Overview: NLO and beyond

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Topics

* Quick reminder on ideas behind NLO+PS
* Going beyond NLO+PS
  * Beyond NLO: NNLO calculations
  * NNLO+PS in simple cases
  * Alternative to parton shower: analytic resummation
  * Towards NNLO+NNLL′+PS event generators

Ingredients of NLO+PS

- NLO hard matrix elements
  - Recursion relations for tree amplitudes
  - **New methods and tools for one-loop amplitudes**
  - Automation of infrared subtraction methods
- Matching and merging

Talks of Mawatari and Re

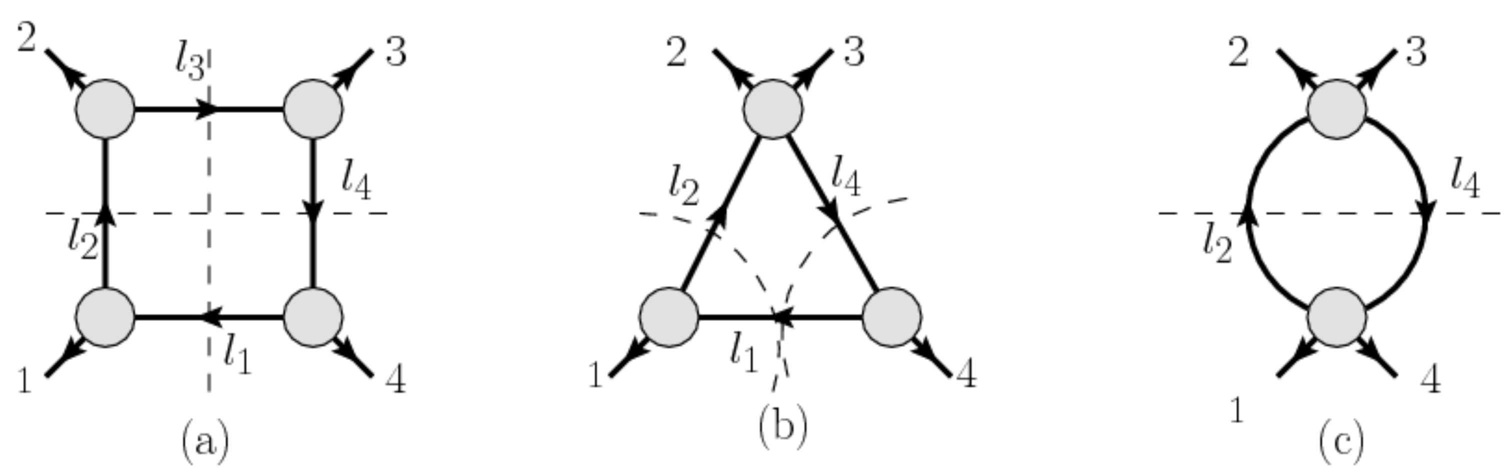
One-loop integrals

Basis of one-loop integrals known since Passarino-Veltman!

$$\mathcal{M} = \sum_i a_i\, A_i + \sum_i b_i\, B_i + \sum_i c_i\, C_i + \sum_i d_i\, D_i + R$$

