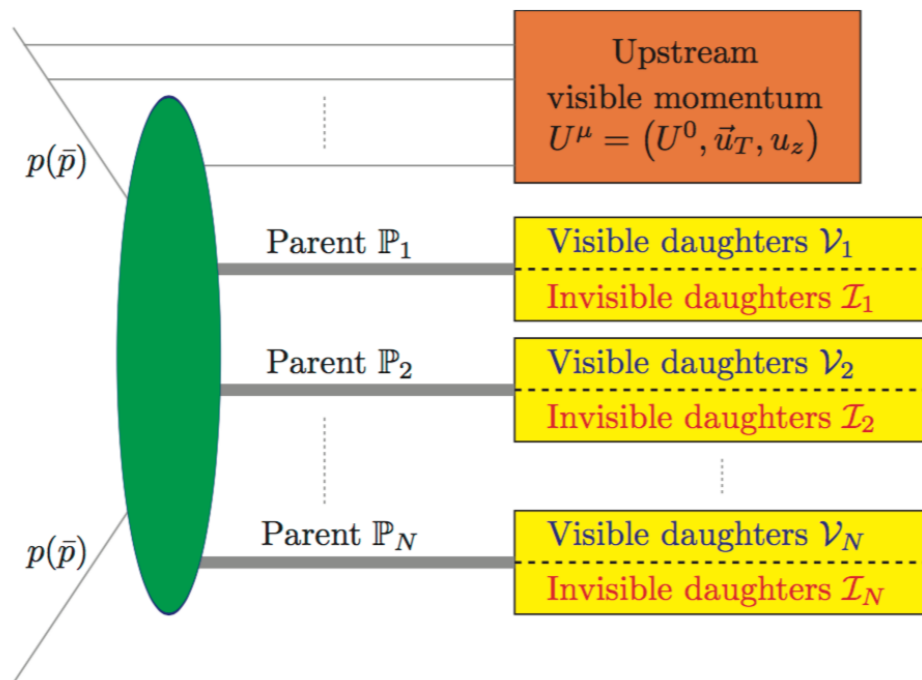


MT2 is **NOT** sensitive (good) variable to represent “kinematics”  
when decaying particles are **different**



# MET variables

- How to cluster visible (& invisible) particles



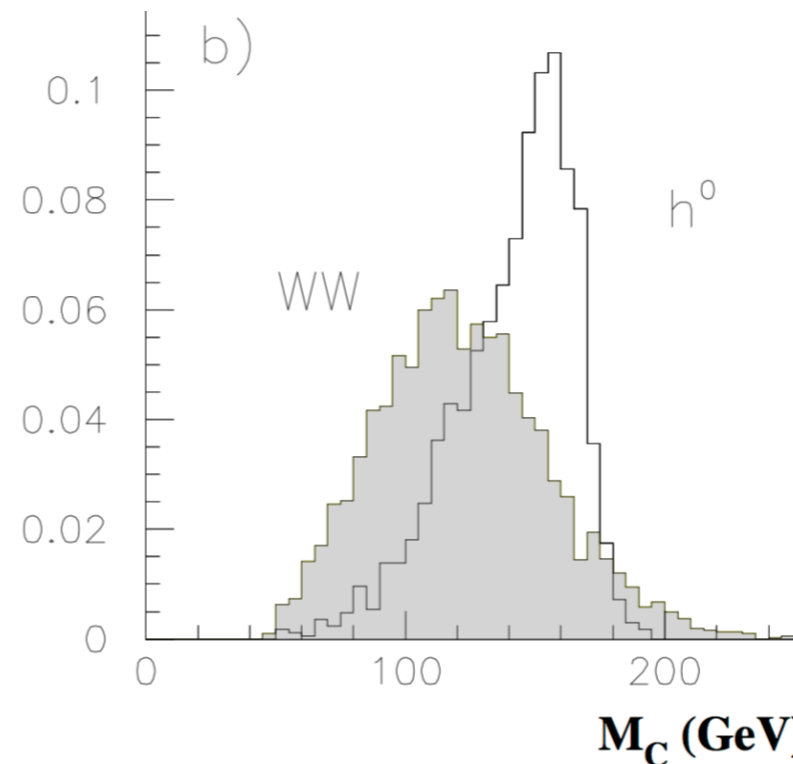
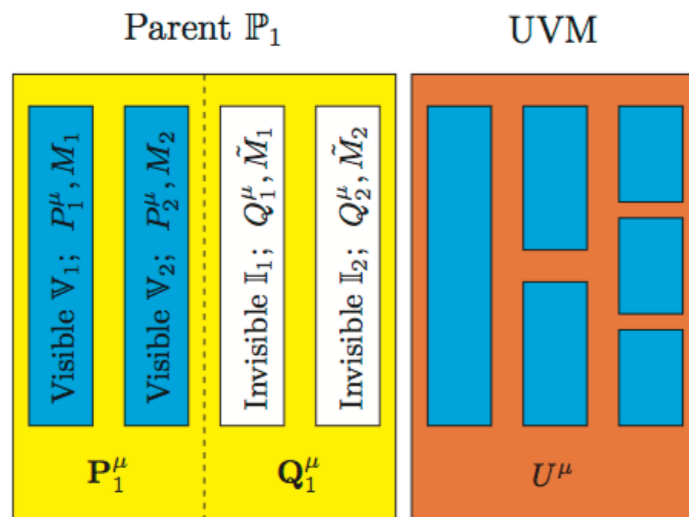
# MET variables

