Matching and Merging

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Introduction

- this talk is about methods aiming at improving the accuracy of "LO" Monte Carlo event generators:
 - ★ when is this really important / needed?
- "matching" and "merging" are the keywords used to identify developments that took place in the last ~ 10 years:
 - ★ overview, with some details, of (some of) these methods
 - not enough time to discuss all possible approaches. I've made a selection.
- event generators: simulate BSM signals and SM backgrounds:
 - ► so far "matching" and "merging" applied mostly to SM processes:
 - the theory uncertainty of SM predictions is (or will soon be) a limiting factor for "precision Physics", i.e. find a significant deviation from a very precise experimental measure.
 - (part of) current effort is to apply/automatise these methods also to BSM processes. I will show some examples. Some MC developers heavily involved in this task are at this workshop!
- I'm here till Friday evening: any question, don't hesitate!
- if you want to contact me by email: emanuele.re AT lapth.cnrs.fr
- Later today I'll add a slide with a list of references.

matching and merging: when and why ?

Introduction: bump search



- s-channel resonance "easy" to discover:
 - Higgs discovery in $\gamma\gamma$ and ZZ
 - the "750 GeV diphoton bump"
- for discovery, one needs denough ata (possibly on both side of the bump) and (maybe) a fixed-order (N)LO prediction for the background.
- after discovery, characterization requires more theory input (rates, shapes, jet-binned x-sections), hence also more precise tools:

...let's see this with an example...

- need to know expected signal events, precisely and with an associated theory uncertainty:
 - higher-order corrections





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- ⇒ NLO+PS "matching" methods include both effects and allow for flexible and fully ^N_{jets} differential simulations.
- ⇒ for Higgs studies, NNLO+PS would be desirable, and it is available ...to this end merging NLO+PS computations for different multipicities is necessary...

Introduction: excess in p_T tail

- ► ME+PS merging is particularly important to model "S+jets" processes, where:
 - $S = \text{hard system} = \{\ell, \nu, V, t\}$
 - jets are from QCD emissions (as opposed to jets from SUSY cascades)
- it becomes crucial to model kinematics regions characterized by variable number of jets:
 - cuts on $H_T = ... + \sum_{\text{all jets}} |\vec{p}_{T,j}|$ and/or tails of p_T distributions



NLO+PS matching



$$d\sigma_{\text{SMC}} = \underbrace{B(\Phi_n) \, d\Phi_n}_{d\sigma_B} \left\{ \Delta(t_{\max}, t_0) \right\}$$



$$\Delta(t_{\max}, t) = \exp\left\{-\int_{t}^{t_{\max}} d\Phi'_r \; \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z')\right\}$$

SMC Sudakov form factor

$$d\sigma_{\text{SMC}} = \underbrace{B(\Phi_n) \, d\Phi_n}_{d\sigma_B} \left\{ \Delta(t_{\max}, t_0) + \Delta(t_{\max}, t) d\mathcal{P}_{\text{emis}}(t) \right.$$



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emission probability at scale t

- shapes change (all-order effect!), but overall normalization fixed: it stays LO (unitarity)
- they are only LO+LL accurate (whereas we want (N)NLO QCD corrections)

 $\alpha_{\rm S}\sim 0.1 \Rightarrow$ to improve the accuracy, use exact perturbative expansion

$$d\sigma = d\sigma_{\rm LO} + \left(\frac{\alpha_{\rm S}}{2\pi}\right) d\sigma_{\rm NLO} + \left(\frac{\alpha_{\rm S}}{2\pi}\right)^2 d\sigma_{\rm NNLO} + \dots$$

LO: Leading Order NLO: Next-to-Leading Order

...