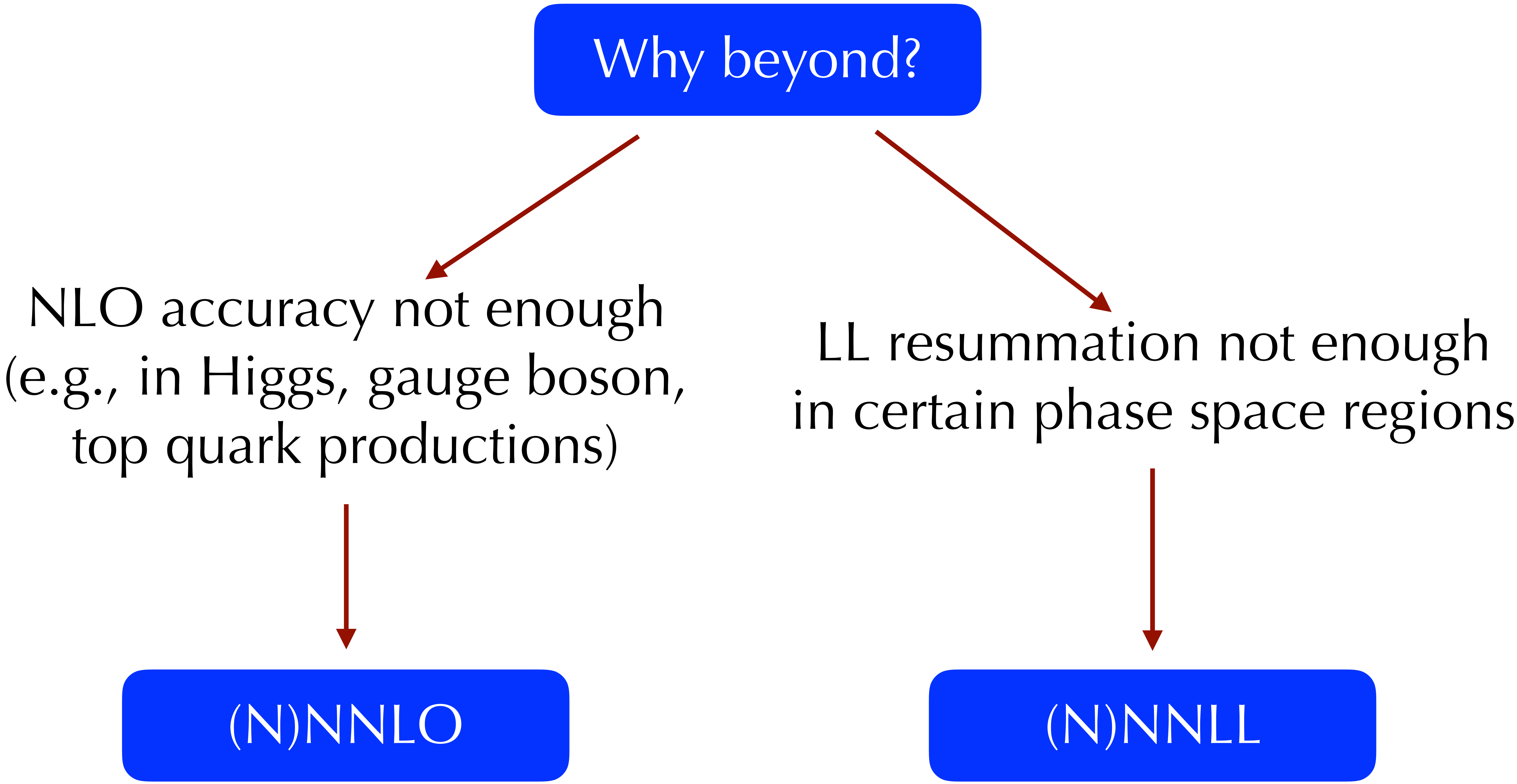
We only need to know the coefficients…

Generalized unitarity + tree-level techniques!



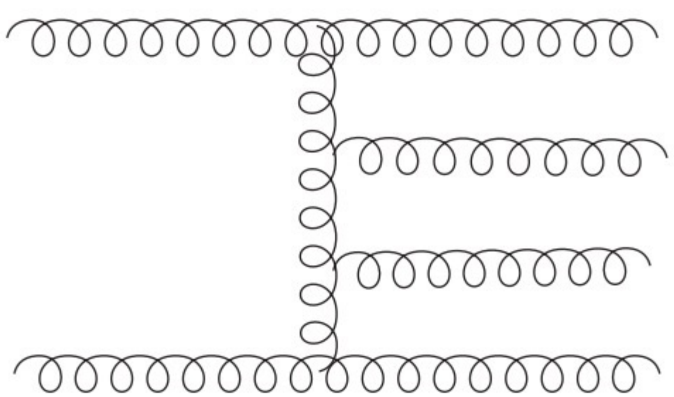
Automated tools: Blackhat, CutTools, MadLoop, GoSam, OpenLoops, Sherpa, Samurai, Grace, Rocket, Whizard, FDC, …

Beyond NLO+PS

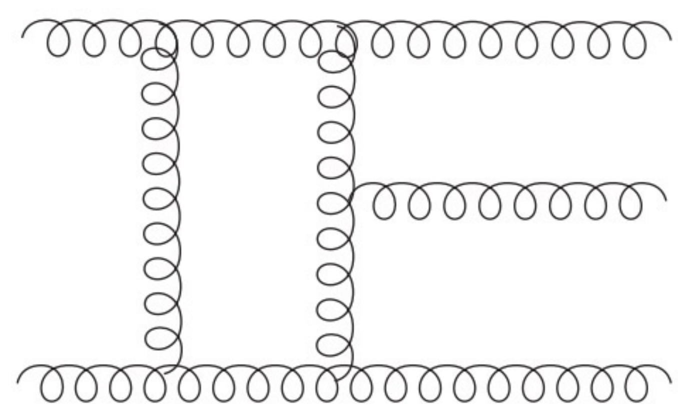


NNLO in a nutshell

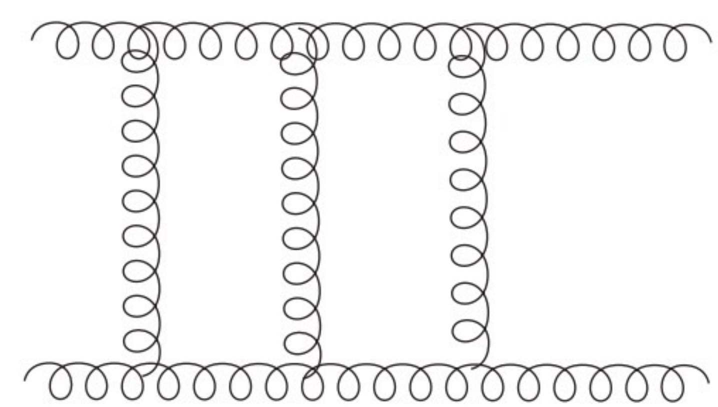
Combining 3 contributions to cancel infrared divergences



double real



virtual+real



double virtual

Bottlenecks:

- Two-loop integrals
- IR cancellation (subtraction/slicing)

Two-loop integrals

- Unlike NLO, generic basis of integrals not known (yet)!
- Case-by-case reduction to “master integrals” via IBP relations (Laporta algorithm) Laporta (hep-ph/0102033)
  - Public tools: Air, Fire, LiteRed, Reduze, …
  - Time-consuming for complicated problems!
- Computation of master integrals highly non-trivial!