- How to cluster visible (& invisible) particles

Tao Han: hep-ph/0508097

$$\underline{H \rightarrow W_1 W_2 \rightarrow \ell_1 \nu_1 \ell_2 \nu_2:}$$

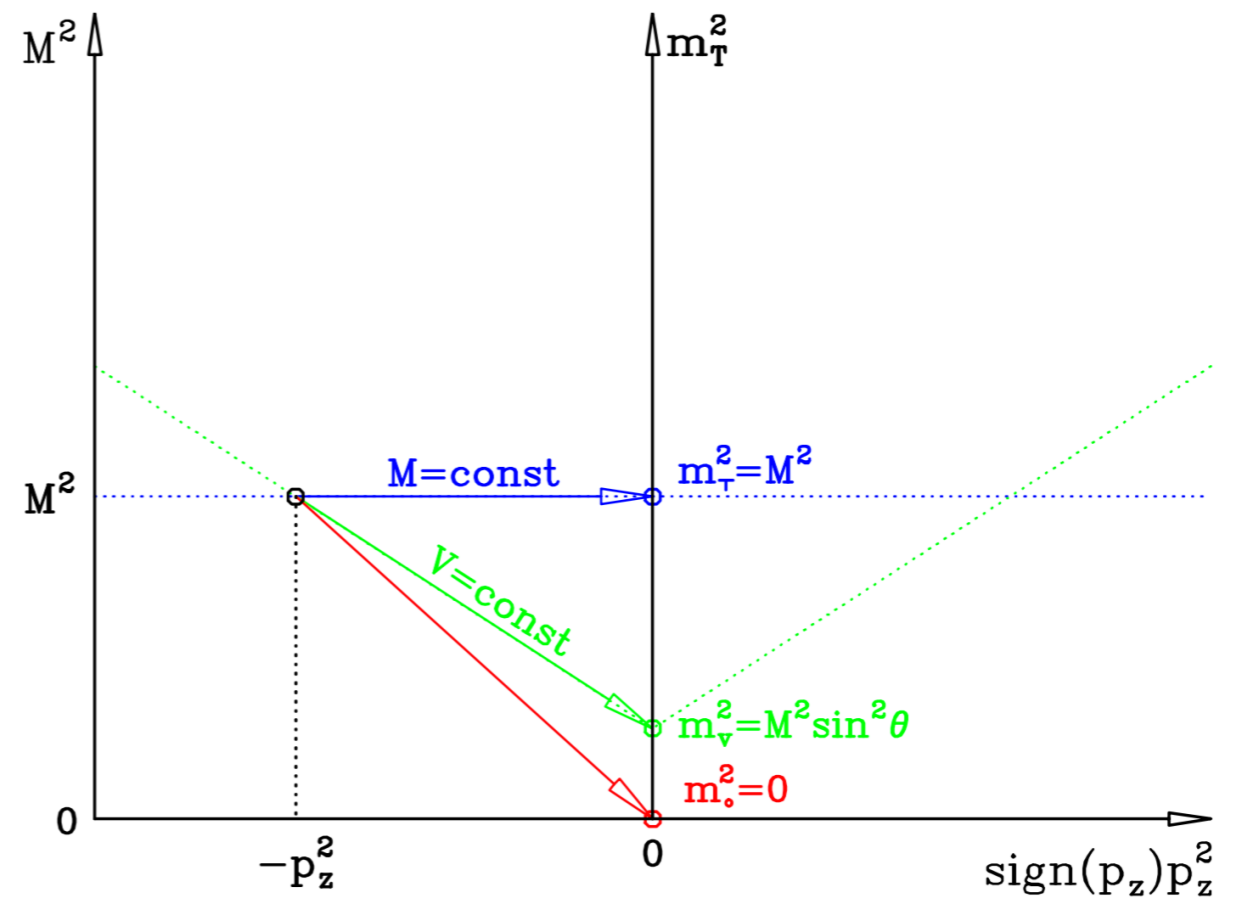
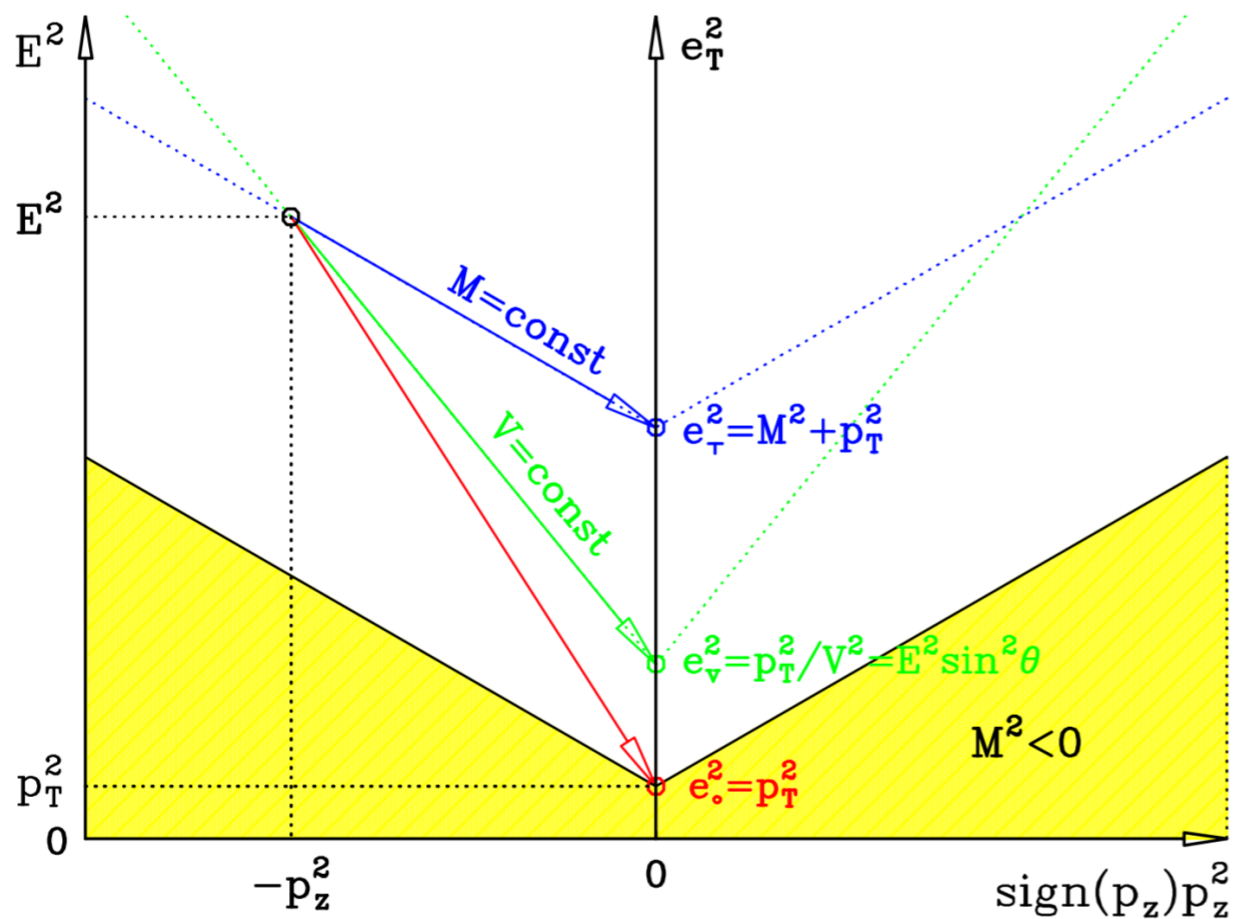
$$M_{C,WW}^2 = \left( \sqrt{p_{T,\ell\ell}^2 + M_{\ell\ell}^2} + \cancel{p}_T \right)^2 - (\vec{p}_{T,\ell\ell} + \vec{p}_T)^2$$



V. Barger, T. Han, and J. Ohnemus, Phys. Rev. D37, 1174 (1988)


# MET variables

- How to project four-vector into transverse (to beam) plane.