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$$\text{LO: Leading Order NLO: Next-to-Leading Order ...}$$

$$d\sigma = d\Phi_n \left\{ \begin{array}{c} B(\Phi_n) \\ LO \end{array} + \frac{\alpha_s}{2\pi} \left[\underbrace{V(\Phi_n) + R(\Phi_{n+1}) d\Phi_r}_{\text{NLO}} \right] \right\}$$

 $\alpha_{\rm S} \sim 0.1 \Rightarrow$ to improve the accuracy, use exact perturbative expansion LO: Leading Order NLO: Next-to-Leading Order $d\sigma = d\sigma_{\rm LO} + \left(\frac{\alpha_{\rm S}}{2\pi}\right) d\sigma_{\rm NLO} + \left(\frac{\alpha_{\rm S}}{2\pi}\right)^2 d\sigma_{\rm NNLO} + \dots$... + $d\sigma = d\Phi_n \left\{ \underbrace{B}(\Phi_n) \right\}$ $\frac{\alpha_s}{2\pi} \left[\underbrace{V(\Phi_n) + \mathbf{R}(\Phi_{n+1}) \, d\Phi_r}_{\mathbf{K}} \right]$ LO NLO

in reality, the above equation is implemented as follows:

$$\begin{split} d\sigma &= d\Phi_n \left\{ B(\Phi_n) + \frac{\alpha_s}{2\pi} \Big[V(\Phi_n) + \int d\Phi_r C(\Phi_n, \Phi_r) \Big] \right\} \\ &+ \frac{\alpha_s}{2\pi} d\Phi_{n+1} \Big[R(\Phi_{n+1}) - C(\Phi_n, \Phi_r) \Big] \end{split}$$

where $C(\Phi_n, \Phi_r)$ has the same soft/collinear singular behaviour of R, and it can be integrated explicitly over Φ_r .

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- Why NLO is important?
- first order where rates are reliable
- shapes are, in general, better described
- possible to attach sensible theoretical uncertainties [done typically by changing ren. and fac. scales]



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- first order where rates are reliable
- shapes are, in general, better described
- possible to attach sensible theoretical uncertainties [done typically by changing b ren. and fac. scales]
- When NNLO is needed?
- NLO corrections large
- very high-precision needed
 - \Rightarrow Drell-Yan, Higgs, $t\bar{t}$ production



...

plot from [Anastasiou et al., '03]

NLO

precision

- / nowadays this is the standard
- × limited multiplicity
- X (fail when resummation needed)

parton showers

- ✓ realistic + flexible tools
- ✓ widely used by experimental coll's
- X limited precision (LO)
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Problem: - overlapping regions!

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Problem: - overlapping regions!



- double-counting also for virtual corrections: first order expansion of Sudakov FF for fully unresolved emission $\Delta(t,t_0)$

✓ many proposals, 2 well-established methods available to solve this problem: MC@NLO and POWHEG [Frixione-Webber '03, Nason '04]

$$d\sigma_{\text{LOPS}} = d\Phi_n \quad B(\Phi_n) \quad \left\{ \Delta(t_{\max}, t_0) + \Delta(t_{\max}, t) \frac{\alpha_s}{2\pi} \frac{1}{t} P(z) \ d\Phi_r \right\}$$

$d\sigma_{\mathrm{MC@NLO}} = d\sigma_{\mathbb{S},n}(\Phi_n) \otimes PS(\Phi_n) + d\sigma_{\mathbb{H},n}(\Phi_{n+1}) \otimes PS(\Phi_{n+1})$

NLOPS: MC@NLO

$$d\sigma_{\mathbb{S},n} = d\Phi_n \Big\{ B(\Phi_n) + \frac{\alpha_s}{2\pi} \Big[V(\Phi_n) + \int R_{MC}(\Phi_{n+1}) \, d\Phi_r \Big] \Big\}$$
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$$R_{MC} \ d\Phi_{n+1} \simeq B(\Phi_n) \frac{1}{t} P(z) \ d\Phi_n \ d\Phi_r$$

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$$d\sigma_{\mathbb{H},n} = d\Phi_{n+1} \frac{\alpha_s}{2\pi} \Big[R(\Phi_{n+1}) - R_{MC}(\Phi_{n+1}) \Big]$$

$$R_{MC} d\Phi_{n+1} \simeq B(\Phi_n) \frac{1}{t} P(z) d\Phi_n d\Phi_n$$

$$d\sigma_{\text{LOPS}} = d\Phi_n \quad B(\Phi_n) \quad \left\{ \Delta(t_{\max}, t_0) + \Delta(t_{\max}, t) \frac{\alpha_s}{2\pi} \frac{1}{t} P(z) \ d\Phi_r \right\}$$
$$d\sigma_{\rm POW} = d\Phi_n \ \bar{B}(\Phi_n) \ \left\{ \Delta(\Phi_n; k_{\rm T}^{\rm min}) + \Delta(\Phi_n; k_{\rm T}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \ d\Phi_r \otimes PS(\Phi_{n+1}) \right\}$$