Two-loop master integrals

- Simplifications of analytic calculations (and results)
  - Canonical basis for differential equations

    Henn (1304.1806)
  - Mathematical structures of iterated integrals

    TASI 2014 lecture by Duhr (1411.7538)
  - Application to production of vector bosons

    VVamp: Gehrmann, von Manteuffel, Tancredi (1503.04812); vM, T(1503.08835)
- Progresses in numeric evaluations

Numeric loop integrals

* The only possibility for complicated processes!
* Numerically solving differential equations: top pair

  Bärnreuther, Czakon, Fiedler (1312.6279)
* Sector decomposition

  Binoth, Heinrich (hep-ph/0004013, hep-ph/0305234)
  * Public codes: FIESTA, SecDec, ...
* Mellin-Barnes

Sector decomposition

Factorization of singularities in Feynman parameters

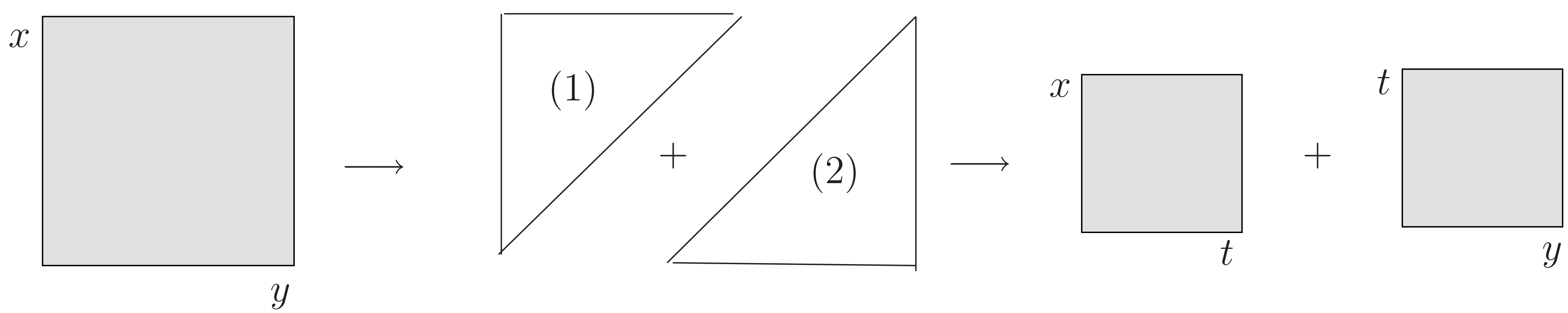


Figure from Heinrich (0803.4177)

- A recent remarkable application: HH production with full top mass dependence

Borowka, Greiner, Heinrich, et al. (1604.06447)

- A new efficient code from a Chinese group

Li, Wang, Yan, Zhao (1508.02512)

Two-loop amplitudes: analytic IR structure

Generic formula from soft-collinear effective theory (SCET):

$$Z^{-1}(\epsilon_{\rm IR})\mathcal{M}(\epsilon_{\rm IR}) = \mathcal{O}(\epsilon_{\rm IR}^0)$$

$$\begin{aligned}
\boldsymbol{Z} = 1 &+ \frac{\alpha_s^{\rm QCD}}{4\pi}\left(\frac{\Gamma_0'}{4\epsilon^2}+\frac{\boldsymbol{\Gamma}_0}{2\epsilon}\right) \\
&+ \left(\frac{\alpha_s^{\rm QCD}}{4\pi}\right)^2 \left\{ \frac{(\Gamma_0')^2}{32\epsilon^4} + \frac{\Gamma_0'}{8\epsilon^3}\left(\boldsymbol{\Gamma}_0 - \frac{3}{2}\beta_0\right) + \frac{\boldsymbol{\Gamma}_0}{8\epsilon^2}(\boldsymbol{\Gamma}_0 - 2\beta_0) + \frac{\Gamma_1'}{16\epsilon^2} + \frac{\boldsymbol{\Gamma}_1}{4\epsilon} \right. \\
&\left. - \frac{2T_F}{3}\sum_{i=1}^{n_h}\left[\Gamma_0'\left(\frac{1}{2\epsilon^2}\ln\frac{\mu^2}{m_i^2} + \frac{1}{4\epsilon}\left[\ln^2\frac{\mu^2}{m_i^2}+\frac{\pi^2}{6}\right]\right) + \frac{\boldsymbol{\Gamma}_0}{\epsilon}\ln\frac{\mu^2}{m_i^2}\right]\right\} + \mathcal{O}(\alpha_s^3)
\end{aligned}$$

Universal two-loop result:

Ferroglia, Neubert, Pecjak, **LLY**: 0907.4791

$$\begin{aligned}
\boldsymbol{\Gamma}(\{\underline{p}\},\{\underline{m}\},\mu) &= \sum_{(i,j)} \frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) \\
&- \sum_{(I,J)} \frac{\boldsymbol{T}_I\cdot\boldsymbol{T}_J}{2}\,\gamma_{\rm cusp}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s) + \sum_{I,j} \boldsymbol{T}_I\cdot\boldsymbol{T}_j\,\gamma_{\rm cusp}(\alpha_s)\,\ln\frac{m_I\mu}{-s_{Ij}} \\
&+ \sum_{(I,J,K)} if^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_K^c\,F_1(\beta_{IJ},\beta_{JK},\beta_{KI}) \\
&+ \sum_{(I,J)}\sum_k if^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_k^c\,f_2\Big(\beta_{IJ},\ln\frac{-\sigma_{Jk}\,v_J\cdot p_k}{-\sigma_{Ik}\,v_I\cdot p_k}\Big) + \mathcal{O}(\alpha_s^3)\,.
\end{aligned}$$

(5

IR subtraction at NNLO

- A couple of methods become mature and productive
  - Antenna subtraction: ZJ, HJ, JJ, …

    Gehrmann-De Ridder, Gehrmann, Glover (hep-ph/0505111)
  - Q_T subtraction: color-neutral final states, e.g., VV