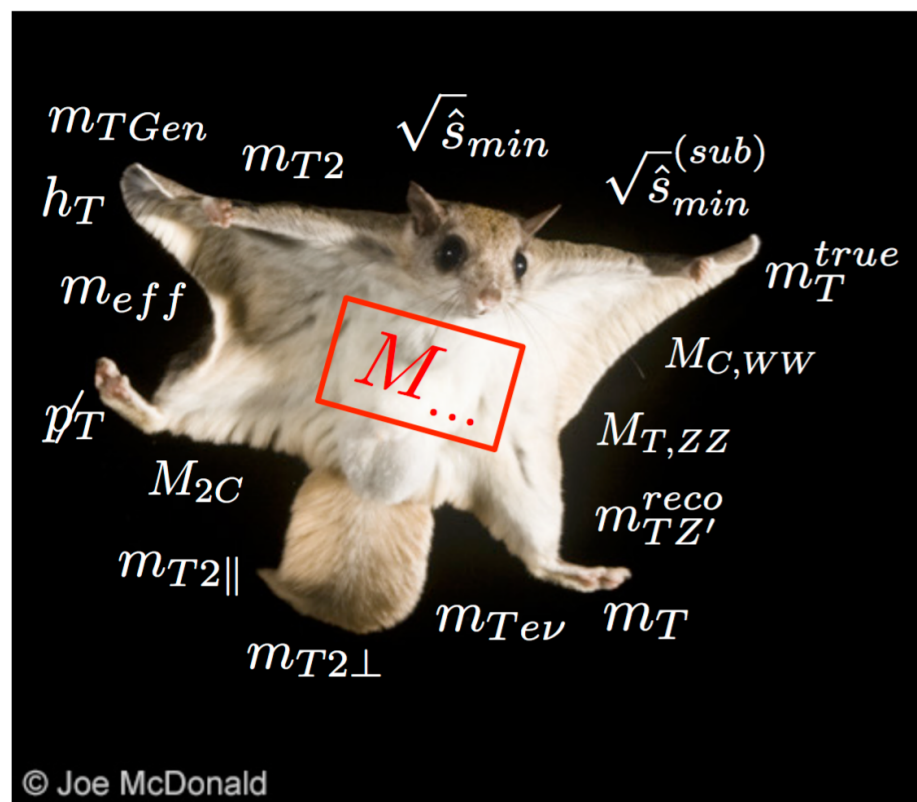
# MET variables

- How to project four-vector into transverse (to beam) plane.

Type of variables	Operations			 Notation
	First	Second	Third	
Unprojected	Partitioning	Minimization	—	$M_N$
Early partitioned (late projected) $M_{NT}$	Partitioning	$T = \top$ projection	Minimization	$M_{N\top}$
	Partitioning	$T = \vee$ projection	Minimization	$M_{N\vee}$
	Partitioning	$T = \circ$ projection	Minimization	$M_{N\circ}$
Late partitioned (early projected) $M_{TN}$	$T = \top$ projection	Partitioning	Minimization	$M_{\top N}$
	$T = \vee$ projection	Partitioning	Minimization	$M_{\vee N}$
	$T = \circ$ projection	Partitioning	Minimization	$M_{\circ N}$

# MET variables

- How to cluster visible (& invisible) particles
- How to project four-vector into transverse (to beam) plane.



More details:

$$M_{1\top}^2(\mathbf{M}_1) \equiv \left( \sqrt{\mathbf{M}_1^2 + \mathbf{p}_{1T}^2} + \sqrt{\mathbf{M}_1^2 + \cancel{p}_T^2} \right)^2 - u_T^2$$

$$M_{1\top}^2(0) = \left( \sqrt{M_{e^+e^-}^2 + \vec{p}_{T,e^+e^-}^2} + \cancel{p}_T \right)^2 - \vec{u}_T^2,$$

$$M_{1\top}(0) = M_{C,ww}$$

$$m_{T2} \equiv \min_{\sum \vec{q}_{iT} = \cancel{p}_T} [\max [\mathcal{M}_{1\top}, \mathcal{M}_{2\top}]]$$

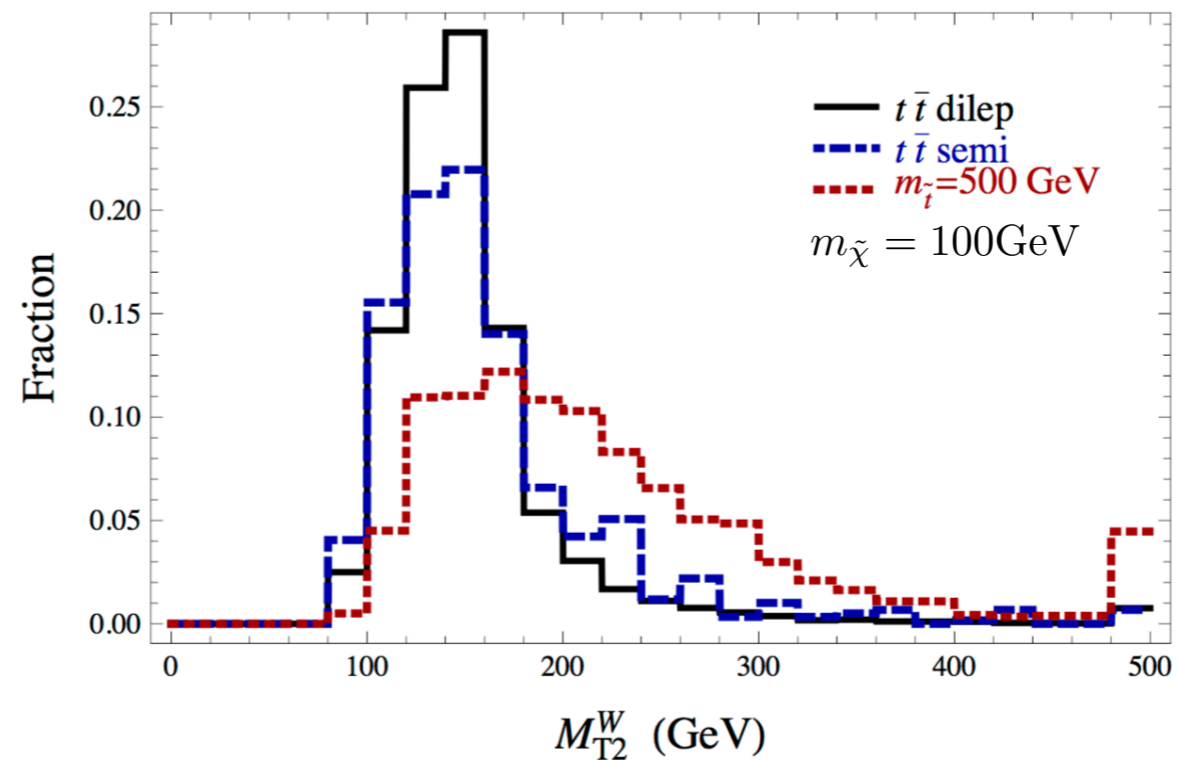
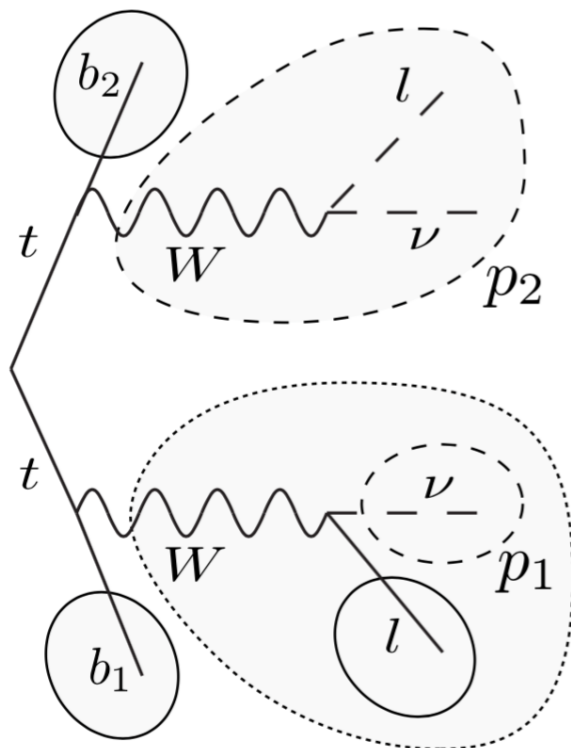
$$m_{T2}^{(1+3)}(\mathbf{M}) \equiv M_{2\top}(\mathbf{M}) = M_2(\mathbf{M}).$$