[+ p_{T} -vetoing subsequent emissions, to avoid double-counting]

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$$B(\Phi_{n}) \Rightarrow \overline{B}(\Phi_{n}) = B(\Phi_{n}) + \frac{\alpha_{s}}{2\pi} \Big[V(\Phi_{n}) + \int R(\Phi_{n+1}) d\Phi_{r} \Big]$$

$$d\sigma_{\text{POW}} = d\Phi_{n} \quad \overline{B}(\Phi_{n}) \quad \Big\{ \Delta(\Phi_{n}; k_{\text{T}}^{\min}) + \Delta(\Phi_{n}; k_{\text{T}}) \frac{\alpha_{s}}{2\pi} \frac{R(\Phi_{n}, \Phi_{r})}{B(\Phi_{n})} d\Phi_{r} \otimes PS(\Phi_{n+1}) \Big\}$$

$$[+ p_{\text{T}} \cdot vetoing \ ubsequent \ emissions, \ to \ avoid \ double-counting]$$

$$\Delta(t_{\text{m}}, t) \Rightarrow \Delta(\Phi_{n}; k_{\text{T}}) = \exp \Big\{ -\frac{\alpha_{s}}{2\pi} \int \frac{R(\Phi_{n}, \Phi'_{r})}{B(\Phi_{n})} \theta(k'_{\text{T}} - k_{\text{T}}) \ d\Phi'_{r} \Big\}$$

NLOPS: summary



- inclusive observables: @NLO
- first hard emission: full tree level ME
- (N)LL resummation of collinear/soft logs
- extra jets in the shower approximation (LL)



NLOPS: summary

By now NLO+PS tools are well established:

- POWHEG and MC@NLO usually agree well. When differences arise, they are usually understood, and they are typically due to terms beyond the nominal accuracy.
- NLO+PS is not yet a closed chapter; some important issues are still being addressed
 W⁺W⁻bb @ NLOPS [Jezo,Nason, et al, this week!]
- ▶ in general, however, any process $pp \rightarrow X$ can be simulated at NLO+PS accuracy
 - $\blacktriangleright\,$ X can contain jets. If it contains N jets, it's not possible to describe observables with n < N jets.

available tools:

- POWHEG based: POWHEG-BOX, PowHel, Matchbox/Herwig++
- MC@NLO based: MG5_aMC@NLO, Sherpa-MC@NLO, Matchbox/Herwig++
- other methods: Geneva, KrK-NLO

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multijet merging [(N)LO+PS merging]

multijet merging

typical background for many BSM signatures is "heavy object" + many jets



- relying on PS for tail of distributions is very dangerous, especially in a multijet environment
- CKKW(-L) and MLM methods address this issue at LO:
 - merge exact LO matrix elements for different multiplicities
 - very important for observables like ${\cal H}_T$ especially when not possible to use data-driven methods
- ▶ ME generators: Alpgen, MadGraph, Sherpa
 - for at least one of them (typically both), interface/implementation available in general-purpose parton-shower program

- - what is the theoretical uncertainty of backgrounds?
 - extending merging to NLO becomes important...

Fix a merging scale $Q_{\rm MS}$, to separate ME and PS domains.



► start from ME weight B(Φ_n), respecting Q_{MS} constraint

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 find "most-likely" shower history (via k_T-algo)



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- clustering scale
 q₁ = k_T

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Fix a merging scale $Q_{\rm MS}$, to separate ME and PS domains.