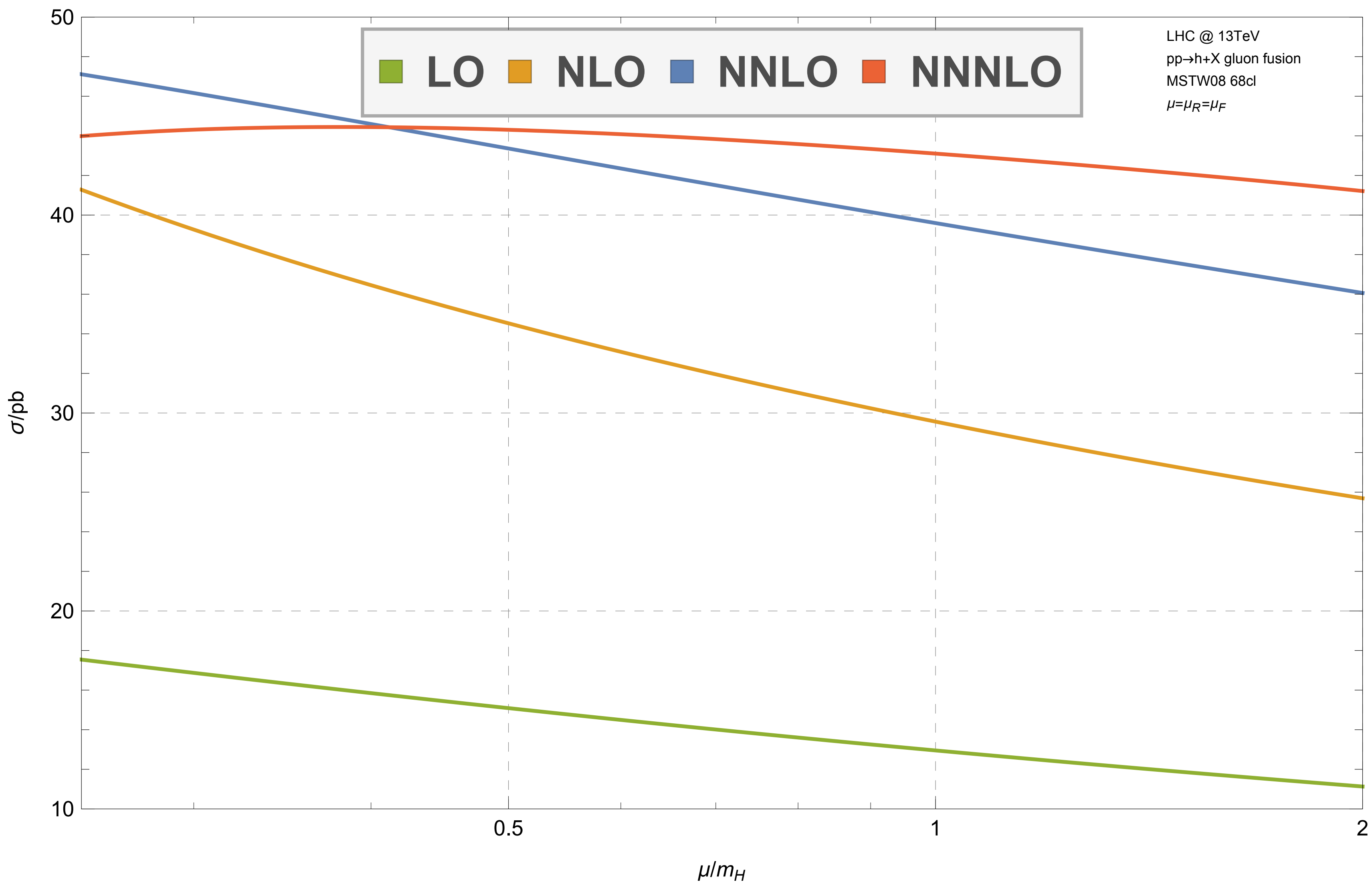
    Catani, Grazzini (hep-ph/0703012)
  - Sector improved: top pair, HJ, …

    Czakon (1005.0274)
  - N-jettiness subtraction: VV, VH, HJ, …

    Boughezal, Focke, Liu, Petriello (1504.02131)
    Gaunt, Stahlhofen, Tackmann, Walsh (1505.04794)

NNNLO Higgs production

Anastasiou, Duhr, Dulat, Herzog, Mistlberger (1503.06056)



First NNNLO result!

NNLO+PS

* Simple cases work: proof of concept
  * POWHEG+MiNLO Hamilton, Nason, Re, Zanderighi (1309.0017)
  * UNNLOPS Hoeche, Li, Prestel (1405.3607, 1407.3773)
  * GENEVA (SCET based) Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi (1311.0286)
* With the sparkle of NNLO calculations, extensions should be possible
* Main obstacle towards a general-purpose NNLO+PS event generator: two-loop integrals

NNLO for e^+e^- colliders?

* In the context of Higgs factories (CEPC, FCC-ee, ILC)
* Most important process: ZH production (per-mille experimental precision!)
* A lot of discussions on how to use ZH to search for BSM physics
* How about SM predictions?
  * NLO EW: about -3% Denner, Kühlbeck, Mertig, Böhm (1992)
  * NNLO EW and QCD+EW?