On-shell constrained

M2

# Put “assumed” constraints

- One can describe SM Background more by using additional constraints

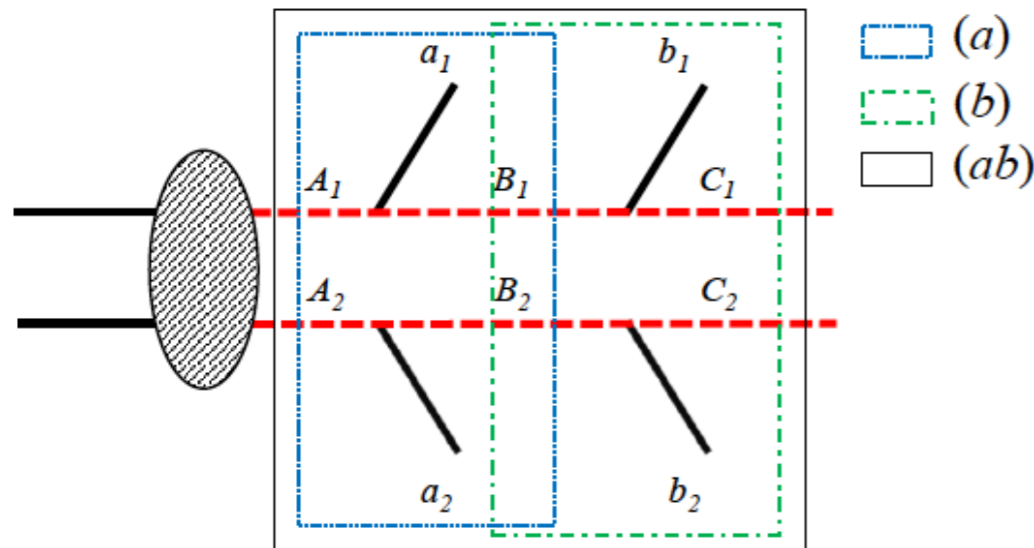


$$M_{T2}^W = \min \left\{ m_y \text{ consistent with: } \left[ \begin{array}{l} \vec{p}_1^T + \vec{p}_2^T = \vec{E}_T^{\text{miss}}, \quad p_1^2 = 0, \quad (p_1 + p_\ell)^2 = p_2^2 = M_W^2, \\ (p_1 + p_\ell + p_{b_1})^2 = (p_2 + p_{b_2})^2 = m_y^2 \end{array} \right] \right\}$$



# Constrained M2

- Mass on-shell constraints



Subsystem	Parents $P_i$	Daughters $D_i$	Relatives $R_i$
(ab)	$A_i$	$C_i$	$B_i$
(a)	$A_i$	$B_i$	$C_i$
(b)	$B_i$	$C_i$	$A_i$

$$M_2(\tilde{m}) \equiv \min_{\vec{q}_1, \vec{q}_2} \{ \max [M_{P_1}(\vec{q}_1, \tilde{m}), M_{P_2}(\vec{q}_2, \tilde{m})] \}$$

$$\vec{q}_{1T} + \vec{q}_{2T} = \vec{P}_T \quad \text{with constraints:}$$

Subsystem (ab)		Subsystem (a)		Subsystem (b)	
variable	constraints	variable	constraints	variable	constraints
$M_{2XX}(ab)$	—	$M_{2XX}(a)$	—	$M_{2XX}(b)$	—
$M_{2CX}(ab)$	$M_{A_1}^2 = M_{A_2}^2$	$M_{2CX}(a)$	$M_{A_1}^2 = M_{A_2}^2$	$M_{2CX}(b)$	$M_{B_1}^2 = M_{B_2}^2$
$M_{2XC}(ab)$	$M_{B_1}^2 = M_{B_2}^2$	$M_{2XC}(a)$	$M_{C_1}^2 = M_{C_2}^2$	$M_{2XC}(b)$	$M_{A_1}^2 = M_{A_2}^2$
$M_{2CC}(ab)$	$M_{A_1}^2 = M_{A_2}^2$ $M_{B_1}^2 = M_{B_2}^2$	$M_{2CC}(a)$	$M_{A_1}^2 = M_{A_2}^2$ $M_{C_1}^2 = M_{C_2}^2$	$M_{2CC}(b)$	$M_{B_1}^2 = M_{B_2}^2$ $M_{A_1}^2 = M_{A_2}^2$

- **Power of constrained minimization (I) :**  
enhanced event saturation to the target mass scale to be measured