 Hard process scale Q

Fix a merging scale $Q_{\rm MS}$, to separate ME and PS domains.



 most-likely shower history

CKKW in a nutshell II

• original weight $B(\mathbf{\Phi}_n) \Rightarrow$ "most-likely" shower history (via k_T -algo): $Q > q_3 > q_2 > q_1$



New weight:

$$\begin{aligned} \alpha_{\rm S}^5(Q)B(\mathbf{\Phi}_3) &\to & \alpha_{\rm S}^2(Q)B(\mathbf{\Phi}_3)\frac{\Delta_g(Q_0,Q)}{\Delta_g(Q_0,q_2)}\frac{\Delta_g(Q_0,Q)}{\Delta_g(Q_0,q_3)}\frac{\Delta_g(Q_0,q_3)}{\Delta_g(Q_0,q_1)}\\ & & \Delta_g(Q_0,q_2)\Delta_g(Q_0,q_2)\Delta_g(Q_0,q_3)\Delta_g(Q_0,q_1)\Delta_g(Q_0,q_1)\\ & & \alpha_{\rm S}(q_1)\alpha_{\rm S}(q_2)\alpha_{\rm S}(q_3) \end{aligned}$$

where $Q_0 \equiv Q_{\rm MS}$ and typically

$$\log \Delta_{\rm f}(q_T, Q) = -\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_{\rm S}(q^2)}{2\pi} \Big[A_{1,\rm f} \log \frac{Q^2}{q^2} + B_{1,\rm f} \Big]$$

 Fill phase space below Q₀ with vetoed shower (for highest multiplicity sample Q₀ = q₁; PS initial scale should be nodal scale at which parton was "created")

• This procedure guarantees that dependence upon $Q_{\rm MS}$ is beyond NLL (proved for e^+e^-)

LO+PS merging: a BSM example

- DM production at the LHC: scalar/pseudoscalar mediator. Usually: monojet search. these are "QCD" jets !
- ▶ analysis based on variable number of jets (*H*_T based) are potentially very powerful





[Buchmueller,Malik,McCabe,Penning '15]

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for loop-induced processes: automation is now available

[Hirschi,Mattelaer '15, Mattelaer,Vryonidou '15 - also earlier studies with Sherpa]

- . the formal accuracy is leading-order
- . theoretical uncertainties will not be in general very small
- . however, shapes will be modelled properly!
- a-priori, dangerous to rely on a description mostly done by parton-shower

multijet merging at NLO

- ▶ multijet merging at NLO is more complicated than at LO, and more subtle: the matrix element " $pp \rightarrow S + (n + 1)$ partons" enters in
 - real emission for " $pp \rightarrow S + n$ partons" @ NLO
 - Born contribution for " $pp \rightarrow S + (n+1)$ partons" @ NLO
- methods: MEPS@NLO, FxFx, UNLOPS, Geneva, POWHEG+MiNLO, Vincia
- similarly to LO, many of these methods use a merging scale (Q_{MS}): a bad choice of merging scale can spoil the formal accuracy
 - typically this can happen if $\alpha_{\rm S} \log^2 (Q_{\rm MS}/Q) \simeq 1$: when $L \simeq 1/\sqrt{\alpha_{\rm S}}$, uncontrolled NNLL logs $\alpha_{\rm S}^2 L$ scale as $\alpha_{\rm S}^{1.5}$ (and not as $\alpha_{\rm S}^2$).
 - to avoid any formal issue, one needs either to not have Q_{MS} at all, or have a very precise control
 of logarithmic structure (beyond the PS accuracy)
 - not having $Q_{\rm MS}$ requires control of NNLL terms (or at least part thereof)
 - if $Q_{\rm MS}$ is present, include the uncertainty due to its choice
- for simple processes (color-singlet production), the development of these techniques lead to match PS with NNLO computations (<u>NNLO+PS</u>)

"FxFx" method

[Frixione, Frederix, '12]

$$\begin{aligned} d\bar{\sigma}_{\mathbb{S},0} &= B_0 + V_0 + B_0 \mathcal{K}_{\mathrm{MC}} \Theta(d_1 < Q_{\mathrm{MS}}) \\ d\bar{\sigma}_{\mathbb{H},0} &= \left[B_1 - B_0 \mathcal{K}_{\mathrm{MC}} \right] \Theta(d_1 < Q_{\mathrm{MS}}) \\ d\bar{\sigma}_{\mathbb{S},1} &= \left[B_1 + V_1 + B_1 \mathcal{K}_{\mathrm{MC}} \right] \Theta(Q_{\mathrm{MS}} < d_1) \end{aligned}$$

$$d\bar{\sigma}_{\mathbb{H},1} = |B_2 - B_1 \mathcal{K}_{\mathrm{MC}}| \Theta(Q_{\mathrm{MS}} < d_1)$$