QCD+EW for ZH

Gong, Li, Xu, **LLY** (to appear)

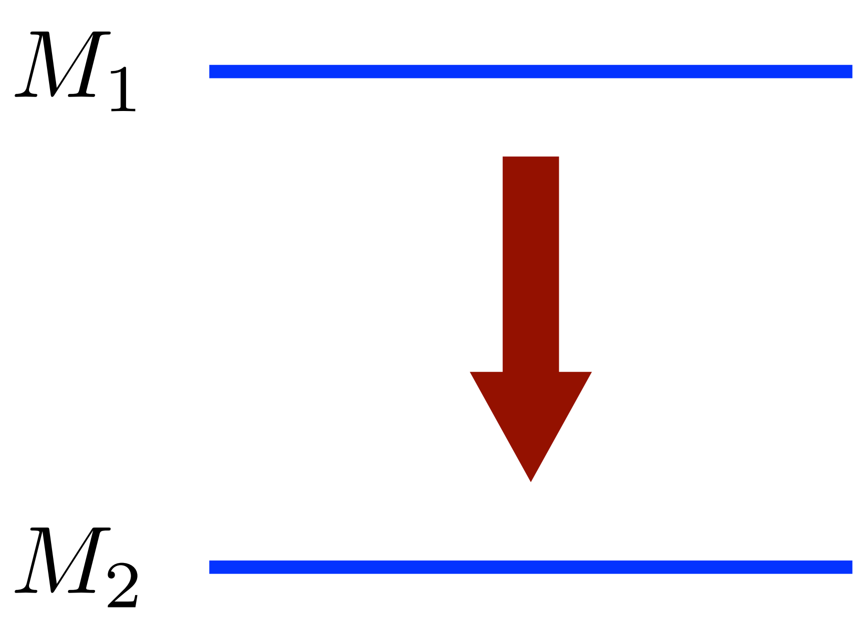
* 41 master integrals, most involve 4 mass scales
* Two methods:
  * Expansion in $1/m_t$
  * Numeric evaluation using sector decomposition
* Preliminary result: about -0.4% correction, similar in size to experimental precision!

Alternative to PS: analytic resummation

* Direct analytic formulas for jet cross sections
* Can achieve high logarithmic accuracies (NNNLL)
* Cross-validation of parton shower results
* Hints for scale choices in event generators

Large corrections and scale hierarchy

Resummation is necessary whenever an observable involves several very different mass scales



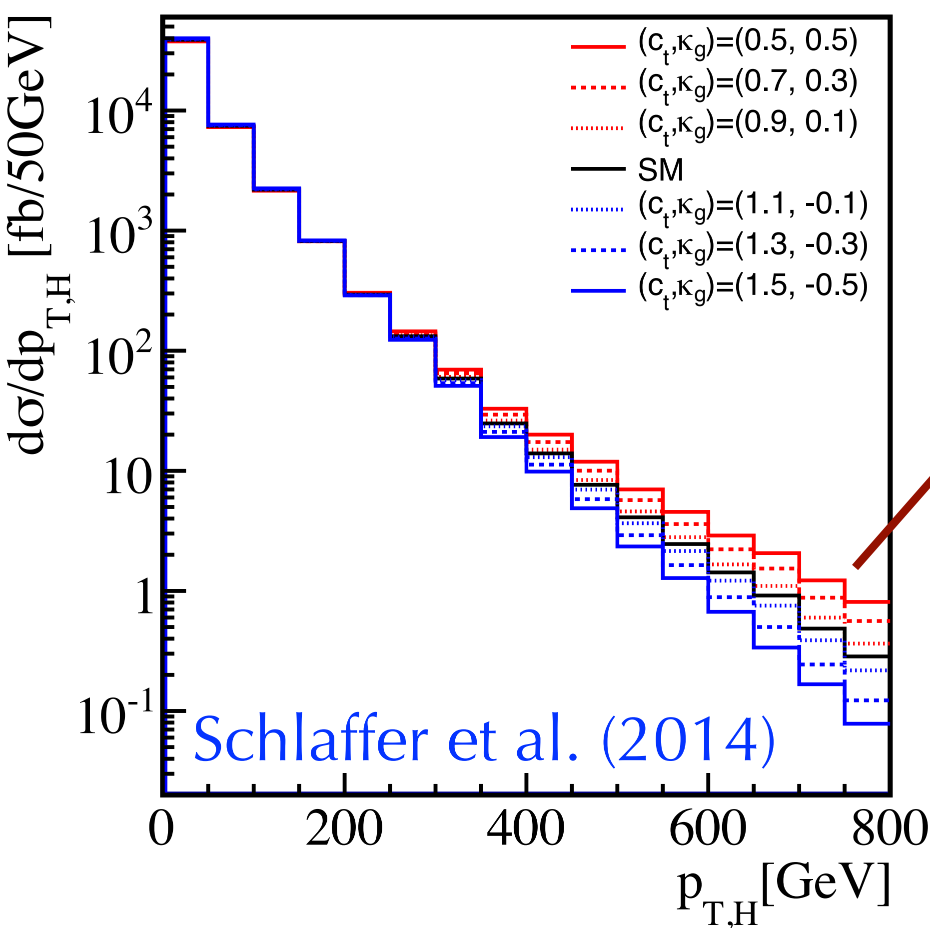
$$\alpha_s^n \ln^m (M_1/M_2)$$

An example: boosted tops

Pecjak, Scott, Wang, **LLY**: 1601.07020

Boosted kinematics

$$\mathcal{L} = \mathcal{L}_0 + \sum_{n,i} \frac{c_{n,i}}{\Lambda^{4+n}} O_{n,i}$$



- Tails of distributions sensitive to new physics
- Testing the SM in the energy frontier
- Important background to BSM scenarios