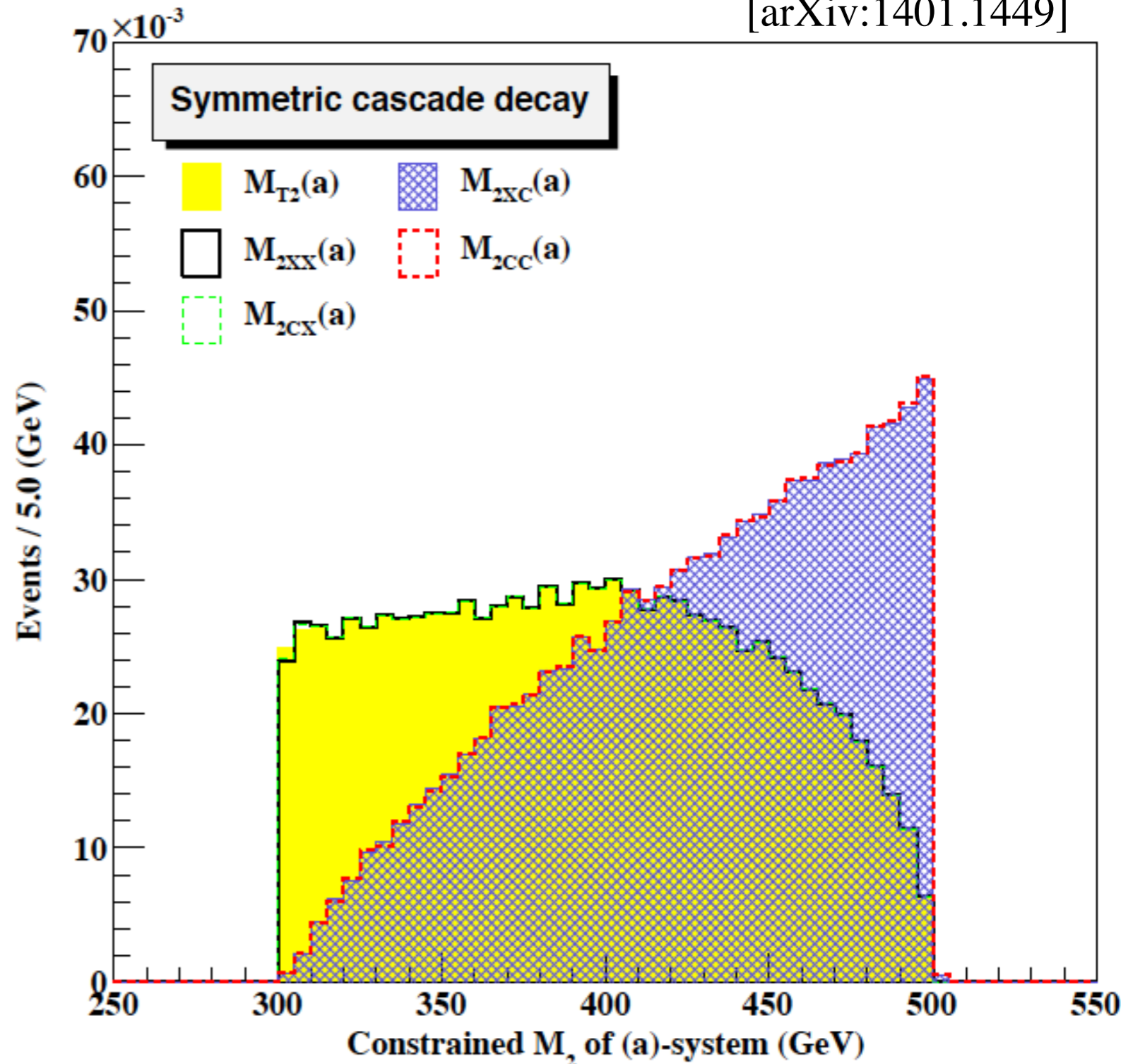
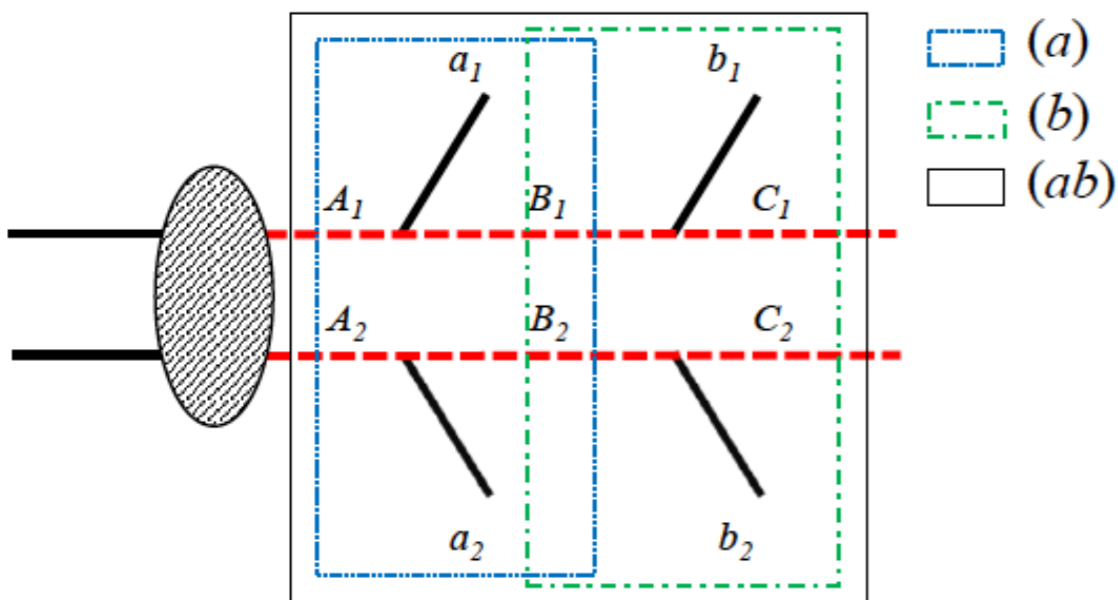
$$M_{2CC} \equiv \min_{\vec{q}_1, \vec{q}_2} \{ \max [M_{P_1}(\vec{q}_1, \tilde{m}), M_{P_2}(\vec{q}_2, \tilde{m})] \}$$