- ► limit contribution of (III, 0) events to region below Q_{MS}
- prescriptions for shower starting scale
- possible to include Sudakov reweighting á la CKKW
- "unitarity" not imposed
- possible to iterate

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- differences typically $\lesssim 1\%$ among different merging scales

- quite good agreement with inclusive NLO+PS too



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when Sudakov reweighting applied:

$$\begin{aligned} d\hat{\sigma}_{\mathbb{S},1} &= \left(d\bar{\sigma}_{\mathbb{S},1} + d\sigma_1^{(\Delta)} \right) \Delta(\Phi_1 \to \Phi_0) \\ d\hat{\sigma}_{\mathbb{H},1} &= d\bar{\sigma}_{\mathbb{H},1} \Delta(\Phi_1 \to \Phi_0) \end{aligned}$$

where

$$d\sigma_1^{(\Delta)} = -B_1 \Delta^{(1)}(\Phi_1 \to \Phi_0),$$

 Δ are CKKW Sudakov factors, and $\Delta^{(1)}$ is the Sudakov expanded at 1st order.

- Above $Q_{\rm MS}$ the tail is NLO accurate. For not-too-small $Q_{\rm MS}$, the integral is NLO accurate.
- merging NLO+PS for V production with MINLO for V + 1 jet, at "merging scale" Q_{MS} .

MadGrapg5_aMC@NLO: FxFx merging

★ V + 0,1,2,(3,4) jets: extensive phenomenological study published recently

[Frederix, Frixione, Papaefstathiou, Prestel, Torrielli '15]



- estimation of perturbative uncertainty + shower "uncertainty"
- 1. $Q_{\rm MS}$ dependence is at most 1.5%. FxFx total typically 3-6% larger than exact inclusive NLO+PS
- 2. once V+2 jets at NLO+PS is included, also higher jet multiplicities are described reasonably well
- 3. the inclusive NLO+PS result depends much more on the PS used

MadGrapg5_aMC@NLO: FxFx merging

★ V + 0,1,2,(3,4) jets: extensive phenomenological study published recently

[Frederix, Frixione, Papaefstathiou, Prestel, Torrielli '15]



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Sherpa: MEPS@NLO

- similar to FxFx: generate MC@NLO samples, and separate their domain of validity using merging scale Q_{MS}
- $d\Phi_{n+1}$ receives contribution from H_n -events below Q_{MS} and from S_{n+1} above Q_{MS}
- procedure can be iterated

Uncertainties

- μ_R and μ_F scale variation
- shower ("resummation") scale: upper limit of parton evolution
- merging scale



Multiscale Improved NLO

- original goal: method to a-priori choose scales in multijet NLO computation
- non-trivial task, since phase space is by construction probed also in presence of widely separated energy scales

Multiscale Improved NLO

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- how: correct weights of different NLO terms with CKKW-inspired approach (without spoiling formal NLO accuracy)
 - for each point sampled, build the "more-likely" shower history that would have produced that kinematics (can be done by clustering kinematics with k_T -algo, then, by undoing the clustering, build "skeleton")
 - "correct" original NLO à la CKKW:
 - $\rightarrow \alpha_{\rm S}$ evaluated at nodal scales
 - \rightarrow Sudakov FFs

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$$\bar{B}_{\rm MiNLO} = \alpha_{\rm S}^2(\boldsymbol{m_h}) \alpha_{\rm S}(\boldsymbol{q_T}) \Delta_g^2(\boldsymbol{q_T}, \boldsymbol{m_h}) \Big[B \left(1 - 2\Delta_g^{(1)}(\boldsymbol{q_T}, \boldsymbol{m_h}) \right) + \alpha_{\rm S} V(\bar{\mu}_R) + \alpha_{\rm S} \int d\Phi_{\rm r} R \Big]$$