Producing boosted tops

Two dangerous contributions

soft gluons

$$\ln \frac{\hat{s} - M_{t\bar{t}}^2}{M_{t\bar{t}}^2}$$

quasi-collinear gluons

$$\ln \frac{m_t^2}{M_{t\bar{t}}^2}$$

A tale of three scales

$$\hat{\sigma}\left(M_{t\bar{t}}^2, \hat{s} - M_{t\bar{t}}^2, m_t^2, \mu_f^2\right)$$

Mellin/Laplace transform

$$\hat{\sigma}\left(M_{t\bar{t}}^2, M_{t\bar{t}}^2/\bar{N}^2, m_t^2, \mu_f^2\right) \ni \ln\frac{M_{t\bar{t}}^2}{\mu_f^2}, \ln\frac{M_{t\bar{t}}^2}{\bar{N}^2\mu_f^2}, \ln\frac{m_t^2}{\mu_f^2}$$

Question: what should μ_f be?

No good answer!

Factorization and resummation!

Double factorization

Boosted limit: $M \gg M/N,\ m_t$

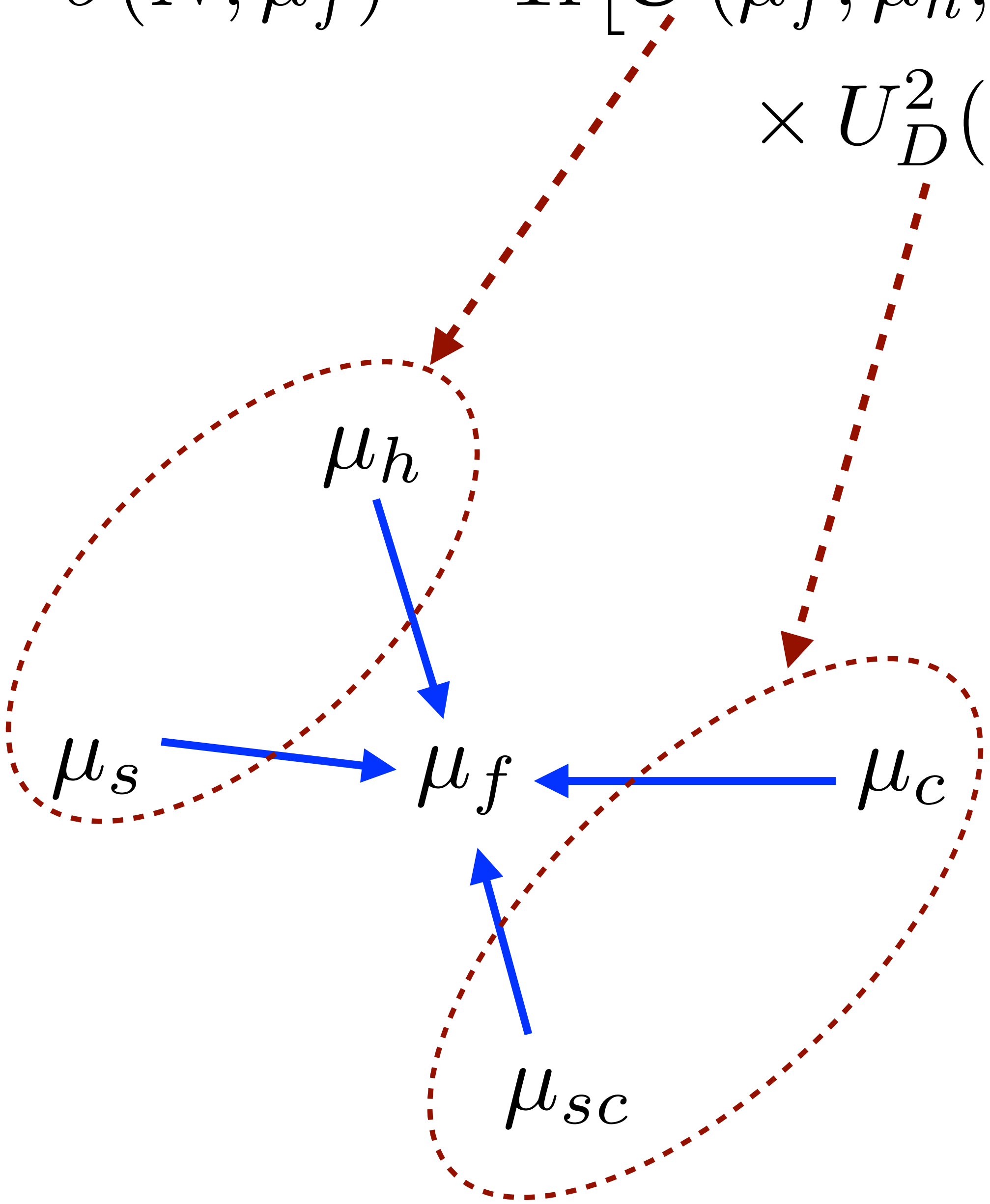
$$\hat{\sigma}(N,\mu_f) \sim \mathrm{Tr}\big[\boldsymbol{H}(L_h,\mu_f)\,\boldsymbol{S}(L_s,\mu_f)\big]\,C_D^2(L_c,\mu_f)\,S_D^2(L_{sc},\mu_f)$$

$$\ln\frac{M^2}{\mu_f^2} \qquad \ln\frac{M^2}{\bar{N}^2\mu_f^2} \qquad \ln\frac{m_t^2}{\mu_f^2} \qquad \ln\frac{m_t^2}{\bar{N}^2\mu_f^2}$$

Emergence of a soft-collinear scale m_t/N!