$$\vec{q}_{1T} + \vec{q}_{2T} = \vec{P}_T$$

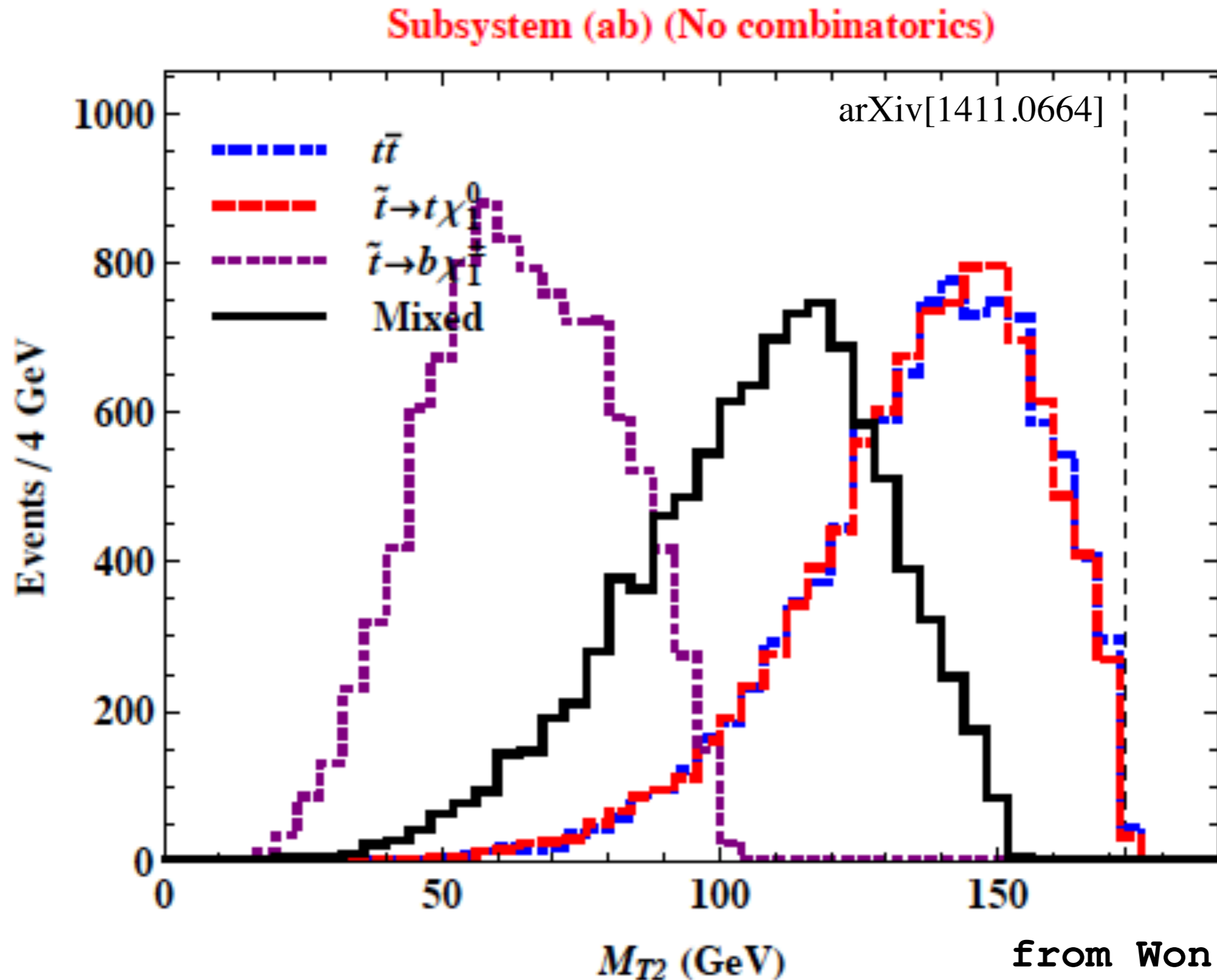
$$M_{P_1} = M_{P_2}$$

$$M_{R_1}^2 = M_{R_2}^2$$

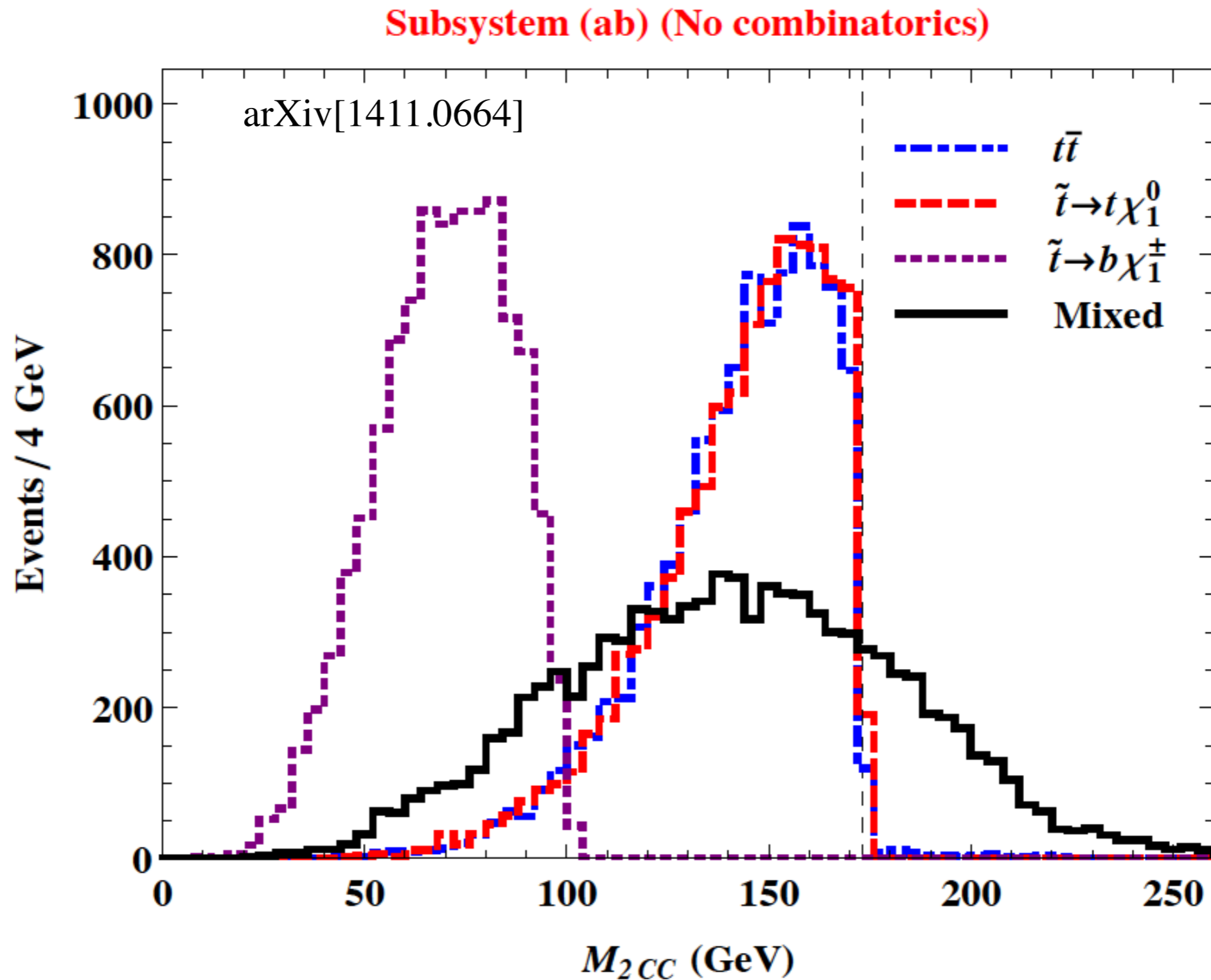
[arXiv:1401.1449]



- Power of constrained minimization for signal discovery (ex:  $M_{T2}$  vs  $M_{2CC}$ )



- Power of constrained minimisation for signal discovery (ex: MT2 vs M2CC)



from Won Sang Cho

# Constrained Minimization

- 1) of **mass functions** of mother particle masses :

$$\tilde{M}(p, q) \quad / . \quad p: \text{visible}, q: \text{invisible four momenta}$$

- 2) **over invisible momentum d.o.f**  $\dot{q}$
- 3) **subject to constraint functions**  $\dot{c}_i(p, q)$   
involved with on-shell / endpoint relations

$$\bar{M} = \min_{q \in R^n} \tilde{M}(p, q) \quad \text{subject to} \quad c_{i=1..m}(p, q) = 0$$

- **For example) MT2**

- =>  $\tilde{M}^2 \equiv \max [(p_1 + q_1)^2, (p_2 + q_2)^2]$

- => subject to **minimal constraints with PT conservation.**



# Numerical Algorithm

- Augmented Lagrangian Method

- Modify the problem

- **Constrained Minimisation** (in  $x$ ,  $\lambda$ )

TO

- **A series of Unconstrained Minimisation** (in  $x$ )

- while the constraint conditions are satisfied by the convexification by penalty-terms

- simultaneously, the Lagrange multipliers get updated and evolved, iteration by iteration !!