$$\begin{array}{c} & \bar{\mu}_{R} = (m_{h}^{2}q_{T})^{1/3} \\ & \Delta(q_{T}, m_{h}) \\ & q_{T} \quad \Delta(q_{T}, q_{T}) \\ & \mu_{F} = q_{T} \end{array} \right) \\ & \delta(q_{T}, m_{h}) \\ & \delta(q_{T}, m_{h}) \\ & \delta(q_{T}, m_{h}) \end{array} \\ & \delta(q_{T}, m_{h}) \\ & \delta(q_{T}, m_{h}) \\ & \delta(q_{T}, q_{T}) \\ & \mu_{F} = q_{T} \end{array} \right) \\ & \bar{\mu}_{F} = q_{T} \\ & \bar{\mu}_{R} = (m_{h}^{2}q_{T})^{1/3} \\ & \delta(q_{T}^{2}, m_{h}) = -\int_{q_{T}^{2}} \frac{dq^{2}}{q^{2}} \frac{\alpha_{S}(q^{2})}{2\pi} \Big[A_{f} \log \frac{m_{h}^{2}}{q^{2}} + B_{f} \Big] \\ & \delta(q_{T}, m_{h}) \\ & \delta(q_{T}, m_{h}) \\ & \delta(q_{T}, q_{T}) \\ & \mu_{F} = q_{T} \end{array}$$

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- Minlo-improved HJ yields finite results also when 1st jet is unresolved $(q_T \rightarrow 0)$
- \bar{B}_{MiNLO} ideal to extend validity of HJ-POWHEG [called "HJ-MINLO" hereafter]

"Improved" MiNLO & NLOPS merging

- ► formal accuracy of HJ-MiNLO for inclusive observables carefully investigated
- ▶ HJ-MiNLO describes inclusive observables at order $\alpha_{\rm S}$
- ► to reach genuine NLO when fully inclusive (NLO⁽⁰⁾), "spurious" terms must be of <u>relative</u> order a²_S, *i.e.*

 $O_{\rm HJ-MiNLO} = O_{\rm H@NLO} + O(\alpha_{\rm S}^{2+2})$ if O is inclusive

• "Original MiNLO" contains ambiguous " $\mathcal{O}(\alpha_{\rm S}^{2+1.5})$ " terms

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- ▶ Possible to improve HJ-MiNLO such that inclusive NLO is recovered (NLO⁽⁰⁾), without spoiling NLO accuracy of *H*+*j* (NLO⁽¹⁾).
- accurate control of subleading NNLL small- p_T logarithms is needed (scaling in low- p_T region is $\alpha_{\rm S}L^2 \sim 1$, *i.e.* $L \sim 1/\sqrt{\alpha_{\rm S}}$!)

Effectively as if we merged NLO⁽⁰⁾ and NLO⁽¹⁾ samples, without merging different samples (no merging scale used: there is just one sample).

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these terms are process dependent, and not known analytically for complex processes:
 for non-color-singlet production, possible to effectively extract them numerically
 [Frederix,Hamilton '15]
MiNLO merging: results

[[]Hamilton et al., 1212.4504]



- "H+Pythia": standalone POWHEG ($gg \rightarrow H$) + PYTHIA (PS level) [7pts band, $\mu = m_H$]
- ► "HJ+Pythia": HJ-MINLO* + PYTHIA (PS level) [7pts band, µ from MINLO]
- very good agreement (both value and band)

 \square Notice: band is $\sim 20 - 30\%$...this is Higgs at NLO!

[1]

UNLOPS

- keyword: "unitarity" (preserve NLO inclusive cross section) [Lonnblad, Prestel '12, Platzer '12]
- method: promote to NLO accuracy an "unitarised" CKKW approach, by carefully adding higher order contributions, and removing the pre-existing approximate as terms
 - 1. start from UMEPS merging at LO
 - 2. remove terms that will be included exactly, and add NLO (exclusive) computations
 - unitarise
- can be iterated to higher multiplicities
- by construction, essentially no dependence on merging scale on inclusive cross section
- full exploitation will also be the main focus in Herwig 7 in the near future

[Platzer et al.]