NNLL′ resummation

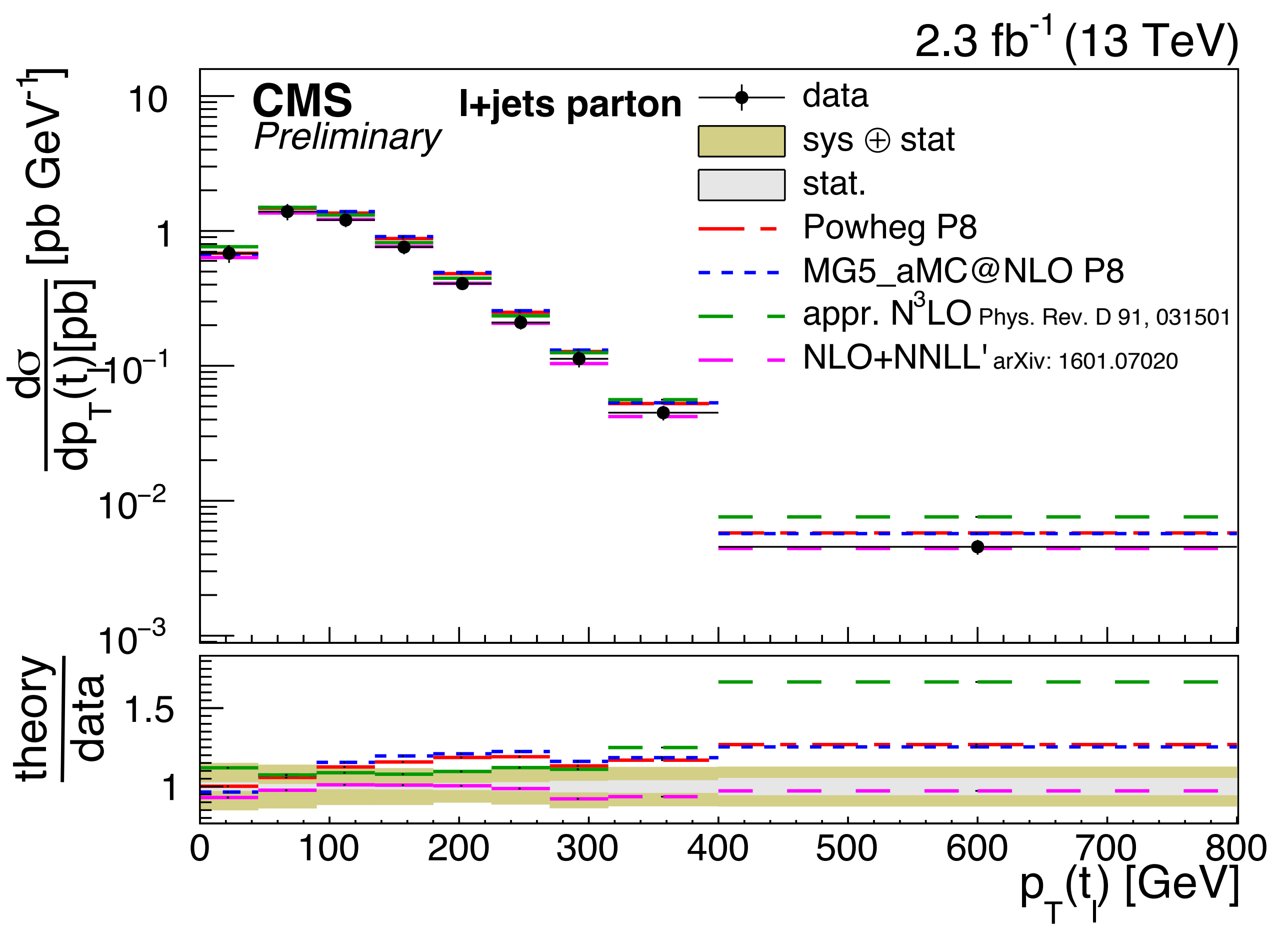
$$\hat{\sigma}(N,\mu_f) \sim \mathrm{Tr}\left[\boldsymbol{U}(\mu_f,\mu_h,\mu_s)\boldsymbol{H}(L_h,\mu_h)\boldsymbol{U}^\dagger(\mu_f,\mu_h,\mu_s)\boldsymbol{S}(L_s,\mu_s)\right]$$
$$\times\, U_D^2(\mu_f,\mu_c,\mu_{sc})C_D^2(L_c,\mu_c)S_D^2(L_{sc},\mu_{sc})$$



- Combined with NNLL threshold resummation
- Combined with NLO result
- Applicable not only in the boosted region!