# Numerical Algorithm

- Augmented Lagrangian with ..

- 1) penalty parameter ( $\mu$ )

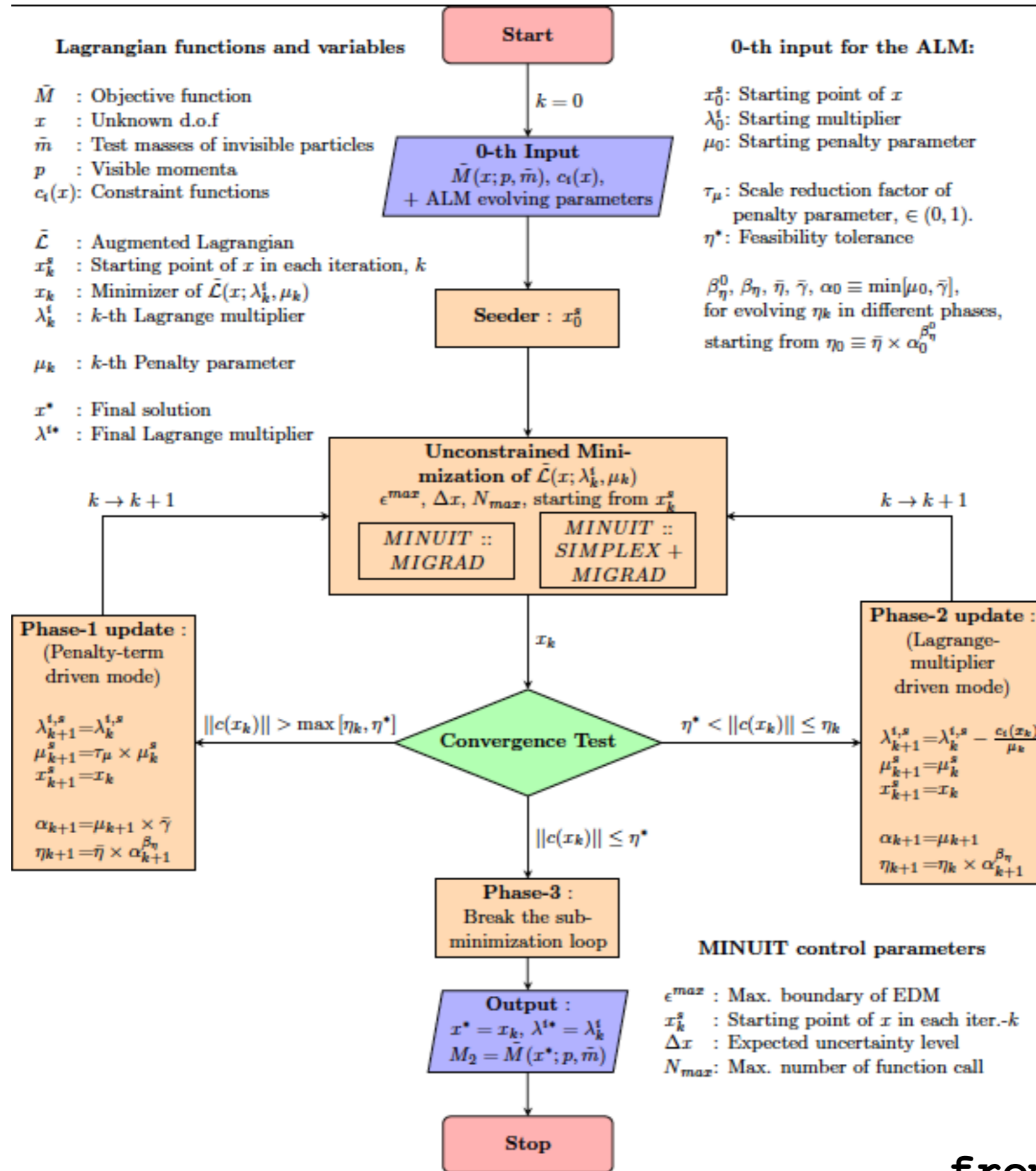
- 2) augmented Lagrange parameter ( $\lambda$ ):

$$\tilde{\mathcal{L}}(\vec{x}; \lambda, \mu) \equiv f(\vec{x}) - \sum_a \lambda_a c_a(\vec{x}) + \frac{1}{2\mu} \sum_a c_a^2(\vec{x})$$