Geneva

- new approach, SCET inspired [Alioli, Bauer, Berggren, Hornig, Tackmann, Vermilion, Walsh, Zuberi '12]
- idea: separate exclusive N-jet and inclusive (N + 1)-jet regions using variable whose resummation is known at high order ("n-jettiness")

$$\sigma_{\geq N} = \int \mathrm{d}\Phi_N \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_N} (\mathcal{T}_N^{\mathrm{cut}}) + \int \mathrm{d}\Phi_{N+1} \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{N+1}} (\mathcal{T}_N) \, \theta(\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}})$$

where

$$\begin{split} \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) &= \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) + \left[\frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}}) - \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\Phi_N}(\mathcal{T}_N^{\mathrm{cut}})\right|_{\mathrm{FO}}\right],\\ \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N) &= \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\Phi_{N+1}}(\mathcal{T}_N) \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\Phi_N \mathrm{d}\mathcal{T}_N} \middle/ \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\Phi_N \mathrm{d}\mathcal{T}_N} \middle|_{\mathrm{FO}}\right], \end{split}$$

- no "dangerous" merging scale dependence, thanks to higher-order resummation for τ_N
- to retain formal accuracy, PS evolution very constrained: au_N has to stay \sim unchanged
- can be extended to higher multiplicities
- implemented for e^+e^- and for Drell-Yan
- the method was also formulated to achieve NNLO+PS accuracy (results shown later)

NNLO+PS

- some of the above approaches allow(ed) to achieve NNLO+PS matching!
 - "just" NLO sometimes is not enough
 - NNLO is the frontier



- these developments don't have an immediate application for direct BSM searches
- however important for "indirect searches", through precise measurements of SM and Higgs processes:
 - ► large NLO K-factors (Higgs production → Higgs characterization)
 - precision Physics (PDF extraction, W-mass measurement)

NNLO+PS Higgs production [POWHEG+MiNLO]



[Hamilton, Nason, ER, Zanderighi, 1309.0017]

uncertainty band is 10% (at NLO it was ~ 20-30% !)

 nice agreement also with NNLL jet-veto resummed result, differences never more than 5-6%

NNLO+PS Drell-Yan [UNNLOPS]

NNLOPS obtained also upgrading UNLOPS to UNNLOPS

[Hoeche, Li, Prestel '14]



NNLO+PS Drell-Yan [Geneva]

[Alioli,Bauer,Berggren,Tackmann,Walsh, '15]



conclusions

- Monte Carlo tools play a major role for LHC searches
- especially if no "smoking gun" new-Physics around the corner, precision will be the key to maximise impact of LHC results
- huge amount of improvements over the last few years in the community
- NLO+PS tools are by now well established and very mature
 - by now they are basically automated also for BSM processes
- major developments in last 3-4 years: NLOPS multijet merging
 - it might play a very important role in absence of smoking-gun BSM signal
- ▶ NNLO+PS is doable, at least for color-singlet production.

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Thank you for your attention!

Extra slides

"Improved" MiNLO & NLOPS merging: details

Resummation formula

$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0 \frac{d}{dq_T^2} \left\{ [C_{ga} \otimes f_a](x_A, q_T) \times [C_{gb} \otimes f_b](x_B, q_T) \times \exp S(q_T, Q) \right\} + R_f$$
$$S(q_T, Q) = -2 \int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \left[A_f \log \frac{Q^2}{q^2} + B_f \right]$$

• If $C_{ij}^{(1)}$ included and R_f is LO⁽¹⁾, then upon integration we get NLO⁽⁰⁾

Take derivative, then compare with MiNLO:

$$\sim \sigma_0 \frac{1}{q_T^2} [\alpha_{\rm S}, \alpha_{\rm S}^2, \alpha_{\rm S}^3, \alpha_{\rm S}^4, \alpha_{\rm S} L, \alpha_{\rm S}^2 L, \alpha_{\rm S}^3 L, \alpha_{\rm S}^4 L] \exp S(q_T, Q) + R_f \qquad L = \log(Q^2/q_T^2)$$

highlighted terms are needed to reach NLO⁽⁰⁾:

$$\int^{Q^2} \frac{dq_T^2}{q_T^2} L^m \alpha_{\rm S}{}^n(q_T) \exp S \sim \left(\alpha_{\rm S}(Q^2)\right)^{n-(m+1)/2}$$

(scaling in low- p_T region is $\alpha_{\rm S} L^2 \sim 1!$)

- if I don't include B_2 in MiNLO Δ_g , I miss a term $(1/q_T^2)$ $\alpha_s^2 B_2 \exp S$
- upon integration, violate NLO⁽⁰⁾ by a term of <u>relative</u> $\mathcal{O}(\alpha_s^{3/2})$