Compare to data

CMS PAS TOP-16-008



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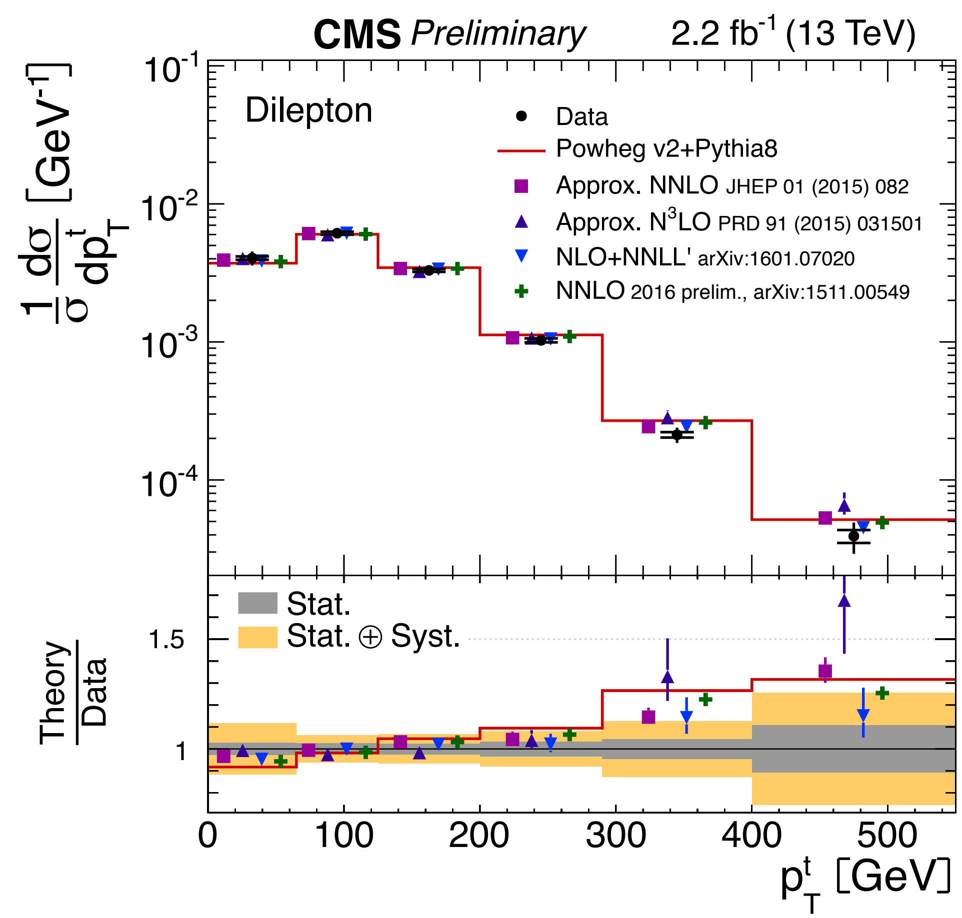
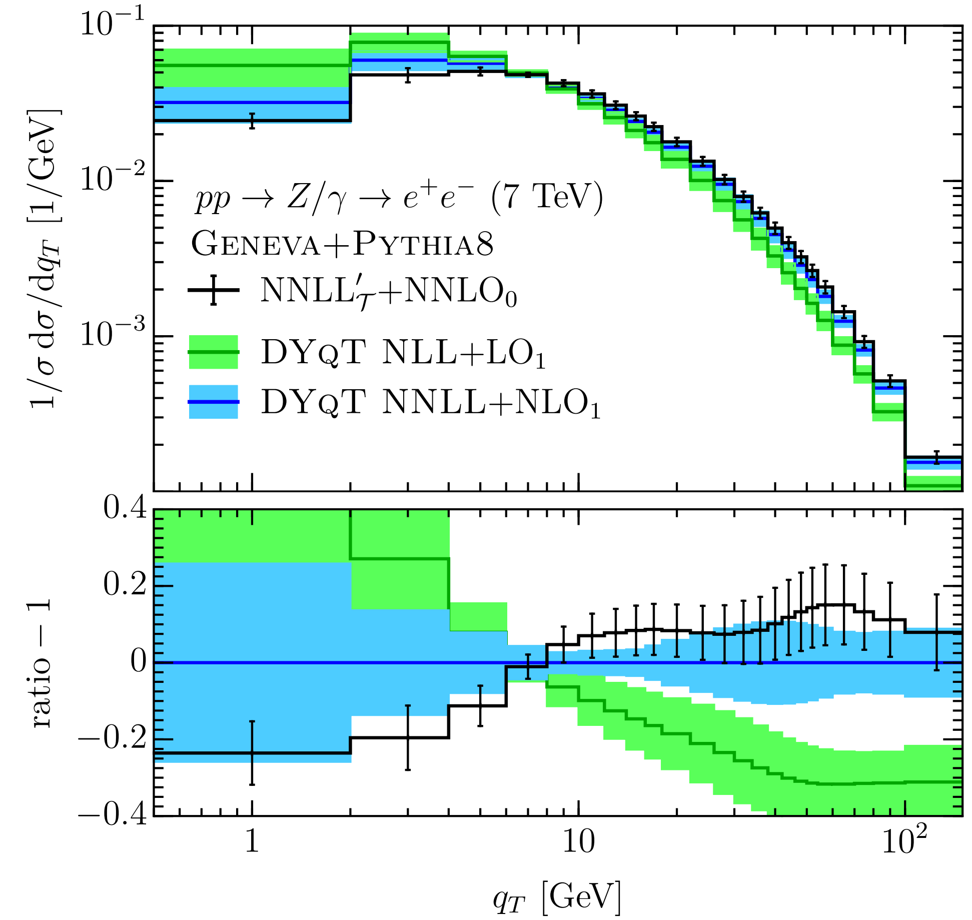


Perfect agreements!

Towards NNLO+NNLL′+PS event generators?

* GENEVA: an ambitious SCET-based approach

Alioli, Bauer, Berggren, Tackmann, Walsh (1508.01475)



* NNLO+NNLL′ for Drell-Yan

* Not clear how to extend to, e.g., top pair

Summary

* NLO+PS now standard in MC community (thanks in part to advances in loop calculations)

* NNLO+PS for simple processes emerging

  * Still far from general-purpose tools (requires major breakthroughs in loop calculations!)

* In certain cases it is necessary to go beyond LL: analytic resummation (boosted tops as an example)

* Possible to combine high-accuracy resummation and parton shower!

Thank you!