$$\lambda_a^{k+1} = \lambda_a^k - \frac{c_a(\vec{x}_k)}{\mu_k}$$



# Flowchart



# Validation

- Example) M2CC of  $t\bar{t}b\bar{b}$  dileptonic decay

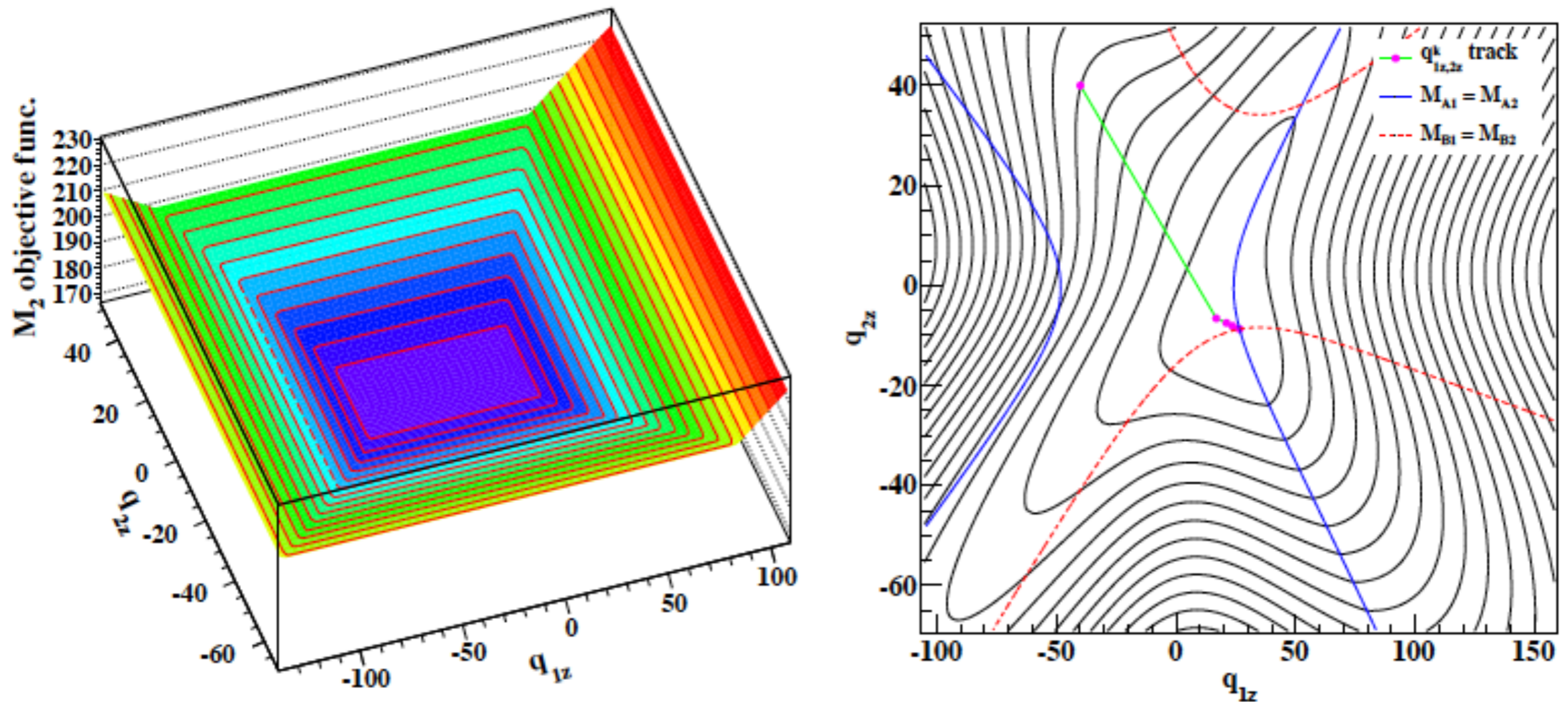


Figure 10. The same as figures 2 and 4, but for the single event considered in section 4.3. Since the objective function has four independent arguments, in order to visualize the evolution of the minimizer, we plot  $q_{1z}$  and  $q_{2z}$ , having fixed the other two variables,  $q_{1x}$  and  $q_{1y}$ , to the values which minimize the objective function for the given choice of  $q_{1z}$  and  $q_{2z}$ .

# OPTIMASS-v1 Released!

- Language : C++, Python
- Requirements : gcc(>4.4),  
Python(>2.6), ROOT with MINUIT2
- Webpage (for download and installation guide):
  - <http://hep-pulgrim.ibs.re.kr/optimass>

# List of Collaborators

- Cho, Won Sang ([wonsang.cho@gmail.com](mailto:wonsang.cho@gmail.com))
- Gainer, James S. ([gainer@phys.ufl.edu](mailto:gainer@phys.ufl.edu))
- Kim, Doojin ([immworry@ufl.edu](mailto:immworry@ufl.edu))
- Lim, Sung Hak ([sunghak.lim@kaist.ac.kr](mailto:sunghak.lim@kaist.ac.kr))
- Matchev, Konstantin T. ([matchev@phys.ufl.edu](mailto:matchev@phys.ufl.edu))
- Moortgat, Filip ([filip.moortgat@cern.ch](mailto:filip.moortgat@cern.ch))
- Pape, Luc ([luc.pape@cern.ch](mailto:luc.pape@cern.ch))
- Park, Myeonghun ([parc.ctpu@gmail.com](mailto:parc.ctpu@gmail.com))

CMS

 Main code developers



# OPTIMASS interface for user's complicated decay topology

- [Full Decay System] Define any number of decay chains, and any type of decay vertices using user's own labelling scheme!

Listing 1: Cards/ttbar-ab.xml

```
1 # XML
2 ---
3 <?xml version='1.0' encoding='utf-8'?>
4 <ProcessCard classname="TTbar_AB" debug="false" version="1.0">
5     <!-- ===== -->
6     <!-- Define event decay chain -->
7     <!-- ===== -->
8     <DecayChains>
9         <DecayChain>
10             t1 - b1 w1 , w1 - e1 v1
11         </DecayChain>
12         <DecayChain>
13             t2 - b2 w2 , w2 - e2 v2
14         </DecayChain>
15     </DecayChains>
```

- **[Subsystem-Mothers]** Define your subsystem's head nodes easily just by listing the names of (intermediate) mother particles defined in the full decay system!

```
16      <!-- ===== -->
17      <!-- Mother node particle in each decay chain to define objective function -->
18      <!-- ===== -->
19      <ParticleMassFunction>
20          <ParticleGroup mass_function="M2" group_function="max">
21              <Particle label="t1" />
22              <Particle label="t2" />
23          </ParticleGroup>
24      </ParticleMassFunction>
```

- **[Subsystem-Effective Invisibles]** Define the effective invisible nodes by simply tagging it in the full decay system!

```
41      <ParticleProperties>
42          <Particle name="top" mass="173." />
43          <Particle name="bottom" mass="4.18" />
44          <Particle name="wboson" mass="80.419" optimize_target="True" />
45          <Particle name="electron" />
46          <Particle name="neutrino" invisible="True" />
47      </ParticleProperties>
```

- **[Kinematic Constraint Functions]** Using the particle names in the full decay chains, their Lorentz 4 momentum d.o.f. (ROOT::TLorentzVector) can freely be used to define constraint functions.

```
58      <!-- ALM Constraints Configuration -->
59      <!-- ===== -->
60      <Constraints penalty_init="1.">
61          <Constraint multiplier_init="0" type="equal">
62              w1.M() - w2.M()
63          </Constraint>
64          <Constraint multiplier_init="0" type="equal">
65              t1.M() - t2.M()
66          </Constraint>
67      </Constraints>
```

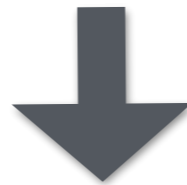
- **[Combined-Events System Support]** Define multiple PT conservation systems using the full system

```
48      <!-- ===== -->
49      <!-- Subchains for MET conditions -->
50      <!-- ===== -->
51      <ParticleInvisibleSubsystem>
52          <Subsystem set_value="manual" >
53              <Particle label="t1" />
54              <Particle label="t2" />
55          </Subsystem>
56      </ParticleInvisibleSubsystem>
```



**OPTIMASS as a mass and event  
reconstructor for  
hypothetical event topologies.**

DATA:  $[i, j] \Rightarrow \{??\} \Rightarrow [\text{visibles}] + \{\text{invisibles}\}$



**OPTIMASS** with (general hypothesis  
- 'model\_card.xml' for  $\{??\}$ )



Physical / Unphysically reconstructed  
 $\{\text{invisibles}\}$  &  $\{\text{node masses}\}$

**$\Rightarrow$  Better discrimination power!**

# Summary

- We have studied kinematics systematically
  - Understanding relations among various variables
  - Understanding properties of variables if “assumed” assumptions are not correct
- One can add additional constraints to describe given kinematics more precisely.
- Dr. Wonsang Cho will provide a tutorial for OPTIMASS today.